Network Theory

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Gyan Ranjan Biswal received his B.E. in Electronics Engineering from the Pt. Ravishankar Shukla University, India in 1999 and M. Tech. (Honors) in Instrumentation & Control Engineering from the Chhattisgarh Swami Vivekananda Technical University, India in 2009 followed by Ph.D. in Electrical Engineering, specialized in the area of Power System Instrumentation (Power Generation Automation) from the Indian Institute of Technology Roorkee, India in 2013.

He is expertise in Design and Development of cooling systems for large size electrical generators, and the C&I of process industries. He has been in academia for about twelve years. Presently, he is with VSS University of Technology, Burla, India at the capacity of Head and Associate Professor, EEE from Dec. 2016. He has more than 65 publications in various Journals and Conferences of Internationally repute to his credit. He also holds a patent as well, and filed one more. He also adapted one international edition book published by Pearson India. He received research grants of US$90,000 (INR 50 lakhs). He has been supervised 09 Masters' theses, and registered 04 PhD theses. He has also been recognized with many national and international awards by elite bodies. He has been awarded with CICS award under the head of Indian National Science Academy for travel support to USA, MHRD Fellowship by Govt. of India, and Gopabandhu Das Scholarship in his career. His major areas of interests are Power System Instrumentation, Industrial Automation, Robust and Intelligent Control, the Smart Sensors, IoT enabled Smart Sensors, the Smart Grid, Fuel Cell lead Sustainable Sources of Energy, and System Reliability.

Dr. Biswal is a Fellow IE (India), Senior Member of IEEE, USA, and Life Member of ISTE, India. He is actively involved in review panels of different societies of international repute viz. IEEE, IFAC, and the ISA. Currently, he is also actively involved as a Member of IEEE–SA (Standards Association) working groups; IEEE P1876 WG, IEEE P21451-001 WG, and IEEE P1415. He has also been invited for delivering guest lectures at World Congress on Sustainable Technologies (WCST) Conf. 2012, London, UK, INDICON 2015, New Delhi, India, National Power Training Institute (NPTI), Nangal, India, and G.B. Pant Engineering College, Pauri, Gharwal, India, Surendra Sai University of Technology (formerly UCE), Burla, and as a guest expert in 2016 IEEE PES General Meeting Boston, MA, USA.

Syllabus

Network Theory

MODULE-I (9 HOURS) [Online mode: 5 HOURS + 1 Test]

Analysis of Coupled Circuits: Self-inductance and Mutual inductance, Coefficient of coupling, Series connection of coupled circuits, Dot convention, Ideal Transformer, Analysis of multi-winding coupled circuits, Analysis of single tuned and double tuned coupled circuits.

Transient Response: Transient study in series RL, RC, and RLC networks by time domain and Laplace transform method with DC and AC excitation. Response to step, impulse and ramp inputs of series RL, RC and RLC circuit.

MODULE-II (7 HOURS) [Online mode: 5 HOURS + 1 Test]

Two Port networks: Types of port Network, short circuit admittance parameter, open circuit impedance parameters, Transmission parameters, Condition of Reciprocity and Symmetry in two port network, Inter-relationship between parameters, Input and Output Impedances in terms of two port parameters, Image impedances in terms of ABCD parameters, Ideal two port devices, ideal transformer. Tee and Pie circuit representation, Cascade and Parallel Connections.

MODULE-III (8 HOURS) [Online mode: 5 HOURS + 1 Test]

Network Functions & Responses: Concept of complex frequency, driving point and transfer functions for one port and two port network, poles & zeros of network functions, Restriction on Pole and Zero locations of network function, Time domain behavior and stability from pole-zero plot, Time domain response from pole zero plot.

Three Phase Circuits: Analysis of unbalanced loads, Neutral shift, Symmetrical components, Analysis of unbalanced system, power in terms of symmetrical components.

MODULE-IV (9 HOURS) [Online mode: 5 HOURS + 1 Test]

Network Synthesis: Realizability concept, Hurwitz property, positive realness, properties of positive real functions, Synthesis of R-L, R-C and L-C driving point functions, Foster and Cauer forms.

MODULE-V (6 HOURS) [Online mode: 5 HOURS + 1 Test]

Graph theory: Introduction, Linear graph of a network, Tie-set and cut-set schedule, incidence matrix, Analysis of resistive network using cut-set and tie-set, Dual of a network.

Filters: Classification of filters, Characteristics of ideal filters.

Text and Reference Books

Recommended Text Books:

1. "Introductory Circuit Analysis", Robert L. Boylestad, Pearson, 12th ed., 2012.
2. "Network Analysis", M. E. Van Valkenburg, Pearson, 3rd ed., 2006.
3. "Engineering Circuit Analysis", W. Hayt, TMH, 2006.
4. "Network Analysis & Synthesis", Franklin Fa-Kun. Kuo, John Wiley & Sons.

Reference Books:

* "Basic Circuit Theory, Huelsman, PHI, 3rd ed.,
* "HUGHES Electrical and Electronic Technology", Revised by J. Hiley, K. Brown, and I. M. Smith, Pearson, 10th ed., 2011.
* "Circuits and Networks", Sukhija and Nagsarkar, Oxford Univ. Press, 2012.
* "Fundamentals of Electric Circuits", C. K. Alexander and M. N. O. Sadiku, McGraw-Hill Higher Education, 3rd ed., 2005.
* "Fundamentals of Electrical Engineering", L. S. Bobrow, Oxford University Press, 2nd ed., 2011.
* "Circuit Theory (Analysis and Synthesis)", A. Chakrabarti, Dhanpat Rai pub.

Other Important References

Reference Sites:	
1.	NPTEL, The National Programme on Technology Enhanced Learning (NPTEL): https://nptel.ac.in/
2.	MIT OpenCourseWare : https://ocw.mit.edu/index.htm

Course Outcomes

Upon successful completion of this course, you (students) will be able to

CO1	Analyze coupled circuits and understand the difference between the steady state and transient response of 1st and 2nd order circuit and understand the concept of time constant.
CO2	Learn the different parameters of two port network.
CO3	Concept of network function and three phases circuit and know the difference of balanced and unbalanced system and importance of complex power and its components.
CO4	Synthesis the electrical network.
CO5	Analyse the network using graph theory and understand the importance of filters in electrical system.

lution. The graph of the network is shown in Fig. 2·3 (*b*).

finitions

Branch. It is a line that replaces one network element.

Node. It is the point of intersection of two or more branches.

Tree. It is an interconnected open set of branches which include all the nodes of the given graph. There cannot be any closed loop in a tree of a graph.

Tree branch. It is any branch of a tree.

Tree Link or Link or Chord. It is that branch of the graph which does not belong to the particular tree under consideration.

Tree complement. It is the totality of tree links.

Loop. This is any closed contour selected in a graph.

Illustration. Two possible trees of the graph shown in Fig 2·3 (*b*) is given below in Fig. 2·4 (*a*) and 2·4 (*b*).

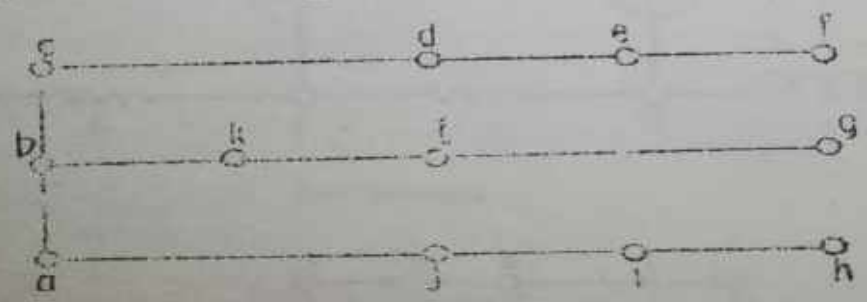
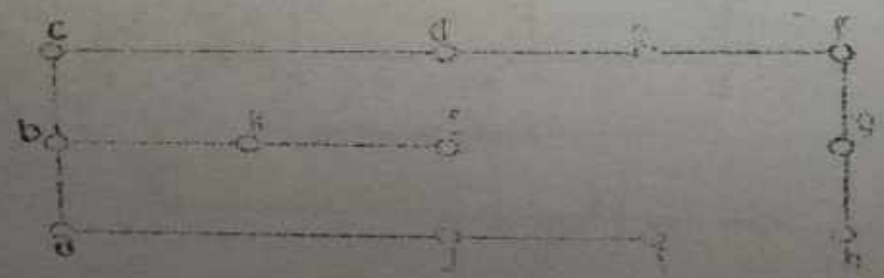


Fig. 2·4 (*a*)



If B be the total number of branches of the graph and L be the total number of links then with N number of nodes the number of tree branches will be $N-1$. Hence the number of links will be $L=B-(N-1)=B-N+1$. This is so because links are those branches of the graph which are eliminated in formulating the tree. Hence the total number of branches of the graph is given by $B=N-1+L$. For the graph shown in Fig. 2·3 (*b*),

$$B=15,\ N=12, \qquad \therefore\ L=15-(12-1)=4$$

This may be verified from the trees shown in Fig. 2·4.

Tie-set. It is a set of branches contained in a loop such that each loop contains only one link and the rest are tree branches as shown in Fig. 2·5 (*a*).

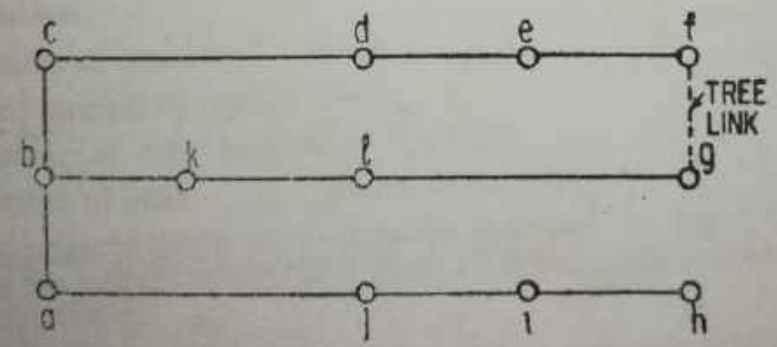


Fig. 2·5 (*a*)

In Fig. 2·5 (*a*) the broken line from node *f* to node *g* is a tree link or link and the branches appearing in the loop that has been formed by inserting the tree link form a *tie-set*.

Fig. 2·4 (*b*)

The tree has the following qualities :

(*a*) It consists of all the nodes of the graph.

