Machine Dynamics – I

Lecture Note

By Er. Debasish Tripathy
(Assist. Prof.
Mechanical Engineering Department,
VSSUT, Burla, Orissa,India)

Syllabus:

Module – I

1. **Mechanisms**: Basic Kinematic concepts & definitions, mechanisms, link, kinematic pair, degrees of freedom, kinematic chain, degrees of freedom for plane mechanism, Gruebler's equation, inversion of mechanism, four bar chain & their inversions, single slider crank chain, double slider crank chain & their inversion.(8)

Module – II

2. **Kinematics analysis**: Determination of velocity using graphical and analytical techniques, instantaneous center method, relative velocity method, Kennedy theorem, velocity in four bar mechanism, slider crank mechanism, acceleration diagram for a slider crank mechanism, Klein's construction method, rubbing velocity at pin joint, coriolli's component of acceleration & it's applications. (12)

Module – III

3. **Inertia force in reciprocating parts**: Velocity & acceleration of connecting rod by analytical method, piston effort, force acting along connecting rod, crank effort, turning moment on crank shaft, dynamically equivalent system, compound pendulum, correction couple, friction, pivot & collar friction, friction circle, friction axis. (6)
4. **Friction clutches**: Transmission of power by single plate, multiple & cone clutches, belt drive, initial tension, Effect of centrifugal tension on power transmission, maximum power transmission(4).

Module – IV

5. **Brakes & Dynamometers**: Classification of brakes, analysis of simple block, band & internal expanding shoe brakes, braking of a vehicle, absorbing & transmission

dynamometers, prony brakes, rope brakes, band brake dynamometer, belt transmission dynamometer & torsion dynamometer.(7)

6. **Gear trains**: Simple trains, compound trains, reverted train & epicyclic train. (3)

Text Book:

Theory of machines, by S.S Ratan, THM

Mechanism and Machines

Mechanism: If a number of bodies are assembled in such a way that the motion of one causes constrained and predictable motion to the others, it is known as a mechanism. A mechanism transmits and modifies a motion.

Machine: A machine is a mechanism or a combination of mechanisms which, apart from imparting definite motions to the parts, also transmits and modifies the available mechanical energy into some kind of desired work.

It is neither a source of energy nor a producer of work but helps in proper utilization of the same.

The motive power has to be derived from external sources.

A slider - crank mechanism converts the reciprocating motion of a slider into rotary motion of the crank or vice versa.

Figure-1

(Available) force on the piston → slider crank + valve mechanism → Torque of the crank shaft (desired).

Examples of slider crank mechanism → Automobile Engine, reciprocating pumps, reciprocating compressor, and steam engines.

Examples of mechanisms: type writer, clocks, watches, spring toys.

Rigid body: A body is said to be rigid if under the action of forces, it does not suffer any distortion.

Resistant bodies: Those which are rigid for the purposes they have to serve.

Semi rigid body: Which are normally flexible, but under certain loading conditions act as rigid body for the limited purpose.

Example: 1. Belt is rigid when subjected to tensile forces. So belt-drive acts as a resistant body. 2. Fluid is resistant body at compressive load.

Link: A resistant body or a group of resistant bodies with rigid connections preventing their relative movement is known as a link.

A link may also be defined as a member or a combination of members of a mechanism, connecting other members and having motion relative to them.

A link is also known as kinematic link or element.

Links can be classified into binary, ternary, quarternary,etc, depending upon their ends on which revolute or turning pairs can be placed.

Figure-2

Kinematic pair:

A kinematic pair or simply a pair is a joint of two links having relative motion between them.

Types of kinematic pairs: Kinematic pairs can be classified according to

(i) Nature of contact

(ii) Nature of mechanical constraint

(iii) Nature of relative motion

Kinematic pairs according to nature of contact

(a) Lower pair: A pair of links having surface or area contact between the members is known as a lower pair. Example: – Nut and screw, shaft rotating in bearing, all pairs of slider crank mechanism, universal joint etc.

