Network Theory

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Gyan Ranjan Biswal received his B.E. in Electronics Engineering from the Pt. Ravishankar Shukla University, India in 1999 and M. Tech. (Honors) in Instrumentation & Control Engineering from the Chhattisgarh Swami Vivekananda Technical University, India in 2009 followed by Ph.D. in Electrical Engineering, specialized in the area of Power System Instrumentation (Power Generation Automation) from the Indian Institute of Technology Roorkee, India in 2013.

He is expertise in Design and Development of cooling systems for large size electrical generators, and the C&I of process industries. He has been in academia for about twelve years. Presently, he is with VSS University of Technology, Burla, India at the capacity of Head and Associate Professor, EEE from Dec. 2016. He has more than 65 publications in various Journals and Conferences of Internationally repute to his credit. He also holds a patent as well, and filed one more. He also adapted one international edition book published by Pearson India. He received research grants of US$90,000 (INR 50 lakhs). He has been supervised 09 Masters' theses, and registered 04 PhD theses. He has also been recognized with many national and international awards by elite bodies. He has been awarded with CICS award under the head of Indian National Science Academy for travel support to USA, MHRD Fellowship by Govt. of India, and Gopabandhu Das Scholarship in his career. His major areas of interests are Power System Instrumentation, Industrial Automation, Robust and Intelligent Control, the Smart Sensors, IoT enabled Smart Sensors, the Smart Grid, Fuel Cell lead Sustainable Sources of Energy, and System Reliability.

Dr. Biswal is a Fellow IE (India), Senior Member of IEEE, USA, and Life Member of ISTE, India. He is actively involved in review panels of different societies of international repute viz. IEEE, IFAC, and the ISA. Currently, he is also actively involved as a Member of IEEE–SA (Standards Association) working groups; IEEE P1876 WG, IEEE P21451-001 WG, and IEEE P1415. He has also been invited for delivering guest lectures at World Congress on Sustainable Technologies (WCST) Conf. 2012, London, UK, INDICON 2015, New Delhi, India, National Power Training Institute (NPTI), Nangal, India, and G.B. Pant Engineering College, Pauri, Gharwal, India, Surendra Sai University of Technology (formerly UCE), Burla, and as a guest expert in 2016 IEEE PES General Meeting Boston, MA, USA.

Syllabus

Network Theory

MODULE-I (9 HOURS) [Online mode: 5 HOURS + 1 Test]

Analysis of Coupled Circuits: Self-inductance and Mutual inductance, Coefficient of coupling, Series connection of coupled circuits, Dot convention, Ideal Transformer, Analysis of multi-winding coupled circuits, Analysis of single tuned and double tuned coupled circuits.

Transient Response: Transient study in series RL, RC, and RLC networks by time domain and Laplace transform method with DC and AC excitation. Response to step, impulse and ramp inputs of series RL, RC and RLC circuit.

MODULE-II (7 HOURS) [Online mode: 5 HOURS + 1 Test]

Two Port networks: Types of port Network, short circuit admittance parameter, open circuit impedance parameters, Transmission parameters, Condition of Reciprocity and Symmetry in two port network, Inter-relationship between parameters, Input and Output Impedances in terms of two port parameters, Image impedances in terms of ABCD parameters, Ideal two port devices, ideal transformer. Tee and Pie circuit representation, Cascade and Parallel Connections.

MODULE-III (8 HOURS) [Online mode: 5 HOURS + 1 Test]

Network Functions & Responses: Concept of complex frequency, driving point and transfer functions for one port and two port network, poles & zeros of network functions, Restriction on Pole and Zero locations of network function, Time domain behavior and stability from pole-zero plot, Time domain response from pole zero plot.

Three Phase Circuits: Analysis of unbalanced loads, Neutral shift, Symmetrical components, Analysis of unbalanced system, power in terms of symmetrical components.

MODULE-IV (9 HOURS) [Online mode: 5 HOURS + 1 Test]

Network Synthesis: Realizability concept, Hurwitz property, positive realness, properties of positive real functions, Synthesis of R-L, R-C and L-C driving point functions, Foster and Cauer forms.

MODULE-V (6 HOURS) [Online mode: 5 HOURS + 1 Test]

Graph theory: Introduction, Linear graph of a network, Tie-set and cut-set schedule, incidence matrix, Analysis of resistive network using cut-set and tie-set, Dual of a network.

Filters: Classification of filters, Characteristics of ideal filters.

Text and Reference Books

Recommended Text Books:

1. "Introductory Circuit Analysis", Robert L. Boylestad, Pearson, 12th ed., 2012.
2. "Network Analysis", M. E. Van Valkenburg, Pearson, 3rd ed., 2006.
3. "Engineering Circuit Analysis", W. Hayt, TMH, 2006.
4. "Network Analysis & Synthesis", Franklin Fa-Kun. Kuo, John Wiley & Sons.

Reference Books:

* "Basic Circuit Theory, Huelsman, PHI, 3rd ed.,
* "HUGHES Electrical and Electronic Technology", Revised by J. Hiley, K. Brown, and I. M. Smith, Pearson, 10th ed., 2011.
* "Circuits and Networks", Sukhija and Nagsarkar, Oxford Univ. Press, 2012.
* "Fundamentals of Electric Circuits", C. K. Alexander and M. N. O. Sadiku, McGraw-Hill Higher Education, 3rd ed., 2005.
* "Fundamentals of Electrical Engineering", L. S. Bobrow, Oxford University Press, 2nd ed., 2011.
* "Circuit Theory (Analysis and Synthesis)", A. Chakrabarti, Dhanpat Rai pub.

Other Important References

Reference Sites:	
1.	NPTEL, The National Programme on Technology Enhanced Learning (NPTEL): https://nptel.ac.in/
2.	MIT OpenCourseWare : https://ocw.mit.edu/index.htm

Course Outcomes

Upon successful completion of this course, you (students) will be able to

CO1	Analyze coupled circuits and understand the difference between the steady state and transient response of 1st and 2nd order circuit and understand the concept of time constant.
CO2	Learn the different parameters of two port network.
CO3	Concept of network function and three phases circuit and know the difference of balanced and unbalanced system and importance of complex power and its components.
CO4	Synthesis the electrical network.
CO5	Analyse the network using graph theory and understand the importance of filters in electrical system.

MID-SESSIONAL TEST
ROLL No.
SEM/BRANCH/SECTION
SUBJECT
Signature of Invigilator
Date

NAS

(System funtn)

Poles & Zeros

let Network funtn / System funtn is N(s) & to expressed in general form

$$N(s) = \frac{Q(s)}{P(s)} = \frac{a_0' s^n + a_1' s^{n-1} + \cdots + a_n'}{b_0' s^m + b_1' s^{m-1} + \cdots + b_m'} = \frac{a_0 (s-a_1)(s-a_2)(s-a_n)}{b_0 (s-b_1)(s-b_2)(s-b_m)}$$

$$\therefore \quad N(s) = \frac{a_0 (s-a_1)(s-a_2)\cdots(s-a_n)}{b_0 (s-b_1)(s-b_2)\cdots(s-b_m)}$$

where $Q(s) = 0$ has n roots & $P(s) = 0$ has m roots.

where $\frac{a_0}{b_0}$ is known as scale factor & is represented by H.

$$N(s) = H \cdot \frac{(s-z_1)(s+z_2)}{(s-p_1)(s-p_2)}$$

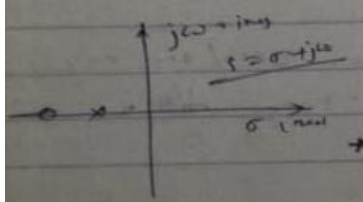
* If the variable s takes value = any of roots of $Q(s) = 0$ namely $a_1, a_2, a_3 \ldots$ N(s) becomes zero.

∴ roots of eqn $Z(s) = 0$ are called zeros of the network funtn

& —"— $P(s) = 0$ —"— poles —"— network funtn

as N(s) becomes ∞.

pole → X , zero → (o)



* These poles & zeros are also known as critical freq'ies.

$N(s) = \frac{p(s)}{q(s)}$

* (to design stable network in terms of impedance function.

Necessary Conditions for driving pt. impedance [Z(s)]

(with common factors in O(s) & P(s) cancelled)

i.e. For $Z(s) = \frac{O(s)}{P(s)}$ = ~~[illegible]~~

1. the co-efficient in polynomials O(s) & P(s) of N(s) = O(s)/P(s) must be real & (+)ive & such fun^n are called real functions
2. poles & zeros must be conjugate if imaginary / complex.
3. (a) the real part of all poles & zeros must be (-)ive / zero &
   (b) if ——"—— is zero, then poles / zeros must be simple pole / zero.

poles/zero which not repeated is known as a simple pole / simple zero.

Ex $Z(s) = \frac{O(s)}{P(s)} =$

$\therefore O(s) = k(s-a-jb)(s-a+jb)$

$= k[(s-a)^2 + b^2]$

$= k[s^2 - 2as + a^2 + b^2]$ $\because k$ = (+)ive const.

now O(s) is to be real fun^n, the co-efficient of s must be (+)ive & real. Hence a must be (-)ive.

4. the polynomials O(s) & P(s) may not have missing terms betn those of highest & lowest degree, unless all even / all odd terms are missing.
5. the degree of O(s) & P(s) may differ by either zero / one only.
6. the terms of lowest degree in O(s) & P(s) may differ in the degree by ~~one~~ one at most.

Restriction on locations of Poles & Zeros for transfer fctn

Necessary conditions for transfer fctn (with common factors in $P(s)$ & $Q(s)$ cancelled)

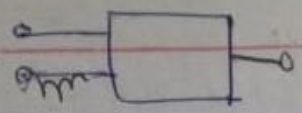
if $N(s) = \frac{P(s)}{Q(s)}$

1. the coefficient in the poly. $P(s)$ & $Q(s)$ of $N(s) = P(s)/Q(s)$ must be real & those for $Q(s)$ must be +ive.
2. poles & zeros must be conjugate if imaginary/complex.
3. (a) the real part of poles must be −ive OR zero &
   (b) if —"— is zero, then poles must be simple. This includes the origin.
4. the polynomials $q(s)$ may not have any missing terms betn that of highest & lowest degree, unless all even/all odd terms are missing.
5. the polynomial $P(s)$ may have terms missing betn the terms of lowest & highest degree, & some of the co-efficient may be −ive.
6. the degree of $P(s)$ may be as small as zero independent of the degree of $q(s)$.
7. (a) For G_{12} & α_{12}: the max. degree of $P(s)$ is the degree of $q(s)$.
8. (b) For Z_{12} & Y_{12}: —"— —"— $P(s)$ —"— $q(s)$ plus one.

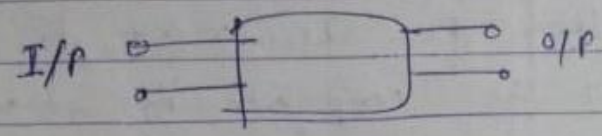
Deno	Num.	
$V_2(s)$	$I_2(s)$	Z_{12} — transfer impedance
$V_1(s)$	$G_{12}(s)$	Y_{12} — " admittance
$I_1(s)$	$Z_{12}(s)$	G_{12} — voltage transfer ratio, α_{12} — current transfer ratio

One Port Network

one port network



~~two~~ port network



$Z(s) \rightarrow$ driving point impedance

$Y(s) \rightarrow$ —"— admittance

$G(s) \rightarrow$ voltage transfer function

$\alpha(s) \rightarrow$ current —"—

Ex

$$F_1(s) = \frac{s^4 + 4s^3 + 2s^2 + 2}{s^3 + 2s^2 + s + 4} = \frac{P_1(s)}{Q_1(s)}$$ (determining terms)

Time Domain Behavior from poles-zeros

Time Domain Behavior from Pole-Zero

$$V(s) = \frac{I(s)}{Y(s)} = \frac{P(s)}{Q(s)} = H\,\frac{(s-a_1)(s-a_2)\cdots(s-a_n)}{(s-b_1)(s-b_2)\cdots(s-b_m)} \quad \text{—(1)}$$

$$= \frac{K_1}{(s-b_1)} + \frac{K_2}{(s-b_2)} + \cdots + \frac{K_m}{(s-b_m)} \quad \text{—(2)}$$

(by partial fraction expansion)

$\because$ $K_1, K_2, \cdots K_m$ are co-efficients, which may be calculated as; &

$$K_r = H\,(s-s_r)\,\frac{P(s)}{Q(s)}\Big]_{s=s_r}$$

$$K_r = H\,\frac{(s_r-a_1)(s_r-a_2)\cdots(s_r-a_n)}{(s_r-b_1)(s_r-b_2)\cdots(s_r-b_m)} \quad \text{—(2)}$$

where $(s_r - a_r)$ & $(s_r - b_r)$ each being a difference of two complex no.s is another complex no. & may be expressed as

$$(a_r - a_y) = A_{ry}\, e^{j\alpha_{ry}} \quad \text{—(}\bigcirc\text{)}$$

A_{ry} → mag. ; α_{ry} → phase angle of $(a_r - a_y)$

similarly

$$(a_r - b_y) = B_{ry}\, e^{j\beta_{ry}}$$

$$\therefore\ K_r = H\,\frac{A_{r1}A_{r2}\cdots A_{rn}}{B_{r1}B_{r2}\cdots B_{rm}}\, e^{j[\alpha_{r1}+\alpha_{r2}\cdots\alpha_{rn}-\beta_{r1}-\beta_{r2}\cdots-\beta_{rm}]} \quad \text{—(3)}$$

now

$$v(t) = \mathcal{L}^{-1}\{V(s)\} = K_1 e^{b_1 t} + K_2 e^{b_2 t} + \cdots + K_n e^{b_n t} \quad \text{—Ans.}$$

Time Domain Behavior

$G_{12}(s), G_{21}(s) \rightarrow$ voltage transfer ratio

$\alpha_{12}(s); \alpha_{21}(s) \rightarrow$ current " "

$Z_{12}(s), Z_{21}(s) \rightarrow$ Transfer impedance $\rightarrow$

$Y_{12}(s), Y_{21}(s) \rightarrow$ " admittance

Ex

$Z_{11} = \frac{V_1(s)}{I_1(s)}\ \Omega$

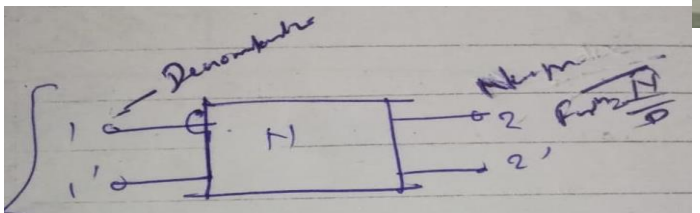
$Z_{22} = \frac{V_2(s)}{I_2(s)}\ \Omega$

$Z_{12} = \frac{V_1(s)}{I_2(s)}\ \Omega$

$Z_{21} = \frac{V_2(s)}{I_1(s)}\ \Omega$

$$G_{12}(s) = \frac{1}{G_{21}(s)}, \quad \alpha_{12}(s) = \frac{1}{\alpha_{21}(s)}$$

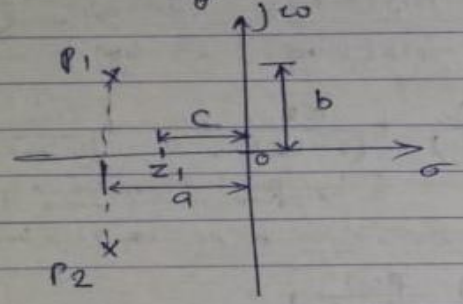
$$Y_{12}(s) = \frac{1}{Y_{21}(s)}, \quad Z_{12}(s) = \frac{1}{Z_{21}(s)}$$



Denominator	Numerator $V_2(s)$	Numerator $I_2(s)$
$V_1(s)$	$G_{12}(s)$	$Y_{21}(s)$
$I_1(s)$	$Z_{12}(s)$	$\alpha_{12}(s)$

Ex ① pole-zero pattern of a net. is ~~given~~ known as . Write an expression for

$Z(s) = ?$



Ans. one zero exists at $(-c, 0)$
& two poles ——— $(-a, \pm jb)$

$$\therefore P(s) = K_1(s+c)$$

$$\& \quad Q(s) = K_2(s+a-jb)(s+a+jb)$$

$$Q(s) = K_2\left[(s+a)^2+b^2\right]$$

$$\therefore Z(s) = \frac{P(s)}{Q(s)} = \frac{K_1(s+c)}{K_2\left[(s+a)^2+b^2\right]}$$

$$\boxed{Z(s) = H\frac{(s+c)}{s^2+2as+a^2+b^2}}$$

(2) An inductor L in series with a resistor R of 9 Ω resistance is connected in ||el to a capacitor C. The driving pt. Impedance of the ||el combination has a zero at $s = -3$ and two poles at $s = -1.5 + j\frac{1}{2}\sqrt{111}$ & $s = -1.5 - j\frac{1}{2}\sqrt{111}$. Calculate the value of L & C.

Ans

$$Z(s) = (R + sL) \parallel \frac{1}{sC}$$

$$= \frac{(R+sL)\frac{1}{sC}}{R + sL + \frac{1}{sC}}$$

$$Z(s) = \frac{R + sL}{s^2LC + sRC + 1}$$

$$Z(s) = \frac{(s + R/L)}{C(s^2 + sR/L + 1/LC)} \quad \text{—— (1)}$$

given that

$$Z(s) = K\frac{(s-z)}{(s-p_1)(s-p_2)}$$

where K is const

$$= K\frac{s+3}{\left(s+1.5 \pm j\frac{\sqrt{111}}{2}\right)}$$

$$= K\frac{(s+3)}{(s+1.5)^2 + (\sqrt{111}/2)^2}$$

$$Z(s) = K\frac{(s+3)}{s^2 + 3s + 30} \quad \text{—— (2)}$$

from eqn (1) & (2)

$$R/L = 3 \quad \& \quad \frac{1}{LC} = 30$$

$$\therefore L = \frac{R}{3} = 3H \quad (R = 9\,\Omega \text{ given})$$

$$\& \quad C = \frac{1}{30} = \frac{1}{3 \cdot L}$$

$$C = \frac{1}{90} F$$

Ans

(3) the laplace transform of a voltage v(t) is given by $V(s) = \frac{s(s+1)(s+2)}{(s+3)(s+4)}$. Draw poles & zeros of the func & det. v(t) using pole-zero diagram.