(*b*) If the graph has N nodes, then its tree will have $(N-1)$ branches.

(*c*) There cannot be any closed path in a tree.

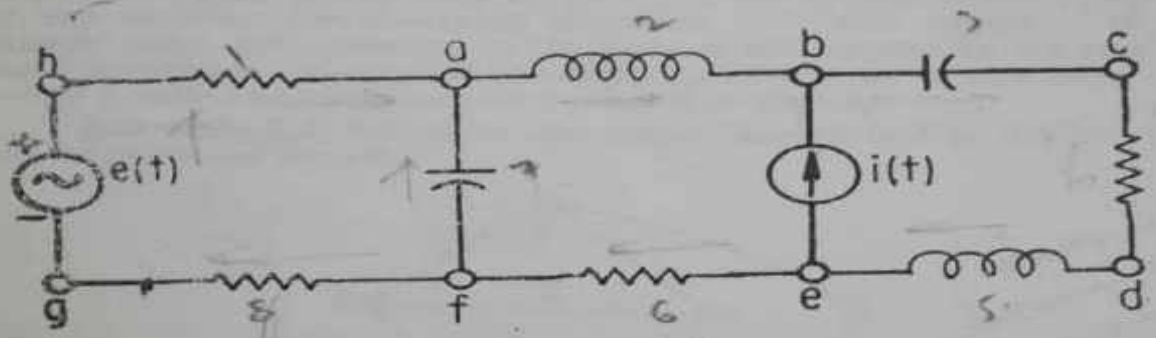
(*d*) There can be many possible different trees for a given graph depending on the number of nodes and branches of the graph.

Classifications of Graph

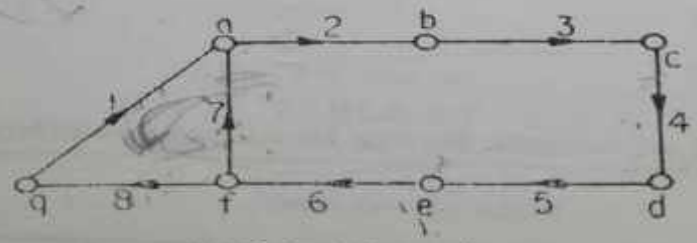
- Tie-set (involving mesh analysis)
- Cut-set (involving nodal analysis)

Tie-set method

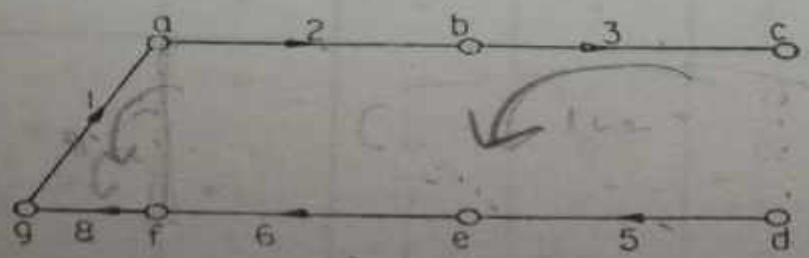
Example 2·3. Develop the Tie-set matrix for the network given in Fig. 2·7.



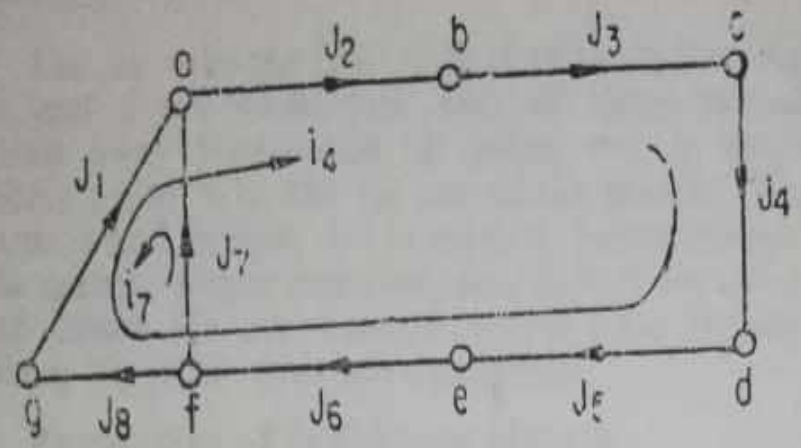
(a) Network



(b) Oriented graph



(c) One possible Tree



(*d*) Link currents in Tree of Fig. (*b*).

Fig. 2·7.

Solution.

Number of branches $B=8$

Total number of nodes $N=7$

Number of tree branches $=N-1=6$

Number of links $L=B-N+1=2$

The oriented graph of the network is drawn in Fig. 2·7 (*b*) by replacing the voltage source by a short circuit and the current source by an open circuit.

One possible tree is shown in Fig. 2·7 (*c*) and the link currents are shown in Fig. 2·7 (*d*).

Tie-Set Matrix

Link or loop current	*Branch Current*							
	1	*2*	*3*	*4*	*5*	*6*	*7*	*8*
7	−1	0	0	0	0	0	+1	−1
4	+1	+1	+1	+1	+1	+1	0	+1

Loop current i_7 is flowing in the same direction as that of the branch current in link 7, and the loop current i_7 is flowing against the branch currents in branches 1 and 8. Hence +1 is entered below branch 7 and −1 is entered each below branch nos. 1 and 8 Below other branches in the 1st row are entered 0 as shown in Table 2·2. The second tie-set is formed by the link 4 and tree branches 5, 6, 8, 1, 2 and 3. The loop current i_4 is flowing in the same direction as branch currents in 1, 2, 3, 4, 5, 6, 8 and hence below each of these branch number +1 is entered in the 2nd row. Also 0 is placed below branch number 7.

2·5. Incidence Matrix

The incidence matrix translates the graphical data of a network into algebraic form. Hence it has great importance in computer applications. This was first introduced by Kirchhoff who suggested a method of relating a rectangular array of numbers 0, +1 and −1 to a given oriented linear graph.

A directed or oriented graph can be traversed in two directions. One of these directions may arbitrarily be selected as the positive direction. Then, a matrix is constructed where rows correspond to different nodes or vertices of the graph and where columns correspond to different branches of the graph. If a branch J of the graph is connected to a node or vertex A with the direction of J converging to the node A, then +1 will appear in the entry AJ of the matrix. For diverging direction −1 will appear. If the branch does not connect to the node, 0 will appear as the entry in the appropriate position of the matrix. This array of numbers +1, −1, 0 is called an incidence or connection matrix.

Incident Matrix

—1, 0 is called an incidence ...

Example 2·4. Consider the graph shown in Fig. 2·8 and find out its incidence matrix.

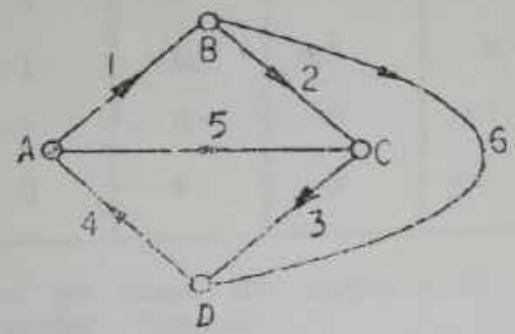


Fig. 2·8. Graph

away −1
incoming +1
not related 0

Solution.

TABLE 2·3

Incidence matrix of graph shown in Fig.

Node or Vertex	Branches or edges					
	1	2	3	4	5	6
A	−1	0	0	+1	−1	0
B	+1	−1	0	0	0	−1
C	0	+1	−1	0	+1	0
D	0	0	+1	−1	0	+1

Let us take the first node A of the table. At node A branches 1, 4 and 5 are connected. Out of these branches 1 and 5 are directed away from node A hence −1 is entered below branch numbers 1 and 5 in the 1st row of the matrix. Branch 4 is directed towards A and hence +1 is entered below branch 4 in the 1st row of the matrix. Other branches, *viz.*, 2, 3, 6 are not connected to node A and hence 0's are entered below these branches in the 1st row. Similarly the other rows are completed.

Properties of Incidence Matrix

2·5-1. (i) Determinant of the incidence matrix of a closed loop is zero.

Let us consider the closed path ABC and tabulate its incidence matrix. A closed path or loop has the same number of nodes or vertices and branches or edges.

TABLE 2·4

Incidence matrix for closed path ABC

Nodes	*Branches*		
	1	*2*	*3*
A	−1	0	−1
B	+1	−1	0
C	0	+1	+1

The incidence matrix is shown in Table 2·4. This is a square matrix.

Now the determinant is

$$\begin{vmatrix} -1 & 0 & -1 \\ +1 & -1 & 0 \\ 0 & +1 & +1 \end{vmatrix}$$

$$= -1\begin{vmatrix} -1 & 0 \\ +1 & +1 \end{vmatrix} - 0\begin{vmatrix} +1 & 0 \\ 0 & +1 \end{vmatrix} - 1\begin{vmatrix} +1 & -1 \\ 0 & +1 \end{vmatrix}$$

$$= 1 - 0 - 1 = 0.$$

(ii) Algebraic sum of the column entries of an incidence matrix is zero.

Every linear graph has an incidence matrix and vice-versa. As each branch has only two nodes, each column of incidence

Example 2·5. Draw the oriented graph from the incidence matrix given in Table 2·5.

TABLE 2·5

Nodes	Branches						
	1	2	3	4	5	6	7
A	+1	0	+1	−1	0	0	−1
B	0	+1	0	+1	0	+1	0
C	−1	−1	0	0	+1	−1	0
D	0	0	−1	0	−1	0	+1

Solution. First jot down the nodes A, B, C, D as shown in Fig. 2·9. Now consider branch number 1. It appears from the incidence matrix entries that this branch is between the nodes A and C and it is going away from node C towards node A as the entry against A is $+1$ and that against C is -1. Hence connect the nodes A and C by a line and give the arrow towards A and call it branch 1 as shown in Fig. 2·9. Similarly draw the other oriented branches.

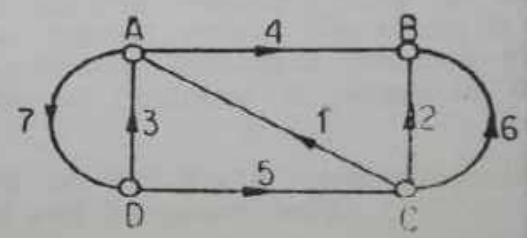


Fig. 2·9

Example 2·6. Find out the number of possible trees of the network graph shown in Fig. 2·10.