(b) Higher pair: When a pair has a point or line contact between the links, it is known as a higher pair. Example: – Wheel rolling on a surface, cam and follower pair, tooth gears, ball and roller bearings.

Kinematic pairs according to nature of mechanical constraint

(a) Closed pair : When the elements of a pair are held together mechanically, it is known as a closed pair. The contact between the two can be broken only by destruction of at least one of the member.

(b) Unclosed pair : When two links of a pair are in contact either due to force of gravity or some spring action, they constitute an unclosed pair.

Kinematic pairs according to nature of relative motion:

(a) Sliding pair: If two links have a sliding motion relative to each other, they form a sliding pair.

(b) Turning pair: When one link has a turning or revolving motion relative to the other, they constitute a turning or revolving pair.

(c) Rolling Pair: When the links of a pair have a rolling motion relative to each other, they form a rolling pair.

(d) Screw pair: If two mating links have a turning as well as sliding motion between them, they form a screw pair. Ex – lead screw and nut.

(e) Spherical pair: When one link in the form of a sphere turns inside a fixed link, it is a spherical pair. Ex – ball and socket joint.

Degrees of freedom:

An unconstrained rigid body moving in space can describe the following independent motions.

1. Translational motion along any three mutually perpendicular axes x, y, z and
2. Rotational motions about these axes. Thus, a rigid body possesses six degrees of freedom.

Figure-3

Degrees of freedom of a pair are defined as the number of independent relative motions both translational and rotational. A pair in space can have,

DOF = 6 – number of restraints.

Classification of kinematic pairs:

Depending upon the number of restraints imposed on the relative motion of the two links connected together, a pair can be classified as

Class	Number of restraints	Form	Restraint on		Kinematic pair	Figure
			Translatory	Rotary		
I	1	1st	1	0	Sphere – plane	
II	2	1st	2	0	Sphere – cylinder	
		2nd	1	1	Cylinder – plane	
III	3	1st	3	0	Spheric	
		2nd	2	1	Sphere – slotted cylinder	
		3rd	1	2	Prism – plane	
IV	4	1st	3	1	Slotted – spheric	
		2nd	2	2	Cylinder – cylinder	
V	5	1st	3	2	Cylinder – collar	
		2nd	2	3	Prismatic bar in prismatic hole	

A particular relative motion between two links of a pair must be independent of the other relative motions that the pair can have. A screw and nut pair permits translational and rotational motions. However as the two motion cannot be accomplished independently, a screw and nut pair is a kinematic pair of the fifth class.

Kinematic chain:

A kinematic chain is an assembly of links in which the relative motions of the links is possible and the motion of each relative to the other is definite.

Non – kinematic chain:

In case the motion of a link results in definite motions of other links, it is a non–kinematic chain.

A redundant chain: A redundant chain does not allow any motion of a link relative to the other.

Linkage:

A linkage is obtained if one of the links of a kinematic chain is fixed to the ground. If motion of any of the movable links results in definite motions of the others the linkage is known as a mechanism.

If one of the links of a redundant chain is fixed, it is known as a structure.

Mobility of mechanisms:

According to the number of general or common restraints a mechanism may be classified into different order.

A sixth order mechanism cannot exist since all the links become stationary and no movement is possible.

Degrees of freedom of a mechanism in space can be determined as follows.

Let N = total number of link in a mechanism
F = degree of freedom.
P_1 = number of pairs having one degree of freedom.
P_2 = number of pairs having two degree of freedom
In mechanism one link is fixed

Number of degrees of freedom of (N-1) movable links = 6(N-1) pair having one degree of freedom imposes 5 restraints on the mechanism reducing its degrees of freedom by $5P_1$.

Thus, $F = 6(N-1) - 5P_1 - 4P_2 - 3P_3 - 2P_4 - 1P_5$

For plane mechanisms, the following relation may be used to find the degree of freedom.

$$F = 3(N-1) - 2P_1 - 1P_2$$ → Gruebler's criterion.

