CLASS NOTES

ON

POWER SYSTEM OPERATION AND CONTROL

A COURSE IN 7TH SEMESTER OF

BACHELOR OF TECHNOLOGY PROGRAMME IN ELECTRICAL ENGINEERING

(COURSE CODE-BEE701)

DEPARTMENT OF ELECTRICAL ENGINEERING

VEER SURENDRA SAI UNIVERSITY OF TECHNOLOGY

BURLA

2015

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FOREWORD BY THE AUTHOR

Modern power system is a complex system, spread over a large geographically area. The power system has been expanded manifold in the recent past due to increased demand and industrial growth. The power system has also seen new emerging trend in its technology, operation and planning. The issues such as, reactive power and active power control, angle stability and voltage stability, inter-area power transfer, power quality, automatic generation and frequency control for multi-machine system, reliability evaluation, operation in competitive environment, are important factors in operation and control of the power system.

Generation, transmission, distribution, and loads make up the fundamental structure of a power system. Generators, substations, transmission circuits, transformers, circuit breakers, metering, demand, load, are the main physical devices of an electric power system. Voltage, current, power, energy, frequency, and impedance are the fundamental terms used in power technology. The operation of power system needs proper coordination between these devices such that the fundamental variables of the system remain within desired limits.

Moreover if the system is subjected to faults, it may become unstable. Therefore it is necessary to study and analyze the system state and to determine the system stability and security. For this purpose, we need to use many numerical techniques and modern stochastic or metaheuristics computational techniques. After studying this subject, one may be able to address and understand the issues related to the power system.

POWER SYSTEM OPERATION & CONTROL (3-1-0)

SYLLABUS

MODULE-I (10 HOURS)

Concept of real and reactive powers, Complex power, Transmission capacity, The static load flow equations(SLFE),Definition of the load flow problem, Network model formulation, A load flow sample study, Computational aspect of the load flow problem. Gauss siedel and Newton Raphson method for power flow fast decoupled load flow, On load tap changing transformer and block regulating transformer, effects of regulating transformers.

MODULE-II (10 HOURS)

Power System Stability: Steady State Stability, Transient stability, Swing equation, Equal area criterion for stability, critical clearing angle, point by point Methods of improvement of transient stability. Voltage stability, concept, causes and countermeasures, Voltage stability indices.

MODULE-III (10 HOURS)

Economic Operation of Power System: Distribution offload between units within a plant, Transmission losses as function of plant generation, Calculation of loss coefficients, Distribution of loads between plants with special reference to steam and hydel plants, Automatic load dispatching.

Z bus Algorithm, Symmetrical and unsymmetrical fault analysis for power system, Z bus method in fault analysis.

MODULE-IV (10 HOURS)

Load frequency control, PF versus QV control, Modelling of speed governing system, Division of power system into control areas, Single area control and two area control.

BOOKS

[1]. John J Grainger, W. D. Stevenson, *"Power System Analysis",* TMH Publication

[2]. P. Kundur, *"Power System Stability and Control",* TMH Publication

[3]. C. L. Wadhwa, *"Electric Power System",* New Age Publishers.

CHAPTER-1

INTRODUCTION TO POWER SYSTEM

PER UNIT SYSTEM

The equipment such as transformers, transmission lines, machines, generators, condenser, etc. used in the power systems are represented by its electrical equivalent circuit consisting of impedances, admittances. The value of these variables are given in terms of its actual value based on the ratings of the equipment. The system uses many equipment with different ratings. This needs the calculation of other variables on the actual basis. Thus to avoid heterogeneity of calculations, a common mode of representing the variables is used. This is known as per unit system.

In per unit system, the power system variables such as voltage, current and impedances etc. are represented on a common base. For this purpose we need to define base quantity of the system. Generally two base quantities *'Base KVA or Base MVA' and 'Base KV'* are defined and others can be calculated.

Let

$Base\,Current = I_{Base}$

$Base\,Impednace = Z_{Base}$

If the voltage and power are given for single phase:

$$I_{Base} = \frac{Base\ KVA_{1\varphi}}{Base\ KV_{LN}} \tag{1.1}$$

$$Z_{Base} = \frac{Base\ KV_{LN}^{2}\ X\ 10^{3}}{Base\ KVA_{1\varphi}} \tag{1.2}$$

Or (1.2) can be written as

$$Z_{Base} = \frac{Base\ KV_{LN}^{2}}{Base\ MVA_{1\phi}} \tag{1.3}$$

If the voltage and power are given for three phase:

$$I_{Base} = \frac{Base\ KVA_{3\phi}}{\sqrt{3}\ X\ Base\ KV_{LL}} \tag{1.4}$$

$$Z_{Base} = \frac{\left(\frac{Base\ KV_{LL}}{\sqrt{3}}\right)^2\ X\ 10^3}{\left(\frac{Base\ KVA_{3\phi}}{3}\right)} \tag{1.5}$$

$$Z_{Base} = \frac{(Base\ KV_{LL})^2\ X\ 10^3}{Base\ KVA_{3\phi}} \tag{1.6}$$

$$Z_{Base} = \frac{(Base\ KV_{LL})^2}{Base\ MVA_{3\phi}} \tag{1.7}$$

The per unit value of any variable is given by (1.8)

$$p.u. Value\ \ of\ \ the\ Variable = \frac{Actual\ Value\ of\ \ the\ Variable}{Base\ Value\ of\ \ the\ Variable} \tag{1.8}$$

Say for voltage (1.9)

$$V_{p.u.} = \frac{V_{Actual}}{V_{Base}} \tag{1.9}$$

Similarly for current, impedance and power it is given by (1.10) to (1.12)

$$I_{p.u.} = \frac{I_{Actual}}{I_{Base}} \tag{1.10}$$

$$Z_{p.u.} = \frac{Z_{Actual}}{Z_{Base}} \tag{1.11}$$

$$MVA_{p.u.} = \frac{MVA_{Actual}}{MVA_{Base}} \tag{1.12}$$

Many a times the per unit quantities need to be calculated to a new base. It can be done by calculating the actual value at first and then converted to per unit on a new base.

Let the old base is given by $BaseKV_{Old}, BaseMVA_{Old}$ and new base is given by $BaseKV_{New}, BaseMVA_{New}$.

The old and new per unit value of impedance is given by $Z_{p.u..(Old)}$ & $Z_{p.u..(New)}$ respectively.

From (1.11) we get at the old base:

$$Z_{Actual} = Z_{p.u.(Old)}\ X\ Z_{Base(Old)} = Z_{p.u.(Old)} X \frac{\left(Base\ KV_{LL(Old)}\right)^2}{Base\ MVA_{3\phi(Old)}} \tag{1.13}$$

But the actual impedance on a new base can be given by

$$Z_{Actual} = Z_{p.u.(New)}\ X\ Z_{Base(New)} = Z_{p.u.(New)} X \frac{\left(Base\ KV_{LL(New)}\right)^2}{Base\ MVA_{3\phi(New)}} \tag{1.14}$$

Equating (1.13) and (1.14)

$$Z_{p.u.(New)} = Z_{p.u.(old)} X \frac{\left(Base\ KV_{LL(Old)}\right)^2}{Base\ MVA_{3\phi(Old)}}\ X\ \frac{Base\ MVA_{3\phi(New)}}{\left(Base\ KV_{LL(New)}\right)^2} \tag{1.15}$$

$$Z_{p.u.(New)} = Z_{p.u.(old)} X \frac{Base\ MVA_{3\phi(New)}}{Base\ MVA_{3\phi(Old)}}\ X\ \frac{\left(Base\ KV_{LL(Old)}\right)^2}{\left(Base\ KV_{LL(New)}\right)^2} \tag{1.16}$$

$$Z_{p.u.(New)} = Z_{p.u.(old)} X \frac{Base\ MVA_{3\phi(New)}}{Base\ MVA_{3\phi(Old)}}\ X \left(\frac{Base\ KV_{LL(Old)}}{Base\ KV_{LL(New)}}\right)^2 \tag{1.17}$$

By (1.17) the per unit impedance can be calculated on the new base.

POWER FLOW IN A TRANSMISSION LINE

The power flow in a transmission line can be calculated by considering the system shown in Fig.-1.1. It consists of a single transmission line connected between two buses. These buses are known as Sending end bus and Receiving end bus.

FIG.-1.1 TRANSMISSION LINE POWER FLOW

The line is characterized by its line constants as follows

$$A = |A|\angle\alpha, B = |B|\angle\beta$$

So that the power received at the receiving end is given by

$$S_R = P_R + jQ_R = V_R I_R^* \tag{1.18}$$

As we know the line equation in terms of $ABCD$ constant are

$$V_S = AV_R + BI_R \tag{1.19}$$

$$I_S = CV_R + DI_R \tag{1.20}$$

From (1.19)

$$I_R = \frac{V_S - AV_R}{B} \tag{1.21}$$

$$I_R^* = \left(\frac{V_S - AV_R}{B}\right)^* \tag{1.22}$$

$$I_R^* = \frac{(|V_S|\angle -\delta) - (|A|\angle -\alpha)(|V_R|\angle 0)}{(|B|\angle -\beta)} \tag{1.23}$$

From (1.18) and (1.23) We have

$$P_R + jQ_R = (|V_R|\angle 0)\frac{(|V_S|\angle -\delta) - (|A|\angle -\alpha)(|V_R|\angle 0)}{(|B|\angle -\beta)} \tag{1.24}$$

(1.24) can be written as

$$P_R + jQ_R = \frac{|V_R||V_S|}{|B|}\angle(\beta - \delta) - \frac{|V_R|^2|A|}{|B|}\angle(\beta - \alpha) \tag{1.25}$$

Equating the real and imaginary part

$$P_R = \frac{|V_R||V_S|}{|B|}Cos(\beta - \delta) - \frac{|V_R|^2|A|}{|B|}Cos(\beta - \alpha) \tag{1.26}$$

$$Q_R = -\left[\frac{|V_R||V_S|}{|B|}Sin(\beta - \delta) - \frac{|V_R|^2|A|}{|B|}Sin(\beta - \alpha)\right] \tag{1.27}$$

For the transmission line series resistance is very less as compared to series reactance and $|A| \approx 1.0, \alpha \approx 0.0, |B| = Z \approx X, \beta \approx 90^\circ$

Hence (1.26) can be fairly approximated as (1.28)

$$P_R = \frac{|V_R||V_S|}{X}Sin\delta \tag{1.28}$$

From (1.28) we conclude that the power transmitted over a transmission line is determined by the voltage at both the ends, the reactance of the line and phase difference between the voltages of both ends. Similarly the power flow between two buses can be given by (1.29)

$$P = \frac{|V_1||V_2|}{X_{12}} Sin(\delta_1 - \delta_2) \quad (1.29)$$

LINE COMPENSATION

As stated above the power flow in a transmission line depends upon the voltage at both the ends, the reactance of the line and phase difference between the voltages. If the power flow has to be increased or decreased then e have to control these variables. It can be controlled by

- Voltage magnitude control
- Transmission line reactance control
- Phase angle control

Once the installation of the transmission line is over its A, B, C, D parameters are constant because these depend upon the size and material of conductors and the configurations of the conductors. And hence the value of reactance and resistance are also fixed. The value of reactance X however can be controlled by providing compensation.

Since the series impedance of the transmission line consists of inductive reactance, the total series reactance can be reduced by connecting a capacitor in series. The shunt admittance of the transmission line consists of capacitive reactance, the effect of which can be compensated by connecting a shunt reactor. These processes are known as providing line compensation. The line compensation are of two types.

- Series compensation
- Shunt compensation

In series compensation as mentioned above, a suitable value of capacitor is connected in series with the transmission line as shown in Fig.-1.2. The location of this capacitor is optional and depend upon the requirement of transmission company.

FIG.-1.2 TRANSMISSION LINE WITH SERIES COMPENSATION

In shunt compensation a shunt reactor of suitable capacity is connected as an shunt element at the required bus of the transmission line as shown in Fig.-1.3.

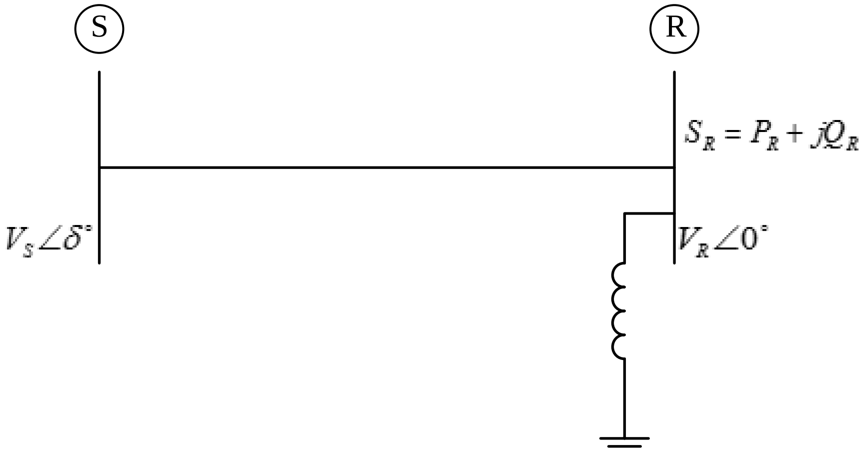


FIG.-1.3 TRANSMISSION LINE WITH SHUNT COMPENSATION

The value of series compensation and shunt compensation known as degree of compensation depend upon operational policy. It is seen that the series compensation is very much effective in controlling the power transfer over the transmission line where as the shunt compensation is most proved way for voltage control at the bus at which the compensation has been provided.

In power system most of the loads are inductive in nature resulting in reducing the voltage at which it is connected, it is well known to connect the capacitor at that voltage. It improves the power factor at that bus by supplying reactive power at the said bus. Thus Fig.-1.3 can be modified to Fig.-1.4.

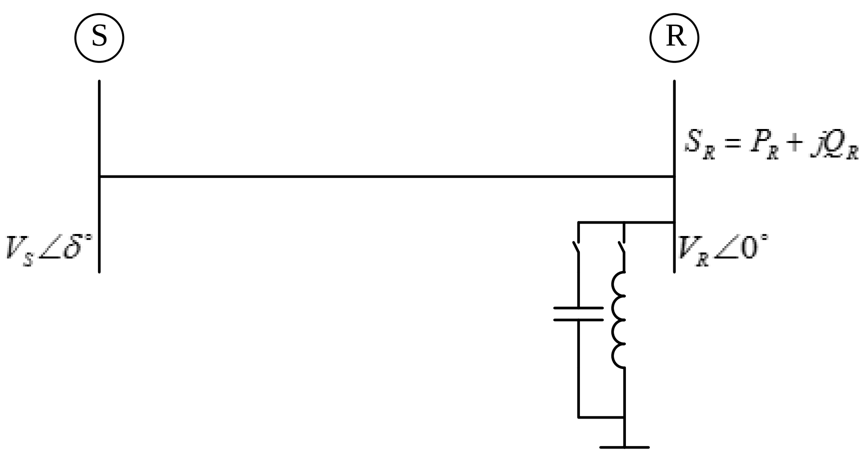


FIG.-1.4 TRANSMISSION LINE WITH SHUNT COMPENSATION

If the magnitude of voltage at the bus is less than the required magnitude then the shunt capacitor is connected. If the magnitude of voltage at the bus is more than the required magnitude then the shunt reactor is connected. Other way we can say, the shunt capacitor is used to improve the voltage and the shunt reactor is used to avoid the overvoltage.

With the insertion of compensation the circuit parameters changes and thus the operation of the power system. The line compensation provides:

- Improvement in power flow
- Power flow control
- Share of power between the transmission lines
- Voltage control
- Improved stability
- Improved security

Modern power system uses flexible AC transmission system (FACTS) devices to achieve required degree of compensation and thus control over the system operation.

CHAPTER-2

BUS IMPEDANCE MATRIX

The bus impedance matrix Z_{BUS} can be determined for a $'N-bus'$ power system by using the algorithm described as below. By using this algorithm Z_{BUS} can also be modified. The existing Z_{BUS} can be modified either by addition of a new bus or a new link. A new bus can be added to the system by connecting it to either reference bus through a link or an existing bus through a link. A new link can be added to the system by connecting it between reference bus and existing bus or between two existing bus. Thus Z_{BUS} can be modified in four ways.

The block diagram of $'N-bus'$ power system is shown in Fig.-2.1.

Where,

$'V_i'$ - Voltages at the $'i^{th}'$ bus

$'I_i'$ - Currents injected at the $'i^{th}'$ bus

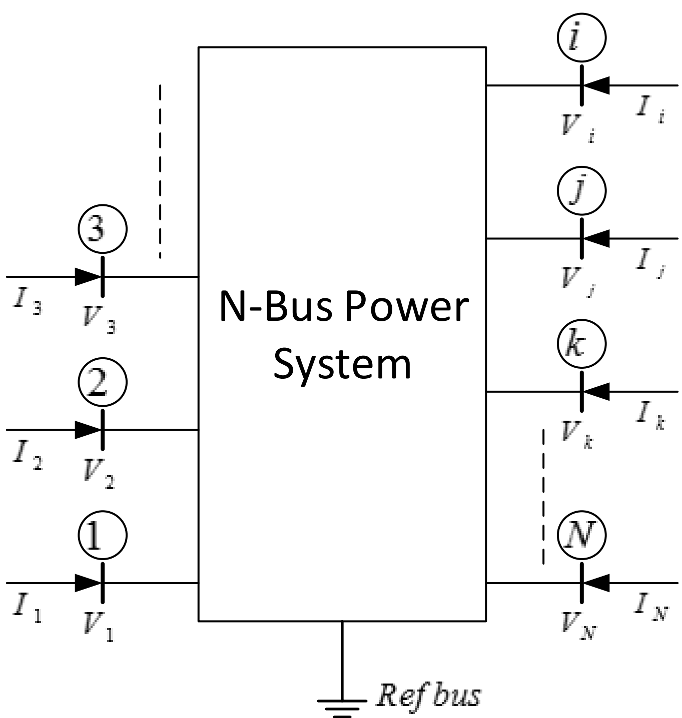


FIG.-2.1 BLOCK DIAGRAM OF N-BUS POWER SYSTEM

The original bus impedance matrix Z_{BUS} can be given by (2.1).

$$Z_{BUS} = \begin{bmatrix} Z_{11} & Z_{12} & \cdots\cdots & Z_{1N} \\ Z_{21} & Z_{22} & \cdots\cdots & Z_{2N} \\ \vdots & \vdots & \vdots & \vdots \\ Z_{N1} & Z_{N2} & \cdots\cdots & Z_{NN} \end{bmatrix} \tag{2.1}$$

The system voltage can be given by (2.2)

$$V_{BUS} = Z_{BUS} I_{BUS} \tag{2.2}$$

Where

$$V_{BUS} = \begin{bmatrix} V_1 \\ V_2 \\ \vdots \\ V_N \end{bmatrix} \tag{2.3}$$

$$I_{BUS} = \begin{bmatrix} I_1 \\ I_2 \\ \vdots \\ I_N \end{bmatrix} \tag{2.4}$$

Therefore the system can be described by (2.5)

$$\begin{bmatrix} V_1 \\ V_2 \\ \vdots \\ V_N \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} & \cdots\cdots & Z_{1N} \\ Z_{21} & Z_{22} & \cdots\cdots & Z_{2N} \\ \vdots & \vdots & \vdots & \vdots \\ Z_{N1} & Z_{N2} & \cdots\cdots & Z_{NN} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ \vdots \\ I_N \end{bmatrix} \tag{2.5}$$

It means

$$V_i = I_1 Z_{i1} + I_2 Z_{i2} + \cdots\cdots + I_i Z_{ii} + I_j Z_{ij} + \cdots\cdots + I_N Z_{iN} \tag{2.6}$$

For all $i = 1,2,3,\cdots\cdots\cdots, N$

CASE-1 ADDITION OF A NEW BUS $'p'$ TO REFERENCE BUS THROUGH LINK Z_b

The system is modified by addition of a new bus $'p'$ to a reference bus through link Z_b as shown in Fig.-2.2.