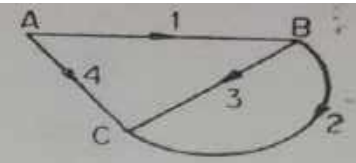


Fig. 2·10

Solution. The incidence matrix $[A]$ is given in Table 2·6.

TABLE 2·6

Nodes	Branches			
	1	2	3	4
A	−1	0	0	−1
B	+1	−1	−1	0
C	0	+1	+1	+1

Therefore, the reduced incidence matrix A_r is obtained by removing the last row matrix $[A]$ and is given by

$$[A_r]=\begin{bmatrix} -1 & 0 & 0 & -1 \\ 1 & -1 & -1 & 0 \end{bmatrix}$$

$$\therefore \quad [A_r']=\begin{bmatrix} -1 & 1 \\ 0 & -1 \\ 0 & -1 \\ -1 & 0 \end{bmatrix}$$

$\therefore$ The number of possible trees

$$=\det [A_r][A_r']$$

$$=\det \begin{bmatrix} -1 & 0 & 0 & -1 \\ 1 & -1 & -1 & 0 \end{bmatrix} \begin{bmatrix} -1 & -1 \\ 0 & -1 \\ 0 & -1 \\ -1 & 0 \end{bmatrix}$$

$$=\det \begin{bmatrix} 2 & -1 \\ -1 & 3 \end{bmatrix}$$

$$=\begin{vmatrix} 2 & -1 \\ -1 & 3 \end{vmatrix}$$

$$=6-1$$

$$=5$$

Properties of Incident Matrix

The trees are shown in Fig. 2·11.

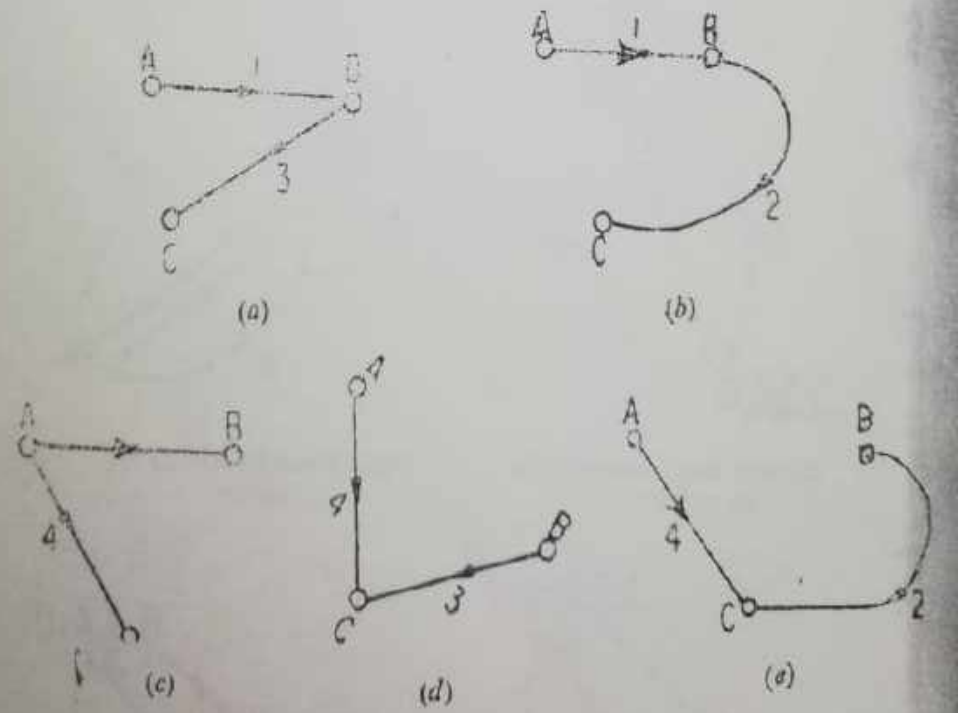


Fig. 2·11

Cut-set. It is that set of elements which separates two main parts of a network such that replacing any one element will destroy this property. Each cut-set contains one and only one tree branch, the remaining elements being tree links as shown in Fig. 2·5 (*b*).

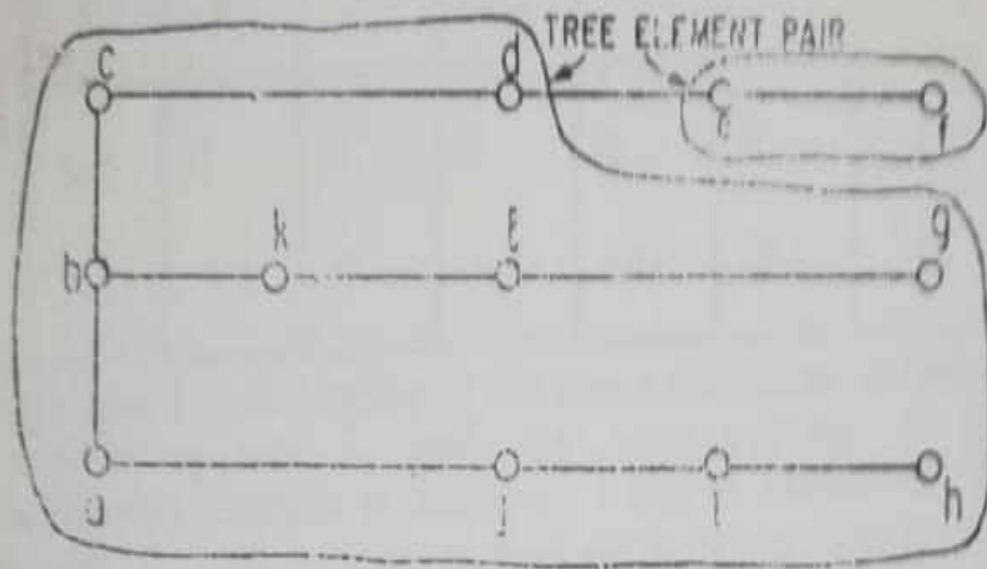


Fig. 2·5 (*b*)

In Fig. 2·5 (*b*) consider any tree branch say between node *d* and node *e*. Remove this branch and draw closed lines around the two remaining portions of the tree. We can refer to these groups as tree elements. Now those link which bridge the two tree elements

Cut Set Matrix and Node-pair Potential

From the tie-set matrix we can have a set of independent loop current variables given by the link currents. From the cut-set matrix we can select a set of independent node pair potential variables. Each cut-set contains only one tree-branch and the remaining are tree links. Tree branches connect all the nodes in the network graph. Hence it is possible to trace the path from one node to any other node by moving along the tree branch only. Therefore, it is possible to uniquely express the potential difference between any two nodes, generally called the 'node-pair voltage' in terms of the tree branch voltages.

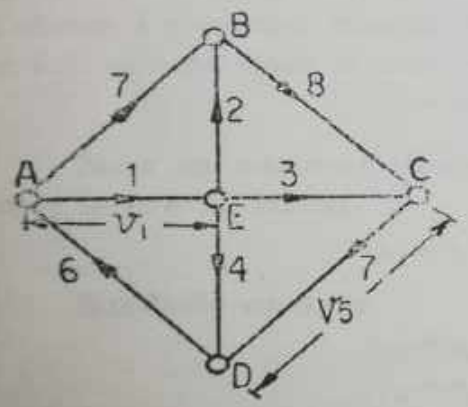
Consider the network graph of Fig. 2·6 (*b*) and the tree of Fig. 2·6 (*c*) redrawn in Fig. 2·12 (*a*) and (*b*) respectively.

Let $v_1, v_2, v_3 \ldots\ldots, v_8$ be the voltages of the branches 1, 2, 3, ..., 8 respectively. Current i_1 flows through branch 1 to make node A positive with respect to the node E Hence v_1 is designated as the voltage of node A with respect to that of node E. Potentials $v_2, v_3, \ldots\ldots, v_8$ are similarly chosen considering the assumed direction of flow of the pertinent branch currents. Branches 1, 2, 3, 4 constitute the tree branches as shown in Fig. 2·12 (*b*) and the voltages v_1, v_2, v_3, v_4 form a set of independent variables. In order to distinguish these independent variables from the dependent branch voltages, let these be denoted as e_1, e_2, e_3 and e_4 respectively. Thus we have

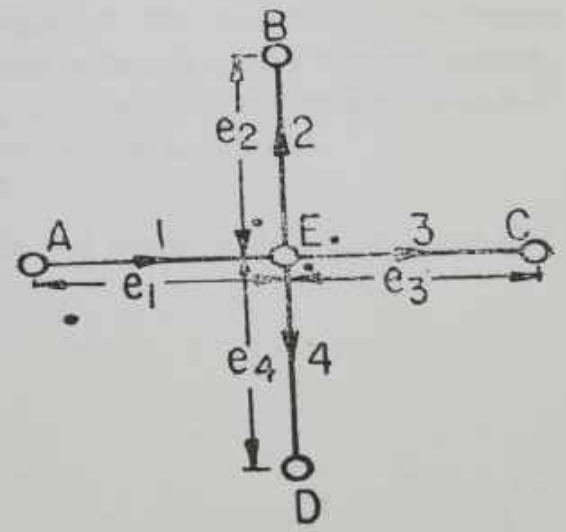
$$e_1 = v_1$$
$$e_2 = v_2$$

and $e_3 = v_3$

$e_4 = v_4$



(a) Graph



(b) Tree

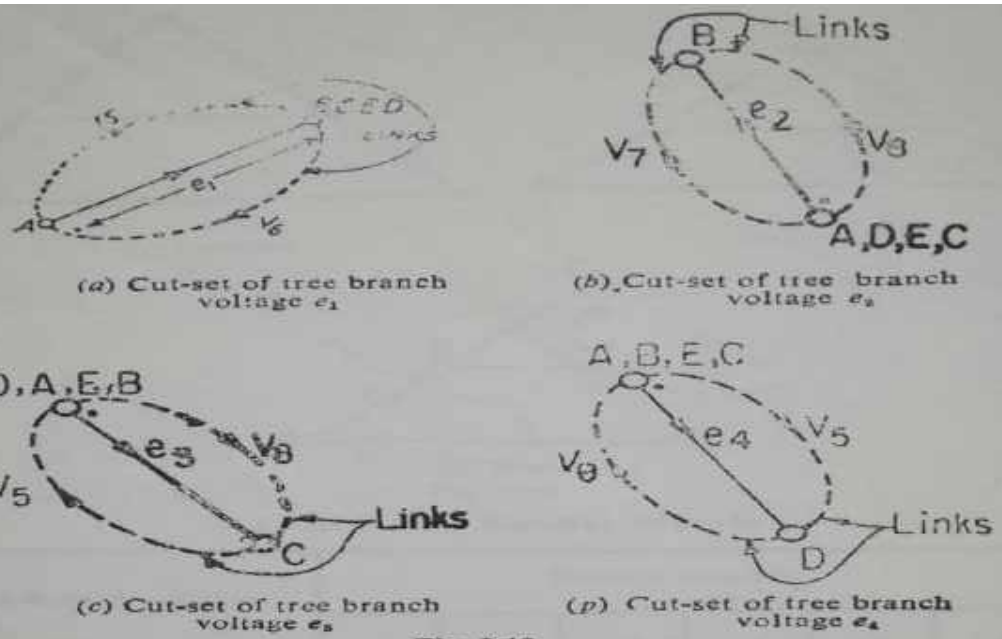
Fig. 2·12

The other branch voltages or the link voltages can be written in terms of the tree branch voltages e_1, e_2, e_3 and e_4 with the help of Fig. 2·12 (a).