If the linkage has single degree of freedom then $P_2 = 0$, Hence

$$F = 3(N-1) - 2P_1$$

Most of the linkage are expected to have one degree of freedom.

$$\Rightarrow 1 = 3(N-1) - 2P_1$$
$$\Rightarrow 2P_1 = 3N - 4$$

As P_1 and N are to be whole numbers, the relation can be satisfied only if as follows

$N = 4, \qquad P_1 = 4$

$N = 6, \qquad P_1 = 7$

$N = 8, \qquad P_1 = 10$

Thus with the increase in the number of links, the number of excess turning pairs goes on increasing. To get required number of turning pairs from the required number of binary links not possible. Therefore the excess or the additional pairs or joints can be obtained only from the links having more than two joining points

Equivalent Mechanisms:

It is possible to replace turning pairs of plane mechanisms by other type of pairs having one or two degrees of freedom, such as sliding pairs or cam pairs.

1. Sliding pair can be replaced as a turning pair with infinite length of radius
2. Two sliding pair can be replaced as two turning pair if their sliding axises intersect.
3. The action of a spring is to elongate or to shorten as it becomes in tension or in compression. A similar variation in length is accomplished by two binary links joined by a turning pair.
4. A cam pair has two degrees of freedom

   $$F = 3(N-1) - 2P_1 - 1P_2$$

   A cam pair can be replaced by one binary link with two turning pairs at each end.

Figure-4

The Four- bar chain:

A link that makes complete revolution is called the crank. The link opposite to the fixed link is called coupler and the fourth link is called lever or rocker if it oscilates or another crank if rotates. Condition for four–bar linkage is

$$d < a + b + c$$

Let $a > d$, then three extreme situations can be possible

(i) $d + a < b + c$

(ii) $d + c < a + b$

(iii) $d + b < c + a$

Adding (i) and (ii) $\Rightarrow 2d < 2b$ $\Rightarrow d < b$

Adding (ii) and (iii) $\Rightarrow 2d < 2a$ $\Rightarrow d < a$

Adding (iii) and (i) $\Rightarrow 2d < 2c$ $\Rightarrow d < c$

Thus the necessary conditions for the link 'a'to be a crank are that the shortest link is fixed and the sum of the shortest and the longest link is less than the sum of other two links.

If 'd' is fixed then a and c can rotate around d and also b; this is called drag – crank mechanism or rotary – rotary converter, or crank – crank or double crank mechanism.

B will rotate about a , if $\angle ABC$ is greater than 180^0 in any case, and b will rotate about c if $\angle DBC$ is more than 180^0 in any case.

Different mechanisms obtained by fixing different links of this kind of chain will be as follows (known as inversion).

1. If any of the adjacent links of link d i.e. a or c is fixed, d can have full revolution and link opposite to it oscillates. It is known as crank – rocker or crank- lever mechanism or rotary – oscillatory converter.
2. If the link opposite to the shortest link, i.e. link b is fixed and the shortest link d is made coupler, the other two links a and c would oscillate. The mechanism is called rocker – rocker or double – rocker or double –lever mechanism or oscilating – oscilating converter.

Shortest + longest < sum of other two → class–I four bar linkage.

Shortest + longest > sum of other two → class–II fourbar linkage.

All inversion s of class–II four bar linkage will give double rocker mechanism.

The above observations are summarized in the Grashof's law, which states that a four bar mechanism has at least one revolving link if the sum of the lengths of the largest and the shortest links is less than the sum of the lengths of the other two links.

Special cases when shortest+ longest = sum of other two.

Parallel – crank four bar: If b // d (two opposite links are parallel) then all the inversions will be crank – crank mechanism. Ex : Parallel mechanism and anti parallel mechanism.