Ans $N(s) = \frac{P(s)}{Q(s)}$ —— (1)

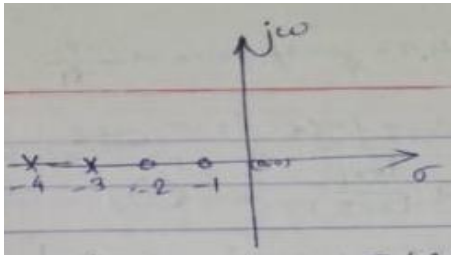
zeros are roots of eqn $P(s) = 0$ —— (2)

(·) $\therefore (s+1)(s+2) = 0$

$s = -1$ & -2

& poles → $Q(s) = (s+3)(s+4) = 0$ —— (3)

(×) $s = -3, -4$



$$V(s) = \frac{5(s+1)(s+2)}{(s+3)(s+4)} = 5\left[1 - 4\left(\frac{s+10/4}{(s+3)(s+4)}\right)\right]$$

$$V(s) = 5\left[1 - 4\left\{\frac{K_1}{s+3} + \frac{K_2}{s+4}\right\}\right]$$

draw phasor at pole $s = -3$ & it is found that

$$K_1 = \frac{ZP_1 \angle ZP_1}{P_2P_1 \angle P_2P_1} = \frac{|3-2.5| \angle 180^\circ}{|4-3| \angle 0^\circ}$$

$$K_1 = -0.5$$

$$\& \quad K_2 = \frac{|ZP_2| \angle ZP_2}{|P_1P_2| \angle P_1P_2} = 3.5$$

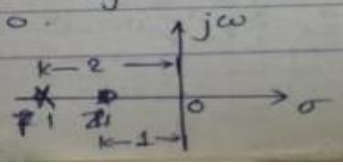
$$V(s) = 5\left[1 - 4\left\{\frac{-0.5}{s+3} + \frac{3.5}{s+4}\right\}\right]$$

$$= 5\left[1 + \frac{2}{s+3} - \frac{6}{s+4}\right]$$

$$\mathcal{L}^{-1}\{V(s)\} = V(t) = 5\left[\delta(t) + 2e^{-3t} - 6e^{-4t}\right] \text{ for } t > 0$$

Ans

(4) the pole-zero pattern of a network function $H(s)$ is shown in fig. find out $H(s)$ if $H(s) = 1/2$ at $s = 0$.



Not fn $N(s)$ is given by $N(s) = H\frac{s-z_1}{s-p_1}$

Ans

Here $z_1 = -1$ & $p_1 = -2$

$$N(s) = H\frac{(s+1)}{(s+2)} \quad \text{--- (1)}$$

Now $N(s)$ at $s=0$ is $N(s)\big|_{s=0} = \frac{1}{2}$

$$\therefore \frac{1}{2} = H\cdot\frac{(0+1)}{(0+2)} = \frac{H}{2}$$

$$\therefore H = 1$$

$$\therefore N(s) = \frac{s+1}{s+2} \quad \text{--- Ans}$$

(5) The differential eqn of a system is given as;

$$4\frac{d^2e_o}{dt^2} + 2\frac{de_o}{dt} + e_o = 3\frac{de_i}{dt}$$

$e_i \rightarrow$ i/p, $e_o \rightarrow$ o/p

(a) find the system fn

(b) Plot the pole zero configuration

(c) A potential of the form $2e^{jt}$ is applied to this system. Find the o/p = ?

Ans

(a) Assume that excitation is exponential & the system is at rest initially.

i/p $\rightarrow$ $E_i e^{st}$

o/p $\rightarrow$ $E_o e^{st}$

$$\therefore (4s^2 + 2s + 1) E_o e^{st} = (3s - 1) E_i e^{st}$$

$$\therefore N(s) = \frac{E_o}{E_i} = \frac{P(s)}{Q(s)} = \frac{3s-1}{4s^2+2s+1}$$

Ans

(b) zeros are roots of eqn

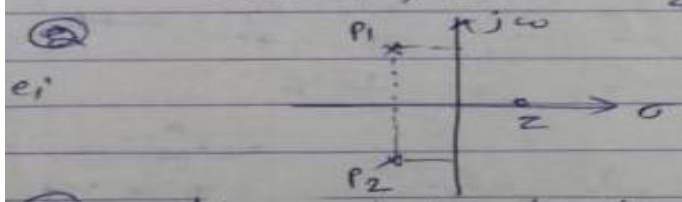
$$P(s) = 0 = 3s - 1$$

$$\therefore s = 1/3 = z \ (\bullet)$$

poles → $Q(s) = 0 = 4s^2 + 2s + 1$

$$\Rightarrow (\times) P_1, P_2 = -\frac{1}{4} \pm j\frac{\sqrt{3}}{4}$$

(a)



(c) the applied stimulus corresponds to $s = j$, hence o/p is given by;

$$e_o = \cancel{2N(j)}\ 2N(j) e^{jt}$$

$s = j$

$$e_o = 2\left[\frac{3j-1}{4j^2+2j+1}\right] e^{jt}$$

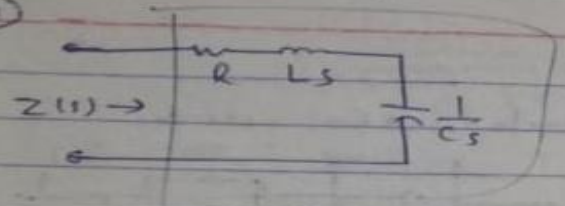
$$e_o = 2\left[\frac{3j-1}{2j-3}\right] e^{jt}$$

$$e_o = 1.75\, e^{j(t - 37.9^\circ)}$$ Ans.

R-L-C Network

RLC n-port network with transfer impedances

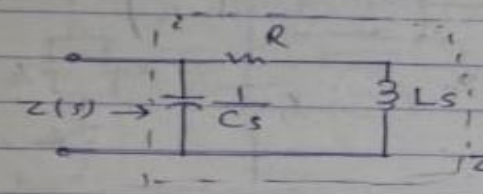
(1)



$$Z(s) = R + Ls + \frac{1}{Cs}$$

$$= L\,\frac{s^2 + (R/L)s + \frac{1}{LC}}{s}$$

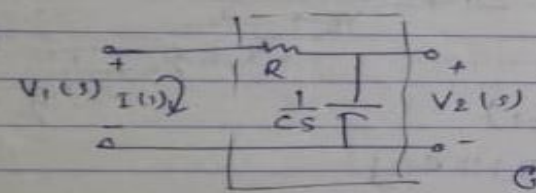
(2)



$$Z(s) = \frac{1}{Cs + \frac{1}{(R + Ls)}}$$

$$Z(s) = \frac{1}{C}\,\frac{s + R/L}{s^2 + (R/L)s + \frac{1}{LC}}$$

(3)



$$V_1(s) = R\,I(s) + \frac{1}{Cs}\,I(s) \quad \text{—(1)}$$

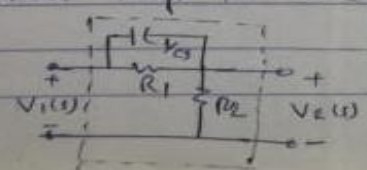
$$\frac{1}{Cs}\,I(s) = V_2(s) \quad \text{—(2)}$$

$$G_{12}(s) = \frac{V_2(s)}{V_1(s)}$$

$$\therefore\ G_{12}(s) = \frac{(1/Cs)}{\left(R + \frac{1}{Cs}\right)} \quad \text{— Ans}$$

$$\&\quad Y_{11}(s) = \frac{I(s)}{V_1(s)} = \frac{1}{R + \frac{1}{Cs}} = \frac{1}{R}\,\frac{s}{s + 1/RC} \quad \text{— Ans}$$

(4) A voltage divider network ckt has more than one current loop in the network. find out $G_{12}(s) = ?$



HPF

Ans

$$G_{12}(s) = \frac{V_2(s)}{V_1(s)} \quad \text{——— (1)}$$

let $Z_{eq}(s) = R_1 \parallel \frac{1}{Cs}$

$$Z_{eq}(s) = \frac{\frac{R_1}{Cs}}{R_1 + \frac{1}{Cs}} = \frac{R_1}{1 + R_1 C s}$$

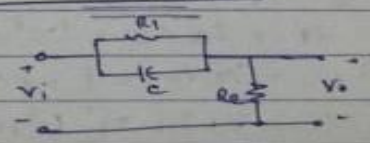
Now $G_{12}(s) = \frac{V_2(s)}{V_1(s)} = \frac{R_2}{R_2 + Z_{eq}(s)}$

$$G_{12}(s) = \frac{R_2}{R_2 + Z_{eq}}$$

$$\therefore G_{12}(s) = \frac{s + 1/R_1 C}{s + (R_1 + R_2)/R_1 R_2 C}$$

Ans

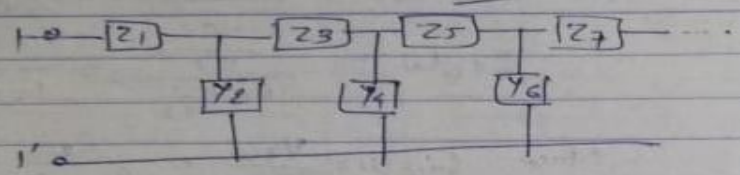
* degree of numerator & denominator are same. This network is widely used in automatic control system where it is known as lead network (phase lead / HPF)



$$Z = Z_1 + \cfrac{1}{Y_2 + \cfrac{1}{Z_3 + \cfrac{1}{Y_4 + \cfrac{1}{Z_5 + \cfrac{1}{Y_6}}}}} \cdots$$

Ladder Network

(simple ladder)

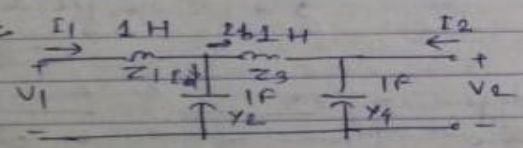


Continuous fraction

$$Z = Z_1 + \cfrac{1}{Y_2 + \cfrac{1}{Z_3 + \cfrac{1}{Y_4 + \cfrac{1}{Z_5 + \cfrac{1}{Y_6}}}}}$$

continued ~~fracto~~ fraction

Ex



$$Z_{11} = \frac{s^4 + 3s^2 + 1}{s^2 + 2s}$$

$$I_b = Y_4 V_2 = s V_2$$

$$V_a = V_2 + I_b Z_3 = (s^2+1) V_2$$

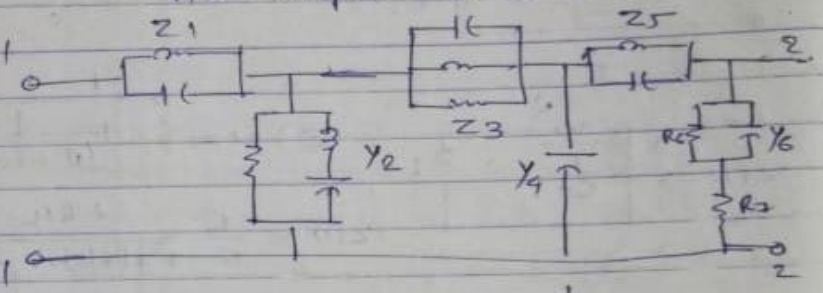
$$I_1 = I_b + Y_2 V_a = [s + s(s^2+1)] V_2$$

$$V_1 = V_a + Z_1 I_1 = [(s^2+1) + s(s^3+2s)] V_2$$

$$\frac{V_2}{I_1} = \frac{1}{s^3+2s} \quad \& \quad \frac{V_2}{V_1} = \frac{1}{s^4+3s^2+1}$$ Ans

non-simple Ladder Network

non-simple ladder network



$$Z_{eq} = Z_1 + \cfrac{1}{Y_2 + \cfrac{1}{Z_3 + \cfrac{1}{Y_4 + \cfrac{1}{Z_5 + \cfrac{1}{Y_6}}}}}$$

$Z_1 = (L_1 \| C_1)$

$Y_2 = [(L_2 + C_2) \| R_2]$

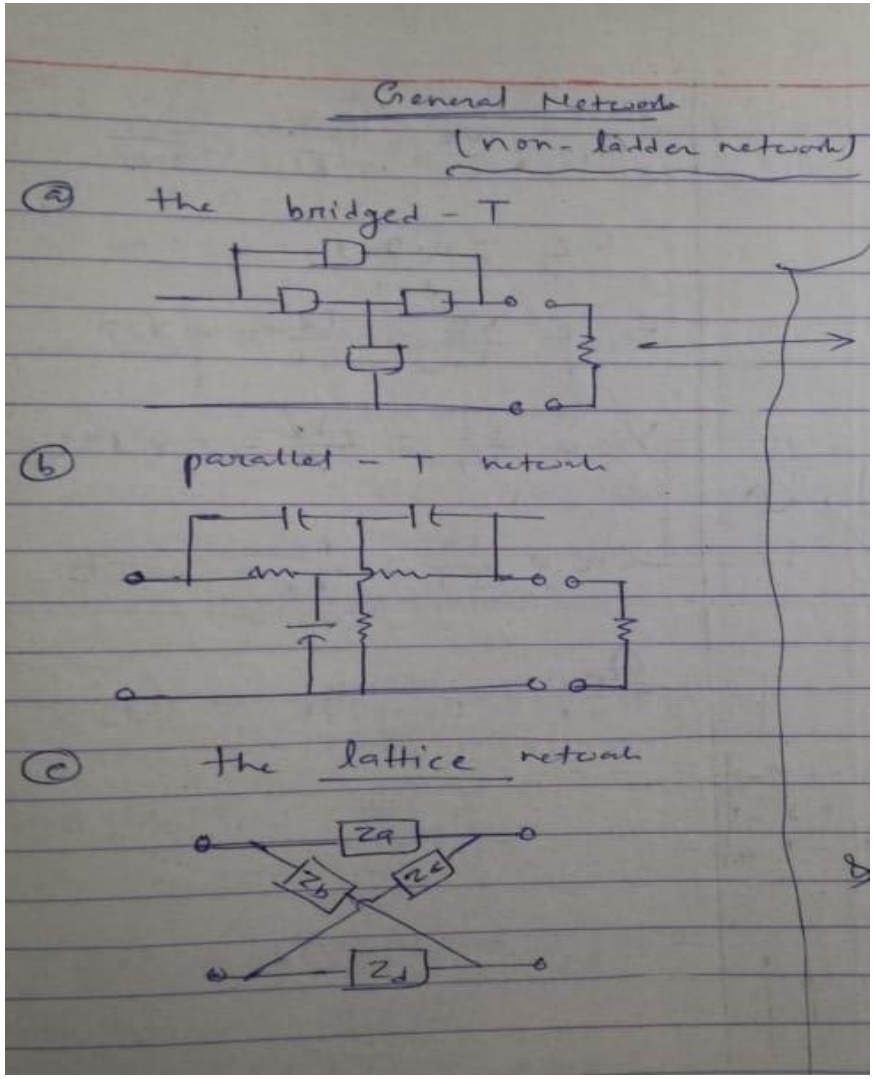
$Z_3 = (C_3 \| L_3 \| R_3)$

$Y_4 = C_4$

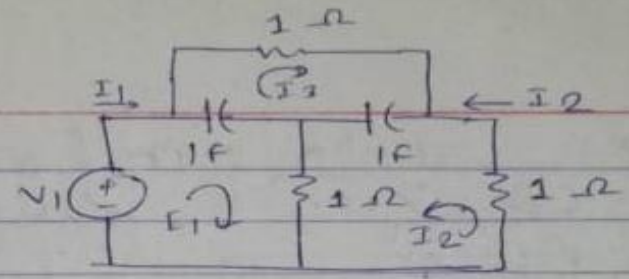
$Z_5 = (L_5 \| C_5)$

$Y_6 = (R_6 \| C_6) + R_7$

Non-Ladder Network



Non-Ladder Network



$$\Delta = \begin{vmatrix} \frac{1}{s}+1 & 1 & -1/s \\ 1 & 1/s+2 & 1/s \\ -1/s & 1/s & 2/s+1 \end{vmatrix}$$

$$\Delta = 2/s^2 + 5/s + 1$$

$$Y_{11} = \frac{I_1}{V_1} = \frac{\Delta_{11}}{\Delta} = \frac{2s^2+5s+1}{s^2+5s+2} \quad \text{— Ans}$$

$$Y_{12} = \frac{I_2}{V_1} = \frac{\Delta_{21}}{\Delta} = -\frac{(s^2+2s+1)}{(s^2+5s+2)} \quad \text{— Ans}$$

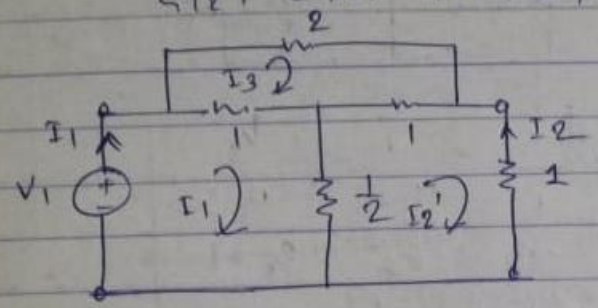
$$\Delta_{11} = \begin{vmatrix} V_1 & 1 & -1/s \\ 0 & 1/s+2 & 1/s \\ 0 & 1/s & \frac{2}{s}+1 \end{vmatrix}$$

$$\Delta_{21} = \begin{vmatrix} 1+1/s & V_1 & -1/s \\ 1 & 0 & 1/s \\ -1/s & 0 & 1+2/s \end{vmatrix} = 1 + \frac{2}{s} + \frac{1}{s^2}$$