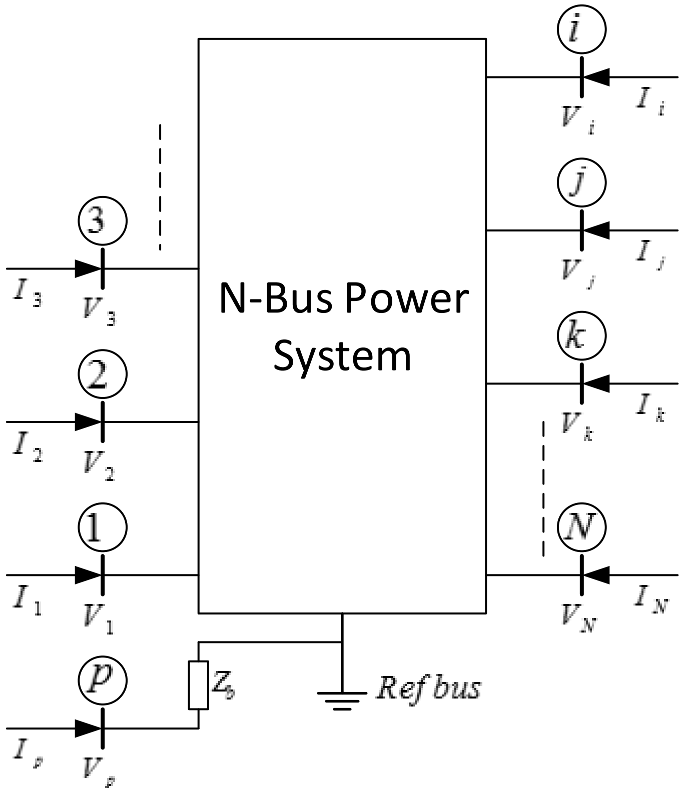


FIG. 2.2 ADDITION OF NEW BUS $'p'$ TO REFERENCE BUS

The system voltage equation of (2.5) is modified as a new bus $'p'$ is added because the current injected at bus $'p'$ introduces the voltage drop I_pZ_b and the voltage of bus $'p'$, V_p equals this voltage drop. It implies the system is modified by the addition of a new row in (2.5) as follows.

$$\begin{bmatrix} V_1 \\ V_2 \\ \vdots \\ V_N \\ \cdots \\ V_p \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} & \cdots\cdots & Z_{1N} & \vdots & 0 \\ Z_{21} & Z_{22} & \cdots\cdots & Z_{2N} & \vdots & 0 \\ \vdots & \vdots & \cdots\cdots & \vdots & \vdots & \vdots \\ Z_{N1} & Z_{N2} & \cdots\cdots & Z_{NN} & \vdots & 0 \\ \cdots\cdots & \cdots\cdots & \cdots\cdots & \cdots\cdots & \cdots\cdots & \cdots\cdots \\ 0 & 0 & \cdots\cdots & 0 & \vdots & Z_b \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ \vdots \\ I_N \\ \cdots \\ I_p \end{bmatrix} \tag{2.7}$$

$$Z_{BUS,New} = \begin{bmatrix} Z_{11} & Z_{12} & \cdots\cdots & Z_{1N} & \vdots & 0 \\ Z_{21} & Z_{22} & \cdots\cdots & Z_{2N} & \vdots & 0 \\ \vdots & \vdots & \cdots\cdots & \vdots & \vdots & \vdots \\ Z_{N1} & Z_{N2} & \cdots\cdots & Z_{NN} & \vdots & 0 \\ \cdots\cdots & \cdots\cdots & \cdots\cdots & \cdots\cdots & \cdots\cdots & \cdots\cdots \\ 0 & 0 & \cdots\cdots & 0 & \vdots & Z_b \end{bmatrix} \tag{2.8}$$

$$Z_{BUS,New} = \begin{bmatrix} & & & & \vdots & 0 \\ & & & & \vdots & 0 \\ & & Z_{BUS,Original} & & \vdots & \vdots \\ & & & & \vdots & 0 \\ \cdots\cdots & \cdots\cdots & \cdots\cdots & \cdots\cdots & \cdots\cdots & \cdots\cdots \\ 0 & 0 & \cdots\cdots & 0 & \vdots & Z_b \end{bmatrix} \qquad (2.9)$$

CASE-2 ADDITION OF A NEW BUS $'p'$ TO AN EXISTING BUS $'k'$ THROUGH Z_b

The bus impedance matrix can be modified with the addition of a new bus to an existing bus through a link as shown in Fig.-2.3.

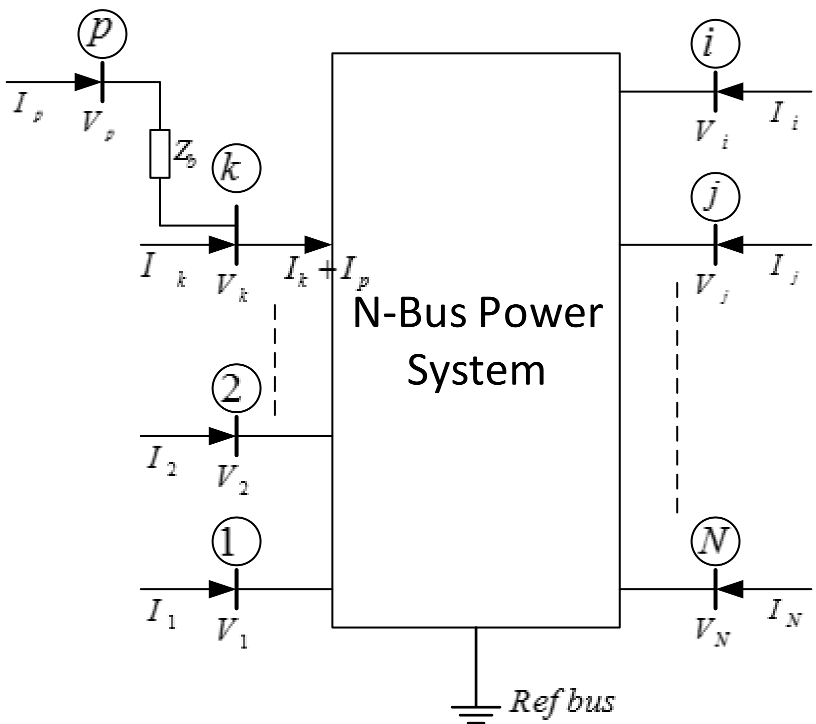


FIG. 2.3 ADDITION OF NEW BUS $'p'$ TO AN EXISTING BUS $'k'$

With the addition of a link, the voltage at the $k^{th}-bus$ is updated by the voltage drop of $I_p Z_{pp}$ due to current injection of $'i_p'$ at the $p^{th}-bus$ and given by

$$V_{k,new} = V_{k,original} + I_p Z_{kk} \qquad (2.10)$$

Where

$$V_{k,original} = I_1 Z_{k1} + I_2 Z_{k2} + \cdots\cdots + I_k Z_{kk} + \cdots\cdots + I_N Z_{kN} \qquad (2.11)$$

The voltage at the $p^{th}-bus$ thus is given by

$$V_p = V_k + I_p Z_b = V_{k,original} + I_p Z_{kk} + I_p Z_b \tag{2.12}$$

$$V_p = \underbrace{I_1 Z_{k1} + I_2 Z_{k2} + \cdots\cdots + I_N Z_{kN}}_{V_{k,original}} + I_p \left(Z_{kk} + Z_b\right) \tag{2.13}$$

Thus the system voltage as mentioned in (2.5) can be modified as per (2.14) and so is the bus impedance matrix (2.15).

$$\begin{bmatrix} V_1 \\ V_2 \\ \vdots \\ V_N \\ \cdots \\ V_p \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} & \cdots\cdots & Z_{1N} & \vdots & Z_{1k} \\ Z_{21} & Z_{22} & \cdots\cdots & Z_{2N} & \vdots & Z_{2k} \\ \vdots & \vdots & \cdots\cdots & \vdots & \vdots & \vdots \\ Z_{N1} & Z_{N2} & \cdots\cdots & Z_{NN} & \vdots & Z_{Nk} \\ \cdots\cdots & \cdots\cdots & \cdots\cdots & \cdots\cdots & \cdots\cdots & \cdots\cdots \\ Z_{k1} & Z_{k2} & \cdots\cdots & Z_{kN} & \vdots & Z_{kk} + Z_b \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ \vdots \\ I_N \\ \cdots \\ I_p \end{bmatrix} \tag{2.14}$$

$$Z_{BUS,New} = \begin{bmatrix} Z_{11} & Z_{12} & \cdots\cdots & Z_{1N} & \vdots & Z_{1k} \\ Z_{21} & Z_{22} & \cdots\cdots & Z_{2N} & \vdots & Z_{2k} \\ \vdots & \vdots & \cdots\cdots & \vdots & \vdots & \vdots \\ Z_{N1} & Z_{N2} & \cdots\cdots & Z_{NN} & \vdots & Z_{Nk} \\ \cdots\cdots & \cdots\cdots & \cdots\cdots & \cdots\cdots & \cdots\cdots & \cdots\cdots \\ Z_{k1} & Z_{k2} & \cdots\cdots & Z_{kN} & \vdots & Z_{kk} + Z_b \end{bmatrix} \tag{2.15}$$

$$Z_{BUS,New} = \begin{bmatrix} & & & & \vdots & Z_{1k} \\ & & & & \vdots & Z_{2k} \\ & & Z_{BUS,Original} & & \vdots & \vdots \\ & & & & \vdots & Z_{Nk} \\ \cdots\cdots & \cdots\cdots & \cdots\cdots & \cdots\cdots & \cdots\cdots & \cdots\cdots \\ Z_{k1} & Z_{k2} & \cdots\cdots & Z_{kN} & \vdots & Z_{kk} + Z_b \end{bmatrix} \tag{2.16}$$

ADDITION OF NEW LINK HAVING IMPEDANCE Z_b BETWEEN THE EXISTING BUS $'k'$ AND REFERENCE BUS

The addition of new link having impedance Z_b between the existing bus $'k'$ and reference bus can be achieved as Fig.-2.4.

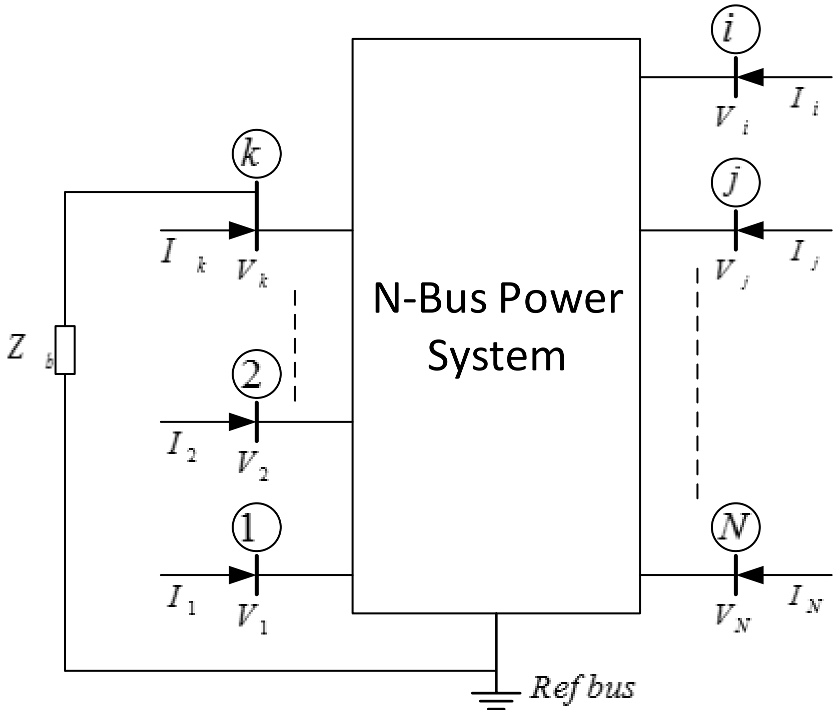


FIG.-2.4 ADDITION OF NEW LINK BETWEEN THE EXISTING BUS $'k'$ AND REFERENCE BUS

The addition of new link having impedance Z_b between the existing bus $'k'$ and reference bus can be regarded as the addition of a new bus $'p'$ to an existing bus $'k'$ through Z_b. Then new bus $'p'$ is shorted with the reference bus. That means the voltage equations shall be updated by as per (2.14) then V_p equals to zero as follows.

$$\begin{bmatrix} V_1 \\ V_2 \\ \vdots \\ V_N \\ \cdots \\ 0 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} & \cdots\cdots & Z_{1N} & \vdots & Z_{1k} \\ Z_{21} & Z_{22} & \cdots\cdots & Z_{2N} & \vdots & Z_{2k} \\ \vdots & \vdots & \cdots\cdots & \vdots & \vdots & \vdots \\ Z_{N1} & Z_{N2} & \cdots\cdots & Z_{NN} & \vdots & Z_{Nk} \\ \cdots\cdots & \cdots\cdots & \cdots\cdots & \cdots\cdots & \cdots\cdots & \cdots\cdots \\ Z_{k1} & Z_{k2} & \cdots\cdots & Z_{kN} & \vdots & Z_{kk}+Z_b \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ \vdots \\ I_N \\ \cdots \\ I_p \end{bmatrix} \tag{2.17}$$

$$\begin{bmatrix} V_1 \\ V_2 \\ \vdots \\ V_N \\ \cdots \\ 0 \end{bmatrix} = \begin{bmatrix} & & & & \vdots & Z_{1k} \\ & & & & \vdots & Z_{2k} \\ & & Z_{Bus,Original} & & \vdots & \vdots \\ & & & & \vdots & Z_{Nk} \\ \cdots\cdots & \cdots\cdots & \cdots\cdots & \cdots\cdots & \cdots\cdots & \cdots\cdots \\ Z_{k1} & Z_{k2} & \cdots\cdots & Z_{kN} & \vdots & Z_{kk}+Z_b \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ \vdots \\ I_N \\ \cdots \\ I_p \end{bmatrix} \tag{2.18}$$

The new bus impedance matrix can be calculated after reducing the Z_{BUS} in (2.18) by eliminating the last row and last column using Kron's reduction.

$$Z_{hi,new} = Z_{hi,old} - \frac{Z_{h(n+1)} Z_{(n+1)i}}{Z_{kk} + Z_b} \tag{2.19}$$

The resulting $Z_{hi,new}$ shall be new bus impedance matrix $Z_{BUS,new}$.

ADDITION OF NEW LINK HAVING IMPEDANCE Z_b BETWEEN TWO EXISTING BUSES $'j'$ AND $'k'$

A new link having impedance Z_b can be added between two existing buses $'j'$ and $'k'$ as shown in Fig.-2.5

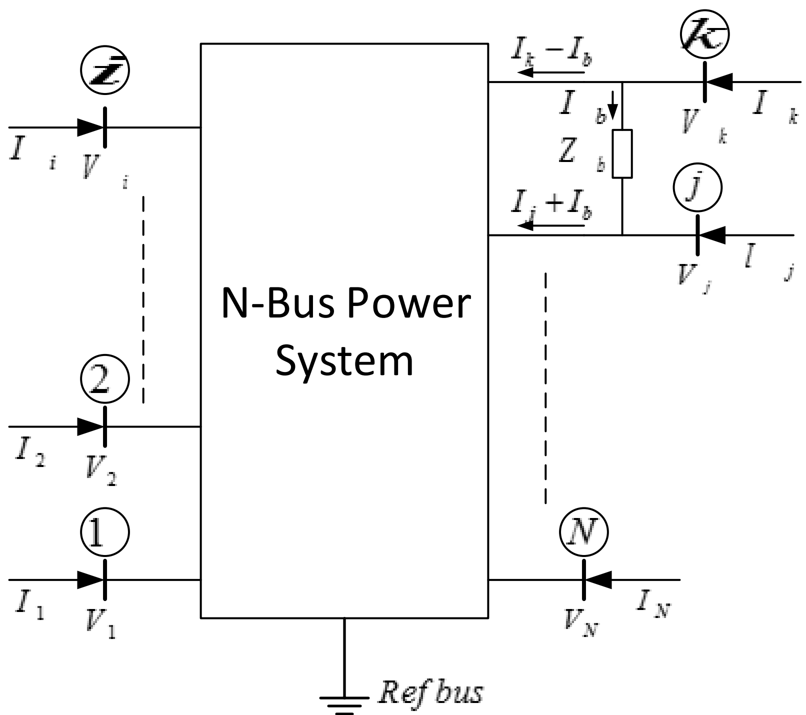


FIG.-2.5 ADDITION OF NEW LINK BETWEEN TWO EXISTING BUSES

By adding a link between $j^{th} - bus$ and $k^{th} - bus$ the current injected at the $j^{th} - bus$ and $k^{th} - bus$ are modified to $I_j + I_b$ and $I_k - I_b$ where I_b is current flowing through the link Z_b.

Therefore the voltage at the bus-1 can be updated as follows.

$$V_1 = I_1 Z_{11} + I_2 Z_{12} + \cdots\cdots + (I_j + I_b) Z_{1j} + (I_k - I_b) Z_{1k} + \cdots\cdots + I_N Z_{1N} \tag{2.20}$$

$$\Rightarrow V_1 = I_1 Z_{11} + I_2 Z_{12} + \cdots\cdots + I_j Z_{1j} + I_k Z_{1k} + \cdots\cdots + I_N Z_{1N} + I_b (Z_{1j} - Z_{1k}) \tag{2.21}$$

$$\Rightarrow V_1 = \underbrace{I_1 Z_{11} + I_2 Z_{12} + \cdots\cdots + I_j Z_{1j} + I_k Z_{1k} + \cdots\cdots + I_N Z_{1N}}_{V_1^0} + \underbrace{I_b\left(Z_{1j} - Z_{1k}\right)}_{\Delta V_1} \tag{2.22}$$

$$\Rightarrow V_1 = V_1^0 + \Delta V_1 \tag{2.23}$$

Similarly

$$V_j = V_j^0 + \Delta V_j$$

$$V_k = V_k^0 + \Delta V_k$$

$$V_j = I_1 Z_{j1} + I_2 Z_{j2} + \cdots\cdots + I_j Z_{jj} + I_k Z_{jk} + \cdots\cdots + I_N Z_{jN} + I_b\left(Z_{jj} - Z_{jk}\right) \tag{2.24}$$

$$V_k = I_1 Z_{k1} + I_2 Z_{k2} + \cdots\cdots + I_j Z_{kj} + I_k Z_{kk} + \cdots\cdots + I_N Z_{kN} + I_b\left(Z_{kj} - Z_{kk}\right) \tag{2.25}$$

Subtracting (2.25) from (2.24)

$$\begin{aligned} V_j - V_k = {} & \left(Z_{j1} - Z_{k1}\right)I_1 + \left(Z_{j2} - Z_{k2}\right)I_2 + \cdots\cdots + \left(Z_{jj} - Z_{kj}\right)I_j + \left(Z_{kj} - Z_{kk}\right)I_k \\ & + \cdots\cdots + \left(Z_{jN} - Z_{kN}\right)I_N + \left(Z_{jj} - 2Z_{jk} + Z_{kk}\right)I_b \end{aligned} \tag{2.26}$$

But the addition of link Z_b has resulted in the network equation (2.26)

$$V_k - V_j = I_b Z_b \tag{2.27}$$

$$\Rightarrow I_b Z_b + V_j - V_k = 0 \tag{2.28}$$

$$\begin{aligned} I_b Z_b + {} & \left(Z_{j1} - Z_{k1}\right)I_1 + \left(Z_{j2} - Z_{k2}\right)I_2 + \cdots\cdots + \left(Z_{jj} - Z_{kj}\right)I_j + \left(Z_{kj} - Z_{kk}\right)I_k \\ & + \cdots\cdots + \left(Z_{jN} - Z_{kN}\right)I_N + \left(Z_{jj} - 2Z_{jk} + Z_{kk}\right)I_b = 0 \end{aligned} \tag{2.29}$$

$$\begin{aligned} & \left(Z_{j1} - Z_{k1}\right)I_1 + \left(Z_{j2} - Z_{k2}\right)I_2 + \cdots\cdots + \left(Z_{jj} - Z_{kj}\right)I_j + \left(Z_{kj} - Z_{kk}\right)I_k + \cdots\cdots \\ & + \left(Z_{jN} - Z_{kN}\right)I_N + \left(Z_{jj} - 2Z_{jk} + Z_{kk} + Z_b\right)I_b = 0 \end{aligned} \tag{2.30}$$

(2.30) suggests addition of new row and new column in (2.5) as follows.

$$\begin{bmatrix} V_1 \\ V_2 \\ \vdots \\ V_N \\ \cdots \\ 0 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} & \cdots\cdots & Z_{1N} & \vdots & \left(Z_{1j} - Z_{1k}\right) \\ Z_{21} & Z_{22} & \cdots\cdots & Z_{2N} & \vdots & \left(Z_{2j} - Z_{2k}\right) \\ \vdots & \vdots & \cdots\cdots & \vdots & \vdots & \vdots \\ Z_{N1} & Z_{N2} & \cdots\cdots & Z_{NN} & \vdots & \left(Z_{Nj} - Z_{Nk}\right) \\ \cdots\cdots & \cdots\cdots & \cdots\cdots & \cdots\cdots & \cdots\cdots & \cdots\cdots \\ \left(Z_{j1} - Z_{k1}\right) & \left(Z_{j2} - Z_{k2}\right) & \cdots\cdots & \left(Z_{jN} - Z_{kN}\right) & \vdots & \left(Z_{jj} - 2Z_{jk} + Z_{kk} + Z_b\right) \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ \vdots \\ I_N \\ \cdots \\ I_b \end{bmatrix} \tag{2.31}$$

$$\begin{bmatrix} V_1 \\ V_2 \\ \vdots \\ V_N \\ \cdots \\ 0 \end{bmatrix} = \begin{bmatrix} & & & & \vdots & (Z_{1j} - Z_{1k}) \\ & & & & \vdots & (Z_{2j} - Z_{2k}) \\ & Z_{BUS,Original} & & & \vdots & \vdots \\ & & & & \vdots & (Z_{Nj} - Z_{Nk}) \\ \cdots\cdots & \cdots\cdots & \cdots\cdots & \cdots\cdots & \cdots\cdots & \cdots\cdots \\ (Z_{j1} - Z_{k1}) & (Z_{j2} - Z_{k2}) & \cdots\cdots & (Z_{jN} - Z_{kN}) & \vdots & (Z_{jj} - 2Z_{jk} + Z_{kk} + Z_b) \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ \vdots \\ I_N \\ \cdots \\ I_b \end{bmatrix} \quad (2.32)$$

The new bus impedance matrix can be calculated after reducing the Z_{BUS} in (2.32) by eliminating the last row and last column using Kron's reduction.

$$Z_{hi,new} = Z_{hi,old} - \frac{Z_{h(n+1)} Z_{(n+1)i}}{(Z_{jj} - 2Z_{jk} + Z_{kk} + Z_b)} \quad (2.33)$$

The resulting $Z_{hi,new}$ shall be new bus impedance matrix $Z_{BUS,new}$.