$$v_5 = e_4 - e_3$$
$$v_6 = -e_1 - e_4$$
$$v_7 = e_3 + e_2$$
$$v_8 = e_3 - e_2$$

In case of complicated network, this method of writing the branch voltages in terms of tree-branch voltages may become quite cumbersome and confusing. Hence as in the case of tie-set matrix we may form cut-set matrix.

Cut Set Matrix and Node-pair Potential



(a) Cut-set of tree branch voltage e_1

(b) Cut-set of tree branch voltage e_2

(c) Cut-set of tree branch voltage e_3

(p) Cut-set of tree branch voltage e_4

Fig. 2·13

TABLE 2·7

Cut-Set Matrix of Graph shown in Fig. 2·12 (a)

Tree branch voltage	*Branch number*							
	1	*2*	*3*	*4*	*5*	*6*	*7*	*8*
e_1	+1	0	0	0	0	−1	+1	0
e_2	0	+1	0	0	0	0	+1	−1
e_3	0	0	+1	0	−1	0	0	+1
e_4	0	0	0	+1	+1	−1	0	0

Cut Set Matrix and Node-pair Potential

Now from this cut-set matrix we can write the equations giving the branch voltages in terms of the tree-branch voltages e_1, e_2, e_3 and e_4 by simply reading the columns of the cut-set matrix. Column 1 gives the equation for v_1 in terms of tree-branch voltages. In this column there is one $+1$ entry in row 1. Hence

$$v_1 = e_1$$

Now in column 5 there is one -1 entry in row 3 and one $+1$ entry in row 4. Hence

$$v_5 = e_4 - e_3$$

Similarly we have

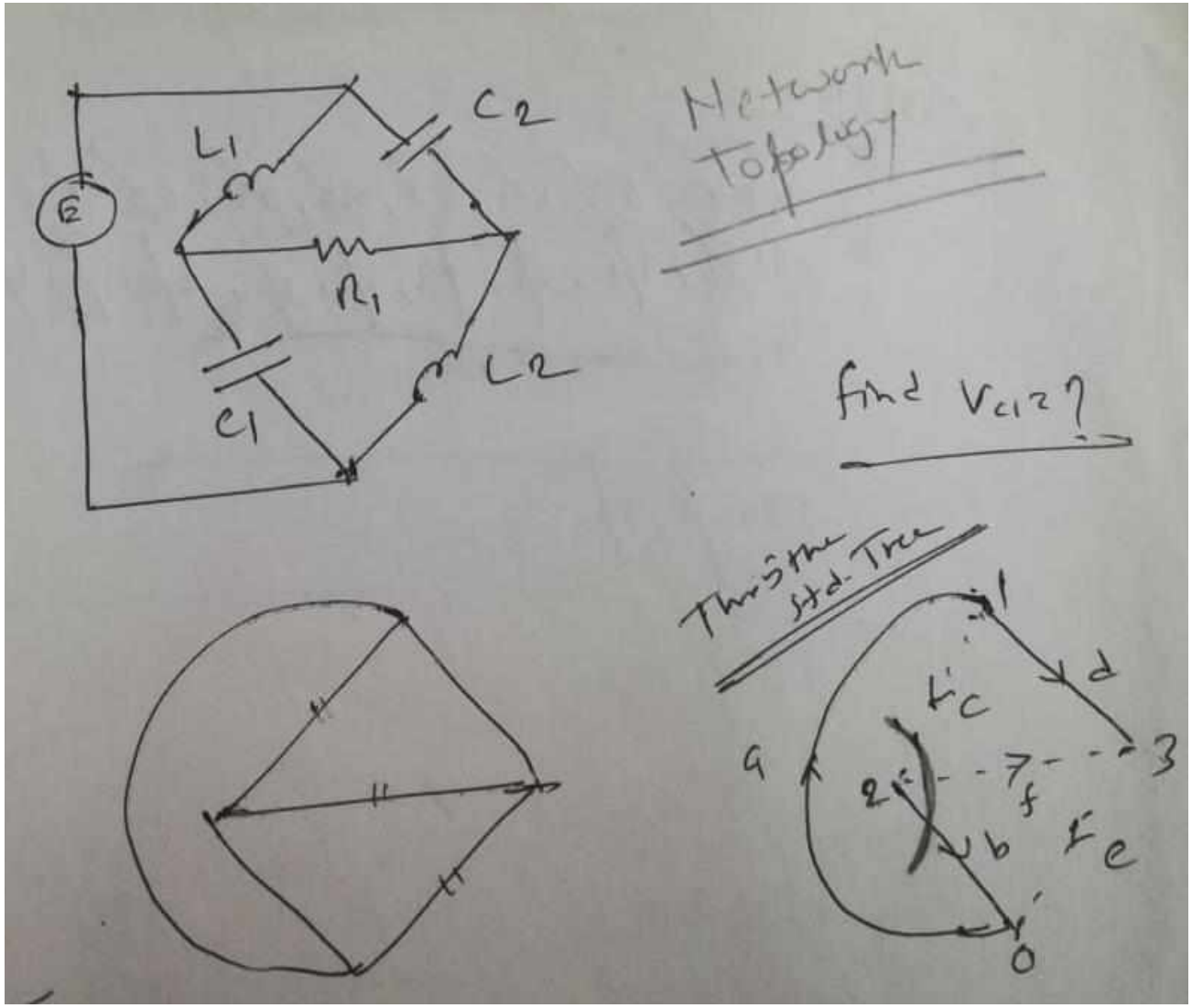
$$v_2 = e_2$$
$$v_3 = e_3$$
$$v_4 = e_4$$
$$v_5 = e_4 - e_3$$
$$v_6 = -e_1 - e_4$$
$$v_7 = e_1 + e_2$$
$$v_8 = e_3 - e_2$$

These equations agree with the equations derived earlier by classical method.

The cut-set matrix provides a compact and effective means of writing algebraic equations giving branch voltages in terms of tree-branch voltages.

The number of independent node-pair potentials is equal to the number of tree-branches. develope suitable matrix

Graph Theory Approach to Solve the Network Problems



State variable
Tree capacitor &
Co-tree inductors

$\rightarrow$ $v_{c1}, v_{c2}, i_{L1}, i_{L2}$

$$i_b + i_f - i_c = 0$$

$$C_1 \frac{dv_{c1}}{dt} + \frac{v_f}{R} - i_{L1} = 0$$

$$\frac{dv_1}{dt} = -\frac{1}{C_1 R} v_f + \frac{1}{C_1} i_{L1}$$

now v_f is voltage which is obtained from basic loop

$0 - 2 - 3 - 1 - 0$; loop into direction

$$v_f - v_d - v_a - v_b = 0$$

$$\Rightarrow v_f = v_d + v_a + v_b$$

$$v_f = v_{c2} - E + v_{c1}$$

$$\Rightarrow \boxed{v_{c1} = E - v_{R1} + v_{c2}}$$

This is how you can get eqn

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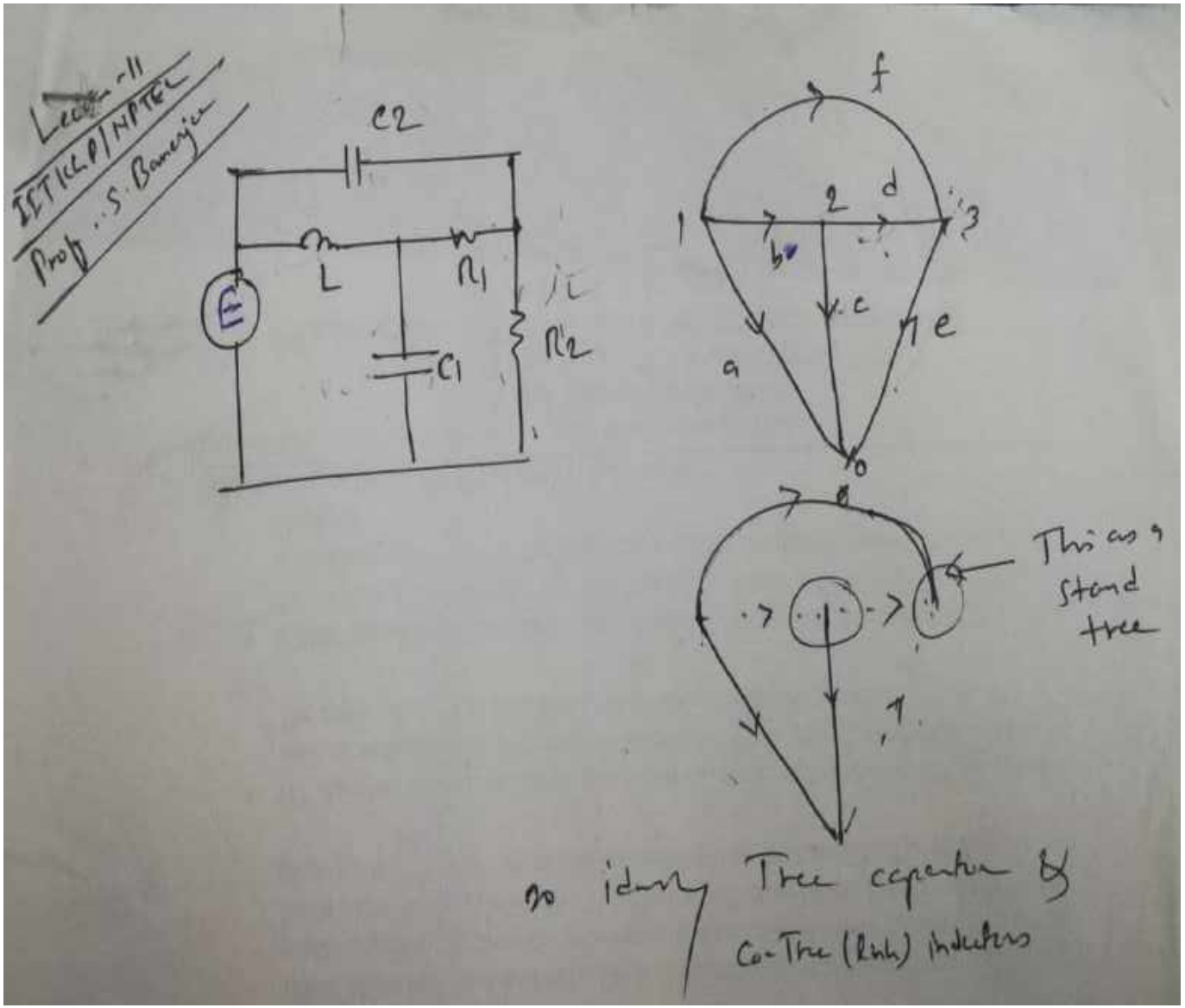
Note (i) If variable is a capacitor voltage, identify a basic cut set containing that capacitor & then write KCL.