Non-Ladder Network

Q resistive bridge T network

$G_{12}, Z_{12}, Y_{12}, \alpha_{12} = ?$



$I_2' = -I_2$

$$\begin{bmatrix} 1.5 & -0.5 & -1 \\ -0.5 & 2.5 & -1 \\ -1 & -1 & 4 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} V_1 \\ 0 \\ 0 \end{bmatrix} \quad \text{—(1)}$$

$$\Delta = \begin{vmatrix} 1.5 & -0.5 & -1 \\ -0.5 & 2.5 & -1 \\ -1 & -1 & 4 \end{vmatrix} = 9$$

$$I_1 = \frac{\Delta_1}{\Delta} = \frac{V_1(9)}{9} = V_1 \quad \text{—(2)} \qquad \because \Delta_1 = \begin{vmatrix} V_1 & -0.5 & -1 \\ 0 & 2.5 & -1 \\ 0 & -1 & 4 \end{vmatrix}$$

$$I_2' = \frac{\Delta_2}{\Delta} = \frac{V_1(1+2)}{9} = \frac{V_1}{3} \quad \text{—(3)}$$

$$\because \Delta_2 = \begin{vmatrix} 1.5 & -V_1 & -1 \\ -0.5 & 0 & -1 \\ -1 & 0 & 4 \end{vmatrix}$$

$$G_{12} = \frac{V_2}{V_1} = \frac{1(I_2')}{V_1} = \frac{V_1}{3V_1}$$

$$G_{12} = 0.333$$

$$Z_{12} = \frac{V_2}{I_1} = \frac{I_2'}{I_1} = 0.333$$

$$Y_{12} = \frac{I_2}{V_1} = \frac{-I_2'}{V_1} = -0.333$$

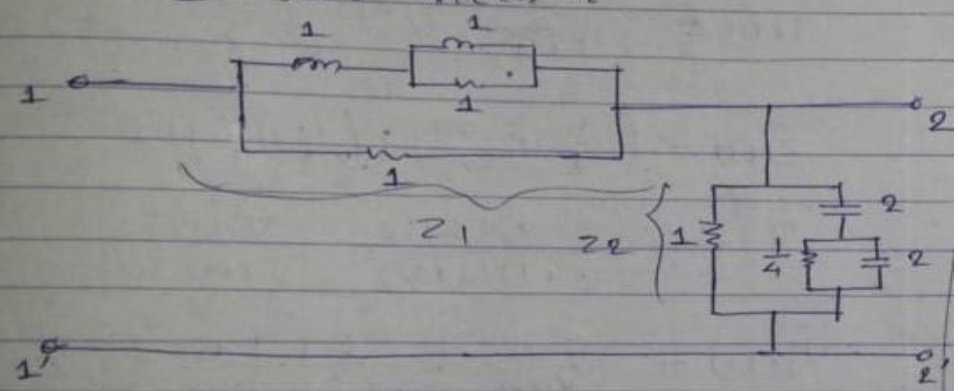
$$\alpha_{12} = \frac{I_2}{I_1} = \frac{-I_2'}{I_1} = -0.333$$

Non-Ladder Network

for a given network

(a) show that with port 2 open, the I/P impedance at port 1 is 1 Ω.

(b) find $G_{12}(s) = ?$



(a)

$$Z_2(s) = 1 \,||\, \left[\frac{1}{2s} + \left(\frac{1}{4} \,||\, \frac{1}{2s}\right)\right]$$

$$Z_2(s) = \frac{s+1}{s^2+3s+1}$$

$$Z_1(s) = \left[(s \,||\, 1) + s\right] \,||\, 1$$

$$Z_1(s) = \frac{s^2+2s}{s^2+3s+1}$$

$$Z(s) = Z_1(s) + Z_2(s) = \frac{s^2+2s+s+1}{s^2+3s+1} = 1\,\Omega$$

Ans

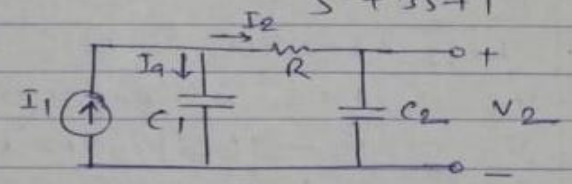
Non-Ladder Network

(b) $G_{12}(s) = \frac{V_2(s)}{V_1(s)} = \frac{Z_2}{Z_1+Z_2}$

voltage divider

$$G_{12}(s) = \frac{s+1}{s^2+3s+1}$$ — Ans

Q



$\alpha_{12}(s) = \frac{I_2(s)}{I_1(s)} = ?$ & $Z_{12}(s) = \frac{V_2(s)}{I_1(s)} = ?$

Ans

$$I_1(s) = I_a(s) + I_2(s) \quad \text{— (1)}$$

$$\because \quad I_2(s) = \frac{\frac{1}{C_1 s}}{R + \frac{1}{C_1 s} + \frac{1}{C_2 s}} \times I_1(s)$$

$$\frac{I_2(s)}{I_1(s)} = \alpha_{12}(s) = \frac{1/C_1 s}{R + 1/C_1 s + 1/C_2 s}$$

$$\therefore \quad \alpha_{12}(s) = \frac{1}{RC_1} \cdot \frac{1}{\left(s + \frac{C_1+C_2}{RC_1C_2}\right)}$$ — Ans

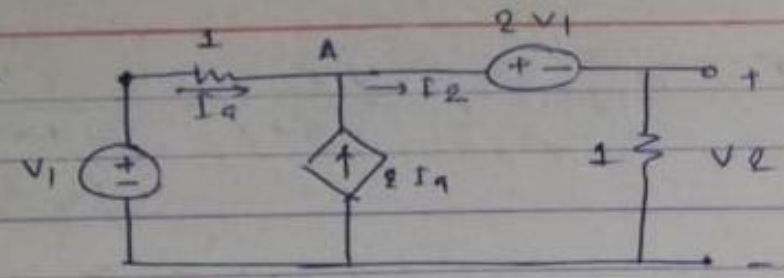
$$V_2(s) = I_2(s) \cdot \frac{1}{C_2 s}$$

$$V_2(s) = \alpha_{12}(s) \cdot I_1(s) \cdot \frac{1}{C_2 s}$$

$$Z_{12}(s) = \frac{V_2(s)}{I_1(s)} = \frac{1}{C_2 s} \cdot \alpha_{12}(s) = \frac{1}{sRC_1C_2\left(s + \frac{C_1+C_2}{RC_1C_2}\right)}$$ — Ans

Non-Ladder Network

Q



$$G_{12} = \frac{V_2}{V_1} = ?$$

Ans

$$I_2 = I_a + 2I_a = 3I_a$$

$$V_2 = 1 \times I_2 = I_2 = 3I_a$$

$$V_a = V_1 - 1(I_a) = V_1 - I_a$$

$$V_a = I_2(1+1) + 2V_1$$

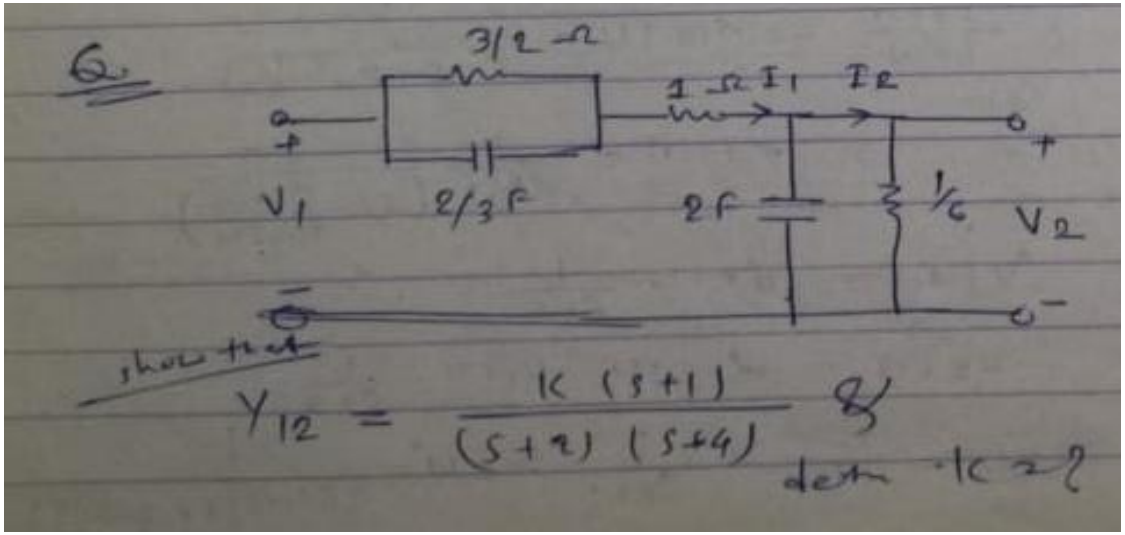
$$V_a = 6I_a + 2V_1$$

now $V_1 - I_a = 6I_a + 2V_1$

or $V_1 = -7I_a$

$$\therefore G_{12} = \frac{V_2}{V_1} = \frac{I_2}{-7I_a} = -\frac{3}{7}$$ Ans

Non-Ladder Network



Non-Ladder Network

Ans

$$I_2(s) = \frac{1/2s}{1/6 + 1/2s} I_1(s)$$

$$I_2(s) = \frac{3}{s+3} I_1(s)$$

now

$$Z(s) = \left(\frac{3}{2} \parallel \frac{3}{2s}\right) + \left(\frac{1}{2s} \parallel \frac{1}{6}\right) + 1$$

$$Z(s) = \frac{(s+4)(s+2)}{(s+1)(s+3)} = \frac{V_1(s)}{I_1(s)}$$

$$Y_{12}(s) = \frac{I_2(s)}{V_1(s)} = \left(\frac{3}{s+3}\right)\frac{I_1(s)}{V_1(s)}$$

$$Y_{12}(s) = \frac{3(s+1)}{(s+2)(s+4)}$$

Hence $k = 3$ Ans

Complex Transform Impedance and Circuit

Concept of complex freq. — we have seen it.

Transform impedance & transform circuit

C

$i_c(t) = C\frac{dv_c(t)}{dt}$

$v_c(t) = \frac{1}{C}\int_{-\infty}^{t} i_c(t)\,dt$

$V_c(s) = \frac{1}{C}\left[\frac{I_c(s)}{s} + \frac{q(0-)}{s}\right]$

$\frac{V_c(s)}{I_c(s)} = Z_c(s) = \frac{1}{Cs}$

R →

$v_R(t) = R\,i_R(t) \rightleftarrows V_R(s) = R\,I_R(s)$ or $I_R(s) = G\,V_R(s)$ ①

Z →

$Z_R(s) = \frac{V_R(s)}{I_R(s)} = R$, $\frac{I_R(s)}{V_R(s)} = G$

L →

$i_L(t) = \frac{1}{L}\int_{-\infty}^{t} v_L(t)\,dt$

$\boxed{v_L(t) = L\frac{di_L}{dt}}$

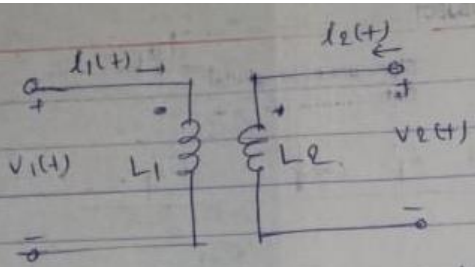
$V_L(s) = L\left[s\,I_L(s) - i_L(0-)\right]$ → initial cond. = 0

$\frac{V_L(s)}{I_L(s)} = Z_L(s) = sL$

$I_L(s) = \frac{1}{L}\left[\frac{V_L(s)}{s} + \frac{V_L^{-1}(0)}{s}\right]$ → $i_L(0-)$

$Z_L(s) = sL$

Complex Transformation



$$v_1(t) = L_1 \frac{di_1(t)}{dt} + M \frac{di_2(t)}{dt} \quad \text{—(1)}$$

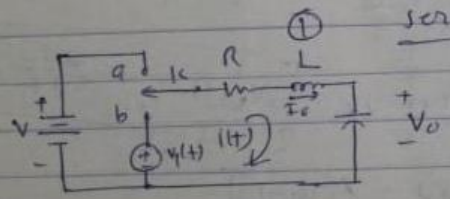
$$v_2(t) = M \frac{di_1(t)}{dt} + L_2 \frac{di_2(t)}{dt}$$

$$V_1(s) = L_1 s I_1(s) - L_1 i_1(0-) + Ms I_2(s) - M i_2(0-)$$

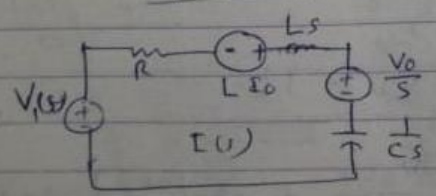
$$V_2(s) = M s I_1(s) - M i_1(0-) + L_2 s I_2(s) - L_2 i_2(0-)$$

transformation of impedance

series ckt



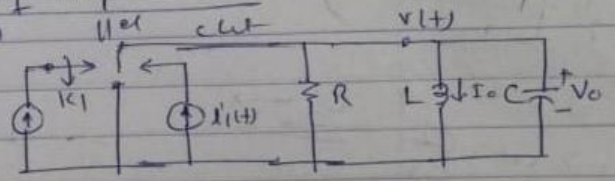
series ckt



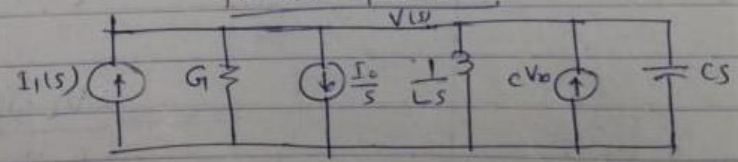
$$I(s) = \frac{V_1(s)}{Z(s)} = \frac{V_1(s) + LI_0 - V_0/s}{R + Ls + \frac{1}{Cs}}$$

$$= \frac{sV_1(s) + LI_0 s - V_0}{Ls^2 + Rs + 1/C}$$

parallel ckt



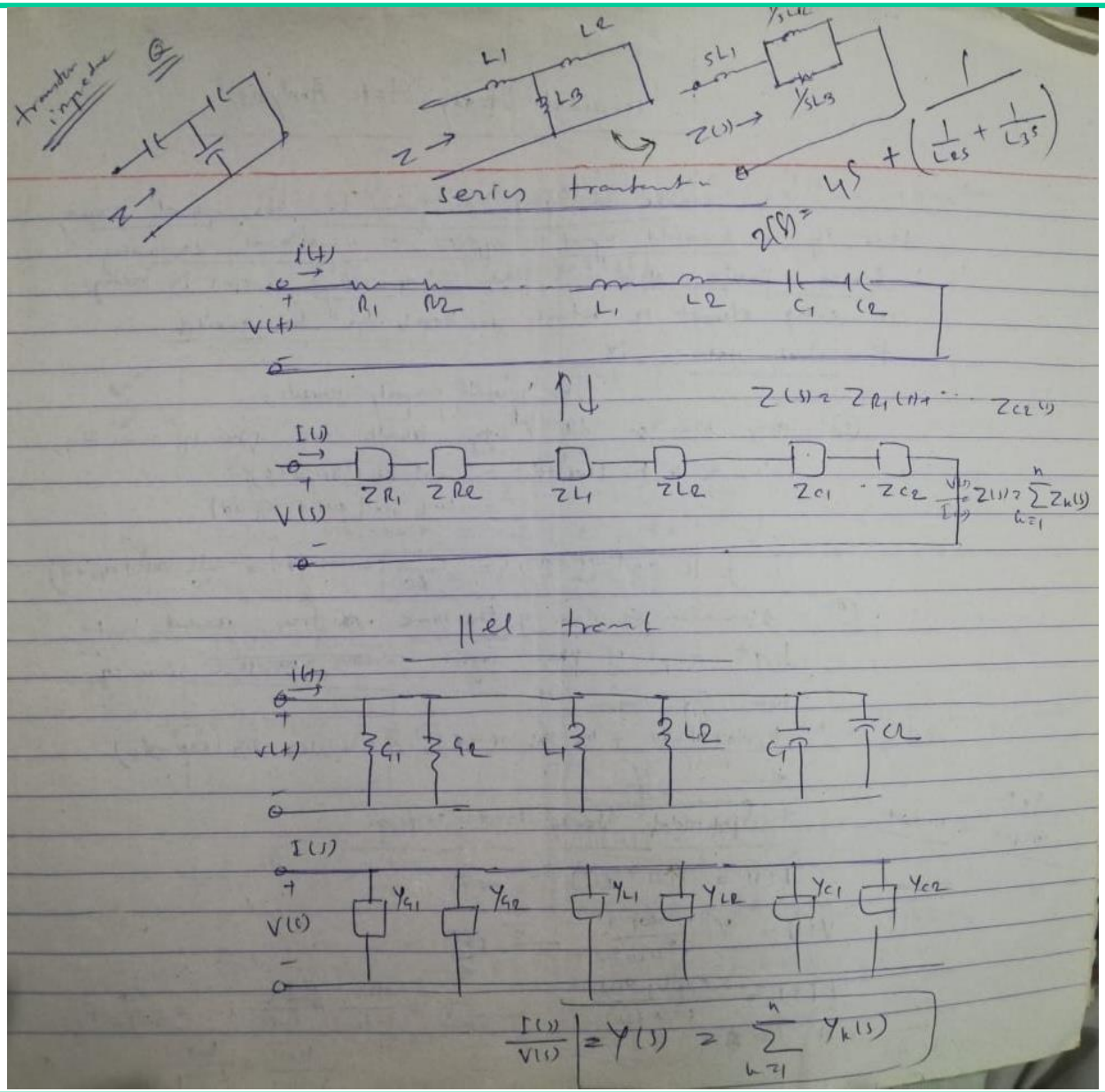
transform of network



$$V(s) = \frac{I(s)}{Y(s)} = \frac{I_1(s) - \frac{I_0}{s} + CV_0}{Cs + \frac{1}{Ls} + G}$$

$$= \frac{s[I_1(s)] + CV_0 s - I_0}{Cs^2 + Gs + 1/L} \quad \text{— Ans}$$

Complex Transformation



Sinusoidal Steady State Analysis

unlike all other waveform has the special property that if a sinusoidal signal is applied to a network containing linear passive element, then resulting current & voltage in every element of network is bound to be sinusoidal in the steady state. &

this special property results -

(1) any sinu. on diff^n / integ results same sinu. of same freq.