As described above the bus impedance matrix for a power system can be formed by using above algorithm by consideration of adding either one bus or one link at a time.

Advantages

- It can be used for fault analysis.
- For any single modification in case of any single outage of a link or a node, new bus impedance matrix can be obtained with less computation time.
- Removal of a branch can be considered by adding negative of link impedance Z_b between the same node.

BUS ADMITTANCE MATRIX

The system voltage for a $'N-bus'$ power system shown in Fig.-2.1 is given in (2.2)

$$V_{BUS} = Z_{BUS} I_{BUS} \tag{2.34}$$

It can be rewritten as

$$I_{BUS} = Y_{BUS} V_{BUS} \tag{2.35}$$

Where Y_{BUS} is the bus admittance matrix and given by (2.36).

$$Y_{BUS} = \begin{bmatrix} Y_{11} & Y_{12} & \cdots\cdots & Y_{1N} \\ Y_{21} & Y_{22} & \cdots\cdots & Y_{2N} \\ \vdots & \vdots & \vdots & \vdots \\ Y_{N1} & Y_{N2} & \cdots\cdots & Y_{NN} \end{bmatrix} \tag{2.36}$$

The bus admittance matrix Y_{BUS} for a $'N-bus'$ power system shown in Fig.-2.1 can be determined by inverting bus impedance matrix Z_{BUS} or the vice versa as in (2.37)

$$Y_{BUS} = [Z_{BUS}]^{-1} \tag{2.37}$$

$$Z_{BUS} = [Y_{BUS}]^{-1} \tag{2.38}$$

The equation (2.35) can be expanded as follow (say for $i^{th}-bus$)

$$I_i = V_1 Y_{i1} + V_2 Y_{i2} + \cdots\cdots + V_i Y_{ii} + V_j Y_{ij} + \cdots\cdots + V_N Y_{iN} \tag{2.39}$$

For all $i = 1,2,3,\cdots\cdots\cdots, N$

(2.39) can be written as

$$I_i = \sum_{i=1}^{N} Y_{ii} V_i \tag{2.40}$$

The diagonal element of the Y_{BUS} such as Y_{ii} is the self-admittance of $i^{th}-bus$. It can be determined by adding all the admittances connected at $i^{th}-bus$.

The off-diagonal element of the Y_{BUS} such as Y_{ij} is the transfer admittance between the $i^{th}-bus$ and $j^{th}-bus$. It can be determined by adding all the admittances connected between the $i^{th}-bus$ and $j^{th}-bus$.

CHAPTER-3

LOAD FLOW ANALYSIS

POWER FLOW

A $'N-bus'$ power system is shown in Fig.-3.1. The power flow at the $i^{th}-bus$ can be given by

$$S_i = V_i I_i^* = P_i + jQ_i \qquad (3.1)$$

For all $i = 1,2,3,\cdots\cdots\cdots, N$

The complex conjugate of the complex power can be written as

$$S_i^* = V_i^* I_i = P_i - jQ_i \qquad (3.2)$$

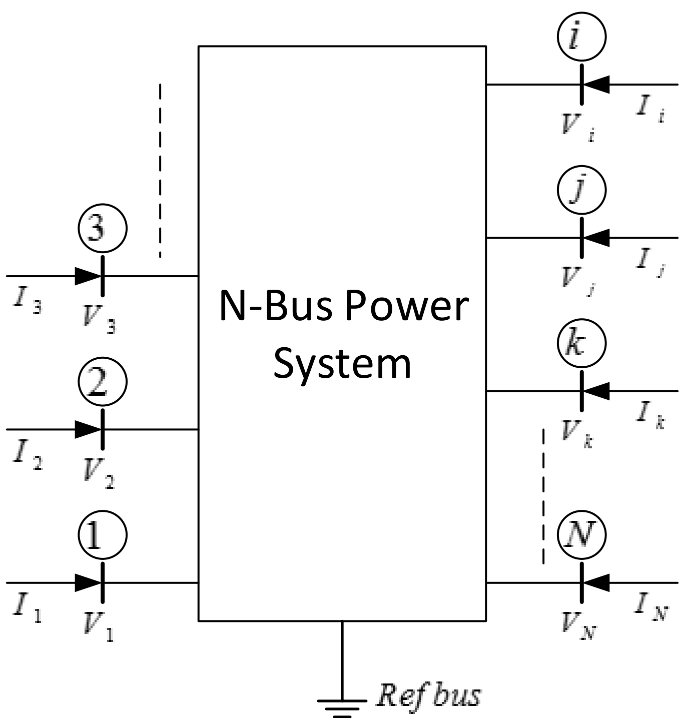


FIG.-3.1 BLOCK DIAGRAM OF N-BUS POWER SYSTEM

In a power system having $'N'$ number of buses the current at the i^{th}-bus can be determined as

$$I_i = \sum_{j=1}^{N} Y_{ij} V_j \tag{3.3}$$

Hence the complex power can be written as

$$S_i^* = V_i^* I_i = P_i - jQ_i = V_i \sum_{j=1}^{N} Y_{ij} V_j \tag{3.4}$$

Since the voltage at the $i^{th} - bus$ can be given by its magnitude and phase angle $V_i = |V_i| \angle \delta_i$ the above equation can be modified as

$$S_i^* = V_i^* I_i = P_i - jQ_i = V_i^* \sum_{j=1}^{N} Y_{ij} V_j = |V_i| \sum_{j=1}^{N} |Y_{ij}| |V_j| \angle(\theta_{ij} + \delta_j - \delta_i) \tag{3.5}$$

Therefore the real and reactive power at the $i^{th} - bus$ can be given by

$$P_i = \sum_{j=1}^{N} |V_i| |Y_{ij}| |V_j| \cos(\theta_{ij} + \delta_j - \delta_i) \tag{3.6}$$

$$Q_i = -\sum_{j=1}^{N} |V_i| |Y_{ij}| |V_j| \sin(\theta_{ij} + \delta_j - \delta_i) \tag{3.7}$$

The real and reactive power at the $i^{th} - bus$ can be given by (3.8) and (3.9) respectively.

$$P_i = P_{Gi} - P_{Di} \tag{3.8}$$

$$Q_i = Q_{Gi} - Q_{Di} \tag{3.9}$$

Where

P_{Gi}, P_{Di} - Real power generation and demand at the $i^{th} - bus$ (Fig.-3.2)

Q_{Gi}, Q_{Di} - Reactive power generation and demand at the $i^{th} - bus$ (Fig.-3.3)

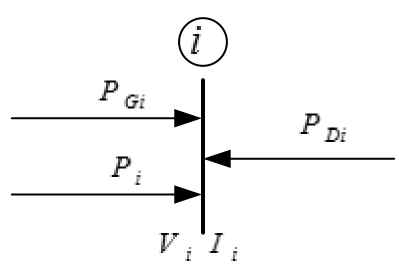


FIG.-3.2 REAL POWER AT THE $i^{th} - bus$

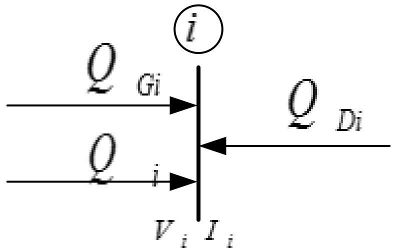


FIG.-3.3 REACTIVE POWER AT THE $i^{th} - bus$

TYPES OF BUSES

In power system the buses can be classified as

Slack bus: At this bus $|V|$ and δ are defined.

Load bus: At this bus P and Q are defined. $(PQ \cdots bus)$

Voltage controlled bus: At this bus P and $|V|$ are defined $(PV \cdots bus)$

GAUSS SIEDEL METHOD FOR POWER FLOW

For all $(PQ \cdots bus)$ from (3.5) we can calculate the voltage at the $i^{th} - bus$ as follows

$$V_i = \frac{1}{Y_{ii}}\left[\frac{P_{i(sch)} - jQ_{i(sch)}}{V_i^*} - \sum_{j=1}^{i-1} Y_{ij}V_j - \sum_{j=i+1}^{N} Y_{ij}V_j\right] \tag{3.10}$$

For all $i = 2,3,\cdots\cdots\cdots, N$

If we are calculating the voltage of $i^{th} - bus$ using (3.10), we have already the calculated values of the previous bus i.e. $(i-1)^{th} - bus$. Hence (3.10) can be modified as follows for $k^{th} - iteration$.

$$V_i^{(k)} = \frac{1}{Y_{ii}}\left[\frac{P_{i(sch)} - jQ_{i(sch)}}{V_i^{(k-1)*}} - \sum_{j=1}^{i-1} Y_{ij}V_j^{(k)} - \sum_{j=i+1}^{N} Y_{ij}V_j^{(k-1)}\right] \tag{3.11}$$

$$\Delta V_i^{(k)} = V_i^{(k)} - V_i^{(k-1)} \tag{3.12}$$

To accelerate convergence, an acceleration factor is proposed to update the voltages calculated in (3.12) as below.

$$V_{i(acc)}^{(k)} = (1-\alpha)V_i^{(k-1)} + \alpha V_i^{(k)} = V_i^{(k-1)} + \alpha(V_i^{(k)} - V_i^{(k-1)}) \tag{3.13}$$

$$\Delta V_{i(acc)}^{(k)} = V_{i(acc)}^{(k)} - V_{i(acc)}^{(k-1)} \tag{3.14}$$

The value of acceleration factor is usually between 1.1 to 1.4. The value of 1.2 is found to be the most perfect.

For all $(PV \cdots bus)$ the voltage magnitude is already specified. Hence at this bus the value of reactive power injected is calculated by(3.16) and checked for limit violence by (3.17).

$$Q_i = -im\left\{V_i^* \sum_{j=1}^{N} Y_{ij} - V_j\right\} \tag{3.15}$$

For $k^{th} - iteration$

$$Q_i^{(k)} = -im\left\{V_i^{(k-1)*}\left[\sum_{j=1}^{i-1} Y_{ij}V_j^{(k)} + \sum_{j=i}^{N} Y_{ij}V_j^{(k-1)}\right]\right\} \tag{3.16}$$

$$Q_{i,\min} \le Q_i^{(k)} \le Q_{i,\max} \tag{3.17}$$

If the limit is violated then the value of Q_i at this bus is set to its lower limit or upper limit as the case may be and the bus is treated as the load bus i.e. $(PQ \cdots bus)$. If the limit is not violated then the revised value of δ_i is calculated using (3.18)

$$\delta_i^{(k)} = \angle V_i^{(k)} \tag{3.18}$$

Set the value of voltage as in (3.19)

$$V_i^{(k)} = \left|V_{i,specified}\right| \angle \delta_i^{(k)} \tag{3.19}$$

After calculation of the voltages of all the buses both $(PQ \cdots bus)$ and $(PV \cdots bus)$, then the difference in the magnitude of voltages (calculated in this iteration) and the magnitude of voltages (calculated in the previous iteration) is calculated using (3.20).

$$\Delta V_i^{(k)} = V_i^{(k)} - V_i^{(k-1)} \tag{3.20}$$

Check $\Delta V_i^{(k)} \le \varepsilon$, (for all $i = 2,3,\cdots\cdots\cdots,N$) (3.21)

$'\varepsilon'$ being a small value such as 0.0001

If (3.21) is satisfied then further iteration for calculation of voltages are stopped and the slack bus power and line flows are calculated using (3.22) and (3.23) or else the next iteration of calculation shall be started.

The slack bus power i.e. for $i = 1$ is given by

$$S_1 = V_1\, I_1^* = P_1 + jQ_1 = V_1 \sum_{j=1}^{N} Y_{1j} V_j^* = \left|V_1\right| \sum_{j=1}^{N} \left|Y_{1j}\right| \left|V_j\right| \angle(\theta_{1j} - \delta_j + \delta_1) \tag{3.22}$$

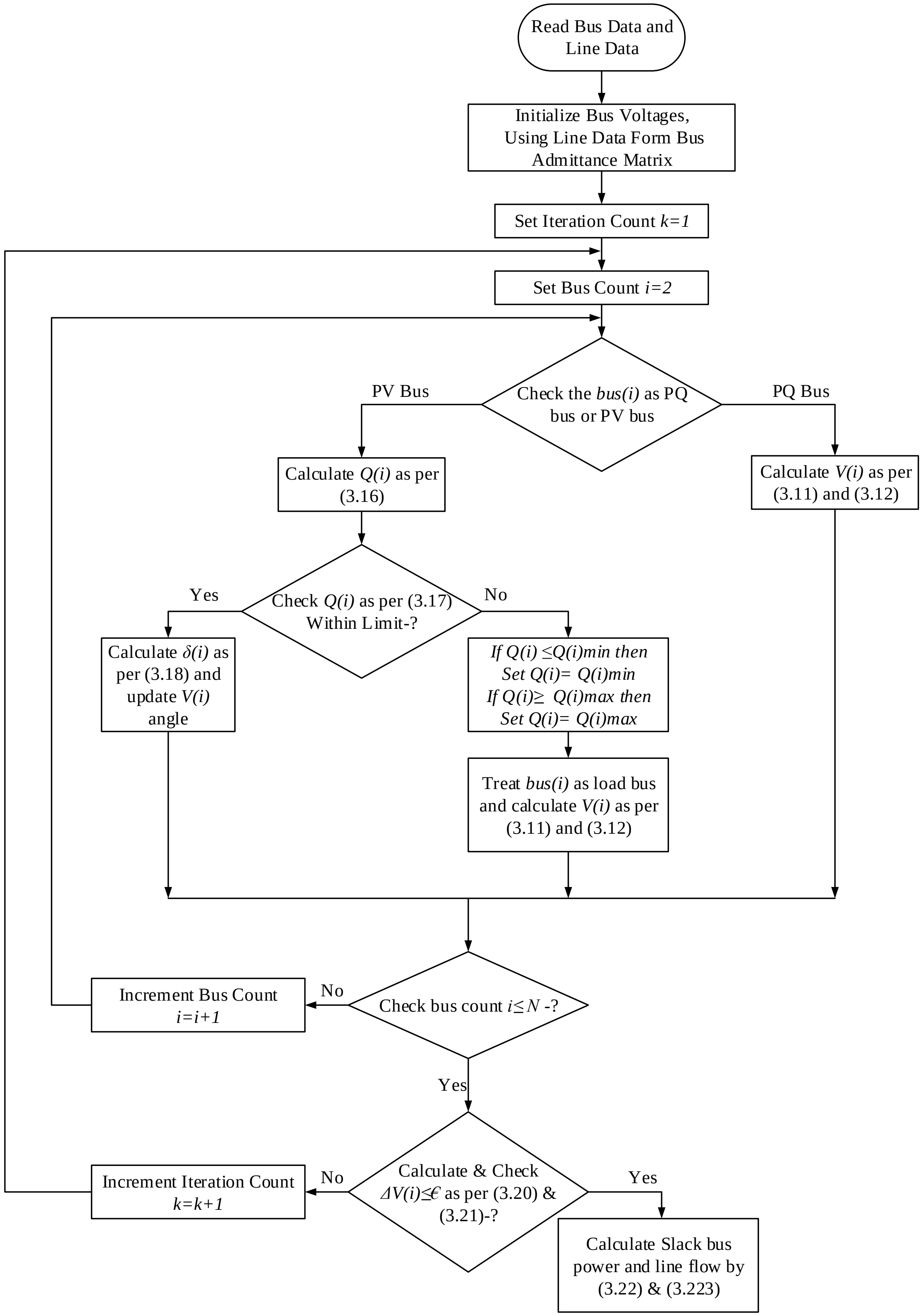


FIG.-3.4 FLOW CHART FOR LOAD FLOW USING GAUSS SEIDEL METHOD

The line power flow between $i^{th}-bus$ and $j^{th}-bus$ can be given by (3.23)

$$S_{ij}=V_iV_jY_{ij}Sin(\delta_i-\delta_j) \tag{3.23}$$

The complete algorithm for load flow calculation has been given in flow chart shown in Fig.-3.4.