Deltoid linkage: If shortest link fixed → a double – crank mechanism is obtained, in which one revolution of the longer link causes two revolutions of the other shorter links.

If any of the longer links is fixed two crank – rocker mechanisms are obtained.

Mechanical advantage:

The mechanical advantage of a mechanism is the ratio of the output force or torque to the input force or torque at an instant. Let friction and inertia forces are neglected.

$$M.A. = \frac{out\ put\ force/torque}{input\ force/torque}$$

Power input = power output (If loss is zero)

$T_2\omega_2 = T_4\omega_4$

$$\Rightarrow M.A. = \frac{T_4}{T_2} = \frac{\omega_2}{\omega_4} = ratiprocal\ of\ velocity\ ratio$$

In case crank rocker mechanism ω_4 of the output link is zero at extreme positions, i.e. when input link is in line with coupler link or $\gamma = 0^0$ or 180^0, the mechanical advantage is infinity. Only a small input torque can overcome a large output torque load. The extreme positions of the linkage are known as toggle positions.

Transmission angle:

The angle μ between the out put link and the coupler is known as transmission angle. The torque transmitted to the output link is maximum when the transmission angle μ is 90^0 . If $\mu = 0^0, 180^0$, the mechanism would lock or jam.

If μ deviates significantly from 90^0 the torque on output link decreases. Hence μ is usually kept more than 45^0 .

Applying cosine law to triangles ABD and BCD,

$$a^2 + d^2 - 2ad\cos\theta = k^2$$

$$b^2 + c^2 - 2bc\cos\mu = k^2$$

$$\Rightarrow a^2 + d^2 - 2ad\cos\theta = b^2 + c^2 - 2bc\cos\mu$$

$$\Rightarrow a^2 + d^2 - b^2 - c^2 - 2ad\cos\theta + 2bc\cos\mu = 0$$

The maximum or minimum values of transmission angle can be found out by putting $d\mu / d\theta$ equal to zero. Differentiating with θ

$$\Rightarrow ad\, sin\,\theta - bc\, sin\,\mu \times \frac{d\mu}{d\theta} = 0$$

$$\Rightarrow \frac{d\mu}{d\theta} = \frac{ad\, sin\,\theta}{bc\, sin\,\mu}$$

$\frac{d\mu}{d\theta}$ is zero when $\theta = 0^0$ or 180^0.

The slider crank chain:

When one of the turning pairs of four bar chain is replaced by a sliding pair, it is called as single – slider crank chain or slider crank chain.

When two of turning pairs of four bar chain is replaced by two sliding pair, it is called as double slider – crank chain.

If the sliding path line passes parallel with the fixed pivot point with some offset then it is called offset slider crank chain.

Inversions of single slider crank chain:

Different mechanisms obtained by fixing different links of a kinematic chain are known as its inversions.

1^{st} Inversion: The inversion is obtained when link 1 is fixed and links 2 and 4 are made the crank and the slider respectively.

Applications:
1. Reciprocating engine.
2. Reciprocating compressor.

2^{nd} Inversion: Fixing of link 2 of a slider – crank chain results in the second inversion. When its link 2 is fixed instead of link 1, link 3 along with the slider at its end B becomes a crank. This makes link 1 to rotate about o along with the slider which also reciprocates on it.

Applications:
1. White worth quick- return mechanism
2. Rotary engine.

3^{rd} Inversion: By fixing link 3 of the slider crank mechanism, third inversion is obtained. Here link 2 again acts as a crank and link 4 oscillates.

Applications:
1. Oscillating cylinder engine
2. Crank and slotted – lever mechanism.

4th Inversion: If link 4 of the slider – crank mechanism is fixed the fourth inversion is obtained. Link 3 can oscillate about the fixed pivot B on link 4. This makes end A of link2 to oscillate about B and end o to reciprocate along the axis of the fixed link 4.

Applications: Hand pump.