$$\therefore \frac{d}{dt} k \sin(\omega t + \phi) = \omega t \cos(\omega t + \phi)$$
$$= \omega k \sin(\omega t + \pi/2 + \phi)$$

$$\int k \sin(\omega t + \phi) = \frac{-k}{\omega} \cos(\omega t + \phi) = \frac{k}{\omega} \sin(\omega t + \pi/2 + \phi)$$

(2) summation of sinu. of the same freq. results but diff^t amp^s & phase angles results another sinu. of same freq.

$$k_1 \sin(\omega t + \phi) + k_2 \sin(\omega t + \phi) \ldots = k_t \sin(\omega t + \phi_t)$$

let $\therefore v(t) = V_1 \sin \omega_1 t$

Sinusoidal Steady State response

$$I(s) = Y(s) \cdot V(s) \quad \text{—(1)}$$

$$V(s) = V_1 \cdot \frac{\omega_1}{s^2 + \omega_1^2} \quad \text{—(2)}$$

$$I(s) = \frac{\omega_1 V_1 Y(s)}{s^2 + \omega_1^2} \quad \text{—(3)} = \frac{k_1}{s - p_1} + \frac{k_2}{s - p_2} + \ldots + \frac{k_m}{s - p_m} + \frac{k_{\omega 1}}{s + j\omega_1} + \frac{k_{\omega 1}}{s - j\omega_1}$$

$i(t) = V_1 |Y_1| \sin(\omega_1 t + \phi) = K_1 e^{P_1 t} + K_2 e^{P_2 t} + \dots + K_m e^{P_m t}$

&

$$\boxed{i_{ss} = V_1 |Y_1| \sin(\omega_1 t + \phi)}$$

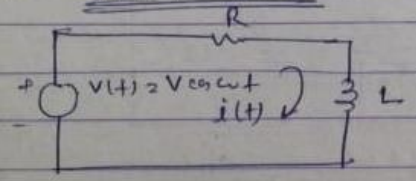
⇒ <u>interms of exponential</u>

$$\left\{ \begin{aligned} \sin\omega t &= \frac{e^{j\omega t} - e^{-j\omega t}}{2j} \\ \& \quad \cos\omega t &= \frac{e^{j\omega t} + e^{-j\omega t}}{2} \end{aligned} \right\}$$

like $v(t)\cos\omega t = \frac{V}{2}\left[e^{j\omega t} + e^{-j\omega t}\right]$

$$\boxed{i_{ss} = i_{ss1} + i_{ss2}}$$

* <u>series - R-L</u> :—



<u>Method 1</u>

$v(t) = V\cos\omega t = \frac{V}{2}\left[e^{j\omega t} + e^{-j\omega t}\right]$

for $\frac{V}{2}e^{j\omega t} \longrightarrow$

$L\frac{di}{dt} + Ri = \frac{V}{2}e^{j\omega t} \longrightarrow \boxed{i_{ss1} = Ae^{j\omega t}}$ ($A \rightarrow$ undetermined coefficient)

$\frac{V}{2} = j\omega LA + RA$, $A = \frac{V/2}{(j\omega L + R)}$

for $\frac{V}{2}e^{-j\omega t} \longrightarrow \boxed{i_{ss2} = Be^{-j\omega t}}$

$\Rightarrow \frac{V}{2} = -j\omega LB + RB \Rightarrow B = \frac{V/2}{(R - j\omega L)}$

$$\boxed{i_{ss} = i_{ss1} + i_{ss2}}$$

$$= \frac{V}{2}\left[\frac{e^{j\omega t}}{R+j\omega L} + \frac{e^{-j\omega t}}{R-j\omega L}\right]$$

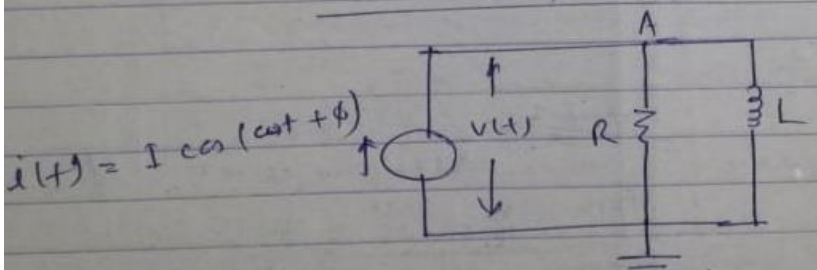
$$= \frac{V}{R^2+\omega^2L^2}\left[R\cos\omega t + \omega L \sin\omega t\right]$$

$$i_{ss} = \frac{V}{\sqrt{R^2+\omega^2L^2}}\cos\left(\omega t - \tan^{-1}\frac{\omega L}{R}\right)$$

— Ans

$$\phi = \tan^{-1}\frac{\omega L}{R}$$

Sinu. Analysis of a ||el R-L Network



$$i(t) = I\cos(\omega t+\phi)$$

$$= \frac{I}{2}\left[e^{j(\omega t+\phi)} + e^{-j(\omega t+\phi)}\right]$$

$$\Rightarrow \frac{V}{R} + \frac{1}{L}\int_{-\infty}^{t} V\,dt = \frac{I}{2}e^{j(\omega t+\phi)}$$

$$V_{ss1} = A e^{j(\omega t+\phi)}$$

affinity

$$A = \frac{I/2}{\frac{1}{R} + \frac{1}{j\omega L}} \quad \&$$

$$V_{ss2} = B e^{-j(\omega t + \phi)}$$

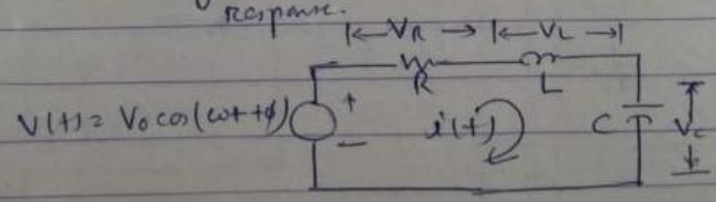
$$B = \frac{I/2}{\frac{1}{R} - \frac{1}{j\omega L}}$$

$$V_{ss} = V_{ss1} + V_{ss2}$$

$$= \frac{I}{2}\left[\frac{e^{j(\omega t+\phi)}}{\frac{1}{R} + \frac{1}{j\omega L}} + \frac{e^{-j(\omega t+\phi)}}{\frac{1}{R} - \frac{1}{j\omega L}}\right]$$

$$\boxed{V_{ss} = \frac{I\omega L R}{\sqrt{R^2+\omega^2 L^2}} \cos\left(\omega t + \phi + \tan^{-1}\frac{R}{\omega L}\right)}$$

Q Voltage $v(t) = V_0 \cos(\omega t + \phi)$ is applied to a series ckt containing R, L & C. Obtain $i(t)$ for steady state response.



Ans apply KVL;

$$V(s) = I(s) \cdot Z(s) \quad \text{—(1)}$$

$$V(s) = I(s)\left[R(s) + Ls + \frac{1}{Cs}\right] \quad \text{—(2)}$$

under sinusoidal steady state eqn ① in phasor form

$$I\left[R + j\omega L + \frac{1}{j\omega C}\right] = V_0 e^{j\phi} \quad \text{—— ③}$$

$$I = I e^{+j\theta_I} = \frac{V_0 e^{j\phi}}{R + j\left(\omega L - \frac{1}{\omega C}\right)}$$

$$|I| = \frac{V_0}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}} \quad \text{(mag)}$$

$$\& \quad \theta_I = \phi - \tan^{-1}\frac{\omega L - \frac{1}{\omega C}}{R} \quad \text{(phase)}$$

$$\therefore \boxed{i(t) = I\cos(\omega t + \theta)}$$

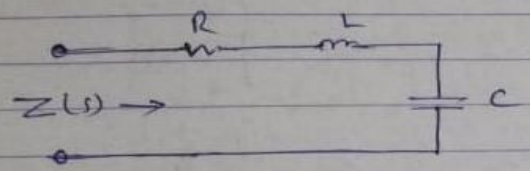
—— Ans

Resonance

Resonant ckt (tuned ckt)

series

$\|^{el}$



$\therefore$ driving pt. impedance $Z(s) = R + Ls + \frac{1}{Cs}$

$$= \frac{s^2 LC + sRC + 1}{Cs}$$

$$= \frac{L\left(s^2 + \frac{R}{L}s + \frac{1}{LC}\right)}{s}$$

$$Z(s) = \frac{L(s-z_1)(s-z_2)}{s}$$

$$\because s^2 + \frac{R}{L}s + \frac{1}{LC} = 0$$

$$z_1, z_2 = -\frac{R}{2L} \pm j\sqrt{\frac{1}{LC} - \left(\frac{R}{2L}\right)^2}$$

freq. at ~~Resonance~~ which damping factor is zero implies that system is called resonance

$s = \sigma + j\omega$, $\sigma = 0$

$\omega = \frac{1}{\sqrt{LC}}$

Critical Resistance (R_c)

$$\sqrt{\frac{1}{LC} - \left(\frac{R}{2L}\right)^2} = 0$$

$$\left(\frac{R}{2L}\right)^2 = \frac{1}{LC}$$

$$\boxed{\therefore R_c = 2\sqrt{\frac{L}{C}} = R_c}$$

Damping ratio

is defined as the ratio of the actual resistance to the critical resistance & defined as ξ

$$\boxed{\xi = \frac{R}{R_c} = \frac{R}{2}\sqrt{\frac{C}{L}}}$$

$\because 0 < \xi \leq 1$

$\therefore R_c = 2\sqrt{\frac{L}{C}}$

~~resonance freq~~ f_r / f_n

Natural (undamped freq.) ω_n / ω_r

→ freq. at which driving point impedance becomes resistive i.e. reactive part = 0

$$\omega_r = \omega_n = 2\pi f_r$$

$$Z(j\omega_r) = R + j\omega_r L + \frac{1}{j\omega_r C} = R$$

$$j\omega_r L + \frac{1}{j\omega_r C} = 0$$

$$\boxed{\begin{aligned} \omega_n = \omega_r &= \frac{1}{\sqrt{LC}} \\ f_n = f_r &= \frac{1}{2\pi\sqrt{LC}} \end{aligned}}$$

$$\xi\omega_r = \frac{R}{2}\sqrt{\frac{C}{L}} \cdot \frac{1}{\sqrt{LC}}$$

$$\boxed{\xi\omega_r = \frac{R}{2L}} = \cancel{\frac{1}{2(\omega_r L/R)}}$$

$$\xi = \frac{R}{2\omega_r L} = \frac{1}{2(\omega_r L/R)} = \frac{1}{2Q}$$

$$\boxed{Q = \frac{\omega_r L}{R}}$$

is known as quality factor

now $s^2 + \frac{R}{L}s + \frac{1}{LC} = 0$

$\therefore s^2 + 2\xi\omega_r s + \omega_r^2 = 0$

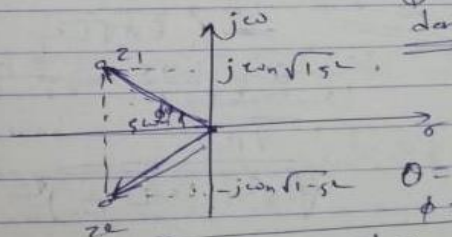
$$z_1, z_2 \Rightarrow -\frac{R}{2L} \pm j\sqrt{\frac{1}{LC} - \left(\frac{R}{2L}\right)^2}$$

$$= -\xi\omega_r \pm \sqrt{(\xi\omega_r)^2 - \omega_r^2}$$

$$= -\xi\omega_r \pm j\omega_r\sqrt{1-\xi^2}$$

$$z_1, z_2 = -\xi\omega_r + j\omega_d$$

↓ damped freq.



$\theta = 180° - \phi$

$\phi = \cos^{-1} z_1 = \cos^{-1}\frac{\xi\omega_r}{\omega_r}$

$$\boxed{\begin{aligned} \phi &= \cos^{-1}\xi \\ \xi &= \cos\phi \end{aligned}}$$

Series Resonance

$$Z(j\omega) = R + j\omega L + \frac{1}{j\omega C}$$

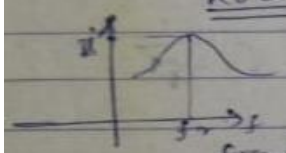
$$= R + j\left(\omega L - \frac{1}{\omega C}\right)$$

$$Z(j\omega) = R + jX \qquad \left\{ \begin{array}{l} X_C = \frac{1}{\omega C} \\ X_L = \omega L \end{array} \right\}$$

$$\therefore X = X_L - X_C$$

$$I = \frac{V}{|Z(j\omega)|} = \frac{V}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}$$

<u>note</u> when $X = 0 = X_L - X_C$



i.e. $\omega L = \frac{1}{\omega C}$

$$\omega_r = \sqrt{\frac{1}{LC}}$$

freq at resonance freq $\quad f_r = \frac{1}{2\pi\sqrt{LC}}$

& $Q = \frac{\omega_r L}{R} = \frac{1}{\omega_r CR}$ (quality factor)

$$V_R = IR = \frac{VR}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}$$

$$V_L = IX_L = \frac{V\omega L}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}$$

$$V_C = IX_C = \frac{V/\omega C}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}$$

— Ans

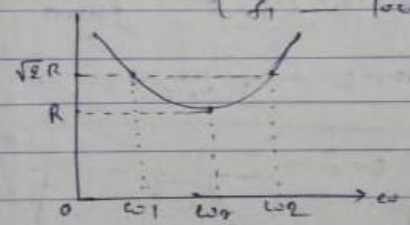
Half Power Freq., BW, Q-factor & Selectivity of a Series resonance

(Cutoff freq. / 3 dB freq.)

* Half Power Freq. :- HP freq. f_1 & f_2 are those freq. at which the power across the R-L-C series ckt is exactly half of the power across the ckt at resonant freq. f_r

$$\Delta = f_2 - f_1$$

$\{ f_2$ — upper power freq.
f_1 — lower power freq. $\}$



Resonant freq. After manipulating series response eqn :-

$$\omega_r = \sqrt{\omega_1 \omega_2} = \sqrt{\omega_r^2\{\xi^2 + 1 - \xi^2\}}$$

or $\boxed{f_r = \sqrt{f_1 f_2}}$ $\quad \because \Delta f = f_2 - f_1 = \frac{\omega_2 - \omega_1}{2\pi}$

* $\Delta f = \frac{\omega_r \xi}{\pi} = \frac{R}{2\pi L}$

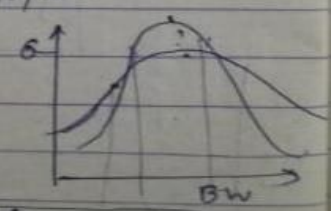
BW Δf is BW of (Net. ckt) system

$$\boxed{\Delta f = f_2 - f_1}$$

Quality factor

$$Q = \frac{\omega_r L}{R}$$

$(Q \uparrow \leftrightarrow BW \downarrow)$



* $\boxed{\Delta f = f_2 - f_1 = BW = \frac{R}{2\pi L} = \frac{R f_r}{2\pi L f_r} = \frac{f_r}{Q}}$

if $Q \uparrow \leftrightarrow BW \downarrow$ & ckt become more selective.

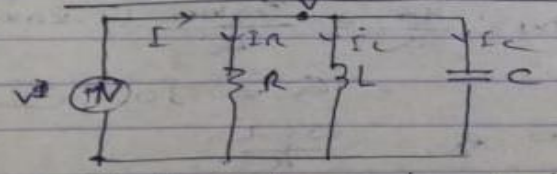
The degree of selectivity of a resonant ckt is represented by the narrowness of the BW Δf & is expressed by

$$\boxed{Q = \frac{f_r}{\Delta f} = \frac{f_r}{BW}}$$

of the inductor of ckt at f_r.

(Hel tuned / tank ckt)

Parallel Resonance ckt



$$Y(j\omega) = \frac{1}{R} + \frac{1}{j\omega L} + j\omega C$$

$$= G + j(B_C - B_L)$$

$$= G + jB$$

B_C – susceptance C

B_L – " L

$G = 1/R$

$$\therefore Y(j\omega) = \sqrt{G^2 + B^2} = \sqrt{G^2 + \left(\omega C - \frac{1}{\omega L}\right)^2}$$

$I_R = V \cdot G = I_r$

$I_L = I_r Q$

$I_C = I_r Q$

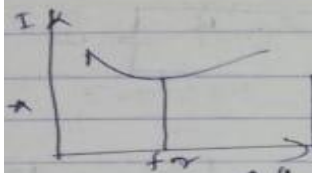
$$\boxed{Q = \frac{R}{\omega_r L} = \omega_r RC}$$

$\therefore I = V \cdot Y = V\sqrt{G^2 + \left(\omega C - \frac{1}{\omega L}\right)^2}$

at resonant freq.