(ii) If the variable is an inductor current, identify a basic loop (tie set) with that inductor & then write KVL.

Graph Theory Approach to Solve the Network Problems



Graph Theory Approach to Solve the Network Problems

State variables: v_{c1}, v_{c2}, i_L

#

let identify $v_{c1} = ?$

so apply basic (F) cut-set;

For v_{c1}:

$$i_b - i_d - i_c = 0$$

$$i_b - C_1 \frac{dv_{c1}}{dt} - i_d = 0$$

$$\Rightarrow \frac{dv_{c1}}{dt} = \frac{1}{C_1} i_L - \frac{v_d}{R_1}$$

$i_b = i_L$

$i_d = \frac{v_d}{R_1}$

now what is $v_d = ?$, we don't know

so v_d is not a state variable.

~~if it should~~ as cut-set yields current, so v_d state obtained from loop.

what is basic loop,

$2 - 3 - 1 - 0$

$$v_d - v_f + v_a - v_c = 0$$

$$\therefore v_d = v_f - v_a - v_c = 0$$

$$\therefore v_d = v_{c2} - E + v_{c1}$$

$v_f = v_{c2}$

$v_a = -E$

$v_c = v_{c1}$

now see $v_a = +/-$;

E → higher potential

Tacen polarity

in ↓ graph theory

∴ substitute v_d & form 2

$$\frac{dv_{c1}}{dt} = \frac{1}{C_1} i_L - \frac{v_f}{R_1}$$

For $v_{c2} = ?$ basic/F-cutset

$$i_f + i_d + i_e = 0$$

$$C_2 \frac{dv_{c2}}{dt} = -\frac{v_d}{R_1} - \frac{v_e}{R_2}$$

here voltage of v_d & v_e are common = 1

$$v_e - v_f + v_a = 0$$

at node 3

$$v_e = v_{c2} - E$$

so substitute v_e & v_d

$$\frac{dv_{c2}}{dt} = \frac{1}{C_2} [\quad]$$

For $i_L = ?$

write KVL eqn:

$$v_b + v_c - v_a = 0$$

$$L \frac{di_L}{dt} + v_{c1} - E = 0$$

$$\Rightarrow \frac{di_L}{dt} = -\frac{1}{L} v_{c1} + \frac{1}{L} E$$

* So MEng is identify std. Tree & so apply advance property of tree.

Example 2·13. Draw the graph of the Lattice network shown in Fig. 2·24. Select a suitable tree.

(*i*) Write the basic cut-set and basic tie-set matrices and write the *KVL* and *KCL* equations.

(*ii*) Find the relationship between independent and dependent variables.

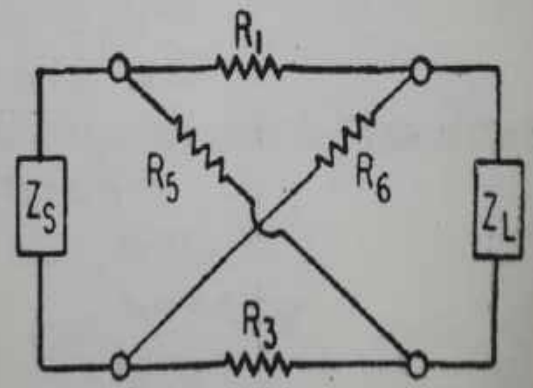


Fig. 2·24

Example 2·10. Draw the graph of the network shown in Fig. 2·20. Select node-pair potential variables and formulate the cut-set matrix. Write the equilibrium equation in matrix form on node-basis.

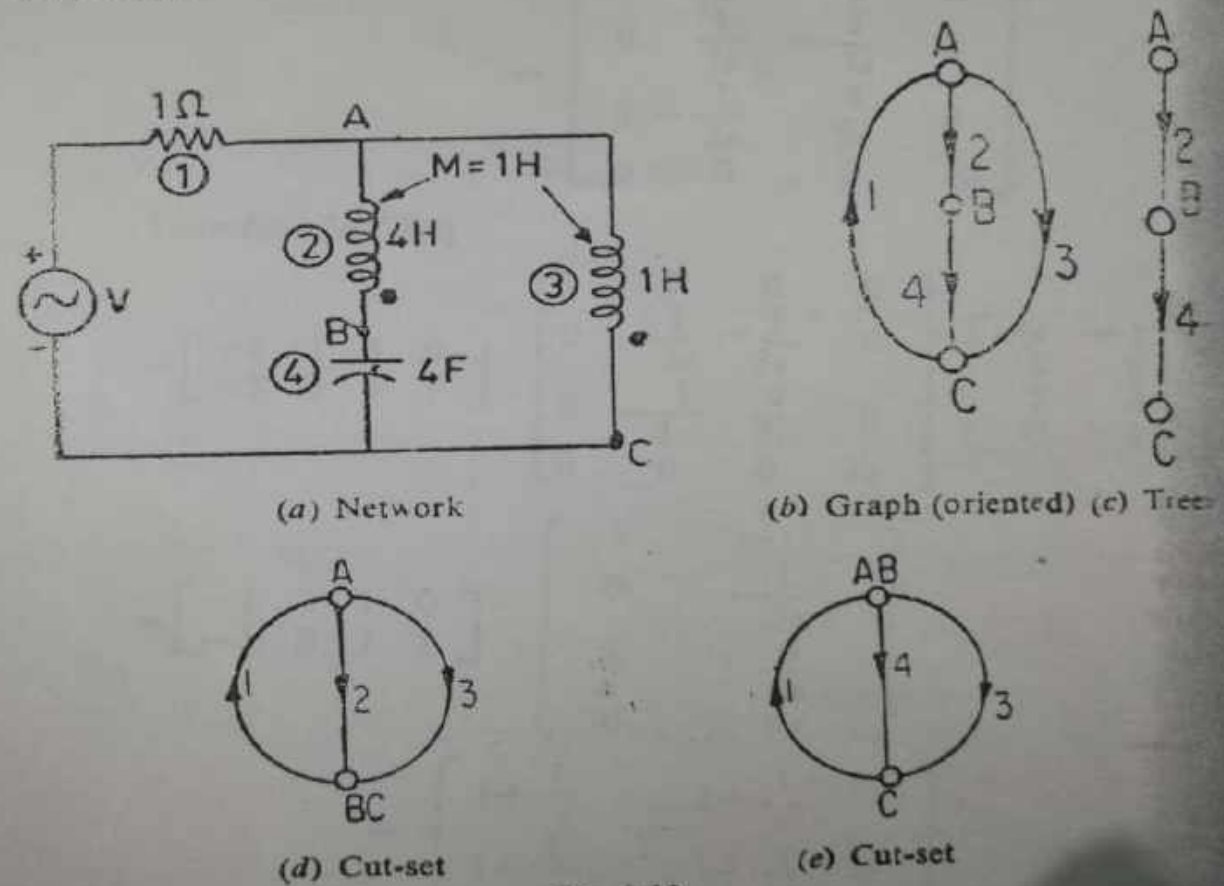


Fig. 2·20

Passive Filter Design

To transmit signals within a specified frequency range, An electrical network is required, which is called filter.

- Passive filter
- Active filter

Classification of filters

(a) As per arm impedances (prototype filter)

- Const. K - filter / ~~(prototype filter)~~ $Z_1 Z_2 = R_0^2$
- m - derived filter
  - the series arm of T-section
  - the shunt arm of π - section

  are multiplied (T) & divided (π) by m respectively.

(b) As per freq. characteristics

- LPF (Low-Pass Filter)
- HPF
- BPF
- BSF

An elect. wave filter/ simply filter is an elect. network which passes or allowed unattenuated tx^n of elect. signal within certain freq. range & stops / disallows tx^n of elect. signal outside this range.