NEWTON RAPHSON METHOD

The set of equations (3.6) and (3.7) for real and reactive power at the $i^{th}-bus$ can be given rewritten as

$$P_i=|V_i|^2G_{ii}+\sum_{\substack{j=1\\j\neq i}}^{N}|V_i||Y_{ij}||V_j|Cos(\theta_{ij}+\delta_j-\delta_i) \tag{3.24}$$

$$Q_i=-|V_i|^2B_{ii}+\sum_{\substack{j=1\\j\neq i}}^{N}|V_i||Y_{ij}||V_j|Sin(\theta_{ij}+\delta_j-\delta_i) \tag{3.25}$$

(for all $i=1,2,3,\cdots\cdots\cdots,N$)

Where

$G_{ii}=|Y_{ii}|Cos(\theta_{ii})$ and $B_{ii}=|Y_{ii}|Sin(\theta_{ii})$

From the above equations real and reactive power are functions of voltage magnitude and its phase angle (3.26) and (3.27)

$$P_i=f(V,\delta) \tag{3.26}$$

$$Q_i=f(V,\delta) \tag{3.27}$$

Differentiating the above equations with respect to its variables

$$\Delta P_i=\sum_{j=1}^{N}\frac{\partial P_i}{\partial\delta_j}\Delta\delta_j+\sum_{j=1}^{N}\frac{\partial P_i}{\partial|V_j|}\Delta|V_j| \tag{3.28}$$

$$\Delta Q_i=\sum_{j=1}^{N}\frac{\partial Q_i}{\partial\delta_j}\Delta\delta_j+\sum_{j=1}^{N}\frac{\partial Q_i}{\partial|V_j|}\Delta|V_j| \tag{3.29}$$

(for all $i=1,2,3,\cdots\cdots\cdots,N$)

Since the value of (V,δ) are defined at the slack bus (3.28) and (3.29) shall be modified to (3.30) and (3.31) because the value of (V,δ) are fixed at the slack bus.

$$\Delta P_i = \sum_{j=1}^{N} \frac{\partial P_i}{\partial \delta_j} \Delta\delta_j + \sum_{j=1}^{N} \frac{\partial P_i}{\partial |V_j|} \Delta |V_j| \qquad (3.30)$$

$$\Delta Q_i = \sum_{j=1}^{N} \frac{\partial Q_i}{\partial \delta_j} \Delta\delta_j + \sum_{j=1}^{N} \frac{\partial Q_i}{\partial |V_j|} \Delta |V_j| \qquad (3.31)$$

(for all $i = 2,3,\cdots\cdots\cdots, N$)

Multiplying and dividing second term of the (3.30) and (3.31) by $\frac{|V_j|}{|V_j|}$

$$\Delta P_i = \sum_{j=1}^{N} \frac{\partial P_i}{\partial \delta_j} \Delta\delta_j + \sum_{j=1}^{N} \left(\frac{|V_j|}{|V_j|}\right) \frac{\partial P_i}{\partial |V_j|} \Delta |V_j| \qquad (3.32)$$

$$\Delta Q_i = \sum_{j=1}^{N} \frac{\partial Q_i}{\partial \delta_j} \Delta\delta_j + \sum_{j=1}^{N} \frac{|V_j|}{|V_j|} \frac{\partial Q_i}{\partial |V_j|} \Delta |V_j| \qquad (3.33)$$

(for all $i = 2,3,\cdots\cdots\cdots, N$)

Or

$$\Delta P_i = \sum_{j=1}^{N} \frac{\partial P_i}{\partial \delta_j} \Delta\delta_j + \sum_{j=1}^{N} |V_j| \frac{\partial P_i}{\partial |V_j|} \frac{\Delta |V_j|}{|V_j|} \qquad (3.34)$$

$$\Delta Q_i = \sum_{j=1}^{N} \frac{\partial Q_i}{\partial \delta_j} \Delta\delta_j + \sum_{j=1}^{N} |V_j| \frac{\partial P_i}{\partial |V_j|} \frac{\Delta |V_j|}{|V_j|} \qquad (3.35)$$

(for all $i = 2,3,\cdots\cdots\cdots, N$)

However for a given value of real and reactive power at the $i^{th} - bus$ we can calculate the deviation from the schedule value to calculated value

$$\Delta P_i = P_{i(sch)} - P_{i(calc)} \qquad (3.36)$$

$$\Delta Q_i = Q_{i(sch)} - Q_{i(calc)} \qquad (3.37)$$

(for all $i = 2,3,\cdots\cdots\cdots, N$) since the value of (P, Q) are not known at the slack bus. Where

$$P_{i(sch)} = P_{Gi} - P_{Di} \qquad (3.38)$$

$$Q_{i(sch)} = Q_{Gi} - Q_{Di} \qquad (3.39)$$

And using (3.24) and (3.25)

$$P_{i(calc)} = |V_i|^2 G_{ii} + \sum_{\substack{j=1 \\ j \neq i}}^{N} |V_i||Y_{ij}||V_j| Cos(\theta_{ij} + \delta_j - \delta_i) \tag{3.40}$$

$$Q_{i(calc)} = -|V_i|^2 B_{ii} + \sum_{\substack{j=1 \\ j \neq i}}^{N} |V_i||Y_{ij}||V_j| Sin(\theta_{ij} + \delta_j - \delta_i) \tag{3.41}$$

(for all $i = 2,3,\cdots\cdots\cdots, N$)

From equations (3.34)-(3.35) and (3.36)-(3.37) we can derive the following equation (3.42) which is main basis of this method.

$$\begin{bmatrix} \frac{\partial P_2}{\partial \delta_2} & \cdots\cdots & \frac{\partial P_2}{\partial \delta_N} & |V_2|\frac{\partial P_2}{\partial V_2} & \cdots\cdots & |V_N|\frac{\partial P_2}{\partial V_N} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \frac{\partial P_N}{\partial \delta_2} & \cdots\cdots & \frac{\partial P_N}{\partial \delta_N} & |V_2|\frac{\partial P_N}{\partial V_2} & \cdots\cdots & |V_N|\frac{\partial P_N}{\partial V_N} \\ \frac{\partial Q_2}{\partial \delta_2} & \cdots\cdots & \frac{\partial Q_2}{\partial \delta_N} & |V_2|\frac{\partial Q_2}{\partial V_2} & \cdots\cdots & |V_N|\frac{\partial Q_2}{\partial V_N} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \frac{\partial Q_N}{\partial \delta_2} & \cdots\cdots & \frac{\partial Q_N}{\partial \delta_N} & |V_2|\frac{\partial Q_N}{\partial V_2} & \cdots\cdots & |V_N|\frac{\partial Q_N}{\partial V_N} \end{bmatrix} \begin{bmatrix} \Delta\delta_2 \\ \vdots \\ \vdots \\ \Delta\delta_N \\ \frac{\Delta V_2}{|V_2|} \\ \vdots \\ \vdots \\ \frac{\Delta V_2}{|V_2|} \end{bmatrix} = \begin{bmatrix} \Delta P_2 \\ \vdots \\ \vdots \\ \Delta P_N \\ \Delta Q_2 \\ \vdots \\ \vdots \\ \Delta Q_N \end{bmatrix} \tag{3.42}$$

The first matrix of the above equation is known as Jacobian matrix $[J]$ of the power system. It has $(2N-2)X(2N-2)$ dimension if all the buses except slack bus are load buses. The Jacobian matrix $[J]$ of the power system shall have four sub-matrices as shown below (3.43)

$$[J] = \begin{bmatrix} J_{11} & J_{12} \\ J_{21} & J_{22} \end{bmatrix} \tag{3.43}$$

The elements of sub-matrices can be evaluated as follows.

Off-Diagonal elements of $[J_{11}]$ for $i \neq j$

$$J_{11} = \frac{\partial P_i}{\partial \delta_j} = -|V_i||Y_{ij}||V_j| Sin(\theta_{ij} + \delta_j - \delta_i) \tag{3.44}$$

Diagonal elements of $[J_{11}]$ for $i = j$

$$J_{11} = \frac{\partial P_i}{\partial \delta_i} = \sum_{\substack{j=1 \\ j \neq i}}^{N} |V_i||Y_{ij}||V_j| Sin(\theta_{ij} + \delta_j - \delta_i) = -Q_i - |V_i|^2 B_{ii} \qquad (3.45)$$

Off-Diagonal elements of $[J_{12}]$ for $i \neq j$

$$J_{12} = \frac{\partial Q_i}{\partial \delta_j} = -|V_i||Y_{ij}||V_j| Cos(\theta_{ij} + \delta_j - \delta_i) \qquad (3.46)$$

Diagonal elements of $[J_{12}]$ for $i = j$

$$J_{12} = \frac{\partial Q_i}{\partial \delta_i} = \sum_{\substack{j=1 \\ j \neq i}}^{N} |V_i||Y_{ij}||V_j| Cos(\theta_{ij} + \delta_j - \delta_i) = P_i - |V_i|^2 G_{ii} \qquad (3.47)$$

Off-Diagonal elements of $[J_{21}]$ for $i \neq j$

$$J_{21} = |V_j| \frac{\partial P_i}{\partial |V_j|} = |V_i||Y_{ij}||V_j| Cos(\theta_{ij} + \delta_j - \delta_i) = -\frac{\partial Q_i}{\partial \delta_j} \qquad (3.48)$$

Diagonal elements of $[J_{21}]$ for $i = j$

$$J_{21} = |V_i| \frac{\partial P_i}{\partial |V_i|} = |V_i| \left[2|V_i| G_{ii} + \sum_{\substack{j=1 \\ j \neq i}}^{N} |Y_{ij}||V_j| Cos(\theta_{ij} + \delta_j - \delta_i) \right] = P_i + |V_i|^2 G_{ii} \qquad (3.49)$$

Off-Diagonal elements of $[J_{22}]$ for $i \neq j$

$$J_{22} = |V_j| \frac{\partial Q_i}{\partial |V_j|} = -|V_i||Y_{ij}||V_j| Sin(\theta_{ij} + \delta_j - \delta_i) = \frac{\partial P_i}{\partial \delta_j} \qquad (3.50)$$

Diagonal elements of $[J_{22}]$ for $i = j$

$$J_{22} = |V_i| \frac{\partial Q_i}{\partial |V_i|} = -|V_i| \left[2|V_i| B_{ii} + \sum_{\substack{j=1 \\ j \neq i}}^{N} |Y_{ij}||V_j| Sin(\theta_{ij} + \delta_j - \delta_i) \right] = Q_i - |V_i|^2 B_{ii} \qquad (3.51)$$

The equation (3.42) can be written as

$$\begin{bmatrix} J_{11} & J_{12} \\ J_{21} & J_{22} \end{bmatrix} \begin{bmatrix} \Delta \delta_i \\ \dfrac{\Delta V_i}{|V_i|} \end{bmatrix} = \begin{bmatrix} \Delta P_i \\ \Delta Q_i \end{bmatrix} \qquad \text{(for all } i = 2,3,\cdots\cdots\cdots, N\text{)} \qquad (3.52)$$

(3.52) can be modified to (3.53)

$$\begin{bmatrix} \Delta\delta_i \\ \dfrac{\Delta V_i}{|V_i|} \end{bmatrix} = \begin{bmatrix} J_{11} & J_{12} \\ J_{21} & J_{22} \end{bmatrix}^{-1} \begin{bmatrix} \Delta P_i \\ \Delta Q_i \end{bmatrix} \text{ (for all } i = 2,3,\cdots\cdots\cdots, N\text{)} \tag{3.53}$$

(3.53) gives the amount by which the voltage magnitude and phase angle has to be updated. Hence the updated value of the voltage magnitude and phase angle becomes

$$\delta_i = \delta_i + \Delta\delta_i \qquad \text{(for all } i = 2,3,\cdots\cdots\cdots, N\text{)} \tag{3.54}$$

$$|V_i| = |V_i| + |\Delta V_i| \qquad \text{(for all } i = 2,3,\cdots\cdots\cdots, N\text{)} \tag{3.55}$$

Test for convergence

Check the corrections obtained in (3.53)

$$\begin{bmatrix} \Delta\delta_i \\ \dfrac{\Delta V_i}{|V_i|} \end{bmatrix} \le \varepsilon \tag{3.56}$$

'ε' being a small value such as 0.0001

If (3.56) is satisfied then further iteration for corrections of voltages are stopped and the slack bus power and line flows are calculated using (3.57) and (3.58) or else the next iteration of calculation shall be started.

The slack bus power i.e. for $i = 1$ is given by

$$S_1 = V_1 I_1^* = P_1 + jQ_1 = V_1 \sum_{j=1}^{N} Y_{1j} V_j^* = |V_1| \sum_{j=1}^{N} |Y_{1j}||V_j| \angle(\theta_{1j} - \delta_j + \delta_1) \tag{3.57}$$

The line power flow between $i^{th} - bus$ and $j^{th} - bus$ can be given by (3.23)

$$S_{ij} = V_i V_j Y_{ij} Sin(\delta_i - \delta_j) \tag{3.58}$$

The complete algorithm for load flow calculation has been given in flow chart shown in Fig.-3.5 only for load buses. This flow chart does not incorporate the voltage controlled bus.

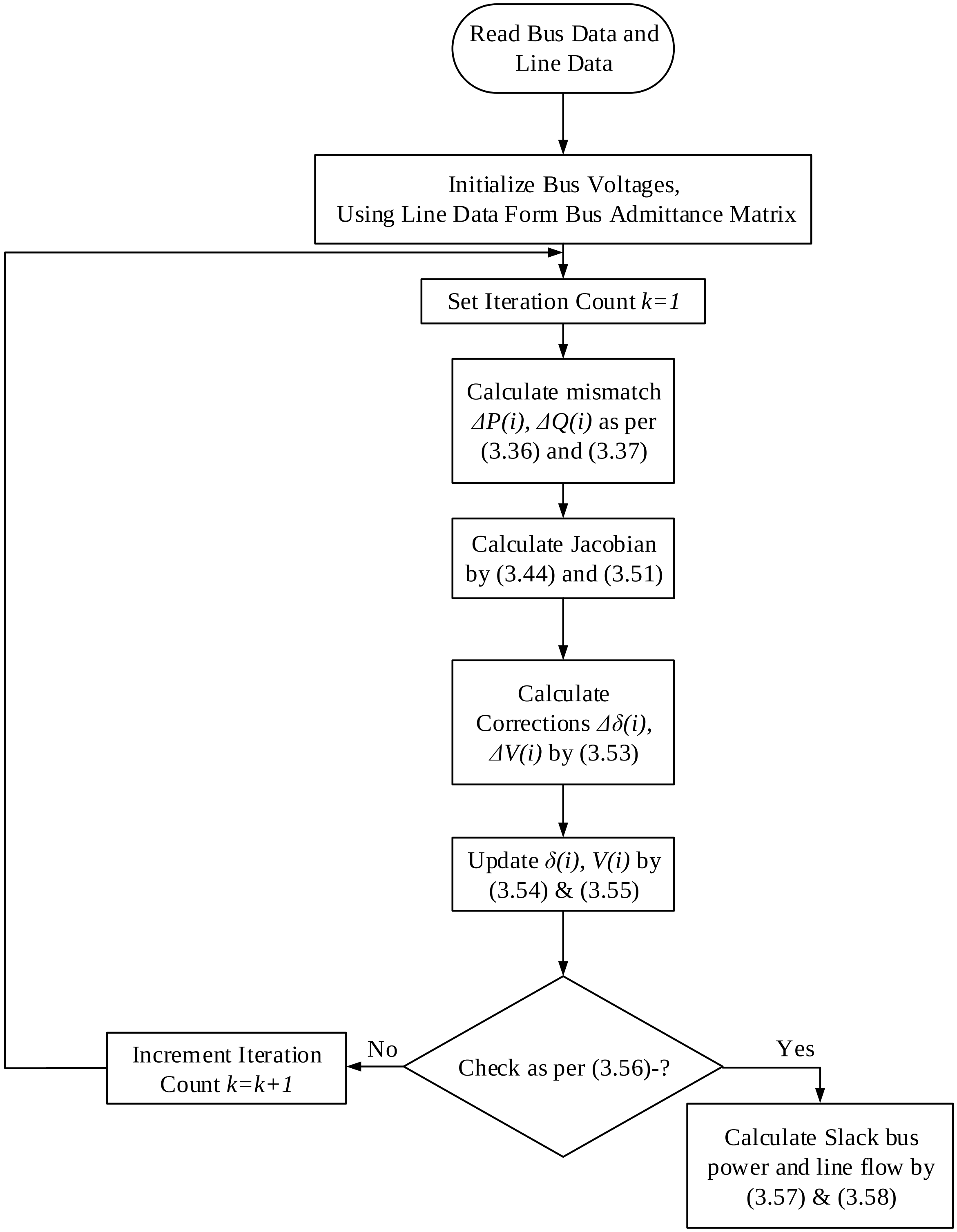


FIG.-3.4 FLOW CHART FOR LOAD FLOW USING NEWTO MN RAPHSON METHOD

(ONLY LOAD BUSES)

VOLTAGE CONTROLLED BUS $(PV \cdots bus)$

At voltage controlled bus of the system i.e. $(PV \cdots bus)$ the voltage magnitude is specified and reactive power is not specified. Hence the correction in the corresponding voltage magnitude shall be zero consequently the corresponding column of the Jacobian sub-matrix shall be omitted. Similarly as the reactive power is not specified at this bus the mismatch in reactive power can not be defined and so the corresponding rows of the Jacobian sub-matrix J_{21}, J_{22} shall be omitted. This shall reduce the dimension of the Jacobian matrix J to $(2N-2-1)X(2N-2-1)$. If there are $'N_G'$ number of voltage controlled bus in the system then the dimension of the Jacobian matrix J to $(2N-2-1)X(2N-2-1)$.

CHAPTER-4

POWER SYSTEM STABILITY

The following definitions provide the foundation for transient stability and small-signal stability analysis.

1) *Disturbance in a Power System.* A disturbance in a power system is a sudden change or a sequence of changes in one or more of the operating parameters of the system, or in one or more of the operating quantities.

2) *Small Disturbance in a Power System.* A small disturbance is disturbance for which the equations that describe the dynamics of the power system may be linearized for the purpose of analysis.

3) *Large Disturbance in a Power System.* A large disturbance is a disturbance for which the equations that describe the dynamics of the power system cannot be linearized for the purpose of analysis.

4) *Steady-State Stability of a Power System.* A power system is steady-state stable for a particular steady-state operating condition if, following any small disturbance, it reaches a steady-state operating condition which is identical or close to the pre-disturbance operating condition. This is also known as small disturbance stability of a power system.

5) *Transient Stability of a Power System.* A power system is transiently stable for a particular steady-state operating condition and for a particular disturbance if, following that disturbance, it reaches an acceptable steady-state operating condition.

The transient stability analysis involves more detailed nonlinear models, solution techniques, and includes steady-state stability analysis of the operating condition that will be reached following the transient. Traditionally, the stability problem has been associated with maintaining synchronous operation. In the evaluation of stability, the concern is the behavior of the power system when subjected to a disturbance. The disturbance may be small or large. This aspect of stability is influenced by the dynamics of the generator rotor angles and the power-angle relationships and is referred to as rotor angle stability.

ANGLE STABILITY

INTRODUCTION

In general, the components of the power system that influence the electrical and mechanical torques of the synchronous machines are listed below.

1) The transmission network before, during, and after the disturbance.

2) The loads and their characteristics.

3) The parameters of the synchronous machines.

4) The control components of the synchronous machines (excitation systems, power system stabilizers).

5) The mechanical turbine and the speed governor.

6) Other power plant components that influence the mechanical torque.

7) Other control devices, such as supplementary controls, special protection schemes, and FACTS (Flexible AC Transmission System) devices that are deemed necessary in the mathematical description of the system.

The power system stability is defined as the property of the system which enables the synchronous machines of the system to respond to a disturbance from a normal operating conditions so as to return to conditions where their operation is again normal.

Transient stability: It is defined as the property of the system which enables the synchronous machines of the system to respond to a major disturbances such as transmission line faults, sudden load changes, loss of generation or line switching, from a normal operating conditions so as to return to conditions where their operation is again normal. Or we can say, "*Transient stability* is the ability of the power system to maintain synchronism when subjected to large disturbances. The system equations for a transient stability study are usually nonlinear".

Dynamic Stability: It is defined as the property of the system which enables the synchronous machines of the system to respond to a disturbance from a normal operating conditions so as to return to conditions where their operation is again normal. Or we can say, "*Small-signal (small-disturbance) stability* is the ability of the power system to maintain synchronism under small disturbances. Instability can result in the form of 1) steady increase in rotor angle due to lack of sufficient synchronizing forces, or 2) rotor oscillations due to lack of sufficient damping forces. In today's power systems, small-signal stability is largely a problem of insufficient damping of oscillations".

FUNDAMENTAL ASSUMPTIONS

- Only system frequency currents and voltages are considered in the stator winding and the power system. Consequently dc offset currents are neglected.
- Symmetrical components are used in the representation of unbalanced faults.
- Generated voltage is considered unaffected by machine speed variation.

ROTOR DYNAMICS AND SWING EQUATION

Elementary principle in dynamics is accelerating torque is the product of moment of inertia of the rotor times its angular acceleration.

$$J\frac{d^2\theta_m}{dt^2} = T_a = T_m - T_e \tag{4.1}$$

J = Moment of Inertia (total) of rotor mass (Kg-m^2)

θ_m = Angular displacement of rotor with respect to a stationary axis on stator

T_m = Mechanical or shaft torque

T_e = Net electrical torque

T_a = Net acceleration torque

Under steady state operation $T_m = T_e \Rightarrow T_a = 0$ and resultant constant speed is synchronous speed.

T_e =Corresponds to net air gap power.