$\omega C = \frac{1}{\omega L}$

$\omega_r^2 = \frac{1}{LC}$



$$\boxed{f_r = \frac{1}{2\pi\sqrt{LC}}}$$

after solving for roots of eqn

$\Delta f = f_2 - f_1 = BW = \frac{G}{2\pi C}$

$$\boxed{Q \text{ of resonant freq.} = \frac{f_r}{\Delta f} \qquad Q = \frac{2\pi f_r C}{G} = \frac{\omega_r C}{G}}$$

The degree of Selectivity :— of a $\|^{el}$ resonant ckt is represented by the narrowness of BW given by $Q = \frac{f_r}{\Delta f}$ as in the case of series resonance.

[Therefore selectivity is det. by the quality factor Q of ckt]

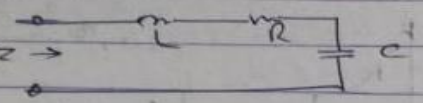
Q is same for both series / $\|^{el}$ resonant ckt.

$$\boxed{R_L = R_C = \sqrt{\frac{L}{C}} \qquad Z = \sqrt{L/C}}$$ of a tank ckt

Q. A series RLC is resonant at 1 MHz. Its BW is 5000 cps & I/P impedance at resonance is 50 Ω. find L, R, C ?

Ans

$R = 50\,\Omega$,



$$f_{resonance} = \frac{1}{2\pi\sqrt{LC}}$$

$$\therefore LC = \frac{1}{4\pi^2 f_r^2} = \frac{1}{4\pi^2 (10^6)^2} = \frac{10^{-12}}{4\pi^2} \quad \text{—(1)}$$

then

$$\Delta f = \frac{f_r}{Q} = BW$$

$$\therefore Q = \frac{\omega_r L}{R}$$

$$\therefore \Delta f = BW = \frac{R f_r}{\omega_r L} = \frac{R}{2\pi L}$$

$$\therefore L = \frac{R}{2\pi \Delta f} = \frac{50}{2\pi \times 5000} = 1.59\ mH$$

from (1)

$$C = \frac{10^{-12}}{4\pi^2 \times L} = 15.9\ pF$$

$\therefore R = 50\,\Omega$, $C = 15.9\ pF$, $L = 1.59\ mH$ — Ans

Q

It is required to construct a series resonant ckt with the following conditions:

① the capacity of the capacitor is 250pF.
② Resonant freq is 600 kHz.
③ BW = 20 kHz

Cal. Q, L, R of ckt & % of current at 500 kHz.

Ans $f_r = \frac{1}{2\pi\sqrt{LC}}$ $C = 250pF$, $f_r = 600kHz$

$\therefore L = 6.2814$ mH

$BW = \frac{f_r}{Q} = \frac{600kHz}{Q} = 20kHz$

$Q = 30$

now $Q = \frac{\omega_r L}{R}$

$R = \frac{\omega_r L}{Q} = \frac{2\pi f_r L}{Q} = 35.3617\Omega$

$$\boxed{I = \frac{I_m}{1 + j\frac{Q2\delta f}{f}}}$$

$\because I_m = \frac{E}{R}$ (max. I at resonance)

$$\therefore \frac{I}{I_m} = \frac{1}{\sqrt{1^2 + \frac{4Q^2\delta f^2}{f^2}}}$$

$\because \delta f = f - f_r = 500 - 600 = -100$

$$\left|\frac{I}{I_m}\right| = \frac{1}{\sqrt{1^2 + \frac{4\times30^2\times(-100)^2\times10^6}{500^2\times10^6}}}$$

$\left|\frac{I}{I_m}\right| = 0.083$ — Ans $= 8.3\%$

$I = 8.3\%$ of I_m

Additional Problem

A series RLC circuit has the following parameters.
$R = 10\,\Omega$, $L = 10\,mH$, $C = 100\,\mu F$. Compute
(i) f_r (ii) Q-factor (iii) BW (iv) f_{lr} & f_{hr} (lower & upper cu

$\therefore$ (i) $\omega_r = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{10\times10^{-3}\times100\times10^{-6}}} = 10^3$ rad/s

$\therefore f_r = \frac{10^3}{2\pi} = 159.16\ Hz$

(ii) $Q = \frac{\omega_r L}{R} = \frac{10^3\times10\times10^{-3}}{10} = 1$

(iii) $BW = \frac{\omega_r}{Q} = \frac{10^3}{1} = 10^3$ rad/sec

or $BW = \frac{10^3}{2\pi} = 159.16\ Hz$

(iv) $BW = 2\Delta f = f_H - f_L = f_{hr} - f_{lr}$

$\therefore \Delta f = \frac{BW}{2} = 79.58\ Hz$

$\therefore f_{hr} = f_r + \Delta f = 159.16 + 79.58$

$f_{hr} = 238.74\ Hz$

& $f_{lr} = f_r - \Delta f = 159.16 - 79.58$

$f_{lr} = 79.58\ Hz$

Analysis of Three Phase Circuits

Power in a Pure Resistive Circuit

When a sinusoidal voltage is applied across a circuit constituted of a pure resistance the current in the circuit is in phase with the voltage, i.e., $\theta_v = \theta_i = 0$ (say), an $\varphi = 0°$. The instantaneous power $p(t)$ given by Eq. (7.4) becomes

$$p(t) = VI\,[1 + \cos 2\omega t] \qquad (7.14$$

and the average power in this case is

$$P = VI \qquad (7.1$$

Figure 7.6 shows the waveforms of the current $i(t)$, voltage $v(t)$, and t instantaneous power $p(t) = v(t)\,i(t)$ for a resistive circuit connected to an source. It may be noted from Fig. 7.6 that the instantaneous power is continua

instantaneous power, and therefore the average power, has a positive value which implies that power can never be obtained from a resistor, but it consumes power which is dissipated in the form of heat.

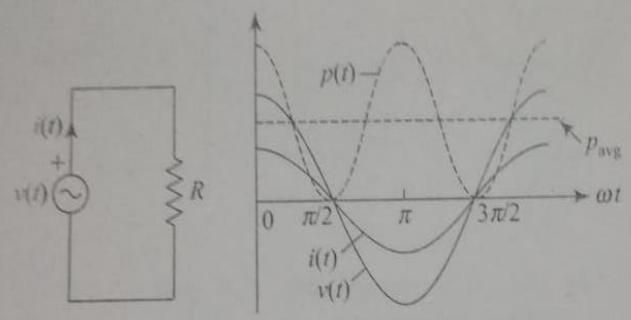


Fig. 7.6 *Waveforms of i(t), v(t), and p(t) for a resistive circuit*

Power in a Pure Inductive Circuit

For a purely inductive circuit, if the phase angle of the voltage $\theta_v = 0°$, then the phase angle of the current $\theta_i = -90°$ and $\varphi = 90°$. The instantaneous power given by Eq. (7.5) may now be written as

$$p(t) = VI\left[\cos 90° \left\{1 + \cos 2(\omega t - 90°)\right\} - \sin 90° \sin 2(\omega t - 90°)\right]$$

$$= VI \sin 2\omega t \tag{7.16}$$

and the average power in this case from Eq. (7.9) is

$$P = VI \cos 90° = 0 \tag{7.17}$$

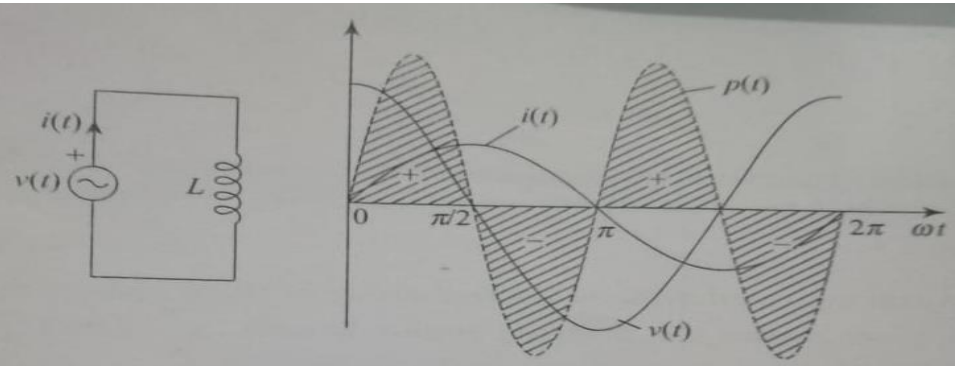


Fig. 7.7 *Waveforms of i(t), v(t), and p(t) for an inductive circuit*

From the power waveform, it can be seen that the average power in a pure inductive circuit is zero. However, a continuous exchange of energy occurs between the energizing source and the inductive circuit at a frequency that is double that of the supply frequency. For positive $p(t)$ the inductor stores electrical energy in its magnetic field and when $p(t)$ is negative an equal amount of energy is returned to the source.

Power in a Pure Capacitive Circuit

When a sinusoidal voltage is connected across a purely capacitive circuit, if $\theta_v = 0°$, then $\theta_i = 90°$ and $\varphi = -90°$. The instantaneous power given by Eq. (7.4) becomes

$$p(t) = VI\left[\cos(-90°)\left\{1+\cos 2(\omega t+90°)\right\}-\sin(-90°)\sin 2(\omega t+90°)\right]$$

$$= -VI \sin 2\omega t \tag{7.18}$$

and the average power in this case from Eq. (7.9) is

$$P = VI \cos 90° = 0 \tag{7.19}$$

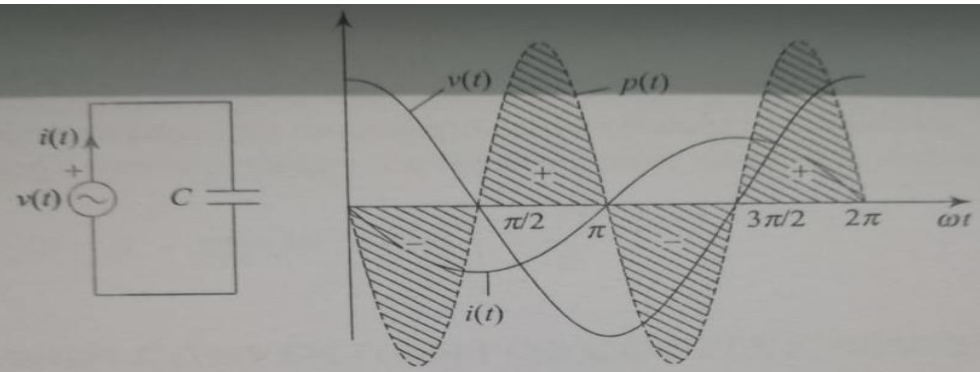


Fig. 7.8 *Waveforms of i(t), v(t), and p(t) for a capacitive circuit*

As in the case of a purely inductive network, the frequency of the power waveform is twice that of either the applied voltage or the current waveforms. Additionally, the average power of the waveform is zero which implies that during a quarter of a cycle of the voltage wave, the capacitor draws power (positive power) from the voltage source to build the charge and during the next quarter of the cycle it returns an equal amount of power (negative power) to the source since it is discharging. Thus, there is a continuous exchange of power between the source and the capacitor.

The product of the voltage and the current of an AC circuit is termed the apparent power. It is usually denoted by the symbol S. Thus,

$$S = VI \text{ volt–ampere or VA} \tag{7.20}$$

Equations (7.12) and (7.13) may now be written as

$$P = VI \cos \varphi = S \cos \varphi \text{ W} \tag{7.21}$$

$$Q = VI \sin \varphi = S \sin \varphi \text{ VAR} \tag{7.22}$$

Then $$S = \sqrt{P^2 + Q^2} \text{ VA} \tag{7.23}$$

This suggests that power can be interpreted by means of the phasor diagram shown in Fig. 7.9(a). The current component $I \cos \varphi$ in phase with the voltage V supplies active power, whereas the current component $I \sin \varphi$ in quadrature with the voltage V supplies the reactive power. The component $I \cos \varphi$ is termed as the active power component of the current, while the component $I \sin \varphi$ is termed as the reactive power component of the current. The complex power $\boldsymbol{S}$ is

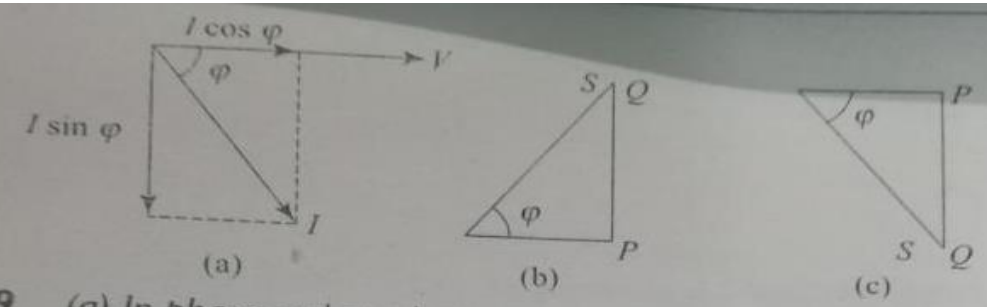


Fig. 7.9 *(a) In phase and quadrature components of current. Complex power triangle for (b) inductive load and (c) capacitive load*

$$S = P + j\,Q \tag{7.24}$$

where Q is positive for an inductive load and negative for a capacitive load.

Complex power in terms of voltage phasor $\boldsymbol{V}$ and current phasor $\boldsymbol{I}$ can be expressed as

$$\boldsymbol{S} = \boldsymbol{V}\boldsymbol{I}^* \tag{7.25}$$

Let $\boldsymbol{V} = V \angle \theta_v$, $\boldsymbol{I} = I \angle \theta_i$, $\boldsymbol{I}$ lags $\boldsymbol{V}$ by $\varphi = \angle \theta_v - \angle \theta_i$

Then, $\boldsymbol{S} = \boldsymbol{V}\boldsymbol{I}^* = VI \angle(\theta_v - \theta_i) = VIe^{j(\theta_v - \theta_i)} = VIe^{j\varphi}$ (7.26a)

$$= (VI \cos \varphi + jVI \sin \varphi) \tag{7.26b}$$

$$= P + j\,Q = S \angle \varphi = S\,(\cos \varphi + j \sin \varphi) \tag{7.26c}$$

While dealing with large powers, the unit of active power is kW/MW, that of reactive power is kVAR/MVAR, and the complex power is kVA/MVA.

Complex Power Computations

Equations developed in the earlier sections are adequate for power computations. However, the following supplementary equations are derived to facilitate complex power computations. The complex power in terms of the RMS voltage V and RMS current I is expressed in Eq. (7.26a) which is reproduced for ready reference:

$$S = \mathbf{V}\mathbf{I}^* = VI\angle(\theta_v - \theta_i) = VIe^{j(\theta_v - \theta_i)} \tag{7.27a}$$

Thus, from Eq. (7.27a) the magnitude of the apparent power is given by

$$|S| = VI \tag{7.27b}$$

If the circuit impedance is $Z = (R + jX)\ \Omega$, then by Ohm's law $\mathbf{V} = Z\mathbf{I}$. Substitution of the voltage for computing $\mathbf{S}$ leads to

$$S = (P + jQ) = Z\mathbf{I} \times \mathbf{I}^* = Z|I|^2 = |I|^2(R + jX) = |I|^2 R + j|I|^2 X \tag{7.28}$$

Thus, from Eq. (7.28) it is seen that

$$P = |I|^2 R = \frac{I_m^2}{2} R \text{ W} \tag{7.29a}$$

And $Q = |I|^2 X = \frac{I_m^2}{2} X$ VAR (7.29b)

Another variation for computing $\mathbf{S}$ can be obtained by eliminating the current. Pursuing a similar procedure, the following expression can be derived:

$$\mathbf{S} = (P + jQ) = \mathbf{V}\mathbf{I}^* = \mathbf{V}\left(\frac{\mathbf{V}}{Z}\right)^* = \frac{|V|^2}{Z^*} \tag{7.30}$$

Power Factor & Power Factor Angle

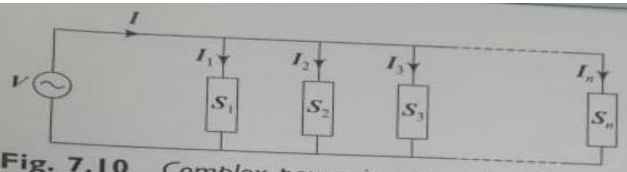


Fig. 7.10 *Complex power in a practical network*

If the current drawn by each load is $I_1, I_2, I_3, \ldots, I_n$ A, the total current I supplied by the utility is given by

$$I = I_1 + I_2 + I_3 + \cdots + I_n$$

The total complex power supplied by the utility is

$$S = VI^* = V\left(I_1^* + I_2^* + I_3^* + \cdots + I_n^*\right) = S_1 + S_2 + S_3 + \cdots + S_n \quad (7.33)$$

Equating the real and reactive power terms in Eq. (7.33) provides the total real and reactive power supplied by the utility as under:

$$P + Q = (P_1 + jQ_1) + (P_2 + jQ_2) + (P_3 + jQ_3) + \cdots + (P_n + jQ_n) \quad (7.34)$$

Hence,

$$P = P_1 + P_2 + P_3 + \cdots + P_n \quad (7.35a)$$

$$Q = Q_1 + Q_2 + Q_3 + \cdots + Q_n \quad (7.35b)$$

Dividing Eq. (7.35b) by Eq. (7.35a) leads to the power factor angle of the utility as follows:

$$\varphi = \tan^{-1}\left(\frac{Q}{P}\right) = \tan^{-1}\left(\frac{Q_1 + Q_2 + Q_3 + \cdots + Q_n}{P_1 + P_2 + P_3 + \cdots + P_n}\right) \quad (7.36)$$

The apparent power supplied by the utility is obtained as

$$S = \frac{P}{\cos\varphi} = \sqrt{P^2 + Q^2} \quad (7.37)$$

From the complex power triangle, (Fig.7.9) the ratio of the real power to the apparent power is defined as power factor (pf). Thus,

$$\cos\varphi = \frac{\text{real power}}{\text{apparent power}} = \frac{P}{S} \quad (7.31)$$

where φ is called the power factor angle since it provides the phase relationship between the applied sinusoidal voltage and the circuit current. The pf angle φ varies between + 90° for a purely capacitive circuit, 0° for a resistive circuit, and – 90° for a purely inductive circuit. The pf, on the other hand, varies between zero and unity. However, for a lagging (inductive) circuit, the pf is negative. It may be emphasized that a leading or a lagging phase angle always refers to the phase of the current with respect to the voltage. Similarly,

$$\sin\varphi = \frac{\text{reactive power}}{\text{apparent power}} = \frac{Q}{S} \quad (7.32)$$

and $\sin\varphi$ is called the reactive factor of the circuit and is denoted as *rf* in its shortened form.