Inversion of double slider – crank chain:

First inversion: The inversion is obtained when link 1 is fixed and the two adjacent pairs 23 and 34 are turning pairs and the other two pairs 12 and 41 sliding pairs.

Application: Elliptical trammel.

Second Inversion: If any of the slide – blocks of the first inversion is fixed, the second inversion of the double – slider – crank chain is obtained. When link 4 is fixed, end B of crank 3 rotates about A and link 1 reciprocates in the horizontal direction.

Application : Scotch yoke.

Third Inversion: This inversion is obtained when link 3 of the first inversion is fixed and link 1 is free to move.

Application: Oldham's coupling.

Oldham coupling is used to connect two parallel shafts when the distance between their axes is small.

Velocity Analysis

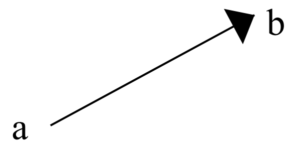
Analysis of mechanisms is the study of motions and forces concerning their different parts.

The study of velocity analysis involves the linear velocities of various points on different links of a mechanism as well as the angular velocities of the links.

When a machine or a mechanism is represented by a skeleton or a line diagram, then it is commonly known as a configuration diagram. Velocity analysis can be done two methods. 1. Analytical and 2. Graphically. Analytical method more convenient by computers. Graphical method is more direct and accurate to an acceptable degree. This graphical approach is done by two methods, i.e. (a) relative velocity method and (b) Instantaneous method.

Vector and vector addition/substraction:

$\vec{V}_{ba} =$



Velocity of a body B relative to A.

$$\vec{V}_{bo} = \vec{V}_{ba} + \vec{V}_{ao}, \Rightarrow ob = oa + ab;$$

$$\Rightarrow \vec{V}_{ba} = \vec{V}_{bo} - \vec{V}_{ao}$$

Motion of a link:

Let a rigid link OA, of length r, rotate about a fixed point o with a uniform angular velocity ω rad/s in the counter – clockwise direction. OA turns through a small angle δθ in a small interval of time δt . Then A will travel along the arc as shown in figure.

Velocity of A relative to O = $\dfrac{ArcAA'}{\delta t}$ or $\vec{V}_{ao} = \dfrac{r\delta\theta}{\delta t} = \omega r$

The direction of $\vec{V}_{ao}$ is along the displacement of A. Also, as δt approaches zero (δt→0), AA′ will be perpendicular to OA. Thus velocity of A is ωr and is perpendicular to OA. This can be represented by a vector **oa**.

Consider a point B on the link OA.

Velocity of B = ω . OB (perpendicular to OB).

If ob represents the velocity of B, it can be observed that

$\dfrac{ob}{oa} = \dfrac{\omega OB}{\omega OA} = \dfrac{OB}{OA}$ i.e. b divides the velocity vector in the same ratio as B divides the link.

Four – link mechanism:

In the four – link mechanism ABCD, AD is fixed, so a & d will be one fixed point in velocity diagram. It is required to find out the absolute velocity of C. Writing the velocity vector equation,

Vel. of C rel. to A = Vel. of C rel. to B + vel. of B rel. to A $\perp$ = Vel. of C rel to D

$V_{ca} = V_{cb} + V_{ba} = V_{cd}$

$\Rightarrow dc = bc + ab$

$\Rightarrow ac = bc + bc$

$\vec{V}_{ba}$ or ab = ω.AB; $\perp$ to AB

$\vec{V}_{cb}$ or bc is unknown in magnitude; $\perp$ to BC.

$\vec{V}_{cd}$ or dc is unknown in magnitude; $\perp$ to DC.

<u>Intermediate point:</u>

For point E on the link BC , $\frac{be}{bc} = \frac{BE}{BC}$, ae represents the absolute velocity of E.

<u>Offset point:</u>

Write the vector equation for point F,

$\vec{V}_{fb} + \vec{V}_{ba} = \vec{V}_{fc} + \vec{V}_{cd}$

$\vec{V}_{ba} + \vec{V}_{fb} = \vec{V}_{cd} + \vec{V}_{fc}$

$ab + bf = cf$

The vector $\vec{V}_{ba}$ and $\vec{V}_{cd}$ are there on the velocity diagram.