* LPF, HPF, BPF, BSF/BRF/BEF

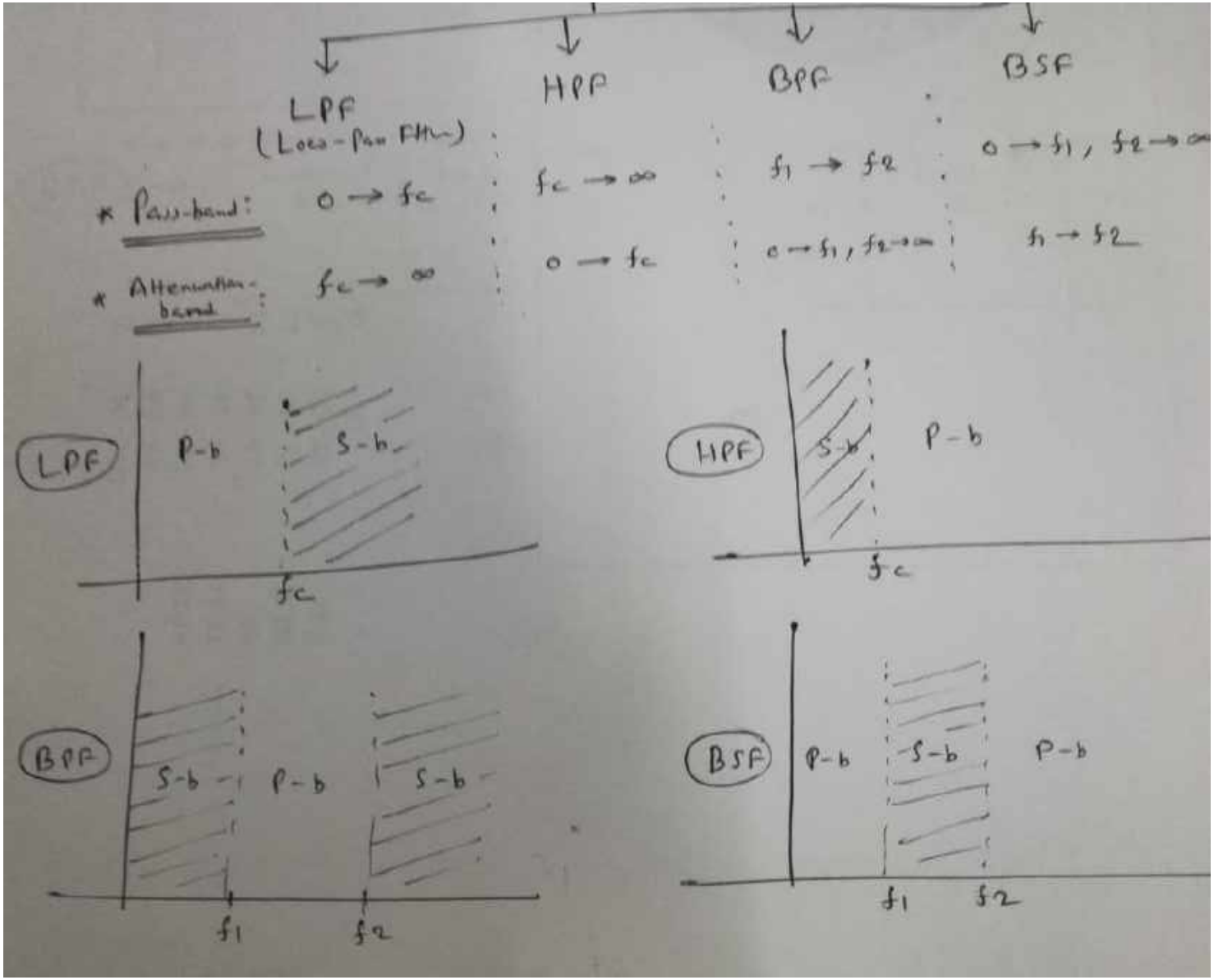
Char. of a filter ⟹

① Char. impedance (Z_0)

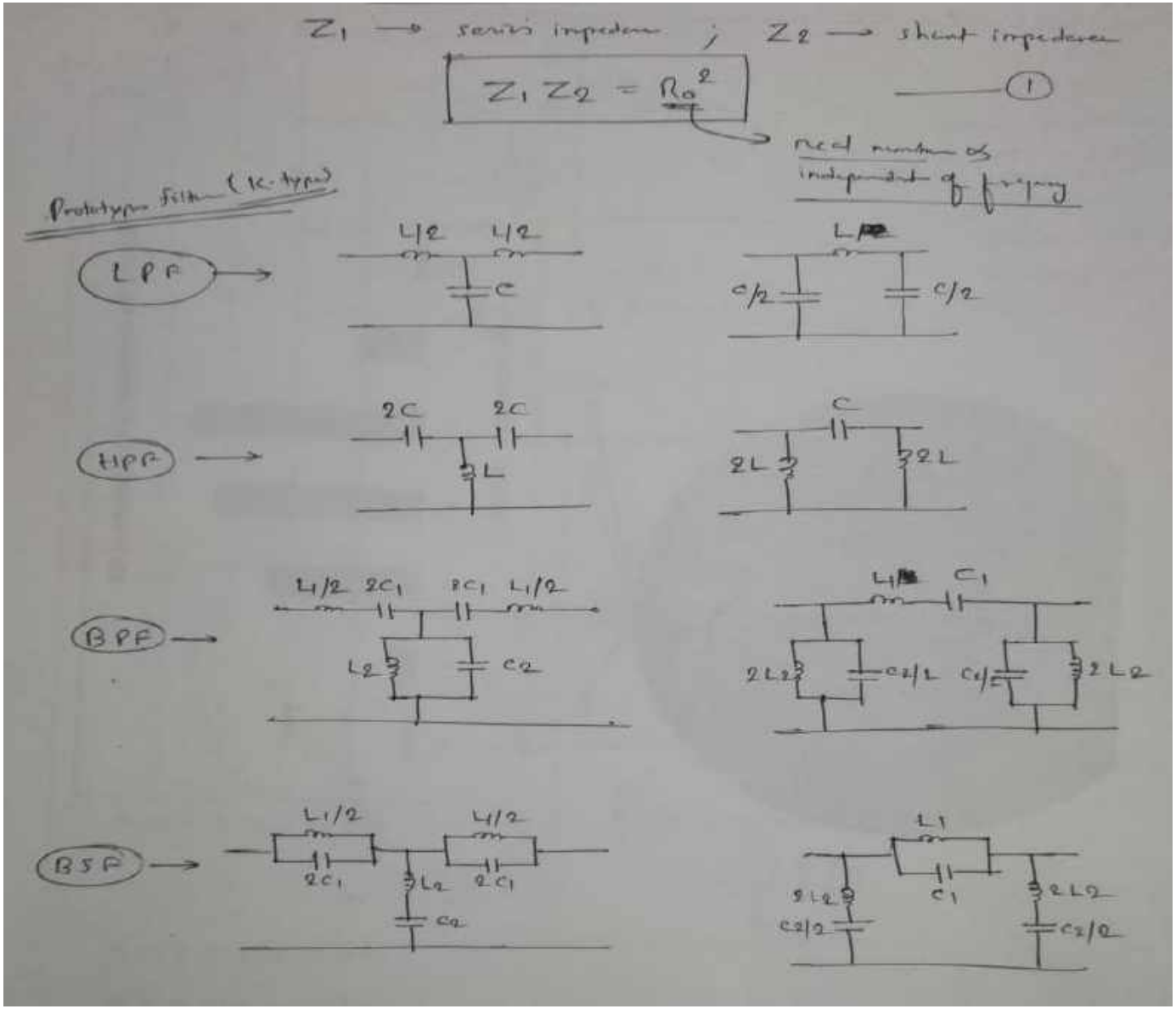
② Pass band → filter should have very low attenuation in passband & " high —"— in stopband.

③ f_c = (cutoff freq.) → demarcates the pass & stop band.

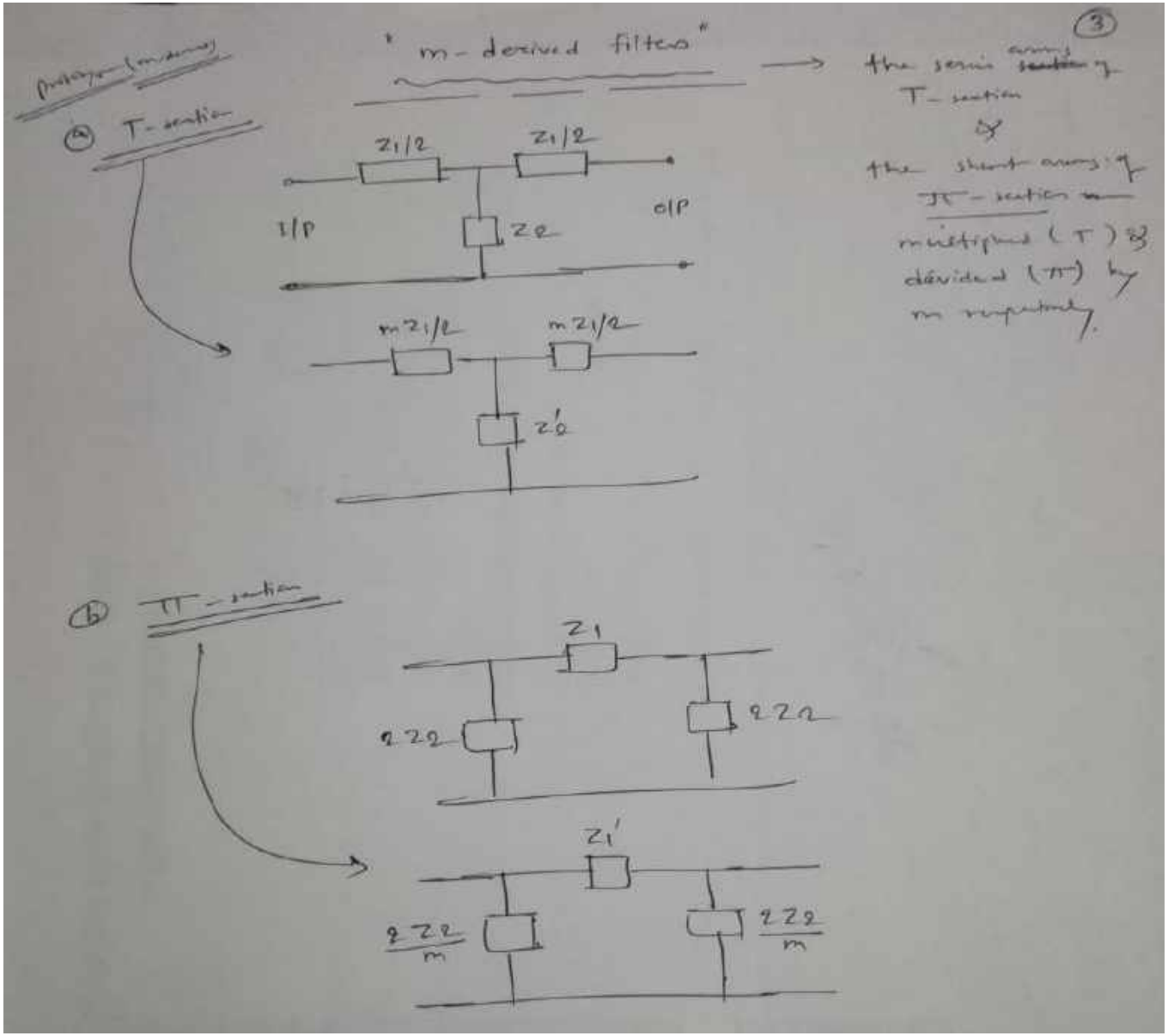
Types of Filter



Constant-K Filters



M-Derived Filters



Bel:-

Power transfer ratio / Power gain in bels $= \log_{10} \frac{P_o}{P_i}$ —— (1)

= Power loss / attenuation

Bel is a large unit.