Significance of Power Factor

From the foregoing discussion it becomes clear that at unity power factor, the amount of apparent power (*VI*) required to be generated is equal to the real power of the load. On the other hand, if reactive loads (which is more of a practice than an exception) are to be serviced, greater volt–amperes are required to be generated to supply the given real watts. Figure 7.11 shows a plot of the *VI* required to be generated to supply 1 W of real power when the pf is varying between unity and 0.2.

It may be seen from the plot that as the pf falls below unity, the amount of apparent power required to be generated increases progressively. Increase in *VI* generation leads to a corresponding increase in the line current which in turn results in increase in line losses, the latter increase being proportional to the square of the line current.

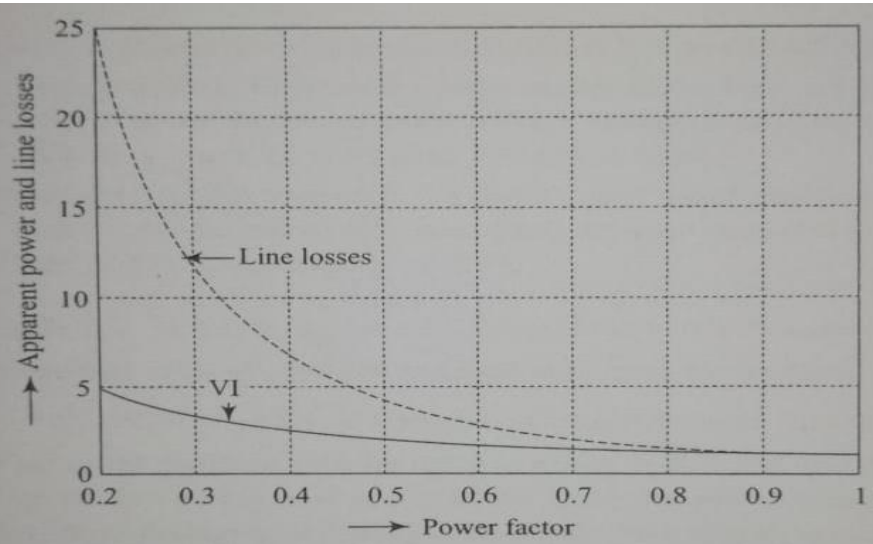


Fig. 7.11 *Variation of apparent power and line losses versus power factor*

From an industrial perspective, a utility is expected to deliver both the real and reactive power load demands at the consumer's terminals. Therefore, a poor pf would mean more expensive power equipment (generators, transformers, etc.), transmission lines, and protective gear which in turn would result in increasing the cost of energy.

Electric loads are a conglomeration of circuits containing (i) purely resistive heating elements such as filament lamps, strip heaters, cooking stoves, etc., which have a pf of unity and (ii) inductive and capacitive elements such as induction and synchronous motors, lamp ballasts, etc., which have typically power factors below unity. Table 7.1 provides power factors of some of the typical electric appliances.

Assume that a capacitor of C F at an RMS voltage of V volts and a frequency of f Hz is employed to supply the required reactive power. From the power triangle, it is seen that

$$Q_{in} = P \tan \varphi_{in} \text{ VAR}$$

and $$Q_{fi} = P \tan \varphi_{fi} \text{ VAR}$$

Leading reactive power required to be injected

$$Q_{in} - Q_{fi} = P(\tan \varphi_{in} - \tan \varphi_{fi}) \text{ VAR} \quad (7.38)$$

Leading power supplied by the capacitor

$$Q = 2\pi \times fCV^2 = \omega CV^2 \text{ VAR} \quad (7.39)$$

Equating Eqs (7.38) and (7.39) and simplifying leads to the required capacitance for improvement of pf as follows:

$$C = \frac{P\left(\tan \varphi_{in} - \tan \varphi_{fi}\right)}{\omega V^2} \text{ F} \quad (7.40)$$

Example 7.3 A 50 Hz, $v_S(t) = 220\sqrt{2}\cos(314t + 10°)$ V source is applied to a two-terminal network, and the resultant current at the terminals is given by $i_S(t) = 5\sqrt{2}\sin(314t - 15°)$ A. Calculate (a) real power and (b) reactive power. (c) What is the frequency of the reactive power? (d) Is the real power being supplied to the two-terminal network? (e) Draw the power triangle and state whether the pf is lagging or leading. (f) Is the network supplying reactive power to the source? (g) What is the reactive factor of the network? $VI\sin\varphi$ $\sin 2\omega$

Solution The RMS value of the applied voltage and the current is 220 V and 5 A, respectively. The current into the network is rewritten to be consistent with Eq. (7.2b) as

$$i_s(t) = 5\sqrt{2}\cos(314t - 105°) \text{ A}$$

Thus, from Eq. (7.2), it is seen that $\theta_v = 10°$ and $\theta_i = -105°$

Phase angle $\varphi = \theta_v - \theta_i = 10° - (-105°) = 115°$ $VI\cos\varphi$ $|\phi|$ θ

(a) Using Eq. (7.12) gives $P = 220 \times 5 \times \cos 115° = -464.8801$ W

(b) Using Eq. (7.13) gives $Q = VI\sin\varphi = 220 \times 10 \times \sin 115° = 996.9386$ VAR

(c) The frequency of variation of the reactive power is twice that of the

Example 7.5 For the circuit shown in Fig. 7.14, calculate (a) real power (b) reactive power, (c) apparent power, and (d) pf. If a capacitor of 0.3 μF is connected in parallel across the terminals AB, how do (e) real power, (f) reactive power, (g) apparent power, and (h) power factor change? What is the magnitude and nature of power contributed by the capacitor? Draw the power triangle. Assume a frequency of 300 Hz and an RMS voltage of 25 V for the supply source.

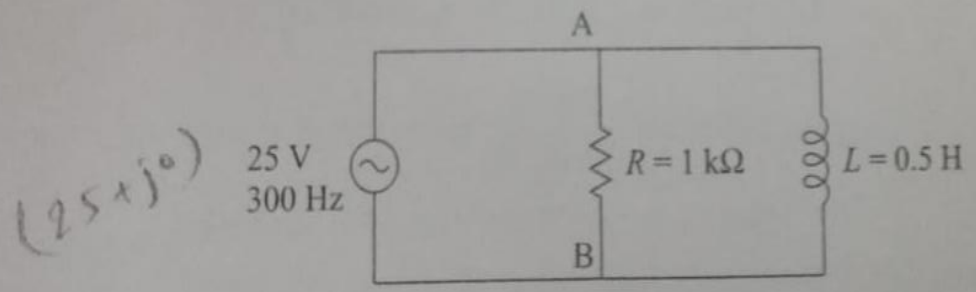


Fig. 7.14

Example 7.9 A 440 V (RMS) generator supplies a load $Z_L = (15 + j20)\ \Omega$ through a transmission line of impedance $Z_{TL} = (1.5 + j4)\ \Omega$ as shown in Fig. 7.19. Calculate the (a) line current I_{TL}, (b) load voltage V_L, (c) real and reactive power loads, (d) apparent power supplied by the generator, and (e) power factor and power factor angle of the generator. (f) What impedance Z should be connected in parallel across the load to ensure that maximum power transfer takes place? (g) What is the power factor of the generator under the condition of the maximum power transfer? (h) What is the maximum power transferred?

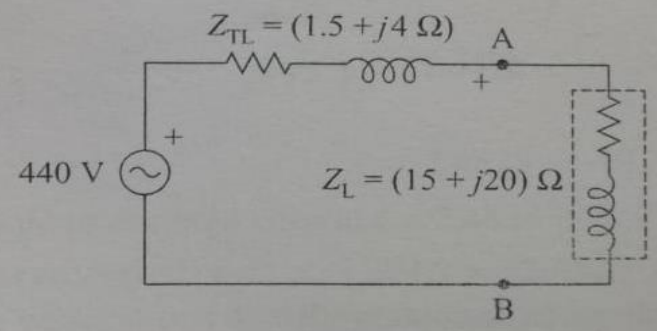


Fig. 7.19

Solution Assume the generator voltage as the reference, i.e., $V = 440\angle 0°$ V

(a) The line current

$$I_{TL} = \frac{440\angle 0°}{Z_{TL} + Z_L} = \frac{440}{(16.5 + j24)} = (8.5588 - j12.4492)$$

$$= 15.1074\angle -55.4915° \text{ A}$$

(b) The load voltage

$$V_L = 440\angle 0 - (8.5588 - j12.4492) \times (1.5 + j4)$$

$$= (377.37 - j15.561) = 377.6895\angle -2.3614°$$

(c) The real and reactive loads

$$S_L = V_L \times I_{TL}^* = (377.37 - j15.561) \times (8.5588 + j12.4492)$$

$$= (3.4235 + j4.5647)\text{kVA}$$

Hence, the real power $P_L = 3.4245$ kW and the reactive power $Q_L = 4.5647$ kVAR (inductive)

(d) The apparent power supplied by the generator $S_G = |V|\,|I_{TL}| = 440 \times 15.1074 = 6.6473$ kVA

(e) The real power supplied by the generator $P_G = (15.1074)^2 \times (1.5 + 15) = 3.7659$ kW

The power factor of the generator

$$\cos\varphi = \frac{P_G}{|S_G|} = \frac{3.7659}{6.6473} = 0.5665,\ \varphi = \cos^{-1}(0.5565)$$

$$= 55.4915° \text{ (lagging)}$$

(f) Looking into the terminals AB, from the load end, the Thevenin's impedance is $Z_{TH} = (1.5 + j4)\ \Omega$. For maximum power transfer, the load impedance must be equal to Z_{TH}^*, i.e., $(1.5 - j4)\ \Omega$. Assume the impedance $Z = (R + jX)\ \Omega$ is connected in parallel with Z_L such that the parallel combination has impedance equal to Z_{TH}^*. Thus,

$$\frac{(R + jX) \times (15 + j20)}{[(R + 15) + j(X + 20)]} = (1.5 - j4) \qquad (7.9.1)$$

Simplification of Eq. (7.9.1) leads to the following two simultaneous equations:

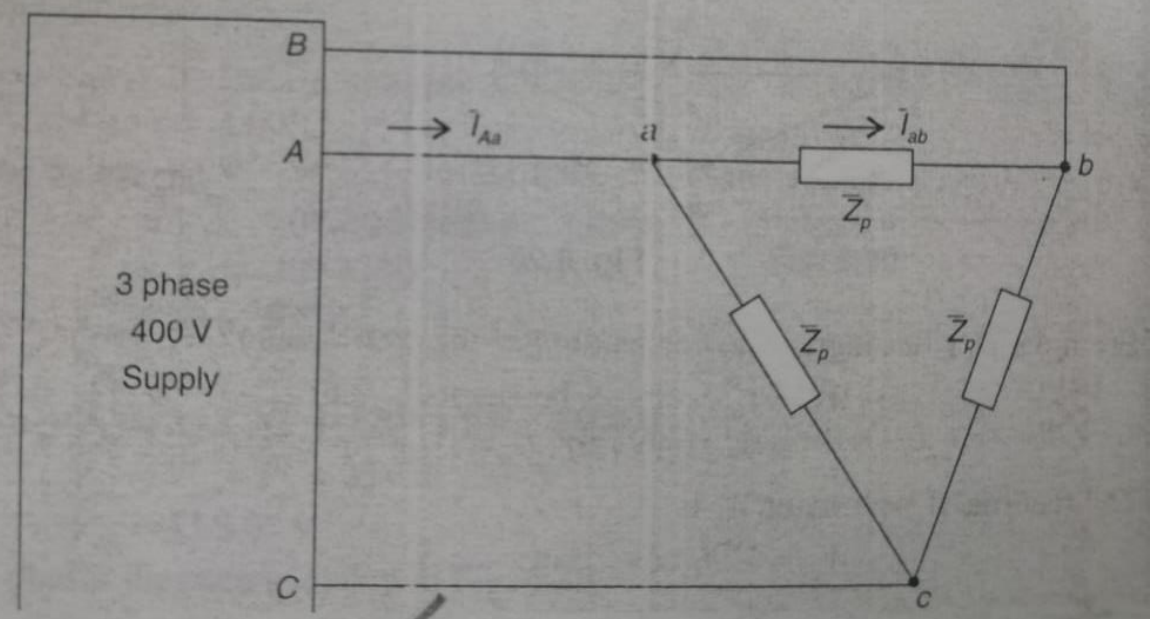
$$13.5\,R - 24\,X = 102.5$$
$$24\,R + 13.5\,X = -30$$

Solving the above two equations simultaneously yields the impedance to be connected in parallel for maximum power transfer, i.e.,

$$Z = (0.8754 - j3.7784)\ \Omega$$

(g) Under the condition of maximum power transfer, the total impedance in the circuit is resistive. Hence, the power factor of the generator is unity.

Example 6.9 A balanced 3-phase 400V supply is connected to balanced 3-phase delta-connected load as shown in Fig. 6.26. It is found by measurement that $\bar{I}_{ab} = 20\angle -30°$ A.



(a) Determine line current $\bar{I}_{Aa}$.
(b) Compute the total value of power received by the load.
(c) Calculate the resistive component of load impedance.
(d) Draw the phasor diagram showing line voltages, phase currents and line currents.

Example 6.12 A 3-phase, 50 Hz, 400 V system feeds a load of 25kW at $pf = 0.7$ lagging as shown in Fig. 6.31 Three capacitors are connected between lines across the load to improve the pf to 0.85. Determine the resultant current drawn from the supply and value of the capacitors.

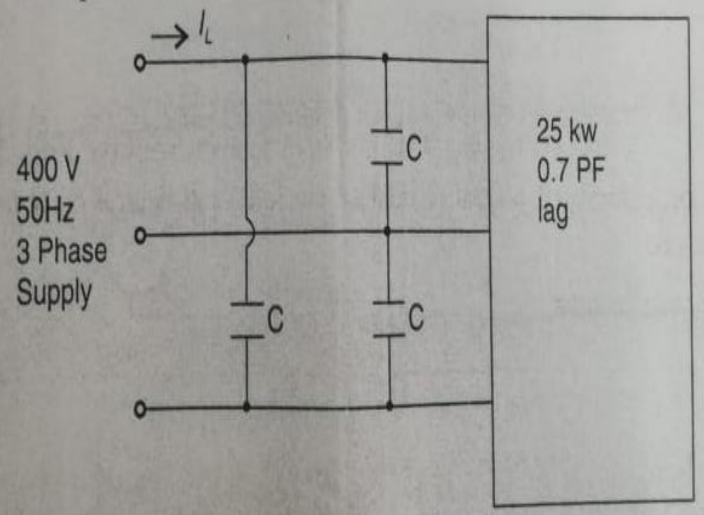
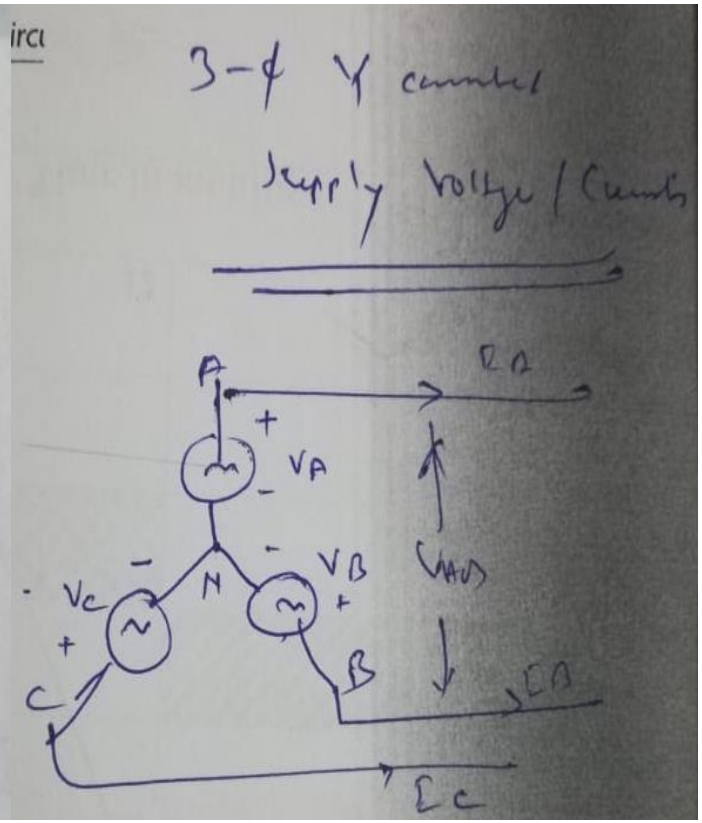