$\vec{V}_{fb}$ is $\perp$ BF; draw a line $\perp$ BF through b.

$\vec{V}_{fc}$ is $\perp$ CF ; draw a line $\perp$ CF through c.

The intersection of the two lines locates the point f. af indicates the velocity of F relative to A or absolute velocity of F.

<u>Velocity Images</u>

Triangle bfc is similar to triangle BFC in which all the three sides bc, cf, fb are perpendicular to BC, CF, and FB respectively. The triangles such as bfc are known as velocity images.

1. Velocity image of a link is a scaled reproduction of the shape of the link in a velocity diagram, rotated bodily through 90^0 in the direction of angular velocity.
2. Order of letter is same as in configuration diagram.
3. Ratios of different images of different links are different.

Angular velocity of links:

1. Angular velocity of BC :

(a) Velocity of C relative to B, $\vec{V}_{cb}$ (upward).
Thus C moves in the counter clockwise direction about B.

$$\vec{V}_{cb} = \omega_{cb} \times BC$$

$$\omega_{cb} = \frac{\vec{V}_{cb}}{CB}$$

(b) Velocity of B relative to C, $\vec{V}_{bc}$ (downward)
i.e. B moves in the counter – clockwise direction about C.

$$\omega_{bc} = \frac{\vec{V}_{bc}}{BC}$$

2. Angular velocity of CD:

Velocity of C relative to D, $\vec{V}_{cd}$ (clockwise)

$$\omega_{cd} = \frac{\vec{V}_{cd}}{CD}$$

Velocity of rubbing:

The rubbing velocity of the two surfaces will depend upon the angular velocity of a link relative to the other.

Pin at A : Let r_a = radius of the pin at A.
Then the velocity of rubbing = $r_a. \omega_{ba}$

Pin at D : Let r_d = radius of the pin at D.

Velocity of rubbing = $r_d . \omega_{cd}$

Pin at B: $\omega_{ba} = \omega_{ab} = \omega$(clockwise), $\omega_{bc} = \omega_{cb} = \frac{\vec{V}_{cb}}{BC}$ (counter clockwise).
Since the directions of the two angular velocities of links AB and BC are in the

opposite directions the angular velocity of one link relative to other is sum of the velocities.

Let r_b = radius of thepin at B , Velocity of rubbing = $r_b(\omega_{ab} + \omega_{bc})$

Pin at C: $\omega_{bc} = \omega_{cb}$ (counter clockwise)

$\omega_{dc} = \omega_{cd}$ (clockwise)

r_c = radius of pin at C.

Velocity of rubbing = $r_c\ (\omega_{bc} + \omega_{dc})$

Slider – crank Mechanism:

Figure shows a slider – crank mechanism in which OA is the crank moving with uniform angular velocity ω rad/s in the clockwise direction. At point B, a slider moves on the fixed guide G. AB is the coupler joining A and Bm. It is required to find out the velocity of slider at B.

Velocity of B relative to O = Velocity of B relative to A + velocity of A relative to O

$\Rightarrow \vec{V}_{bo} = \vec{V}_{ba} + \vec{V}_{ao}$

Take the vector $\vec{V}_{ao}$ which is completely known.

$\vec{V}_{ao} = \omega \cdot OA; \perp$ to OA

$\vec{V}_{ba} \perp AB,$ draw a line parallel to the motion of B.

$\vec{V}_{bo}$ $POG,$ Through g, draw a line parallel to the motion of B.

The intersection of two lines locates ‘b’. $\vec{V}_{bo}$ the slider velocity with respect to G.