Since, rotor speed relative to synchronous speed is of interest, the rotor angle is measured with respect to a reference axis which rotates at synchronous speed therefore

$$\theta_m = \omega_{sm}t + \delta_m \tag{4.2}$$

ω_{sm} -Synchronous speed of machine in mechanical radian

δ_m -Angular displacement in mechanical radian from the synchronously rotating reference axis.

$$\frac{d\theta_m}{dt} = \omega_{sm} + \frac{d\delta_m}{dt} \tag{4.3}$$

$$\frac{d^2\theta_m}{dt^2} = \frac{d^2\delta_m}{dt^2} \qquad (4.4)$$

From equation (4.3)

$\frac{d\theta_m}{dt}$ is constant if $\frac{d\delta_m}{dt} = 0$

$\frac{d\delta_m}{dt}$ Represents the deviation of rotor speed from synchronism

(4.4) and (4.1) yields

$$J\frac{d^2\delta_m}{dt^2} = T_a \qquad (4.5)$$

To introduce $\omega_m = \frac{d\theta_m}{dt}$ for angular velocity of rotor

Power equals torque times angular velocity

$$J\omega_m \frac{d^2\delta_m}{dt^2} = P_a = P_m - P_e \qquad (4.6)$$

P_m Shaft power less rotational losses

$J\omega_m$ =Angular momentum of rotor at synchronous speed ω_{sm} called inertia constant of machine (Joules sec/mech. rad) = M

$$M\frac{d^2\delta_m}{dt^2} = P_a = P_m - P_e \qquad (4.7)$$

In machines

$H = \frac{\text{stored K.E in MJ at synchronous speed}}{\text{machine rating in MVA}}$ (MJ/MVA) is the unit of time in sec.

$$H = \frac{\frac{1}{2}J\omega_{sm}^2}{S_{mach}} = \frac{\frac{1}{2}M\omega_{sm}}{S_{mach}} \qquad (4.8)$$

$$M = \frac{2H}{\omega_{sm}} S_{mach} MJ/mech.rad.$$

$$\frac{2H}{\omega_{sm}}\frac{d^2\delta_m}{dt^2} = \frac{P_a}{S_{mach}} = \frac{P_m - P_e}{S_{mach}} \qquad (4.9)$$

$$\frac{2H}{\omega_s}\frac{d^2\delta}{dt^2} = P_a = P_m - P_e \quad p.u. \qquad (4.10)$$

Subscript $'m'$ stands for mechanical units or else it is electrical units.

$$\frac{H}{\pi f}\frac{d^2\delta}{dt^2} = P_a = P_m - P_e \quad (\delta \text{ - electrical radian}) \tag{4.11}$$

$$\frac{2H}{\omega_s}\frac{d^2\delta}{dt^2} = P_a = P_m - P_e \quad p.u. \tag{4.12}$$

It is the swing equation.

It is the fundamental equation which governs the rotational dynamics of the synchronous machine in stability studies.

A graph of the solution is called the swing curve of the machine which tells whether the machine is in synchronism after disturbance.

This equation (4.12) can also be written as two first order differential equation

$$\frac{2H}{\omega_s}\frac{d\omega}{dt} = P_a = P_m - P_e \quad p.u. \tag{4.13}$$

$$\frac{d\delta}{dt} = \omega - \omega_s \quad p.u. \tag{4.14}$$

$$H_{system} = H_{mach}\frac{S_{mach}}{S_{system}} \tag{4.15}$$

COHERENT MACHINES: MACHINES WHICH SWING TOGETHER

The swing equation for coherent machine can be combined even though rated speeds are different because δ , ω are expressed in electrical degree or radian.

For Machine-1

$$\frac{2H_1}{\omega_s}\frac{d^2\delta_1}{dt^2} = P_{m1} - P_{e1} \tag{4.16}$$

For Machine-2

$$\frac{2H_2}{\omega_s}\frac{d^2\delta_2}{dt^2} = P_{m2} - P_{e2} \tag{4.17}$$

Subtracting (4.17) from (4.16)

$$\frac{d^2\delta_1}{dt^2} - \frac{d^2\delta_2}{dt^2} = \frac{\omega_s}{2}\left(\frac{P_{m1} - P_{e1}}{H_1} - \frac{P_{m2} - P_{e2}}{H_2}\right) \tag{4.18}$$

Multiplying both sides by $\frac{H_1 H_2}{H_1 + H_2}$

$$\frac{2H_{12}}{\omega_s}\frac{d^2\delta_{12}}{dt^2} = P_{m12} - P_{e12} \quad (4.19)$$

Where

$$H_{12} = \frac{H_1 H_2}{H_1 + H_2}$$

$$P_{m12} = \frac{P_{m1} H_2 - P_{m2} H_1}{H_1 + H_2}$$

$$P_{e12} = \frac{P_{e1} H_2 - P_{e2} H_1}{H_1 + H_2}$$

SYNCHRONIZING POWER COEFFICIENTS

A common requirement – generator should loose synchronism

For fixed P_m,

Let $\delta = \delta_0 + \delta_\Delta \quad P_e = P_{e0} + P_{e\Delta}$

Hence $P_{e0} + P_{e\Delta} = P_{\max} Sin(\delta_0 + \delta_\Delta)$ (4.20)

Since δ_Δ is small incremental displacement for which $Sin\delta_\Delta = \delta_\Delta$ and $Cos\delta_\Delta = 1$

$$P_{e0} + P_{e\Delta} = P_{\max} Sin\delta_0 + (P_{\max} Cos\delta_0)\delta_\Delta \quad (4.21)$$

But $P_m = P_{e0} = P_{\max} Sin\delta_0$

$$\therefore P_m - (P_{e0} + P_{e\Delta}) = -(P_{\max} Cos\delta_0)\delta_\Delta \quad (4.22)$$

Substituting the incremental values in basic swing equation

$$\frac{2H}{\omega_s}\frac{d^2(\delta_0 + \delta_\Delta)}{dt^2} = P_m - (P_{e0} + P_{e\Delta}) \quad (4.23)$$

$$\frac{2H}{\omega_s}\frac{d^2\delta_\Delta}{dt^2} + (P_{\max}\cos\delta_0)\delta_\Delta = 0 \quad (4.24)$$

δ_0 is constant value

$P_{\max}\cos\delta_0$ - slope of power angle curve at angle δ_0

$$S_p = \left.\frac{dP_e}{d\delta}\right|_{\delta=\delta_0} = P_{\max} Cos\delta_0$$, Where S_p is the synchronizing power coefficient.

$$\frac{d^2\delta_\Delta}{dt^2} + \frac{\omega_s}{2H} S_P \delta_\Delta = 0 \qquad (4.25)$$

It is the linear second order equations and solutions of which depends on algebraic sign of S_p. If S_p is positive then solution shall be simple harmonic motion. If S_p is negative then solution increases exponentially without limit.

The angular frequency of undamped oscillation is given by

$$\omega_n = \sqrt{\frac{\omega_s S_p}{2H}} Elect.rad/\sec \qquad (4.26)$$

$$f_n = \frac{1}{2\pi}\sqrt{\frac{\omega_s S_p}{2H}} Hz \qquad (4.27)$$

EQUAL AREA CRITERIA OF STABILITY

The single machine infinite bus is given in Fig.-4.1. Let us assume a short circuit fault at the point P. The power angle curve for the generator is given in Fig.-4.2

FIG.-4.2 POWER ANGLE CURVE FOR THE GENERATOR

Let us look at the phenomenon following the disturbance at the point P.

Generator is rotating at synchronous speed with a rotor angle of δ_0 and input mechanical power P_m equals the output electrical power P_e.

- Let the fault occurs at δ_0 or $t = 0$.
- The electrical power output is suddenly reduces to Zero whereas the mechanical power input remains the same i.e. unchanged.
- Due to which the difference in power input and power output must be accounted for by a rate of change of Kinetic Energy of the rotor mass.
- This is accomplished by increase in speed which results from the constant accelerated power P_m.
- Let the time to clear the fault is t_c then for $t < t_c$

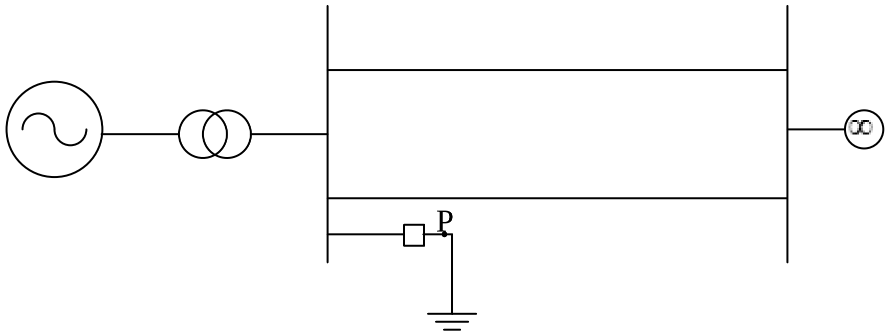
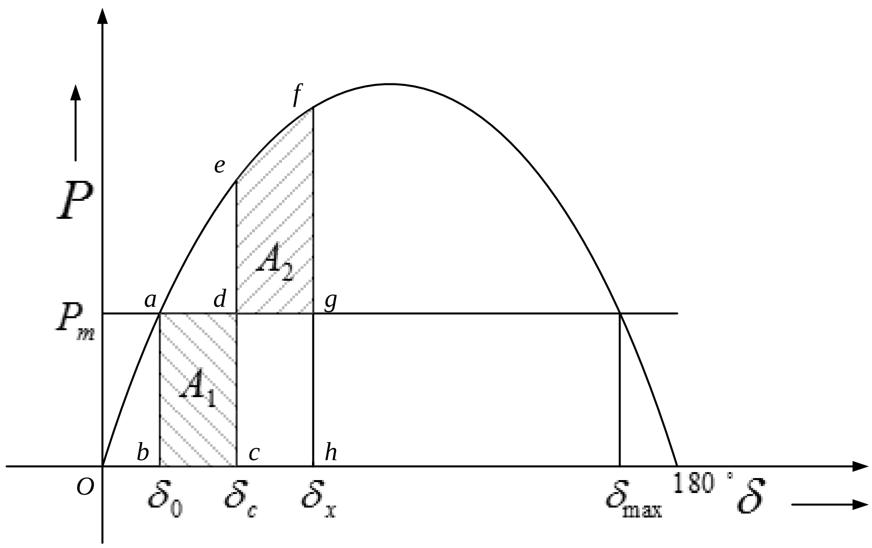


FIG.-4.1 SINGLE MACHINE INFINITE BUS SYSTEM



$$\frac{d^2\delta}{dt^2} = \frac{\omega_s}{2H} P_m \tag{4.28}$$

And the velocity is given by

$$\frac{d\delta}{dt} = \int_0^t \frac{\omega_s}{2H} P_m dt = \frac{\omega_s}{2H} P_m t \tag{4.29}$$

So that $\delta = \frac{\omega_s}{4H} P_m t^2 + \delta_0$ (4.30)

At the instance of fault clearing δ_0 advances to δ_c due to increase in rotor speed and angle is given by

$$\left.\frac{d\delta}{dt}\right|_{t=t_c} = \frac{\omega_s}{2H} P_m t_c \tag{4.31}$$

$$\delta(t)\big|_{t=t_c} = \frac{\omega_s}{4H} P_m t_c^2 + \delta_0 \tag{4.32}$$

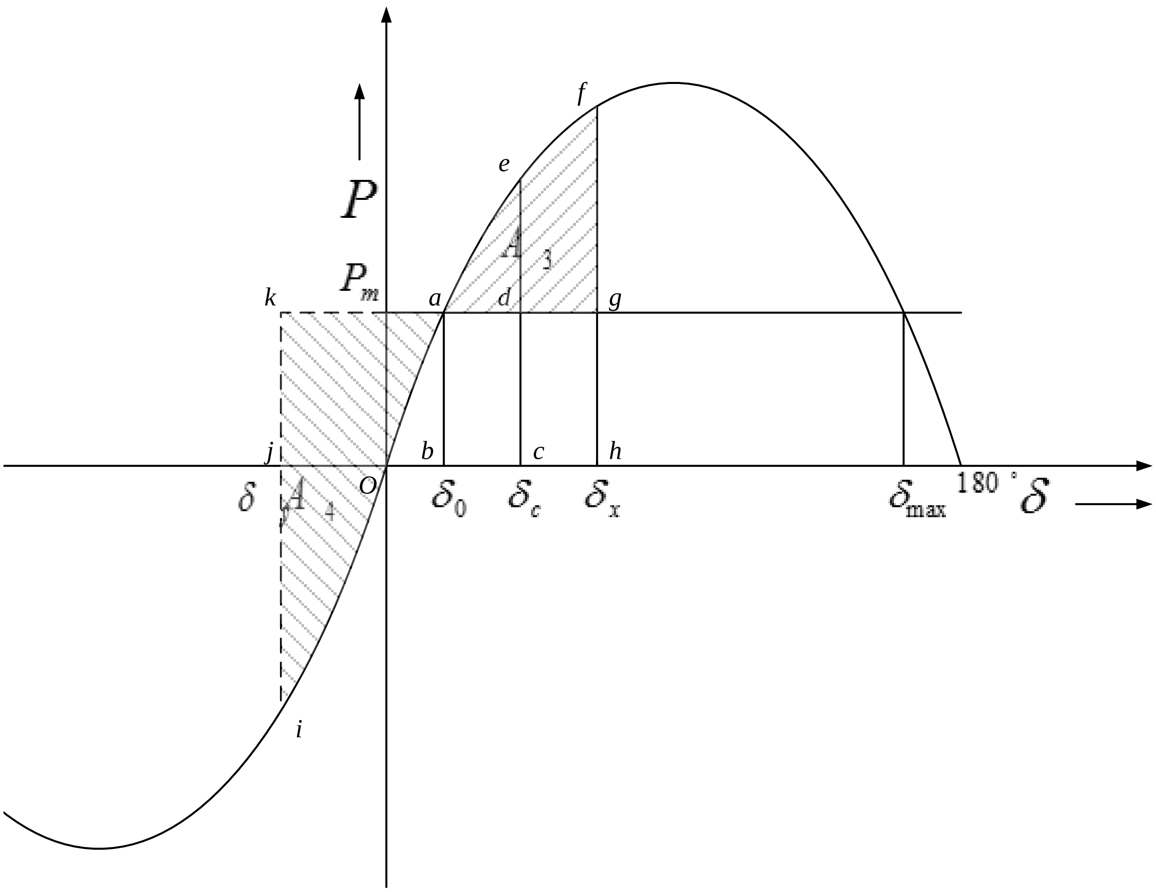


FIG.-4.3 POWER ANGLE CURVE FOR THE GENERATOR (EXTENDED CASE)

- At δ_c the electrical power output abruptly increases to $P_e = P_{\max} Sin\delta_c$ (Point *'e'* on the power angle curve and thus the accelerated power becomes negative.
- As a consequence the rotor slows down as P_e goes through point *'e'* to point *'f'*.
- At *'f'* the rotor speed is again synchronous although rotor angle has advanced to δ_x
- Angle δ_x is determined with the fact that area $'A_1'$ and $'A_2'$ must be equal.
- The accelerated power at *'f'* is still negative and so rotor cannot remain at synchronous speed and continue to slow down.
- The relative velocity is negative.
- Rotor angle moves back from δ_x at *'f'* along the power angle curve to point *'a'*.
- At *'a'* rotor speed is less than the synchronous speed.
- From *'a'* to *'i'* P_m exceeds P_e and the rotor increases speed again until it reaches at *'i'* as shown in Fig.-4.3. Point *'i'* is located so that area A_3 and A_4 are equal.

- In the absence of damping, rotor would continue to oscillate in the sequence *f-a-i*, *i-a-f* with synchronous speed occurring at *'i'* and *'f'*.

When one m/c is swinging with respect to infinite bus we may use this principle of equity of areas called equal area criterion, to determine the stability of system under transient condition, without solving the swing equation.

The swing equation is:

$$\frac{2H}{\omega_s}\frac{d^2\delta}{dt^2} = P_m - P_e \tag{4.33}$$

The angular velocity of rotor relative to synchronous speed is

$$\omega_r = \frac{d\delta}{dt} = \omega - \omega_s \tag{4.34}$$

Differentiating w.r.t. time

$$\frac{d\omega_r}{dt} = \frac{d^2\delta}{dt^2} \tag{4.35}$$

$$\frac{2H}{\omega_s}\frac{d\omega_r}{dt} = P_m - P_e \tag{4.36}$$

If rotor speed is synchronous then ω equals ω_s and ω_r is zero.

Multiplying both side by $\omega_r = \frac{d\delta}{dt}$

$$\frac{2H}{\omega_s}\frac{d\omega_r}{dt}(\omega_r) = (P_m - P_e)\frac{d\delta}{dt} \tag{4.37}$$

$$\frac{H}{\omega_s}\frac{d\omega_r^2}{dt} = (P_m - P_e)\frac{d\delta}{dt} \tag{4.38}$$

Multiplying both side by dt and integrating

$$\int_1^2 \frac{H}{\omega_s} d\omega_r^2 = \int_{\delta_1}^{\delta_2} (P_m - P_e) d\delta \tag{4.39}$$

If the rotor speed is synchronous at δ_1 and δ_2 then $\omega_{r1} = \omega_{r2} = 0$

$$0 = \int_{\delta_1}^{\delta_2} (P_m - P_e) d\delta \tag{4.40}$$

This equation applies to any two points δ_1 and δ_2 on power angle curve provided they are points at which the rotor speed is synchronous.

Performing integration in two steps for points δ_0 and δ_x for Fig.-4.2.

$$\int_{\delta_0}^{\delta_c}(P_m - P_e)d\delta + \int_{\delta_c}^{\delta_x}(P_m - P_e)d\delta = 0 \qquad (4.41)$$

$$\underbrace{\int_{\delta_0}^{\delta_c}(P_m - P_e)d\delta}_{\text{Pre Faulkt Period}} = \underbrace{\int_{\delta_x}^{\delta_c}(P_m - P_e)d\delta}_{\text{Post Fault Period}} \qquad (4.42)$$

This shows area A_5 and A_6 are equal.

If there is a delay in clearing the fault δ_c increases.

Then positive accelerated power is again encountered beyond $\delta_{\max}$ as shown in Fig.-4.4

δ_c will increase without limit and instability results.

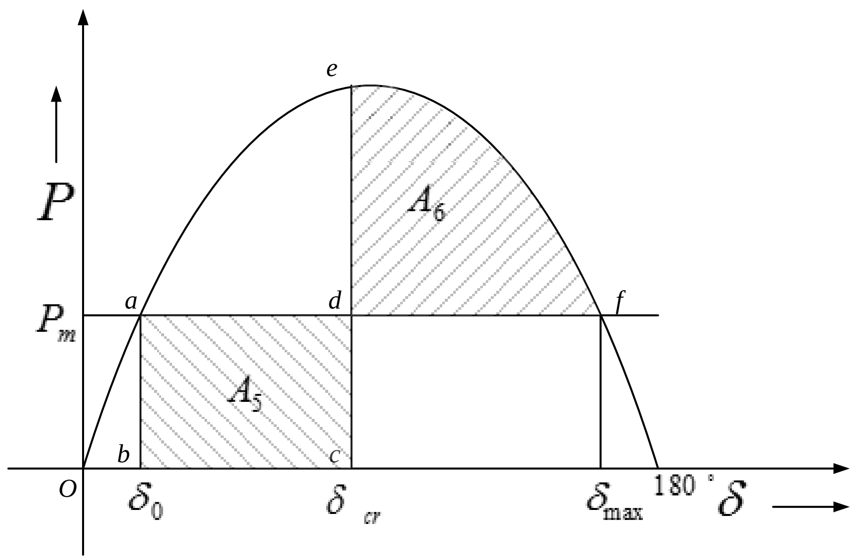


FIG.-4.4 POWER ANGLE CURVE FOR THE GENERATOR (CRITICAL CASE)

The corresponding time for clearing the fault is called critical clearing time t_{cr}

$$A_5 = \int_{\delta_0}^{\delta_{cr}}(P_m - P_e)d\delta \qquad (4.43)$$

$Since(P_e = 0)$ at this instant

$$A_5 = P_m(\delta_{cr} - \delta_0) \qquad (4.44)$$

$$A_6 = \int_{\delta_{cr}}^{\delta_{max}} (P_{max} Sin\delta - P_m) d\delta \tag{4.45}$$

$$A_6 = P_{max}(Cos\delta_{cr} - Cos\delta_{max}) - P_m(\delta_{max} - \delta_{cr}) \tag{4.46}$$

$$Cos\delta_{cr} = \frac{P_m}{P_{max}}(\delta_{max} - \delta_{cr}) + Cos\delta_{max} \tag{4.47}$$

But $\delta_{max} = \pi - \delta_0$ Elect rad and $P_m = P_{max} Sin\delta_0$

$$Cos\delta_{cr} = \frac{P_{max} Sin\delta_0}{P_{max}}(\pi - 2\delta_0) + Cos(\pi - \delta_0) \tag{4.48}$$

$$Cos\delta_{cr} = (\pi - 2\delta_0) Sin\delta_0 - Cos\delta_0 \tag{4.49}$$

Hence critical clearing angle can be calculated by using (4.49)

But $\delta = \frac{\omega_s}{4H} P_m t^2 + \delta_0$

So it can be written for critical clearing angle

$$\delta_{cr} = \frac{\omega_s}{4H} P_m t_{cr}^2 + \delta_0 \tag{4.50}$$

From (4.50) we can calculate critical clearing time as follows in (4.51)

$$t_{cr} = \sqrt{\frac{4H(\delta_{cr} - \delta_0)}{\omega_s P_m}} \tag{4.51}$$

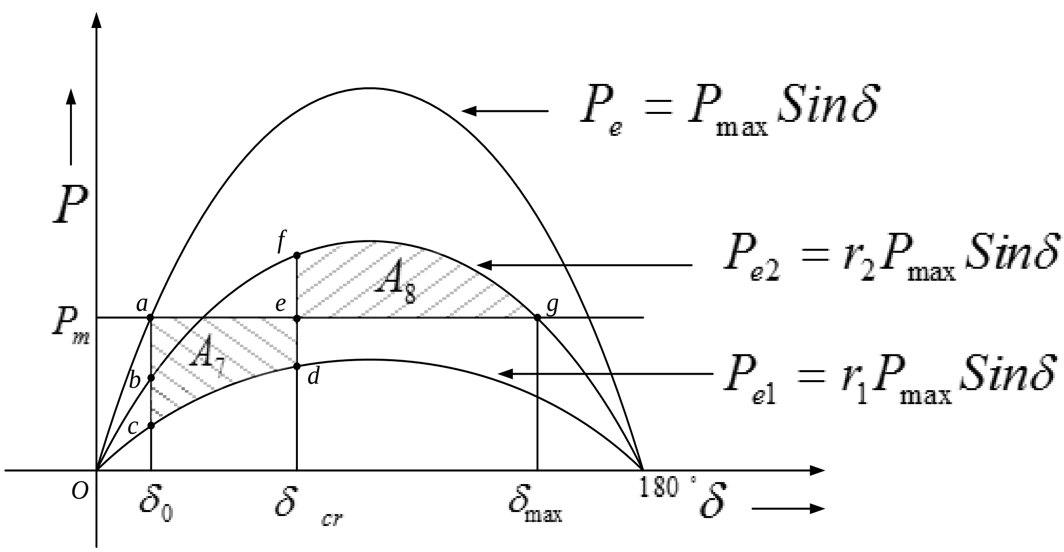


FIG.-4.5 POWER ANGLE CURVE FOR THE GENERATOR (POWER TRANSMITTED DURING FAULT)

In many cases, the power is still transmitted under fault, if the fault is other than short circuit or 3ϕ fault as show in Fig.-4.5.