Fig. 6.31



Phasor Diagram of 3-Phase Star Connected System

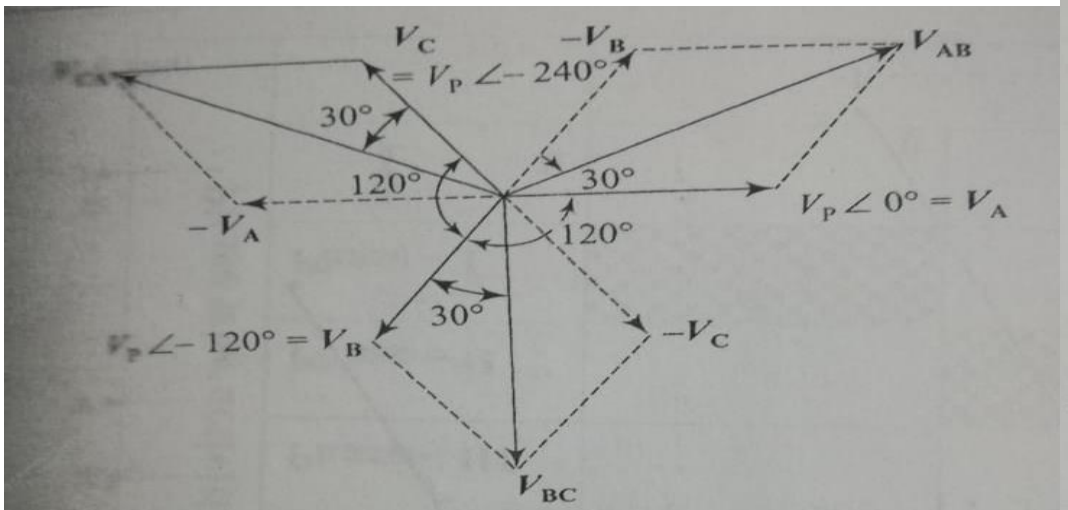
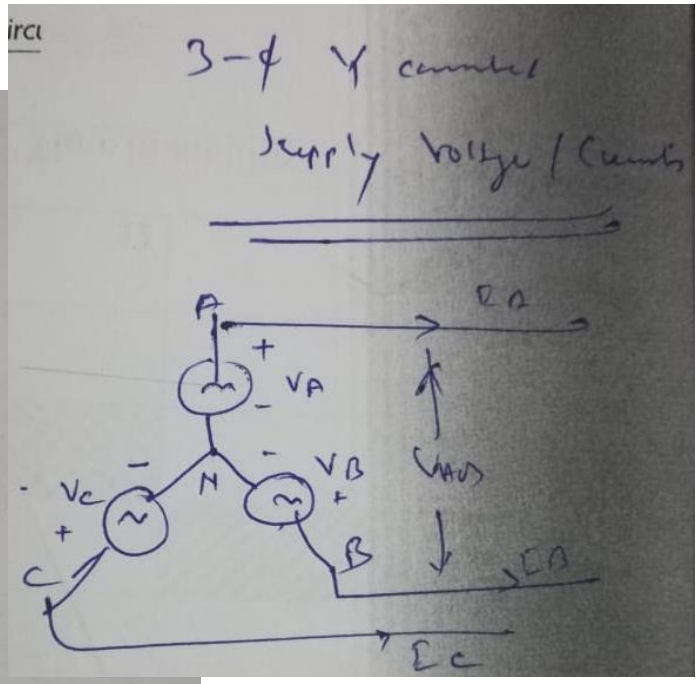


Fig. 8.12 *Phasor diagram for the three-phase star connected supply system*



V_{AB}, V_{BC}, and V_{CA} between the terminals A–B, B–C, and C–
are called the line voltages. The line voltages may be compute
diagram of phase voltages (Fig. 8.12) as follows:

$$= V_A - V_B = V_P \angle 0° - V_P \angle -120° = \sqrt{3}\, V \angle 30° \text{ V} \quad (8.6$$

$$= V_B - V_C = V_P \angle -120° - V_P \angle -240° = \sqrt{3}\, V_P \angle -90° \text{ V} \quad (8.6$$

$$= V_B - V_C = V_P \angle -240° - V_P \angle 0° = \sqrt{3}\, V_P \angle 150° \text{ V} \quad (8.6$$

Phasor Diagram of 3-Phase Star Connected System

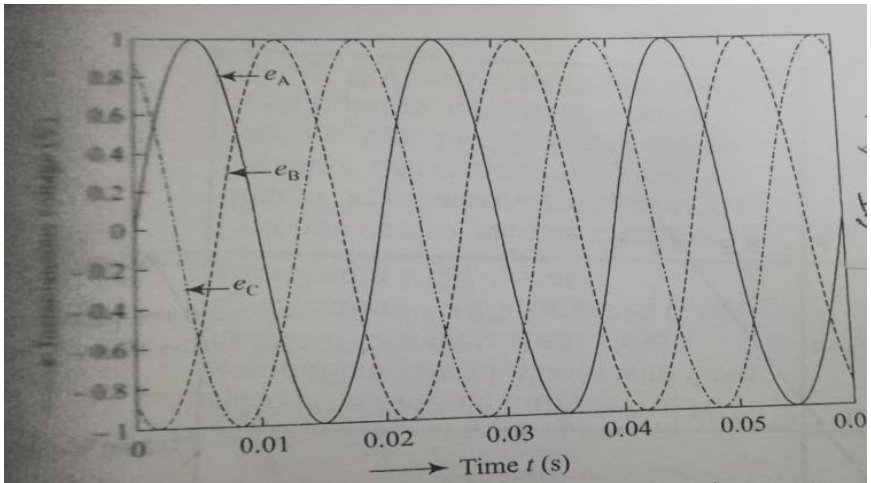
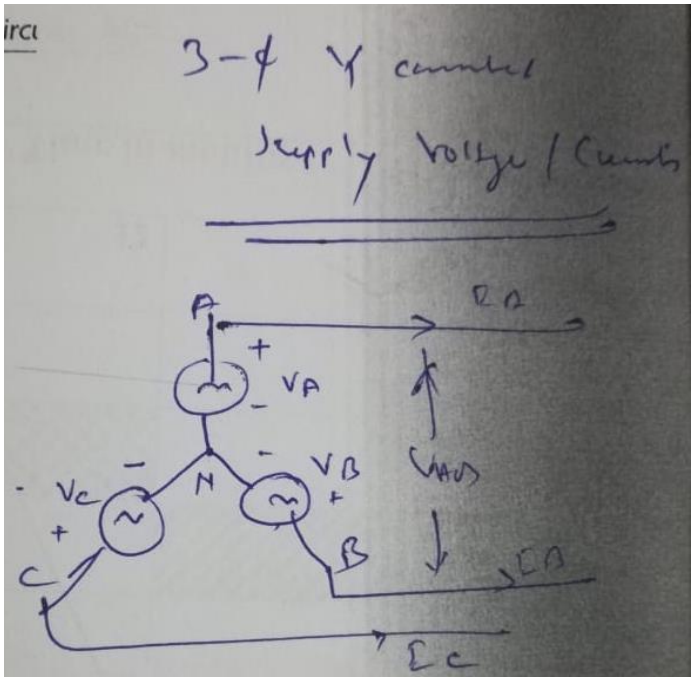


Fig. 8.2 *Generation of three-phase voltages*



noted from Eq. (8.6) that the line voltage V_{AB} leads the phase by 30°, the line voltage V_{BC} leads the phase voltage V_B by 30°, and the V_{CA} leads the phase voltage V_C by 30°. Thus, the system of line constitutes a balanced three-phase voltage system. The magnitude of is $\sqrt{3}$ times the phase voltage. In general, if V_L represents the line

$$V_L = \sqrt{3}\ V_P = 1.732\ V_P \tag{8.7}$$

it is seen that in a star connected three-phase system connected load, the current in a line conductor I_L is identical to the phase

$$I_L = I_P \tag{8.8}$$

of the KCL at the neutral point N in Fig. 8.11 leads to the following

$$I_A + I_B + I_C = 0 \tag{8.9a}$$

Generation of 3-Phase Voltage

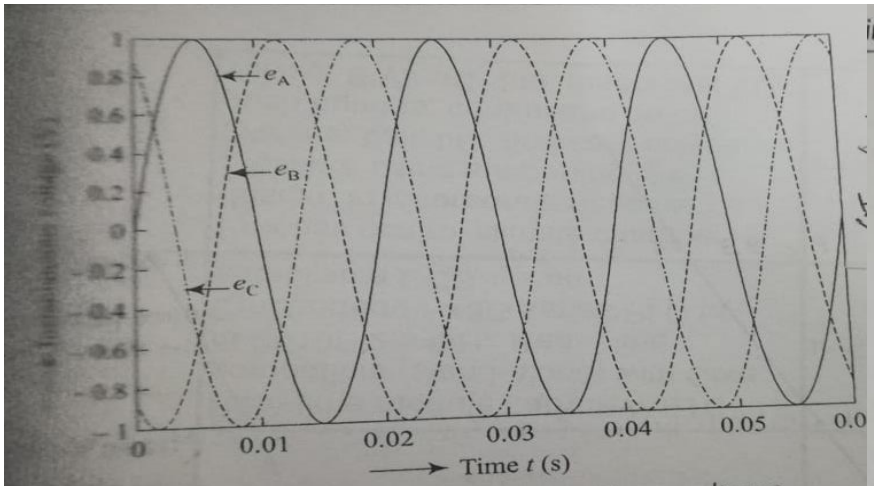
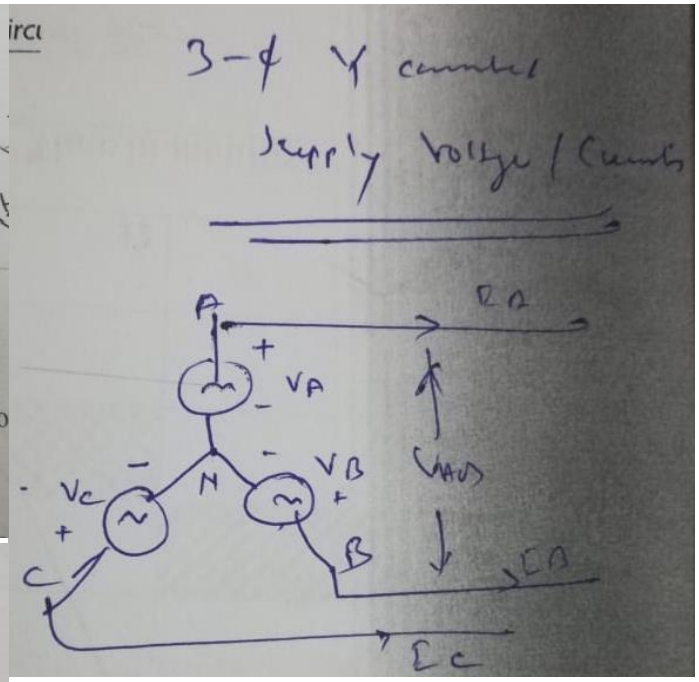


Fig. 8.2 *Generation of three-phase voltages*



speed of the rotor is N rpm and the number of generator poles is P, the frequency of the induced voltages is expressed as

$$f = \frac{NP}{120} \text{ cps or Hz} \tag{8.1}$$

Mathematically, the instantaneous values of the induced voltages are written as

$$e_A = \sqrt{2}\, E \sin \omega t \text{ V} \tag{8.2a}$$

$V_p = \frac{V_{pp}}{\sqrt{2}} = V_{rms}$

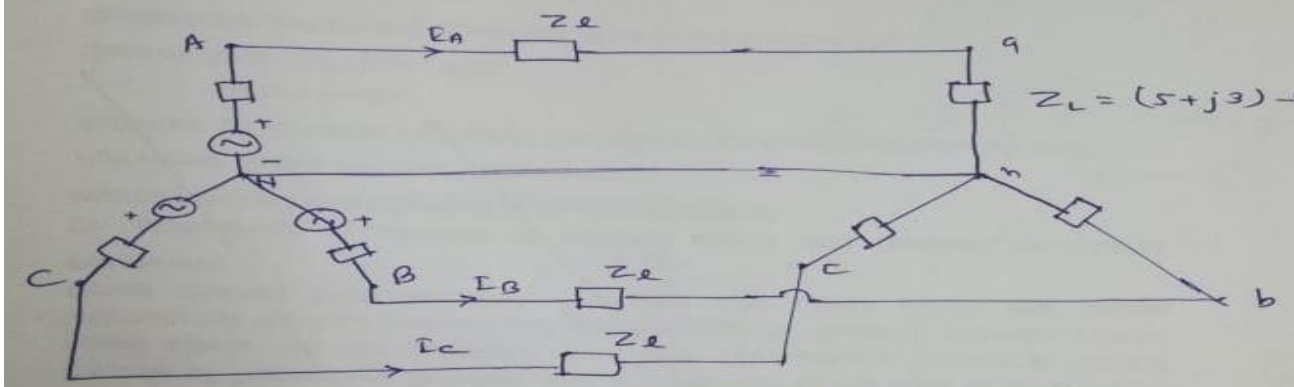
$$e_B = \sqrt{2}\, E \sin(\omega t - 120°) \text{ V} \tag{8.2b}$$

$$e_C = \sqrt{2}\, E \sin(\omega t - 240°) \text{ V} \tag{8.2c}$$

where $E = E_m/\sqrt{2}$ = the RMS value of the voltage, and ω is the angular speed of the rotor in electrical degrees per second. For a two-pole machine, the electrical angle and mechanical angle are the same, while for a P-pole machine, one unit of mechanical angle is equal to $P/2$ units of electrical angle.

For the assumed anti-clockwise direction of rotation, it is seen that the induced emf in winding BB_1 lags that in winding AA_1 by 120°. Similarly, the emf induced in winding CC_1 lags the induced emf in AA_1 by 240°. The induced emfs are

A 3-ϕ Y-connected balanced voltage source with a line voltage of 230 V is supplying a Y-connected balanced load of $(5+j3.0)\ \Omega$ as shown in figure.



If the source impedance is $Z_s = (0.04 + j\,0.20)\ \Omega$ per phase and impedance of each line $Z_l = (0.08 + j\,0.25)\ \Omega$,

Calculate in each-phase;

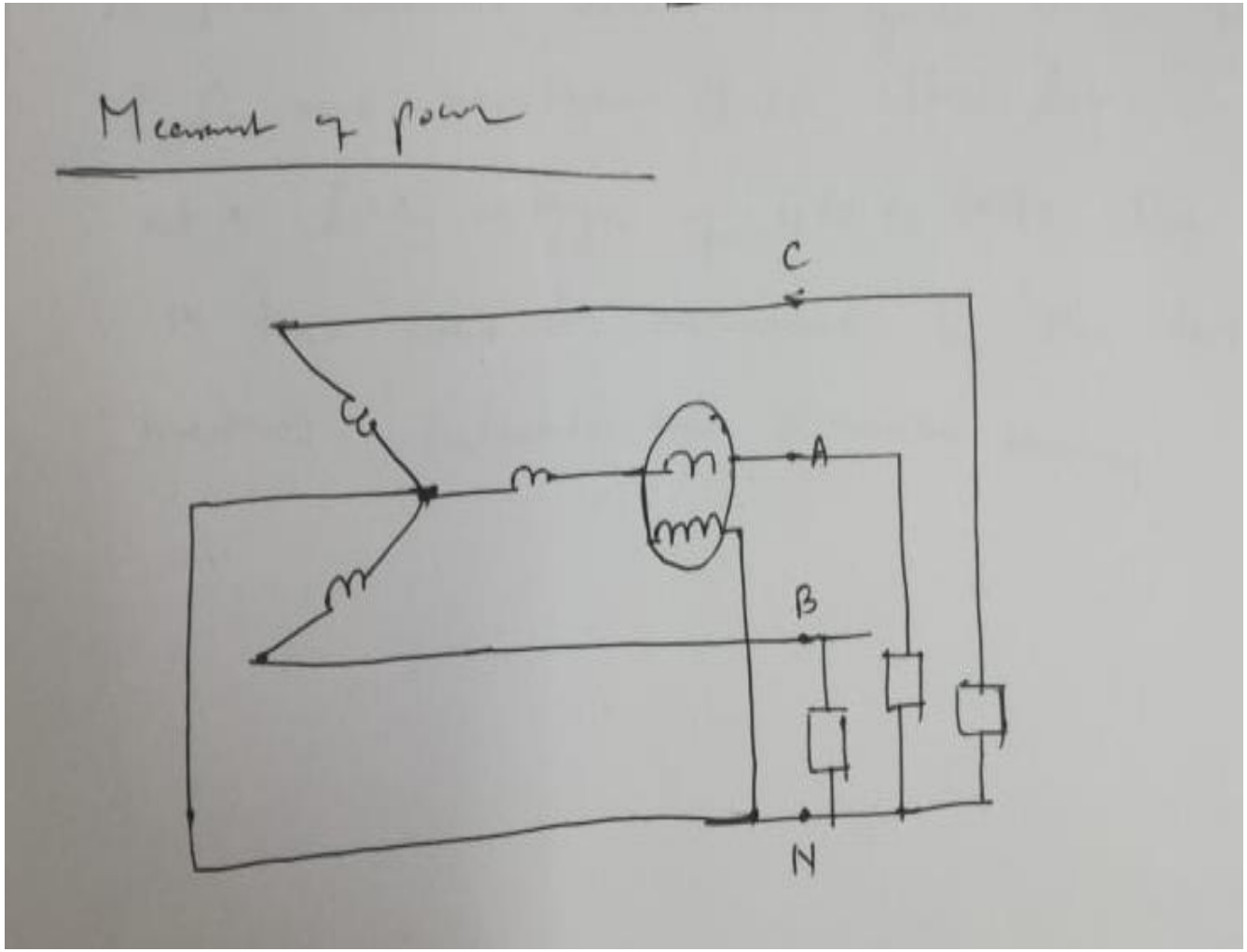
(a) the load currents and

(b) the load voltages.