The coupler AB has angular velocity in counter clockwise direction = $\frac{\vec{V}_{ba}}{AB}$

Crank and slotted lever mechanism:

A crank and slotted – lever mechanism, in which OP is the crank rotating at an angular speed ω rad/s in the clockwise direction about center O. A slider P is pivoted which moves on an oscillating link ASR. P and Q are coincident points. As the crank rotates there is relative movement of the points P and Q along AR.

Writing the velocity vector equation for the mechanism OPA.

Velocity of Q relative to O = velocity of Q relative to P + velocity of P relative to O.

$\Rightarrow \vec{V}_{qo} = \vec{V}_{qp} + \vec{V}_{po}$

$\Rightarrow \vec{V}_{qa} = \vec{V}_{po} + \vec{V}_{qp}$

$\vec{V}_{po} = \omega \cdot OP, \perp$ to OP

$\vec{V}_{qp}$ is unknown in magnitude; P to AR.

$\vec{V}_{qa}$ is unknown in magnitude; $\perp$ to AR.

<u>Construction of velocity diagram</u>:

Draw $\vec{V}_{po}, \vec{V}_{qp}$ P AR through P

$\vec{V}_{qa} \perp AR$, draw a line $\perp$ AR through a.

The intersection point is ‘q’. $\vec{V}_{qp}$ shows the velocity of sliding of the block on link AR. The point r will lie on vector ‘aq’ produced such that $\frac{ar}{aq} = \frac{AR}{AQ}$. To find the velocity of ram S, the velocity vector equation is

$\vec{V}_{so} = \vec{V}_{sr} + \vec{V}_{ro}$,

$\Rightarrow \vec{V}_{sg} = \vec{V}_{ro} + \vec{V}_{sr}$

$\vec{V}_{ro}$ is already in the diagram. Draw a line through r perpendicular to RS for vector $\vec{V}_{sr}$ and a line of motion of the slider S on the guide G, for the vector $\vec{V}_{sg}$. So S is located.

$\vec{V}_{sg}$ = velocity of the slider.

$\omega_{rs} = \frac{\vec{V}_{rs}}{RS}$ (clockwise).

$$\frac{V_s \, max(cutting)}{V_s \, max(returning)} = \frac{c-r}{c+r},$$

Where c = distance between fixed center,

and r = crank length.

<u>Instantaneous center</u>:

The body can be imagined to rotate about some point on the line perpendicular to the velocity vector passing through that point.

The intersection point of two different lines is called instantaneous center of rotation (I – center). An I – center is a center of rotation of a moving body relative to another body. It is named as I_{pq} and it will be in ascending order of the alphabets or digits.

Number of I–centers in a mechanism $= N = \frac{n(n-1)}{2}$ where n = number of links.

Kennedy's theorem:

If three plane bodies have relative motion among themselves, their I–center must lie on a straight line.

Locating I–centers:

A four–link mechanism ABCD has 4 links named 1,2,3,4. The number of I–centers is

$$N = \frac{n(n-1)}{2} = \frac{4(4-1)}{2} = 6$$

I–center 12 and 14 are fixed I–centers.

I–center 23 and 34 are permanent but not fixed I–centers.

I–center 13 and 24 which are neither fixed nor permanent can be located easily by applying Kennedy's theorem as explained below.

I–center 13:

As the three links 1, 2, 3 have relative motions among themselves, their I–centers lie on a straight line. Thus I–center 13 lies on the line joining 12 and 23.

Similarly I–center 13 lies on the line joining 14 and 34. The intersection of the two lines locates the I–center 13.

I–center 24: Considering two sets of links 2, 1, 4 and 2, 3, 4, the I–center would lie on the lines 12–14 and 23–34. The interaction locates the I–center 24.

Rules to Locate I–centers by inspections:

1. In a pivoted joint, the center of the pivot is the I–center for the two links.
2. In a sliding motion, the I–center lies at infinity in a direction perpendicular to the path of motion of the slider.

3. In a pure rolling contact of the two links, the I–center lies at the point of contact at the given instant.