Hence under such case the electric power transmitted during the fault, is given by

$$P_{e1} = r_1 P_{\max} Sin\delta \tag{4.52}$$

And the electric power transmitted after the fault is cleared, is given by

$$P_{e2} = r_2 P_{\max} Sin\delta \tag{4.53}$$

As per equal area criteria area A_7 and A_8 are equal. Thus we can write from (4.

The corresponding time for clearing the fault is called critical clearing time t_{cr}

$$A_7 = \int_{\delta_0}^{\delta_{cr}} \left(P_m - P_{e1}\right) d\delta \tag{4.54}$$

Since $P_{e1} = r_1 P_{\max} Sin\delta$ at this instant

$$A_7 = P_m\left(\delta_{cr} - \delta_0\right) + r_1 P_{\max}\left(Cos\delta_{cr} - Cos\delta_0\right) \tag{4.55}$$

$$A_8 = \int_{\delta_{cr}}^{\delta_{\max}} \left(P_{e2} - P_m\right) d\delta \tag{4.56}$$

Since $P_{e2} = r_2 P_{\max} Sin\delta$ at this instant

$$A_8 = -r_2 P_{\max}\left(Cos\delta_{\max} - Cos\delta_{cr}\right) - P_m\left(\delta_{\max} - \delta_{cr}\right) \tag{4.57}$$

Solving (4.55) and (4.57) we get

$$Cos\delta_{cr} = \frac{\dfrac{P_m}{P_{\max}}\left(\delta_{\max} - \delta_0\right) + r_2 Cos\delta_{\max} - r_1 Cos\delta_0}{r_2 - r_1} \tag{4.58}$$

Hence critical clearing angle can be calculated by using (4.58) for the case when the power is still being transmitted under fault condition.

MULTI-MACHINE STABILITY (CLASSICAL REPRESENTATION)

The equal area criterion can not be used directly in system where three or more machines are represented. When many machines are simultaneously undergoing transient oscillation the swing curve will reflect the combined presence of many such oscillations.

The following additional assumptions are made

- The mechanical power to each machine constant during the entire period of swing curve computation

- Damping power is negligible.
- Each machine is represented by a constant transient reactance in series with a constant transient internal voltage.
- The mechanical rotor angle of each machine coincides with δ, the electrical phase angle of the transient internal voltage.
- All loads may be considered as shunt impedance to ground with values determined by conditions prevailing immediately prior to the transient conditions.

The system stability model based on these assumptions is classical stability model and classical stability studies.

In multi-machine case two preliminary steps are required

- Steady state pre-fault conditions are calculated using a load flow.
- Pre-fault network representation is determined and modified to account for the fault and the post-fault conditions.

STEP BY STEP SOLUTION OF SWING CURVE

The solution of the swing equation can be obtained by conventional step by step method. This is suitable for hand calculation. However with the help of digital computer program, the 5solution can be obtained by using the numerical method such Runge-Kutta method or modified Euler method. In this section step by step method has been described.

Standard interrupting time for circuit breaker and their associated relays are commonly 8, 5, 3 or 2 cycles after the fault occurs thus standard time period of $\Delta t = 0.5$ sec is generally taken as shown in Fig.-4.6.

Assumptions made for this method are-

- The accuracy Power P_a computed at the beginning of an interval is constant from the middle of the preceding interval to the middle of the interval considered Fig.-4.6.
- The angular velocity is constant throughout any interval at the value computed for the middle of the interval Fig.-4.7.

However, neither of the assumptions are true because δ is changing and both P_a and ω are function of δ Fig.-4.8.

The change in speed is the product of the acceleration and time interval and so

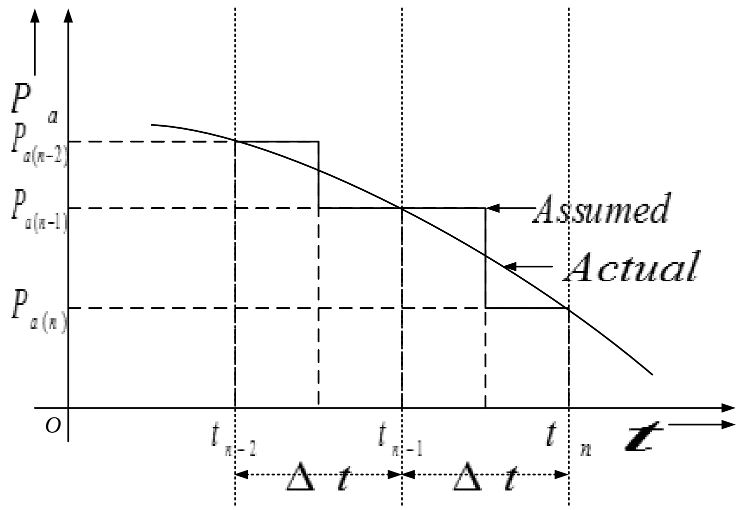


FIG.-4.6 VARIATION OF ACCELERATED POWER

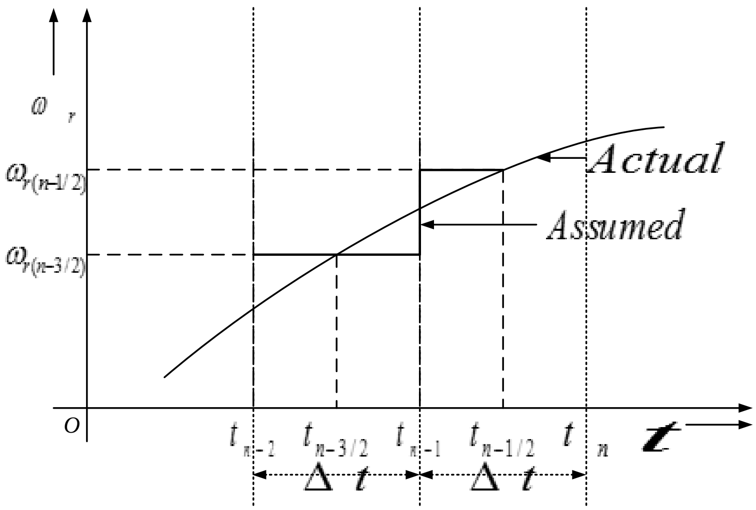


FIG.-4.7 VARIATION OF ANGULAR VELOCITY

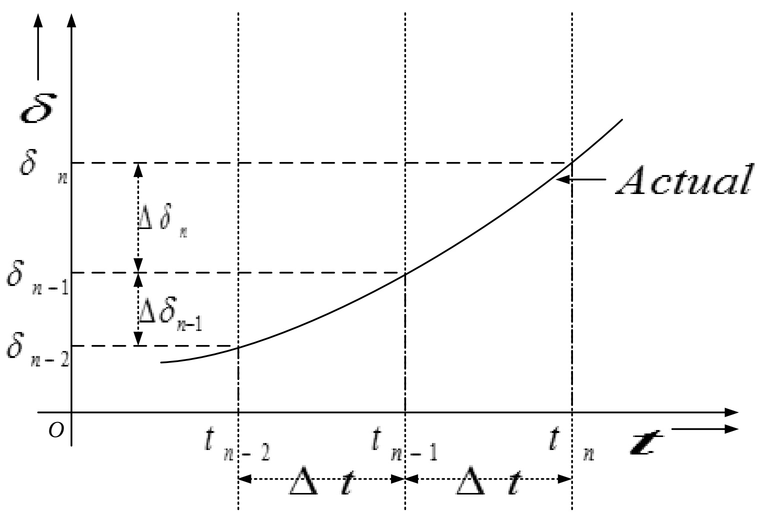


FIG.-4.8 VARIATION OF POWER ANGLE

$$\omega_{r(n-1/2)} - \omega_{r(n-3/2)} = \frac{d^2\delta}{dt^2}\Delta t = \frac{180f}{H}P_{a,n-1}\Delta t \tag{4.59}$$

Change in δ over a period is

$$\Delta\delta_{n-1} = \delta_{n-1} - \delta_{n-2} = \Delta t\omega_{r(n-3/2)} \tag{4.60}$$

$$\Delta\delta_n = \delta_n - \delta_{n-1} = \Delta t\omega_{r(n-1/2)} \tag{4.61}$$

Subtracting (4.60) from (4.61) and using (4.59) to eliminate ω_r

$$\Delta\delta_n = \Delta\delta_{n-1} + KP_{a(n-1)} \tag{4.62}$$

Where, $K = \frac{180f}{H}(\Delta t)^2$

The accelerating power is calculated at the beginning of each new interval. Greater accuracy is obtained when the duration of interval is small.

An interval of 0.05s is usually satisfactory.

FACTORS AFFECTING TRANSIENT STABILITY

There are two guideline criteria

- Angular swing during and following fault conditions.
- Critical clearing time.

H constant and $X_d^{'}$ have direct effect on both of these criteria.

Smaller H means larger swing

$X_d^{'}$ increases - $P_{\max}$ decreases

Smaller $P_{\max}$ means, less difference between δ_0 and δ_{cr}

Small $P_{\max}$ contains the machine to swing

Any measure to lower H and increased $X_d^{'}$ lessens the probability of maintain stability under transient conditions.

Higher rated generators have low H and high $X_d^{'}$

The control schemes are

- Excitation system
- Turbine valve control

- Single pole operation of circuit breakers
- Faster fault clearing time

The system design strategies aimed at lowering system reactance are

- Minimum transformer reactance
- Series capacitor compensation of lines.
- Additional transmission lines

Excitation effect is to reduce the initial rotor angle swing following the fault

- Initial fault
- Voltage reduced
- Excitation system activated
- Voltage applied to field winding boosted by amplifiers
- Increased air gap flux exerts a restraining torque on rotor
- It slows down the rotor
- Modern excitation system can provide from one cycle to $1\frac{1}{2}$ cycle gain in critical clearing time.
- Modern EHG have the ability to provide 1-2 cycle gain.

VOLTAGE STABILITY

INTRODUCTION

Instability may also be encountered without loss of synchronism. A system could become unstable because of the collapse of voltages at certain buses in the system. In this instance, the concern is stability and control of voltage. The analysis in this case deals with the ability of the power system to maintain steady acceptable voltages at all buses in the system under normal operating conditions and after being subjected to disturbances. The main factor causing instability is the inability of the power system to meet the demand. Voltage instability takes on the form of a dramatic drop of transmission system voltages, which may lead to system disruption. During the past two decades it has become a major threat for the operation of many systems and, in the prevailing open access environment, it is a factor leading to limit power transfers.

The transfer of power through a transmission network is accompanied by voltage drops between the generation and consumption points. In normal operating conditions, these drops are in the order of a few percents of the nominal voltage. One of the tasks of power system planners and operators is to check that under heavy stress conditions and/or following credible events, all bus voltages remain within acceptable bounds. In some circumstances, however, in the seconds or minutes following a disturbance, voltages may experience large, progressive falls, which are so pronounced that the system integrity is endangered and power cannot be delivered correctly to customers. This phenomenon is referred to as voltage instability and its result as voltage collapse. This instability stems from the attempt of load dynamics to restore power consumption beyond the amount that can be provided by the combined transmission and generation system.

BASIC THEORY

The following changes in power system contribute to voltage collapse.

- Increase in loading
- Generators or SVC reaching reactive power limits
- Action of tap changing transformers
- Load recovery dynamics
- Line trapping or generator outage

SADDLE NODE BIFURCATION

A stable operating equilibrium disappears and system dynamically collapse

Eigen values at SNB have negative real part

As the system load increases one of the eigen values approaches zero from the left

Two equilibria coalesce one of these equilibria must be unstable

Sensitivity with respect to loading parameters of a typical state variable is infinite

System Jocobian has a zero eigen value

System Jacobian has a zero singular value

Rate of collapse is low first then fast.

INDICES

There are several indices to determine how close the system is to collapse. These indices helps to calculate voltage instability. These indices are the scalar magnitude that is monitored as system parameter changes.

The fundamental theory which monitors the phenomena is explained below.

DAE of the power system is given by

$$\dot{x} = f(x, y, \lambda)$$

$$0 = g(x, y, \lambda)$$

x -state variable,

y -Algebraic variables

λ -parameter that vary

The equilibrium is defined as

$$F(z_0, \lambda_0) = 0$$

$D_z F(z_*, \lambda_*)$ is singular known as singular bifurcation

The power flow model is given by

$$\begin{bmatrix} \Delta P(u, \lambda) \\ \Delta Q(u, \lambda) \end{bmatrix} = F(u, \lambda) = 0$$

Where $u = \begin{bmatrix} V \\ \delta \end{bmatrix}$

SENSITIVITY FACTORS

$$VSF_i = \max\left\{\frac{dV_i}{dQ_i}\right\}$$

$$SF = \left\|\frac{dZ}{d\lambda}\right\|$$

SINGULAR VALUES

$$J = D_z F(z_0, \lambda_0)$$

At equilibrium

$$J = R\sum S^T = \sum_{i=1}^{n} \gamma_i \sigma_i s_i^T$$

$\sum$ -Diagonal Matrix can be given by $\begin{bmatrix} \sigma_1 & & & \\ & \sigma_2 & & \\ & & \cdots & \\ & & & \sigma_n \end{bmatrix} where \sigma_1 \geq \sigma_2 \geq \sigma_n$

γ_i, s_i^T - ith column of the unitary matrices R and S and known as singular values

σ_i -+ve real singular values

$\sum$ - eigen values of matrix JJ^T

Singular value decomposition is used to determine the rank of matrix and equal to the number of no-zero singular values of J.

It monitors smallest singular value

γ_n - left singular vectors

s_n - right singular vectors

Maximum entries in s_n indicates the most sensitive voltage magnitudes (critical buses)

Maximum entries in γ_n indicates the most sensitive direction for changes of power injections.

EIGEN VALUE DECOMPOSITION

$$J = W\Lambda U^T = \sum_{i=1}^{n} w_i \mu_i v_i^T$$

U - left eigen vectors

W - right eigen vectors

Λ - diagonal matrix of complex eigen values

Maximum entries in $'w'$ indicates the most sensitive voltage magnitudes (critical buses)

Maximum entries in $'v'$ indicates the most sensitive direction for changes of power injections.

Second order proximity index

Voltage instability proximity index

Only a ppair of solution remains near the collapse point and then coalesces on it

VIPI uses this pair on rectangular coordinates

LOADING MARGIN

Most basic and widely accepted index of voltage collapse

Not based on particular power system model only requires static power system model

Takes full account of power system non-linearity and limits

Account for the patterns of load increase

Energy Function

Reacitve power margin

Tangent vectors

CHAPTER-5

ECONOMIC OPERATION OF POWER SYSTEM

The proper distribution of the output of a plant between the generator or units within the plant is very much necessary to determine for economic operation of the power system. We can also study a method of expressing transmission loss as a function of the outputs of the various plants. The power system consists of many equipment. The cost involved for installation of these equipment can be regarded as the capital cost. The cost involved for the operation of these equipment is known as variable cost. Further the interest incurred on capital cost can form a part of the variable cost. The variable cost also depend on the power output. Hence the variable operating cost of the units must be expressed in in terms of power output. In general the power output is directly proportional to the fuel input. Hence fuel cost can also be function of output.

$$Fuel\ efficiency = \frac{Output\ (MW)}{Fuel\ input}$$

Maximum η occurs at tangent point of the output vs fuel input.

CRITERION FOR DISTRIBUTION OF THE LOAD

The method described in this section applies to economic distribution of load between the units of a station. The total cost incurred for production of certain amount of power can be economically determined by increasing or decreasing the load on different units of a particular power station. It means the load may be increased on one unit while the load may be decreased on other unit by same amount. It shall results in increasing/ decrease of total cost.

The concept of incremental cost (slope of the curve): Incremental fuel cost of $i^{th}-unit$ is given by $\frac{dF_i}{dP_i}$. Where

F_i - Cost of fuel of the $i^{th}-unit$ in $'Rs./Hr'$

P_i -Output of the $i^{th}-unit$ in $'MW'$

For economic division of load between units within a plant is that all units must operate at the same incremental fuel cost

For a plant with *'N'* units, the total cost of fuel can be given (5.1), whereas the total load is given by (5.2).

$$F_T = F_1 + F_2 + \cdots\cdots\cdots + F_N = \sum_{i=1}^{N} F_i \tag{5.1}$$

$$P_R = P_1 + P_2 + \cdots\cdots\cdots + P_N = \sum_{i=1}^{N} P_i \tag{5.2}$$

Objective is to obtain minimum F_T

Which require $dF_T = 0$

$$dF_T = \frac{\partial F_T}{\partial P_1} dP_1 = \frac{\partial F_T}{\partial P_2} dP_2 + \cdots\cdots\cdots + \frac{\partial F_T}{\partial P_N} dP_N = 0 \tag{5.3}$$

Since P_R remains constant $\Rightarrow dP_R = 0$

$$dP_1 + dP_2 + \cdots\cdots\cdots + dP_N = 0 \tag{5.4}$$

Multiplying (5.4) by λ and subtracting from (5.3)

$$\left(\frac{\partial F_T}{\partial P_1} - \lambda\right) dP_1 + \left(\frac{\partial F_T}{\partial P_2} - \lambda\right) dP_2 + \cdots\cdots\cdots + \left(\frac{\partial F_T}{\partial P_N} - \lambda\right) dP_N = 0 \tag{5.5}$$

It is satisfied if each term is zero,

$$\frac{\partial F_1}{\partial P_1} = \frac{\partial F_2}{\partial P_2} = \cdots\cdots\cdots = \frac{\partial F_N}{\partial P_N} = \lambda \tag{5.6}$$

The cost of fuel of can be given by function of power output as follows

$$F_i = a^2 P_i^2 + bP_i + c \tag{5.7}$$

Where

a, b, c are the coefficients representing the variable and fixed part of the cost components.

In this method only the fuel cost of only one unit will vary if only the power output of that unit is varied. The procedure is known as Lagrangian multiplier.

CRITERION FOR DISTRIBUTION OF THE LOAD CONSIDERING THE LOSSES

If the load has to be distributed between the power stations then the losses incurred in transmission of power has to be taken into account. These losses are also function of plant generation. Let us take an example system shown in Fig.-5.1. In this system two generating stations $'a'$ and $'b'$ are connected by a transmission line having sections as shown. The load is connected at bus $'c'$.