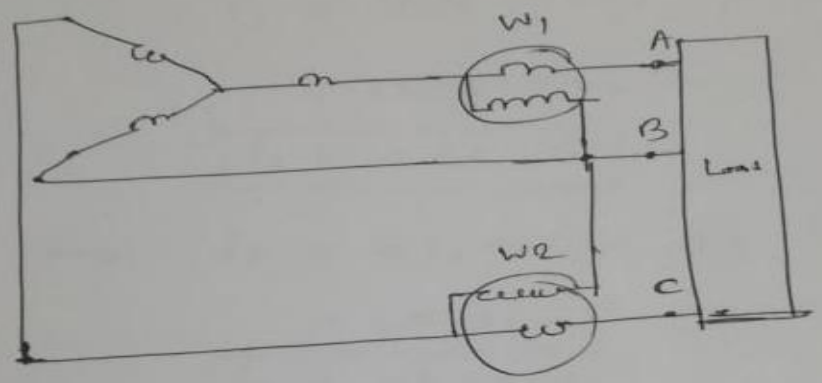
(c) What is the current in the neutral conductor?

(d) Draw the phasor-diagram showing the currents & voltages in phase-A, and 3-ϕ currents & voltages at the load terminals.

(e) How will the load currents and voltages change if the line impedances is made zero?


Measurement of power
C
A
B
N

Two Wattmeter Method



$W_1 = i_A (V_A - V_B)$

$W_2 = i_C (V_C - V_B)$

$\therefore W_1 + W_2$

$= i_A (V_A - V_B) + i_C (V_C - V_B)$

at N; $i_A + i_B + i_C = 0$

$\therefore W_1 + W_2 = i_A V_A + i_C V_C - V_B (i_A + i_C)$

$= i_A V_A + i_C V_C + i_B V_B$

$=$ total $v(t)$

power factor:

$$\boxed{\tan\phi = \sqrt{3}\,\frac{W_1 - W_2}{W_1 + W_2}}$$

$$\boxed{\because P = \sqrt{3}\, I_L V_L \cos\phi}$$

* In a DC ckt, power consumed = 'V' across × 'I' through load

* In a AC ckt, power = the RMS voltage × the RMS current × the power factor of the ckt.

* The instrument which is used to measure power in an AC circuit is called a Wattmeter.

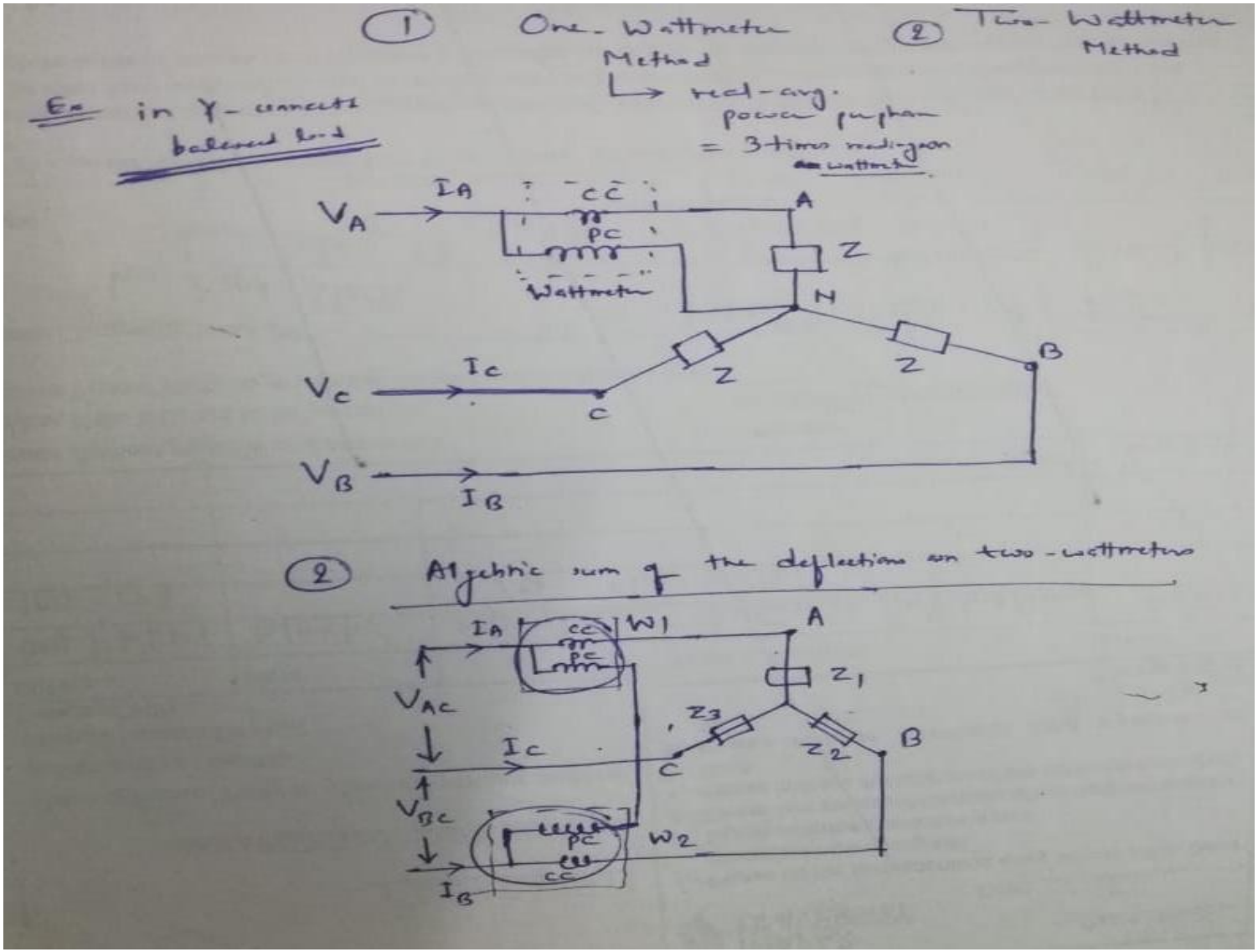
* Two methods of measuring power in AC circuits

(1) One-Wattmeter Method
↳ real-avg. power per phase = 3 times reading on wattmeter.

(2) Two-Wattmeter Method

Ex in Y-connected balanced load

IA : CC : A


1
One-Wattmeter Method
2
Two-Wattmeter Method
Ex in Y-connected balanced load
→ real-avg. power per phase = 3 times reading on wattmeter
I_A
CC
PC
Wattmeter
V_A
A
Z
N
Z
Z
B
I_C
V_C
C
V_B
I_B
2
Algebric sum of the deflections on two-wattmeters
I_A
CC
PC
W1
A
Z_1
V_{AC}
Z_3
Z_2
B
I_C
C
V_{BC}
PC
CC
W2
I_B

Assuming that the instantaneous voltages appearing across the loads Z_1, Z_2, Z_3 at time 't' are, respectively, v_A, v_B, and v_C.

$$\therefore \quad w_1 = v_{AC} \cdot i_A = (v_A - v_C) \cdot i_A \qquad \text{——— (1)}$$

$$w_2 = v_{BC} \cdot i_B = (v_B - v_C) \cdot i_B \qquad \text{——— (2)}$$

(1) + (2)

$$w_1 + w_2 = v_{AC} \cdot i_A + v_{BC} \cdot i_B = (v_A - v_C)\, i_A + (v_B - v_C) \cdot i$$

$$= v_A i_A + v_B i_B - v_C (i_A + i_B) \qquad \text{——— (3)}$$

$$\because \quad i_A + i_B + i_C = i_N = 0$$

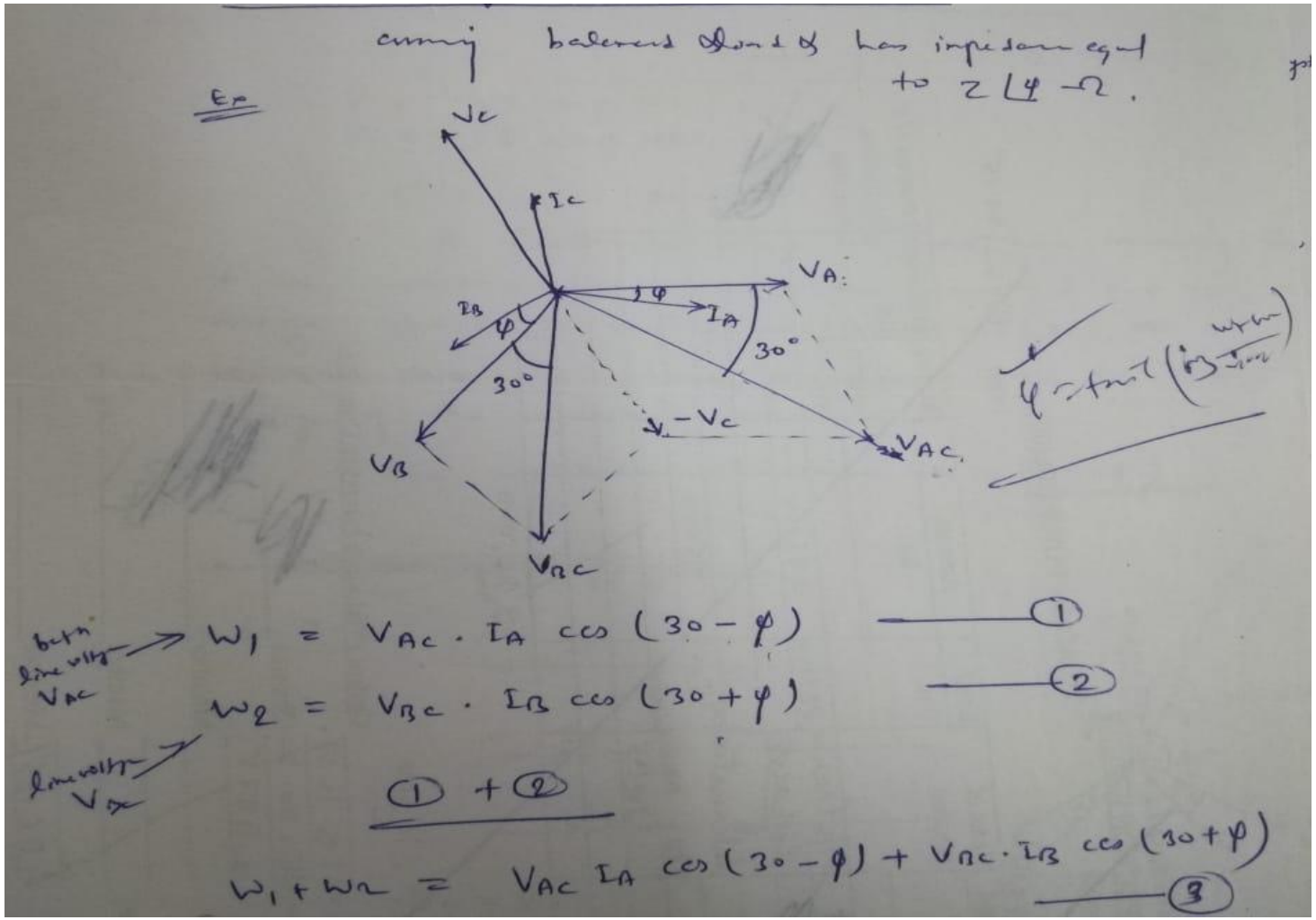
$\therefore$ From eqn (3);

$$w_1 + w_2 = v_A i_A + v_B i_B + v_C i_C \qquad \text{——— (4)}$$

Eqn (4) shows that the two-wattmeter method measure power in a 3-ϕ unbalanced load.

* the same result can be obtained for Δ-connected load as well.

Measurement of Balanced 3-Phase Load



Measurement of Reactive Power

real power – P, reactive power – Q for a 1-ϕ system.

$P = VI\cos\phi$ W

$Q = VI\sin\phi$ VAR

$\because \sin\phi = \cos(90-\phi)$,

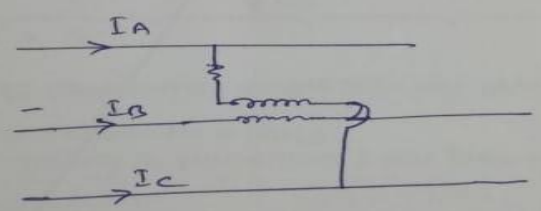
$\therefore$ A wattmeter may be used for measuring Q if the current coil (cc) carries the current I and the voltage applied to the pressure/potential coil (pc), is such that its phase displacement from the actual voltage of the circuit is 90°.

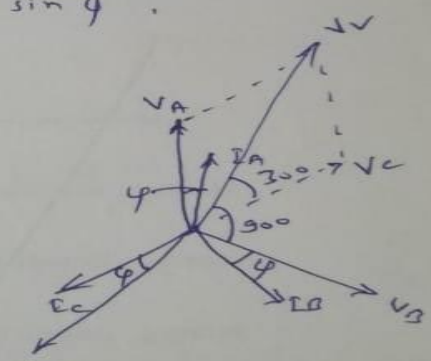
$\therefore$ the wattmeter will read $VI\cos(90-\phi)$

i.e $VI\sin\phi$.

~~In case of 3-ϕ system,~~

In 3-ϕ;





since $V_A = V_B = V_C = V$

$I = I_A = I_B = I_C$ (line current)

$V_L I_B \cos(90+\phi) = \sqrt{3}\, VI\cos(90+\phi)$

$= -\sqrt{3}\, VI\sin\phi = -W_R$

$\therefore Q_{-3-\phi} = 3\, VI\sin\phi = -\sqrt{3}\, W_R$

(1) Three non-reactive loads of 5 kW, 3 kW, and 2 kW are connected between the neutral and the R, Y and B phases, respectively of a three-phase four wire system. The line voltage is 400 V. Find the current in each line conductor and in the neutral wire.

Soln: First of all find out / determine the values of resistances in Y connected load

Since this is a 4-wire system, the

$$V_\phi = \frac{400}{\sqrt{3}} = \frac{V_L}{\sqrt{3}}$$

$$\therefore 5\times10^3 = \frac{(400/\sqrt{3})^2}{R_1} \Rightarrow R_1 = 10.55\,\Omega$$

$$\text{Similarly } 3\times10^3 = \frac{(400/\sqrt{3})^2}{R_2} \Rightarrow R_2 = 17.77\,\Omega$$

$$\text{and } 2\times10^3 = \frac{(400/\sqrt{3})^2}{R_3} \Rightarrow R_3 = 26.66\,\Omega$$

Hence the I_ϕ are

$$V_\phi = \frac{V_L}{\sqrt{3}}$$

$$I_R = \frac{V}{R_1} = \frac{400}{10.55\times\sqrt{3}} = 21.85\,A$$

$$I_Y = \frac{V}{R_2} = \frac{400}{\sqrt{3}\times17.77} = 12.55\,A$$

$$I_B = \frac{V}{R_3} = \frac{400}{\sqrt{3}\times26.66} = 8.66\,A$$

$$\therefore I_N = I_R - \frac{1}{2}[I_Y + I_B]$$

$$I_N = 11.07\,A$$

Practice Problem

A star connected load consists of three coils of resistance 2 Ω and reactance 3 Ω. The load is supplied at a line voltage of 400 V, 50 Hz. The total power in the load is measured by the two-wattmeter method. Calculate the separate readings.

The I_l in Y-connected network/load.

(I_l in the star-connected load) $I_p = \dfrac{V_l/\sqrt{3}}{\sqrt{2^2+3^2}} = 64.05\ A$ —— (1)

and $\phi = \tan^{-1}\left(\frac{X}{R}\right) = \tan^{-1}\left(\frac{3}{2}\right) = 56.31$ —— (2)

$P_1 = V_l I_l \cos(30° + \phi)$ —— (3)

$= 400 \times 64.05 \cos(30° + 56.31)$

$\boxed{P_1 = 1.65\ kW}$

and $P_2 = V_l I_l \cos(30° - \phi)$ —— (4)

$= 400 \times 64.05 \cos(30° - 56.31)$

$\boxed{P_2 = 22.97\ kW}$

$\therefore$ Total power $= \sqrt{3}\, V_l I_l \cos\phi$

$= \sqrt{3} \times 400 \times 64.05 \cos(56.31)$

$\boxed{P_T = 24.63\ kW = P_1 + P_2}$

Practice Problem

3.17 A 'P' amount of Power is to be tx^ed over a power tx^ion line at a line-to-line voltage V and power factor cos θ. This power may be tx^ed by a single-phase (1-ϕ), 2-wire-line or by a three phase (3-ϕ), 3-wire line, the choice depending only upon which line requires the least copper (resistive) loss. Which system will be chosen and what is the approximate saving in copper?

Soln: let I_1 is line current of $1-\phi$
I_3 —"— $3-\phi$

For equal power transmission

$\because P = V I_1 \cos\theta = \sqrt{3}\, V I_3 \cos\theta$ —— (1)

For equal power loss

$2 I_1^2 R_1 = 3 I_3^2 R_3$ —— (2)

(1) & (2)

$\boxed{R_3 = 2R_1}$ —— (3)

Now $R = \frac{\rho \cdot l}{A}$

$\therefore A_1 = 2 A_3$ —— (4)

$\because$ volm of copper $V_1 = 2(lA_1)$ & $V_3 = 3(lA_3)$ —— (5)

Thank you

Tutorial Problems

Ques 1: Describe the interrelating among Real, Reactive and Apparent power.

Ques 2: For the balance source load star-star connected system, the source phase voltage is 200V and load impedance is $100\angle 60^{o}$. Calculate all the phase voltage, line voltage and line currents in phasor form. Also draw a complete phasor diagram.

Ques 3: A 3-Ø power system with a line voltage of 400V is supplying a delta connected load of 1500 W at 0.8 power factor lagging. Determine the phase as line current and also the phase impedance.

Ques 4: A 3-Ø ,400 V source (terminal voltage assumed constant and independent of load) supplies a load with an equivalent star impedance of $(60 + j15)\Omega$ / phase through a transmission line of impedance $(0.3 + j1.0)\Omega$ / line (phase). Compute

(a) The line current

(b) The line voltage

(c) The power ,reactive power and VA consumed by the load

(d) The power as reactive power loss in the line

(e) The power , reactive power and VA supplied by the source