$\underset{(dB)}{\text{decibel}} \Rightarrow 10 \log_{10} \frac{P_o}{P_i}$ —— (2)

(1/10 th of bel)

$$\left.\begin{aligned} &= 20 \log_{10} \left|\frac{V_o}{V_i}\right| \\ &= 20 \log_{10} \left|\frac{I_o}{I_i}\right| \end{aligned}\right\} \text{—— (3)}$$

Neper $\Rightarrow$ is a fundamental unit

eq current/ voltage ratio,

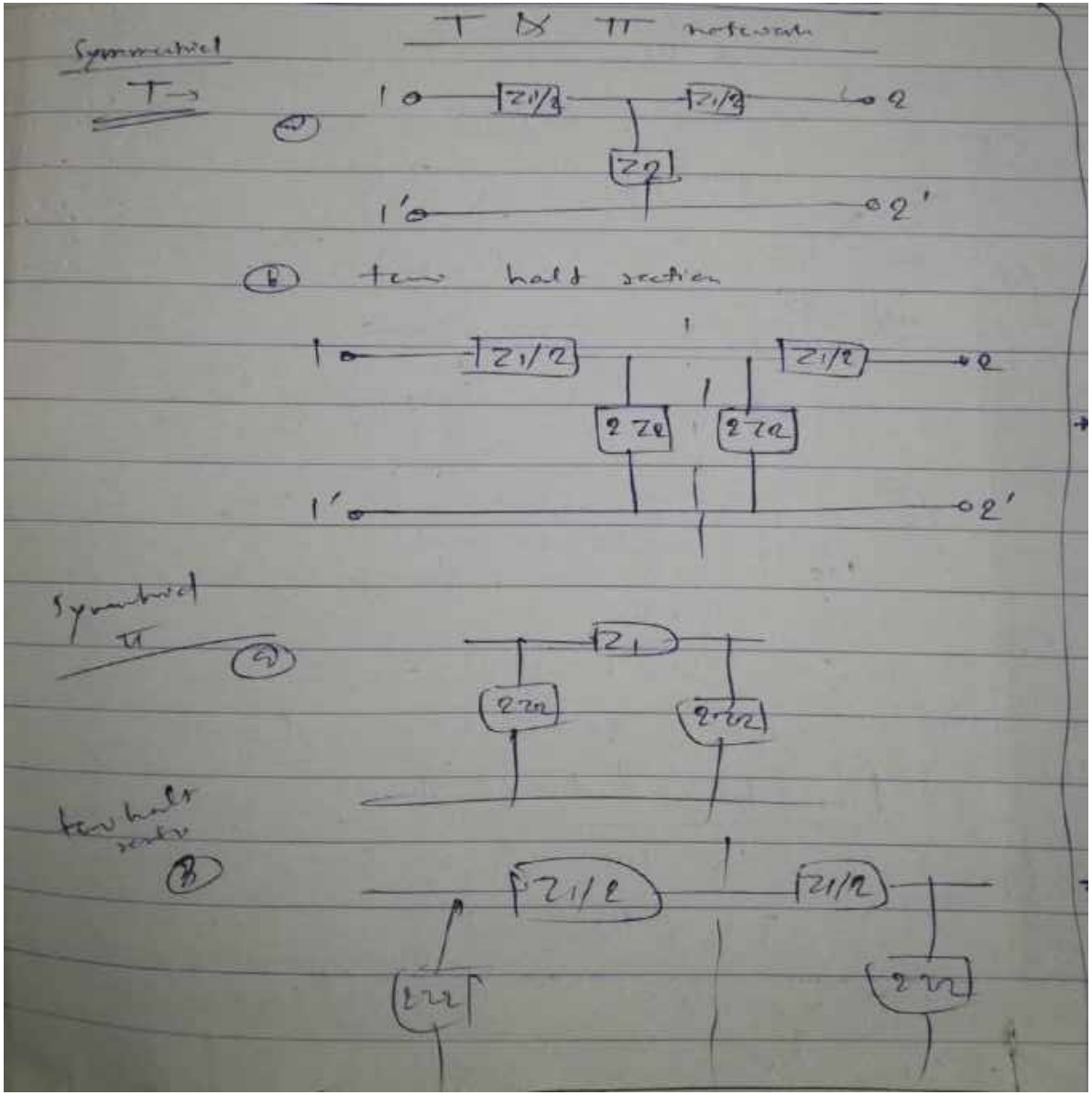
$= 10 \log_e \left|\frac{I_o}{I_i}\right|$

$* \; dB = 20 \log_{10} \left(\frac{I_o}{I_i}\right) = 10 \log_e \frac{I_o}{I_i} * \log_{10}$

$\therefore$ The attenuation in

$$\boxed{dB = 8.686 * \text{attenuation in nepers}}$$

Basic Relations in a Filter



Basic Relations in a Filter

(I) transfer constant 'θ' of a two port network:

in terms of ABCD parameters (two port)

$$\boxed{\theta = \cosh^{-1}\sqrt{AD}} \quad \text{—— (1)}$$

θ = propagation constant (γ)

$$\Rightarrow \therefore \gamma = \underbrace{\alpha}_{\text{attenuation const}} + j\underbrace{\beta}_{\text{phase const.}} = \cosh^{-1}\sqrt{AD} \quad \text{—— (2)}$$

* For T-section $\quad A = \frac{Z_{11}}{Z_{21}}, \; D = \frac{Z_{22}}{Z_{21}}$

fig (a) $\quad Z_{11} = Z_{22}$

$$A = D = \frac{2Z_2 + Z_1}{2Z_2}$$

$$\boxed{A = D = 1 + \frac{Z_1}{2Z_2}} \quad \text{—— (3)}$$

$$\therefore \gamma = \cosh^{-1} A \quad \text{—— (4)}$$

$$\gamma = \cosh^{-1}\left(1 + \frac{Z_1}{2Z_2}\right) \quad \text{—— (5)}$$

$$\cosh\gamma = 1 + \frac{Z_1}{2Z_2}$$

$$= 1 + 2\sinh^2\frac{\gamma}{2}$$

$$\sinh \gamma/2 = \sqrt{\frac{Z_1}{4Z_2}} \quad \text{—— (6)}$$

$$\frac{x}{2} = \sinh^{-1}\sqrt{\frac{Z_1}{4Z_2}}$$

$$\Rightarrow \quad \theta = \gamma = 2\sinh^{-1}\sqrt{Z_1/4Z_2} \quad \text{--- (7)}$$

from eqn (5) / (7)

whenever $\alpha = 0$; $\gamma = \theta = j\beta$,

then filter is said to be attenuated filter or tx line is said to be free.

from eqn (6);

$$\sinh\left(\alpha/2 + j\beta/2\right) = \sqrt{\frac{Z_1}{4Z_2}}$$

$$\sinh\frac{\alpha}{2}\cos\beta/2 + j\cosh\alpha/2\sin\beta/2 = \sqrt{\frac{Z_1}{4Z_2}} \quad \text{--- (8)}$$

Two cases

(A) $\frac{Z_1}{Z_2} > 0$; i.e $\frac{Z_1}{4Z_2}$ is +ve

i.e Z_1 & Z_2 have the same sign.

from eqn (8);

$$\left.\begin{aligned} \sinh\frac{\alpha}{2}\cos\beta/2 &= \sqrt{\frac{Z_1}{4Z_2}} \\ \& \quad \cosh\frac{\alpha}{2}\sin\beta/2 &= 0 \end{aligned}\right\} \text{--- (9)}$$

from eqn (9) ;

$\sin \beta/2 = 0 \quad \therefore \beta/2 = \pm n\pi$ (multiples)

& $\cos \beta/2 = 1$

$\therefore \sinh \alpha/2 = \sqrt{\frac{Z_1}{4Z_2}}$

$$\boxed{\therefore \alpha = 2 \sinh^{-1} \sqrt{\frac{Z_1}{4Z_2}}}$$

attenuation band

attenuation take place at freq. band when $\frac{Z_1}{4Z_2}$ is +ve

(B) $\frac{Z_1}{4Z_2} < 0$

i.e $\frac{Z_1}{4Z_2}$ is imaginary.

$\therefore \sinh \frac{\alpha}{2} = 0 \therefore \alpha = 0$.

i.e attenuation is zero.

called Pass band / tx in band

$\because \sinh \frac{\alpha}{2} = 0 \quad \cosh \frac{\alpha}{2} = 1$

$\therefore j \sin \beta/2 = \sqrt{\frac{Z_1}{4Z_2}}$

$$\Rightarrow \boxed{\beta = 2 \sin^{-1} \sqrt{-\frac{Z_1}{4Z_2}}}$$

i.e Pass free transmission

$$\boxed{-1 < \frac{Z_1}{4Z_2} < 0}$$

because when $\frac{Z_1}{4Z_2}$ is -ve

& $\left|\frac{Z_1}{4Z_2}\right| > 1$

$j\sin\beta/2 = \sqrt{\frac{Z_1}{4Z_2}}$ $\therefore$ $\sin\beta/2 > 1$,

this is absurd. & this signifies that for free transmission

$$-1 < \frac{Z_1}{4Z_2}$$

2. & when $\frac{Z_1}{4Z_2} < -1$

Assuming that elements are pure reactance

i.e $\cos\beta/2 = 0$, $\sin\beta/2 = \pm 1$

$\therefore$ $\beta/2 = (2n-1)\pi/2$

$\because n = 1, 2, \ldots$

in this range by putting $\cos\beta/2 = 0$ i.e $\sin\beta/2 = 1$

$j\cosh\alpha/2 = \sqrt{\frac{Z_1}{4Z_2}}$

$$\boxed{\alpha = 2\cosh^{-1}\sqrt{\left|\frac{Z_1}{4Z_2}\right|}}$$

(fc) Cutoff freq. ⟹ For a given network, as the freq. of applied signal is ~~either~~ altered; the freq. at which conditions changes from tx^ion to attenuation or vice-versa are termed as cutoff freq.