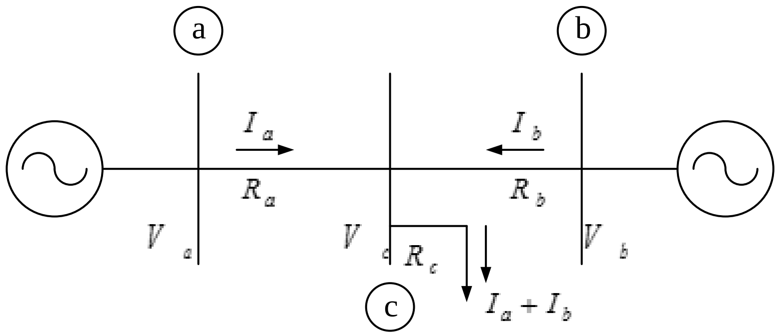


FIG.-5.1 POWER SYSTEM HAVING TWO GENERATING STATION AND A LOAD

The total losses can be given by (5.8)

$$P_L = 3I_a^2 R_a + 3I_b^2 R_b + 3(I_a + I_b)^2 R_c \tag{5.8}$$

$$P_L = 3I_a^2 (R_a + R_c) + 3I_b^2 (R_b + R_c) + 6 I_a I_b R_c \tag{5.9}$$

Since $I_a = \frac{P_a}{\sqrt{3} V_a pf_a}, I_b = \frac{P_b}{\sqrt{3} V_b pf_b}$, (5.9) can be written as

$$P_L = P_a^2 \frac{(R_a + R_c)}{V_a^2 pf_a^2} + 2P_a P_b \frac{R_c}{V_a V_b pf_a pf_b} + P_b^2 \frac{(R_b + R_c)}{V_b^2 pf_b^2} \tag{5.10}$$

$$P_L = P_a^2 B_{aa} + 2P_a P_b B_{ab} + P_b^2 B_{bb} \tag{5.11}$$

The general form of the loss equation (5.11) for the power loss is given by

$$P_L = \sum_m \sum_n P_m B_{mn} P_n \tag{5.12}$$

DISTRIBUTION OF LOAD BETWEEN THE PLANTS

We have for a system with *'N'* plants, the total cost of fuel can be given (5.13), whereas the total load is given by (5.14).

$$F_T = F_1 + F_2 + \cdots\cdots\cdots\cdots + F_N = \sum_{i=1}^{N} F_i \tag{5.13}$$

$$P_R = P_1 + P_2 + \cdots\cdots\cdots\cdots + P_N = \sum_{i=1}^{N} P_i \tag{5.14}$$

The transmission loss shall be added as an additional constraints.

The constraint equation is

$$\sum_{i=1}^{N} P_i - P_L - P_R = 0 \tag{5.15}$$

P_R = Total power received by loads. Since P_R = constant, $dP_R = 0$

$$\sum_{i=1}^{N} dP_i - dP_L = 0 \tag{5.16}$$

For minimum cost $dF_T = 0$

$$dF_T = \sum_{i=1}^{N} \frac{\partial F_T}{\partial P_i} dP_i = 0 \tag{5.17}$$

$$dP_L = \sum_{i=1}^{N} \frac{\partial P_L}{\partial P_i} dP_i = 0 \tag{5.18}$$

$$\sum_{i=1}^{N} \left(\frac{\partial F_T}{\partial P_i} + \lambda \frac{\partial P_L}{\partial P_i} - \lambda \right) dP_i = 0 \tag{5.19}$$

$$\frac{\partial F_T}{\partial P_i} + \lambda \frac{\partial P_L}{\partial P_i} - \lambda = 0 \text{ for every value of } 'i' \tag{5.20}$$

$$\frac{dF_i}{dP_i} \left(\frac{1}{1 - \dfrac{\partial P_L}{\partial P_i}} \right) = \lambda \tag{5.21}$$

$$\frac{dF_n}{dP_n} L_n = \lambda \tag{5.22}$$

Where $L_i = \dfrac{1}{1 - \dfrac{\partial P_L}{\partial P_i}}$ is regarded as the penalty factor of plant $'i'$

For a system with three plants

$$\frac{dF_1}{dP_1} L_1 = \frac{dF_2}{dP_2} L_2 = \frac{dF_3}{dP_3} L_3 = \lambda \tag{5.23}$$

The above method explains how to calculate the economic distribution of load between the plants considering the transmission losses. As we notice, we have to consider the penalty factor to take into account the losses.

CHAPTER-6

FAULT ANALYSIS

SYMMETRICAL COMPONENTS

The power systems are subjected to faults. Most of the faults in power system are unsymmetrical in nature. However symmetrical faults such as short circuits or 3ϕ faults may also occur. The unsymmetrical faults are (i) Line to Ground faults, (ii) Line to Line faults or Double Line faults, (iii) Double Line to Ground faults. C. L. Fortescue has introduced method of symmetrical components to deal with the unbalanced polyphase circuits. This method describes that an unbalanced system of $'n'$ related phasors can be resolved into $'n'$ system of balanced phasors known as symmetrical components of the original systems. Hence a three phase unbalanced systems can be resolved into three balanced systems as shown in Fig.-6.1.

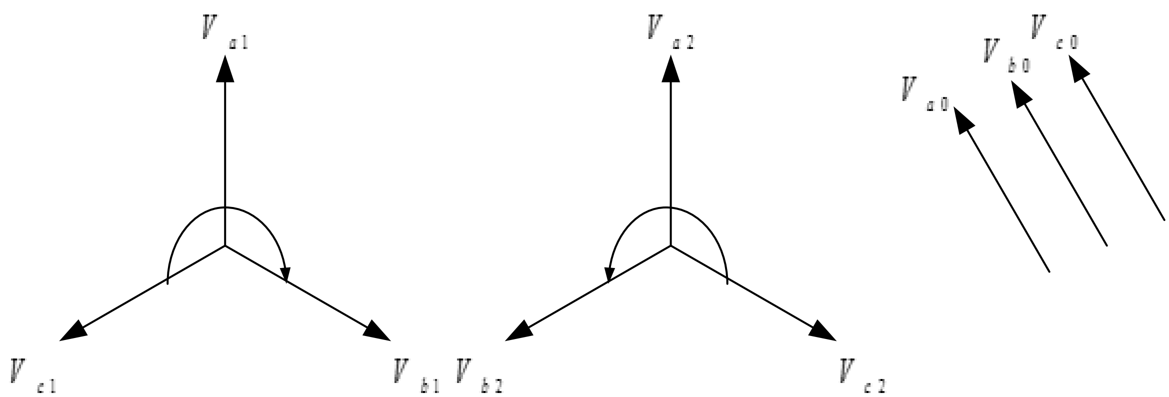


FIG.-6.1 SYMMETRICAL COMPONENTS OF THREE PHASE SYSTEM

V_{a1},V_{b1},V_{c1}-Positive sequence voltages of three phases a,b,c equal in magnitudes and displaced from each other by 120° in phase. Subscript '1' represents positive sequence i.e. having the same phase sequence as the original system.

V_{a2},V_{b2},V_{c2}-Negative sequence voltages of three phases a,b,c equal in magnitudes and displaced from each other by 120° in phase. Subscript '2' represents negative sequence i.e. having the opposite phase sequence as the original system.

V_{a0}, V_{b0}, V_{c0}-Zero sequence voltages of three phases a,b,c equal in magnitudes and displaced from each other by 0° in phase. Subscript '0' represents zero sequence.

Hence the three phasors of original system can be written as (6.1a)-(6.1c)

$$V_a = V_{a1} + V_{a2} + V_{a0} \quad (6.1a)$$

$$V_b = V_{b1} + V_{b2} + V_{b0} \quad (6.1b)$$

$$V_c = V_{c1} + V_{c2} + V_{c0} \quad (6.1c)$$

As shown in the Fig.-6.1 and from the definition above we have

$$V_{b1} = a^2 V_{a1},\ V_{b2} = aV_{a2},\ V_{b0} = V_{a0}$$

$$V_{c1} = aV_{a1},\ V_{c2} = a^2 V_{a2},\ V_{c0} = V_{a0}$$

($a = 1\angle 120^\circ$)

Hence the set of (6.1) can be written as (6.2a)-(6.2c)

$$V_a = V_{a1} + V_{a2} + V_{a0} \quad (6.2a)$$

$$V_b = a^2 V_{a1} + aV_{a2} + V_{a0} \quad (6.2b)$$

$$V_c = aV_{a1} + a^2 V_{a2} + V_{a0} \quad (6.2c)$$

The set of (6.2) in matrix form is given by (6.3) or taking inverse by (6.4)

$$\begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{bmatrix} \quad (6.3)$$

$$\begin{bmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} \quad (6.4)$$

Hence

$$V_{a0} = \frac{1}{3}\left(V_a + V_b + V_c\right) \quad (6.5a)$$

$$V_{a1} = \frac{1}{3}\left(V_a + aV_b + a^2 V_c\right) \quad (6.5b)$$

$$V_{a2} = \frac{1}{3}\left(V_a + a^2 V_b + aV_c\right) \quad (6.5c)$$

Similarly the set of equations (6.1), (6.2), (6.3), (6.4) and (6.5) can be written for the currents of three phases i.e. for I_a, I_b, I_c with the subscripts having the same description as given for the voltages.

SYMMETRICAL COMPONENTS OF UNLOADED GENERATOR

Let us consider an unloaded generator, the terminal voltages of which can be given by V_a, V_b, V_c.

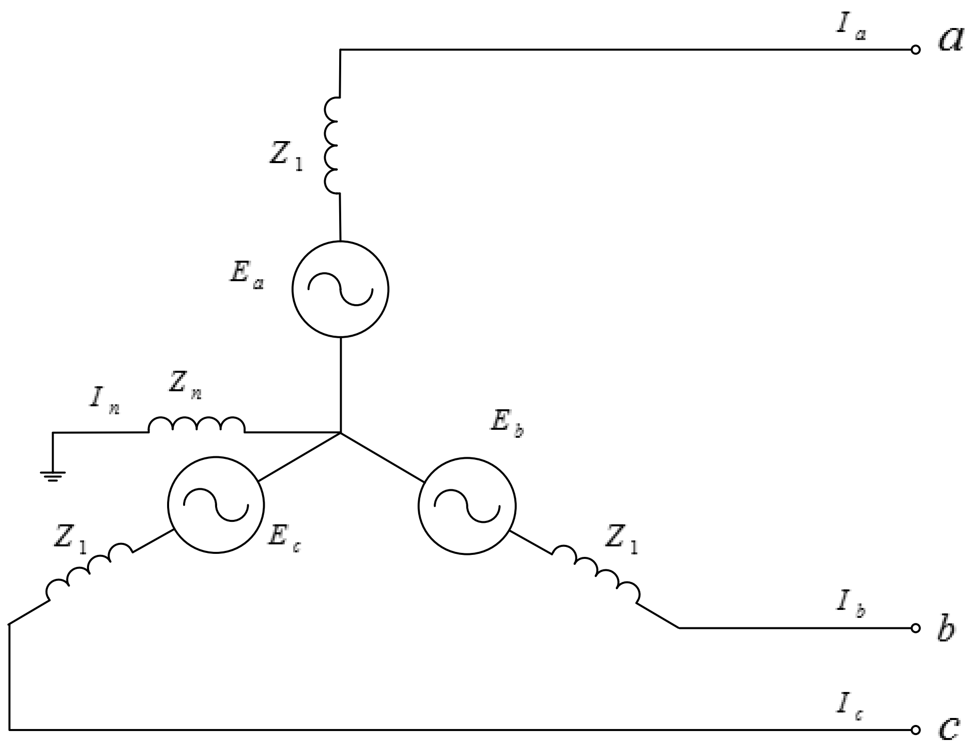


FIG.-6.2 UNLOADED GENERATOR

Current flowing in the neutral is $3I_{a0}$

The voltage drop of zero sequence from point 'p' to ground is $-3I_{a0}Z_n - I_{a0}Z_{go}$

$Z_0 = 3Z_n + Z_{g0}$

The voltage of $phase\text{–}'a'$ can be given by (6.6) and shown in Fig.-6.3

$$\begin{bmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{bmatrix} = \begin{bmatrix} 0 \\ E_a \\ 0 \end{bmatrix} - \begin{bmatrix} Z_0 & 0 & 0 \\ 0 & Z_1 & 0 \\ 0 & 0 & Z_2 \end{bmatrix} \begin{bmatrix} I_{a0} \\ I_{a1} \\ I_{a2} \end{bmatrix} \quad (6.6)$$

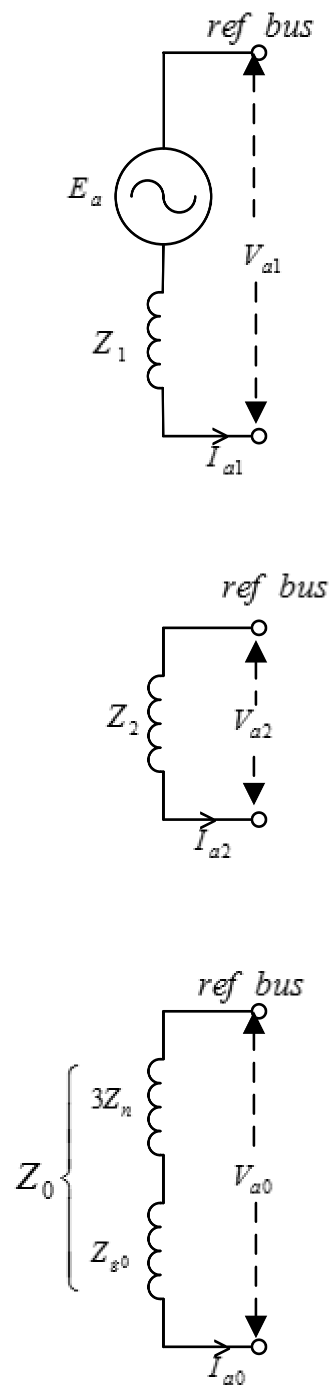


FIG.-6.3 VOLTAGE OF $phase-'a'$ OF UNLOADED GENERATOR

SINGLE LINE TO GROUND FAULT

Let there be a single line to ground fault at terminal of $phase-'a'$ of the unloaded generator (Fig.-6.4).

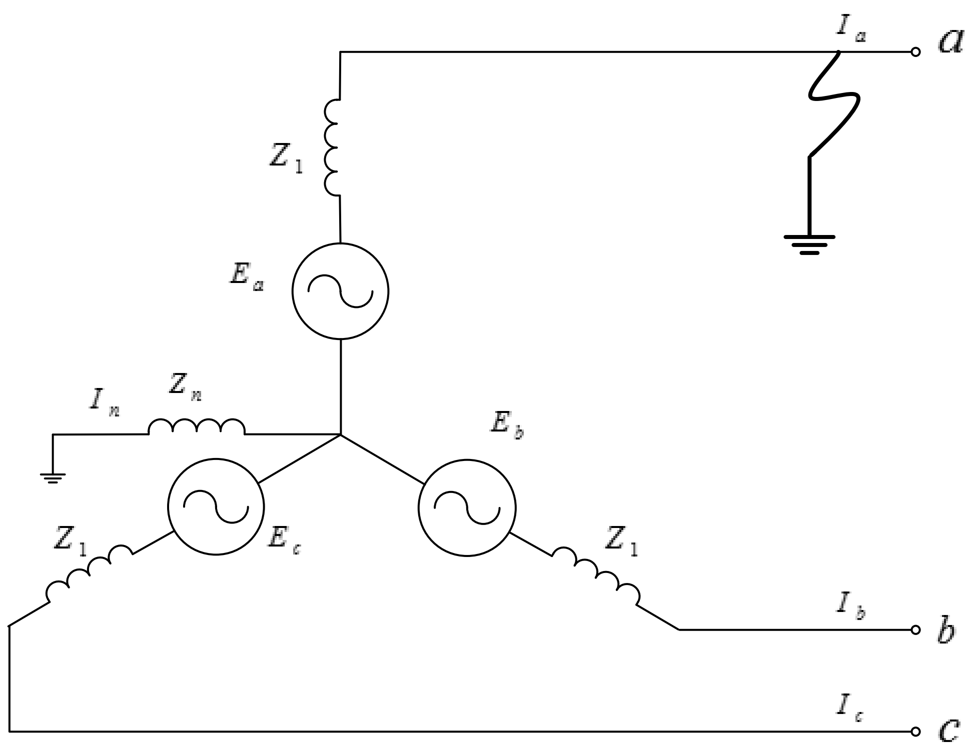


FIG.-6.4 SINGLE LINE TO GROUND FAULT AT TERMINAL $phase-'a'$ OF THE UNLOADED GENERATOR

So that $I_b = I_c = 0, V_a = 0$. As we know from (6.4)

$$\begin{bmatrix} I_{a0} \\ I_{a1} \\ I_{a2} \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix}\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} \tag{6.7}$$

Putting the condition for this fault in (6.7) we have

$$\begin{bmatrix} I_{a0} \\ I_{a1} \\ I_{a2} \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix}\begin{bmatrix} I_a \\ 0 \\ 0 \end{bmatrix} \tag{6.8}$$

Solving (6.8)

$$I_{a1} = I_{a2} = I_{a0} = \frac{1}{3} I_a$$

From (6.6)

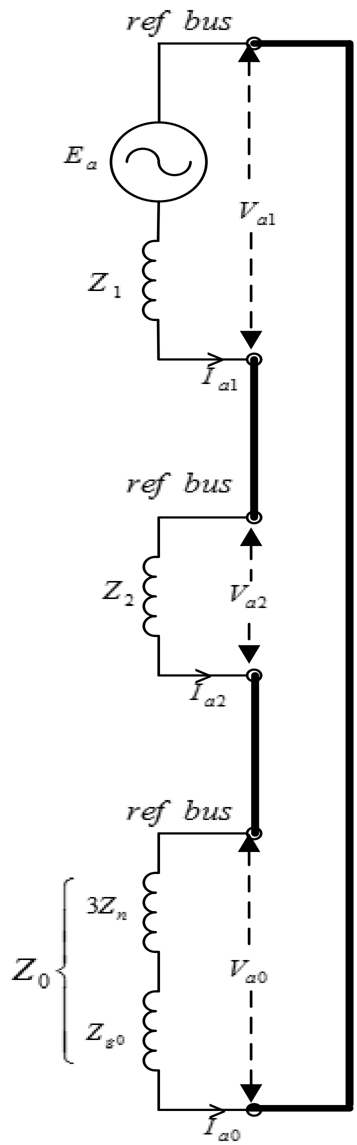


FIG.-6.5 EQUIVALENT NETWORK FOR UNLOADED GENERATOR HAVING SINGLE LINE TO GROUND FAULT

$$\begin{bmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{bmatrix} = \begin{bmatrix} 0 \\ E_a \\ 0 \end{bmatrix} - \begin{bmatrix} Z_0 & 0 & 0 \\ 0 & Z_1 & 0 \\ 0 & 0 & Z_2 \end{bmatrix} \begin{bmatrix} I_{a1} \\ I_{a1} \\ I_{a1} \end{bmatrix} \quad (6.9)$$

Pre multiplying (6.9) with $\begin{bmatrix} 1 & 1 & 1 \end{bmatrix}$ we get

$$V_{a1} + V_{a2} + V_{a0} = -I_{a1}Z_0 + E_a - I_{a1}Z_1 - I_{a1}Z_2 \quad (6.10)$$

$$V_a = -I_{a1}Z_0 + E_a - I_{a1}Z_1 - I_{a1}Z_2 \quad (6.11)$$

$$0 = -I_{a1}Z_0 + E_a - I_{a1}Z_1 - I_{a1}Z_2 \quad (6.12)$$

$$I_{a1} = \frac{E_a}{Z_0 + Z_1 + Z_2} \quad (6.13)$$

$$I_{a1} = I_{a2} = I_{a0} = \frac{E_a}{Z_0 + Z_1 + Z_2} \quad (6.14)$$

The above equation shall be satisfied if the positive sequence, negative sequence and zero sequence impedances are in series across a voltage of $'E_a'$ as shown in Fig.-6.5. This suggests that for a single line to ground fault the positive sequence, negative sequence and zero sequence network shall be connected in series at the point of fault in a power system.

LINE TO LINE FAULT

Let there be line to line fault involving terminal of *phase*–'*b*' and *phase*–'*c*' of the unloaded generator (Fig.-6.6).