As for free transmission

$$-1 < \frac{Z_1}{4Z_2} < 0$$

$$\frac{Z_1}{4Z_2} = 0 \quad \& \quad \frac{Z_1}{4Z_2} = -1$$

(a) (b)

or

$$\left.\begin{aligned} Z_1 &= 0 \\ \& \quad Z_1 + 4Z_2 &= 0 \end{aligned}\right\}$$

Output Impedance of filter in Pass & Stop Band

Z_0 is given by ;

$$Z_0 = \sqrt{B/C} \quad \text{—— (1)}$$

We know that

$$B = \Delta Z / Z_{21}$$

$$\Delta Z = (Z_{11}Z_{22} - Z_{12}Z_{21})$$

&

$$C = 1/Z_{21}$$

$$\therefore \quad Z_0 = \sqrt{Z_{11}Z_{22} - Z_{12}Z_{21}} \quad \text{—— (2)}$$

For the T-section

$$\left.\begin{aligned} Z_{11} = Z_{22} &= Z_2 + \frac{Z_1}{2} \\ \& \; Z_{12} = Z_{21} &= Z_2 \end{aligned}\right\} \quad \text{—— (3)}$$

$$\therefore \quad Z_{0T} = \sqrt{\left(Z_2 + \frac{Z_1}{2}\right)^2 - Z_2^2}$$

after manipulation

$$\Rightarrow \quad Z_{0T} = \sqrt{Z_1 Z_2 \left(1 + \frac{Z_1}{4Z_2}\right)} \quad \text{—— (4)}$$

Let filter network constituted by pure reactances only & Z_1 & Z_2 be reactance of opposite nature

i.e. $\frac{Z_1}{4Z_2} < 0$

$$Z_1 = jX_1, \quad Z_2 = jX_2$$

then

$$Z_{0T} = \sqrt{-X_1X_2\left(1 + \frac{X_1}{4X_2}\right)} \quad \text{—— (5)}$$

$\Rightarrow$ in pass band $\Rightarrow$ Noted that Z_1 & Z_2 (X_1 & X_2) are of opposite sign & further $-1 < \frac{X_1}{4X_2} < 0$.

from eqn (5), In pass band both quantities $-X_1X_2$ & $\left(1 + \frac{X_1}{4X_2}\right)$ are positive i.e. radical sign becomes positive

Output Impedance of filter in Pass & Stop Band

& hence Z_{0T} is real i.e resistive &
let it be indicated by R_0.
According when a network is designed
such a manner that $R_0 = Z_L = Z_{in}$

apply maximum power transfer theorem

i.e the network is pass to resistive load without attenuation (6)

⇒ In stop bands ⇒ with $\frac{X_1}{4X_2} > 0$.
X_1 & X_2 are of the same sign
& $\frac{X_1}{4X_2} > 0$ indicates that
Z_0 is a pure ~~resistance~~ reactance.

⇒ In stop band with X_1 & X_2 of opposite signs
$\frac{X_1}{4X_2} < -1$.

then in this case X_1X_2 & $\left(1+\frac{X_1}{4X_2}\right)$ both terms are (−)ive. that is entire quantity of eqn (4) is (−)ive. & hence
char. impedance is a pure reactance

Output Impedance of filter in Pi-Network

Similarly

$$Z_{11} = Z_{22} = \frac{(Z_2/2)(Z_1 + Z_2/2)}{Z_1 + 4Z_2}$$

$$\therefore \quad Z_{11} = Z_{22} = \frac{Z_1/2 + Z_2}{1 + \frac{Z_1}{4Z_2}} \quad \text{—— (1)}$$

$$\& \quad Z_{12} = Z_{21} = \frac{Z_2}{1 - \frac{Z_1}{4Z_2}} \quad \text{—— (2)}$$

Now we know that

$$Z_{0\pi} = \sqrt{Z_{11}Z_{22} - Z_{12}Z_{21}} \quad \text{—— (3)}$$

after manipulation

$$Z_{0T} = \frac{Z_1 Z_2}{\sqrt{Z_1 Z_2 \left(1 + \frac{Z_1}{4Z_2}\right)}} \quad \text{—— (4)}$$

i.e. $$Z_{0\pi} = \frac{Z_1 Z_2}{Z_{0T}}$$

or $$\boxed{Z_{0\pi} Z_{0T} = Z_1 Z_2} \quad \text{—— (5)}$$

Note: Among both Z_1 & Z_2 one is to be pure resistance & so we also find that

(5a) Z_0 is pure resistive in passband

& Z_0 is pure resistive in stop band

Classification of Filters

(LPF, HPF, BPF, BSF)

(1) According to their freq. char.

(2) Depending upon the relation betn Z_1 & Z_2

(a) const. k filters / prototype filters

(b) m - derived filter

(a) const. k filters :—

$$\boxed{Z_1 Z_2 = k^2}$$

∴ k is a const. independent freq.

& in ~~gen~~ almost all cases

$$\boxed{Z_1 Z_2 = k^2 = R_0^2}$$

R_0 is called "design impedance" of filter section.

thus in const k ; T / π sect. also referred as prototype filters.

(b) m - derived filter —

$Z_1 Z_2 \neq k^2$ but have same char. impe (Z_0) as corresponding to const k section but have much sharper attenuation characteristics

Constant-k Low Pass Filter

$$Z_1 Z_2 = k_0^2 = R_0^2 = \frac{L}{C} \quad \text{—(1)}$$

if $Z_1 = j\omega L$, $Z_2 = \frac{1}{j\omega C}$

~~Design resistance~~ (nominal characteristic impedance)

$$\therefore R_0 = \sqrt{Z_1 Z_2} = \sqrt{L/C} \quad \text{—(2)}$$

Also $\frac{Z_1}{4Z_2} = -\frac{\omega^2 LC}{4}$ —(3)

Now; pass band is given by

$$-1 < \frac{Z_1}{4Z_2} < 0$$

so two ~~fc~~ relations are

$$\frac{Z_1}{4Z_2} = 0 \quad \& \quad \frac{Z_1}{4Z_2} = -1$$

$Z_1 = j\omega L = 0$ i.e. $\omega = 0$ / $f = 0$. this gives first cutoff Freq. —(4)

& $\frac{Z_1}{4Z_2} = -1 = -\frac{\omega^2 LC}{4}$

i.e. $\omega_c = \frac{2}{\sqrt{LC}}$

$f_c = \frac{1}{\pi\sqrt{LC}}$ the second cutoff freq. —(5)

then pass band will of constant value extends from 0 to f_c i.e. 0 to $\frac{1}{\pi\sqrt{LC}}$

Variation of α and β with Frequency

$$\sinh\frac{\gamma}{2} = \sqrt{\frac{Z_1}{4Z_2}} = \sqrt{\frac{-\omega^2 LC}{4}}$$

or

$$\sinh\frac{\gamma}{2} = \frac{j\omega\sqrt{LC}}{2} \quad \text{——} \quad (1)$$

now $f_c = \dfrac{1}{\pi\sqrt{LC}}$ (given)

$$\therefore \sinh\frac{\gamma}{2} = j\frac{f}{f_c} \quad \text{——} \quad (2)$$

now knows that passband extends from

$$-1 < \frac{Z_1}{4Z_2} < 0$$

i.e. $-1 < -\dfrac{\omega^2 LC}{4} < 0$

or $-1 < -\left(\dfrac{f}{f_c}\right)^2 < 0$

or $1 > \left(\dfrac{f}{f_c}\right)^2 > 0$ *

i.e. $\boxed{f_c > f > 0}$ —— (1)

now in the passband attenuation is zero. i.e.

$$\gamma = j\beta \quad (\alpha = 0)$$

now $\sinh\left(\dfrac{j\beta}{2}\right) = \sqrt{\dfrac{Z_1}{4Z_2}} = \dfrac{j\omega\sqrt{LC}}{2}$ —— (2)

or $\sin\dfrac{\beta}{2} = \sqrt{\left|\dfrac{Z_1}{4Z_2}\right|} = \dfrac{\omega\sqrt{LC}}{2} = \dfrac{f}{f_c}$ —— (3)

hence $\beta = 2\sin^{-1}\sqrt{\left|\dfrac{Z_1}{4Z_2}\right|}$

$$\beta = 2\sin^{-1}\left(\frac{f}{f_c}\right)$$

then in passband

$$\begin{cases} \alpha = 0 \ \& \\ \beta = 2\sin^{-1}\left(\dfrac{f}{f_c}\right) \end{cases} \quad \text{——} \quad (4)$$

Variation of α and β with Frequency

$f > f_c$

or $\frac{Z_1}{4Z_2} < -1$

$$\alpha = 2\cosh^{-1}\sqrt{\left|\frac{Z_1}{4Z_2}\right|}$$

$$\alpha = 2\cosh^{-1}\left(\frac{f}{f_c}\right)$$

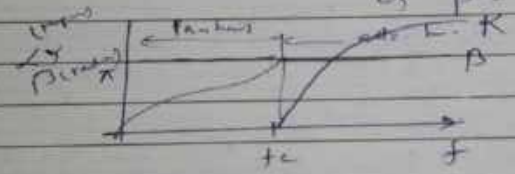
& $\frac{\beta}{2} = (2n-1)\frac{\pi}{2}$ $\quad n = 1, 2, \ldots$

$\beta = \pi$ radian

$\therefore$ in the atte. band

$\because f_c < f$; $\alpha = 2\cosh^{-1}\left(\frac{f}{f_c}\right)$ — (5)

& $\beta = \pi$ radian



Z_0 (Char. impedance)

For T-section filter

$$Z_{0T} = \sqrt{Z_1 Z_2\left(1 + \frac{Z_1}{4Z_2}\right)}$$

$Z_1 = j\omega L$, $Z_2 = \frac{1}{j\omega C}$

$$Z_{0T} = \sqrt{\frac{L}{C}}\sqrt{1-\left(\frac{f}{f_c}\right)^2} \quad \text{— (6)}$$

or

$$Z_{0T} = R_0\sqrt{\left(1-\left(\frac{f}{f_c}\right)^2\right)} \quad \text{— (7)}$$

* In pass band $f < f_c$ & Z_{0T} is real.

& In atte. band $f > f_c$; Z_{0T} becomes imaginary.

Thank you