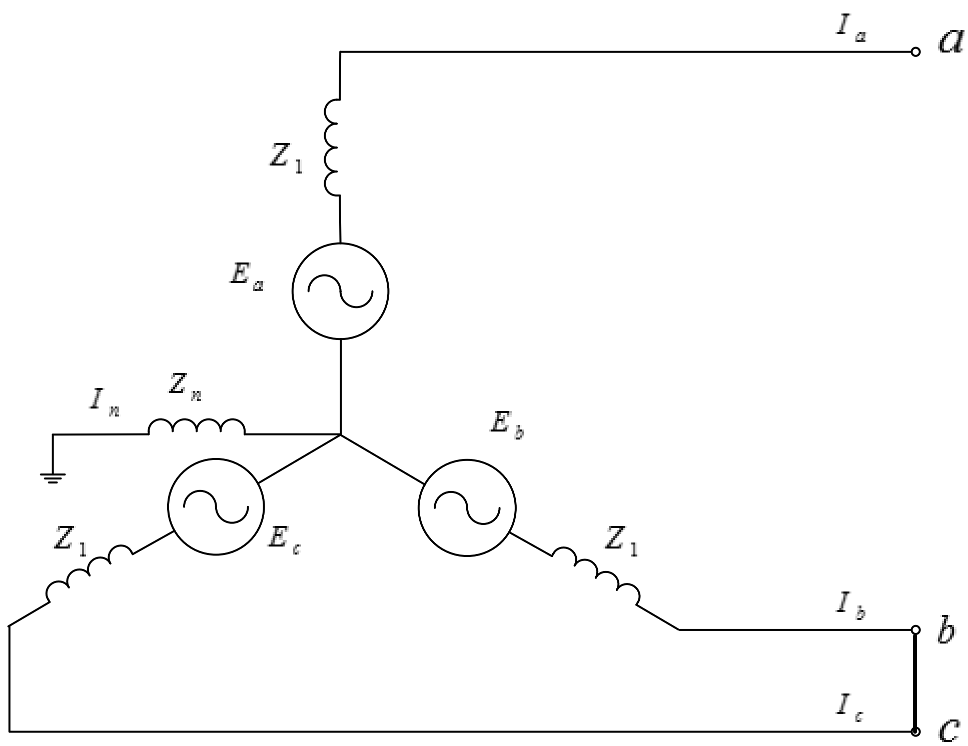


FIG.-6.6 LINE TO LINE FAULT INVOLVING TERMINALS *phase*–'*b*' AND *phase*–'*c*' OF THE UNLOADED GENERATOR

Under this fault $V_b = V_c, I_b = -I_c, I_a = 0$. With these conditions (6.4) modifies to (6.15)

$$\begin{bmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix}\begin{bmatrix} V_a \\ V_b \\ V_b \end{bmatrix} \tag{6.15}$$

$$V_{a1} = V_{a2} \tag{6.16}$$

$$\begin{bmatrix} I_{a0} \\ I_{a1} \\ I_{a2} \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix}\begin{bmatrix} 0 \\ -I_c \\ I_c \end{bmatrix} \tag{6.17}$$

From (6.17)

$$I_{a0} = 0, I_{a2} = -I_{a1} \tag{6.18}$$

From generator to ground Z_0 is finite hence $V_{a0} = 0$ since $I_{a0} = 0$

$$\begin{bmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{bmatrix} = \begin{bmatrix} 0 \\ E_a \\ 0 \end{bmatrix} - \begin{bmatrix} Z_0 & 0 & 0 \\ 0 & Z_1 & 0 \\ 0 & 0 & Z_2 \end{bmatrix}\begin{bmatrix} I_{a0} \\ I_{a1} \\ I_{a2} \end{bmatrix} \tag{6.19}$$

$$\Rightarrow \begin{bmatrix} 0 \\ V_{a1} \\ V_{a1} \end{bmatrix} = \begin{bmatrix} 0 \\ E_a \\ 0 \end{bmatrix} - \begin{bmatrix} Z_0 & 0 & 0 \\ 0 & Z_1 & 0 \\ 0 & 0 & Z_2 \end{bmatrix}\begin{bmatrix} 0 \\ I_{a1} \\ -I_{a1} \end{bmatrix} \tag{6.20}$$

Pre multiplying (6.20) with $\begin{bmatrix} 1 & 1 & -1 \end{bmatrix}$

$$0 = E_a - I_{a1}Z_1 - I_{a1}Z_2 \tag{6.21}$$

$$I_{a1} = \frac{E_a}{Z_1 + Z_2} \tag{6.22}$$

$$I_{a1} = -I_{a2} = \frac{E_a}{Z_1 + Z_2} \tag{6.23}$$

(6.23) shall be satisfied if the positive sequence and negative sequence impedances are in parallel across a voltage of $'E_a'$ as shown in Fig.-6.7. This suggests that for a line to line fault the positive sequence and negative sequence network shall be connected in parallel at the point of fault in a power system.

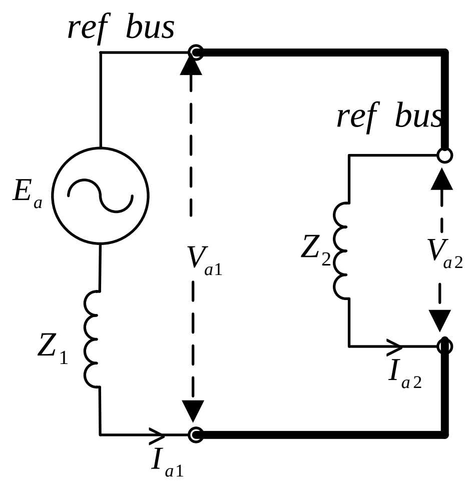


FIG.-6.7 EQUIVALENT NETWORK FOR UNLOADED GENERATOR HAVING LINE TO LINE FAULT

DOUBLE LINE TO GROUND FAULT

Let there be line to line to ground i.e double line to ground fault involving terminal of $phase-'b'$ and $phase-'c'$ of the unloaded generator (Fig.-6.8).

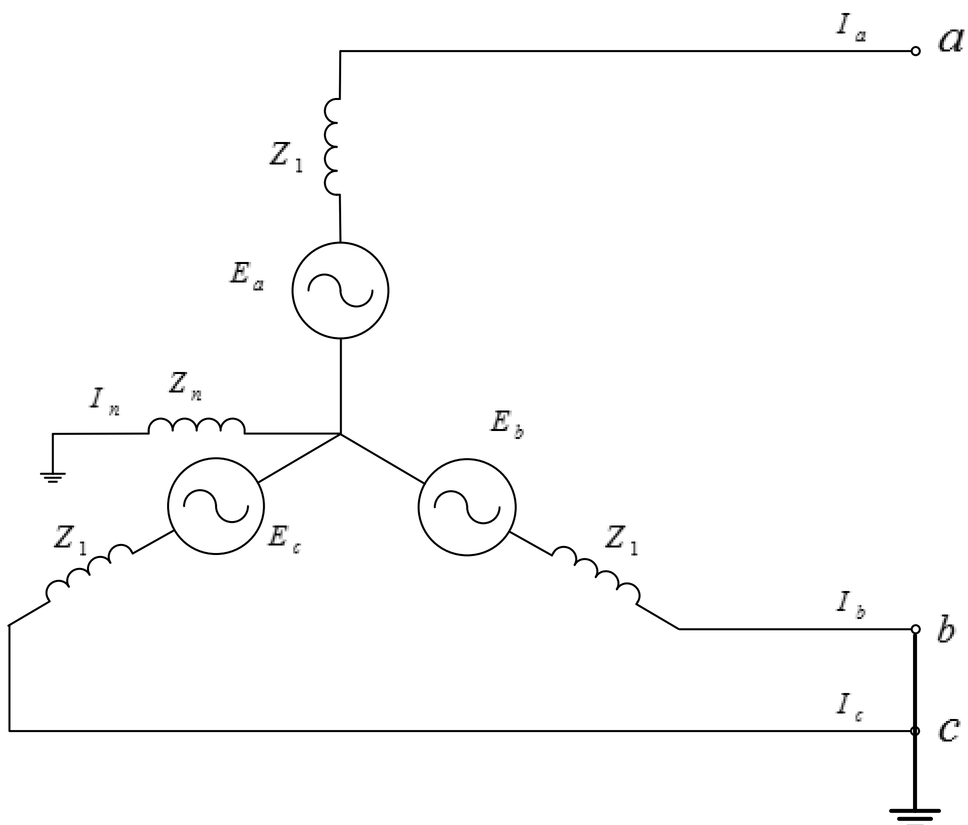


FIG.-6.8 DOUBLE LINE TO GROUND FAULT INVOLVING TERMINALS $phase-'b'$ AND $phase-'c'$ OF THE UNLOADED GENERATOR

Under this fault $V_b = V_c = 0, I_a = 0$. With these conditions (6.4) modifies to (6.24)

$$\begin{bmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix}\begin{bmatrix} V_a \\ 0 \\ 0 \end{bmatrix} \tag{6.24}$$

$$V_{a1} = V_{a2} = V_{a0} = \frac{1}{3}V_a \tag{6.25}$$

$$\begin{bmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{bmatrix} = \begin{bmatrix} 0 \\ E_a \\ 0 \end{bmatrix} - \begin{bmatrix} Z_0 & 0 & 0 \\ 0 & Z_1 & 0 \\ 0 & 0 & Z_2 \end{bmatrix}\begin{bmatrix} I_{a0} \\ I_{a1} \\ I_{a2} \end{bmatrix} \tag{6.26}$$

Pre multiplying (6.26) by

$$Z^{-1} = \begin{bmatrix} \frac{1}{Z_0} & 0 & 0 \\ 0 & \frac{1}{Z_1} & 0 \\ 0 & 0 & \frac{1}{Z_2} \end{bmatrix}$$

We get

$$Z^{-1}\begin{bmatrix} V_{a1} \\ V_{a1} \\ V_{a1} \end{bmatrix} = Z^{-1}\begin{bmatrix} 0 \\ E_a \\ 0 \end{bmatrix} - \begin{bmatrix} I_{a0} \\ I_{a1} \\ I_{a2} \end{bmatrix} \tag{6.27}$$

$$\begin{bmatrix} \frac{1}{Z_0} & 0 & 0 \\ 0 & \frac{1}{Z_1} & 0 \\ 0 & 0 & \frac{1}{Z_2} \end{bmatrix}\begin{bmatrix} V_{a1} \\ V_{a1} \\ V_{a1} \end{bmatrix} = \begin{bmatrix} \frac{1}{Z_0} & 0 & 0 \\ 0 & \frac{1}{Z_1} & 0 \\ 0 & 0 & \frac{1}{Z_2} \end{bmatrix}\begin{bmatrix} 0 \\ E_a \\ 0 \end{bmatrix} - \begin{bmatrix} I_{a0} \\ I_{a1} \\ I_{a2} \end{bmatrix} \tag{6.28}$$

$$\begin{bmatrix} \frac{1}{Z_0} & 0 & 0 \\ 0 & \frac{1}{Z_1} & 0 \\ 0 & 0 & \frac{1}{Z_2} \end{bmatrix}\begin{bmatrix} E_{a1} - I_{a1}Z_1 \\ E_{a1} - I_{a1}Z_1 \\ E_{a1} - I_{a1}Z_1 \end{bmatrix} = \begin{bmatrix} \frac{1}{Z_0} & 0 & 0 \\ 0 & \frac{1}{Z_1} & 0 \\ 0 & 0 & \frac{1}{Z_2} \end{bmatrix}\begin{bmatrix} 0 \\ E_a \\ 0 \end{bmatrix} - \begin{bmatrix} I_{a0} \\ I_{a1} \\ I_{a2} \end{bmatrix} \tag{6.29}$$

Pre multiplying (6.29) with $\begin{bmatrix} 1 & 1 & 1 \end{bmatrix}$

$$\frac{E_a}{Z_0} - I_a\frac{Z_1}{Z_0} + \frac{E_a}{Z_1} - I_a\frac{Z_1}{Z_1} + \frac{E_a}{Z_2} - I_a\frac{Z_1}{Z_2} = \frac{E_a}{Z_1} + \left(\underbrace{I_{a0} + I_{a1} + I_{a2}}_{I_a}\right) \tag{6.30}$$

Solving (6.30) we get

$$I_{a1} = \frac{E_a}{Z_1 + \left(Z_2 \| Z_0\right)} \tag{6.31}$$

The equation (6.31) shall be satisfied if the negative sequence and zero sequence impedances are in parallel and the combined circuit is in series with positive sequence impedance. The resulting circuit is across a voltage of 'E_a' as shown in Fig.-6.9. This suggests that for a double line to ground fault the zero sequence and negative sequence network shall be connected in parallel and the combined circuit is in series with positive sequence network, at the point of fault in a power system.

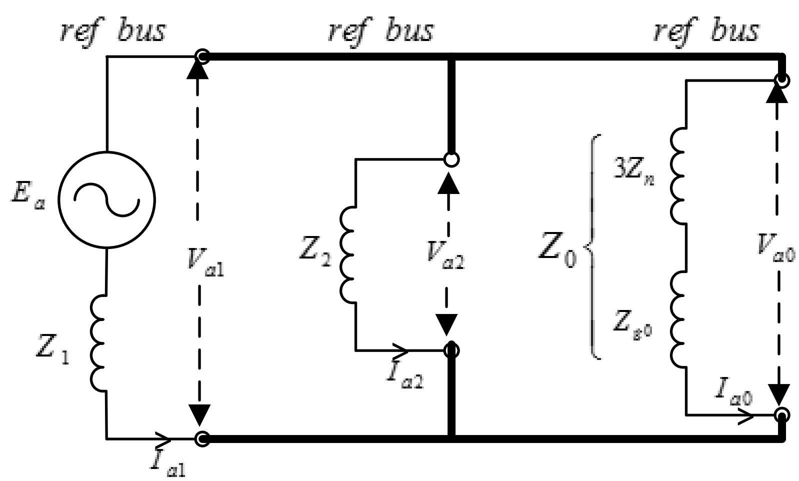


FIG.-6.9 EQUIVALENT NETWORK FOR UNLOADED GENERATOR HAVING DOUBLE LINE TO GROUND FAULT

The method described above can be used for calculation of fault current for power system. A digital computer program can also be written for this algorithm. The students are advised to implement this algorithm through computer program.

CHAPTER-7

CONTROL OF ACTIVE POWER

Active power control is closely related to frequency control and reactive power control is closely related to voltage control.

In a network considerable drop in frequency could result in high magnetizing currents in induction motor and transformer. The frequency of a system depends on active power balance. The control of generation and frequency is commonly referred to as load frequency control (LFC). The conventional generator with governor control is shown in Fig.-7.1.

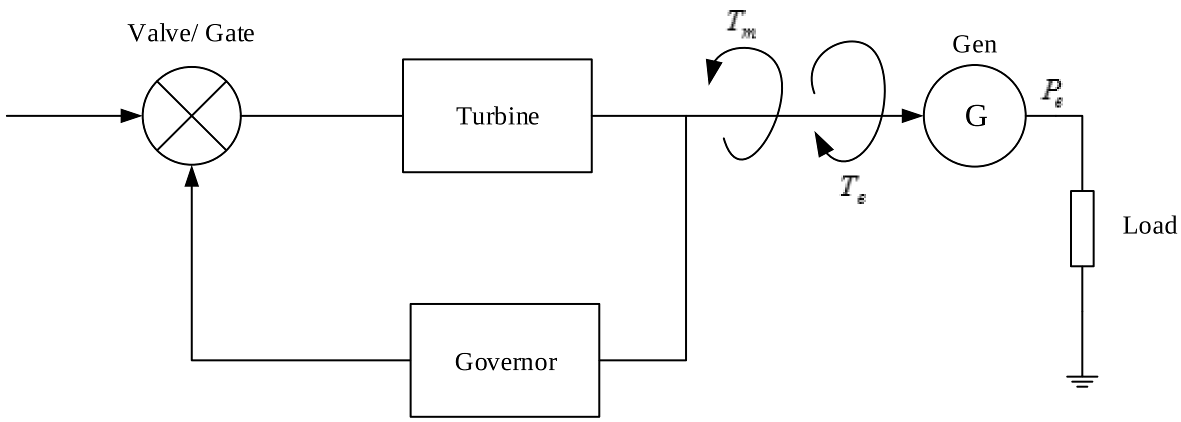


FIG.-7.1 SCHEMATIC BLOCK DIAGRAM OF ACTIVE POWER CONTROL WITH FEEDBACK

GENERATOR RESPONSE TO LOAD CHANGE

The generator model gives the relation between the change in angular speed for a load change. The swing equation represents the change in angular speed with respect to a load change and thus change in accelerating power or accelerating torque. This equation is represented in Fig.-7.2.

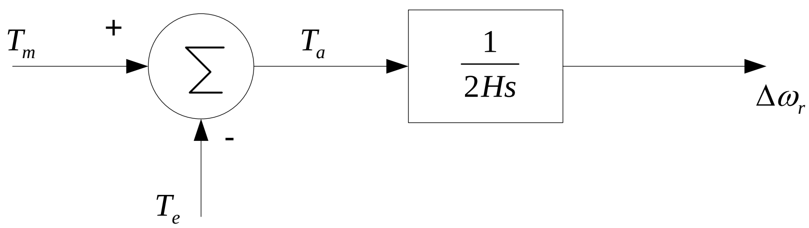


FIG.-7.2 BLOCK DIAGRAM OF GENERATOR

The relation between speed and torque is given by (7.1)

$$P = \omega_r T \tag{7.1}$$

For a change in load by ΔP, the power can be written as (7.2)

$$P = P_0 + \Delta P \tag{7.2}$$

Thus the torque and angular speed is modified as (7.3) & (7.4)

$$T = T_0 + \Delta T \tag{7.3}$$

$$\omega_r = \omega_0 + \Delta\omega_r \tag{7.4}$$

Where P_0, T_0, ω_0 are the initial power, torque and speed respectively. From (7.1) we have

$$P_0 + \Delta P = (\omega_0 + \Delta\omega_r)(T_0 + \Delta T) \tag{7.5}$$

$$\Delta P = \omega_0 \Delta T + T_0 \Delta\omega_r \tag{7.6}$$

$$\Delta P_m - \Delta P_e = \omega_0(\Delta T_m - \Delta T_e) + (T_{m0} - T_{e0})\Delta\omega_r \tag{7.7}$$

Since at steady state electrical and mechanical torques are equal i.e. $T_{m0} = T_{e0}$ and with speed expressed in p.u. i.e. $\omega_0 = 1$

$$\Delta P_m - \Delta P_e = \Delta T_m - \Delta T_e \tag{7.8}$$

Fig.-7.3 represents the equation (7.8).

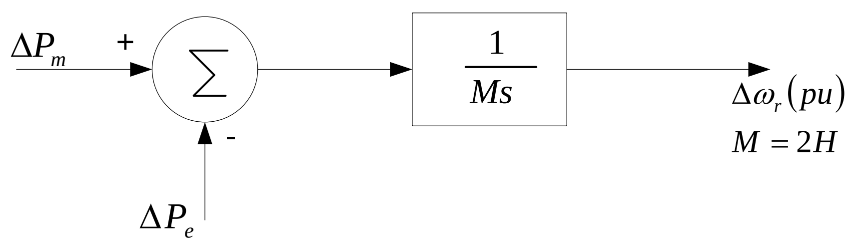


FIG.-7.3 BLOCK DIAGRAM OF GENERATOR

LOAD RESPONSE TO FREQUENCY DEVIATION

Power system loads are composite of a variety of electrical devices. For resistive loads the electrical power is independent of frequency. The overall frequency dependent characteristics of composite load may be expressed as (7.9)

$$\Delta P_e = \Delta P_l + D\Delta\omega_r \tag{7.9}$$

Where,

ΔP_l - max frequency sensitive load changes

$D\Delta\omega_r$ - Frequency sensitive load change

D - load damping constant

$D = 2$ means 1% change in frequency will cause 2% change in load

Combining (7.9) and (7.8) and repreenting in bloack diagram shown in Fig.-7.4. The Fig.-7.4 can be reduced to Fig.-7.5 as shown where load damping constant D is combined with the machine constatnt.

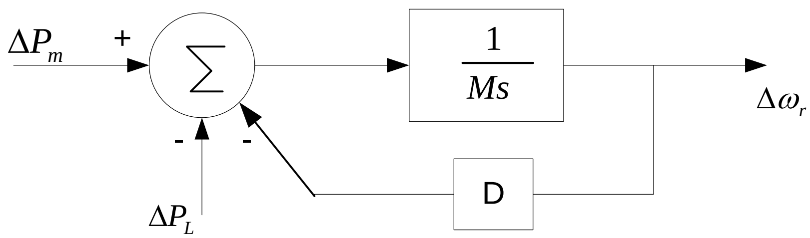


FIG.-7.4 BLOCK DIAGRAM OF GENERATOR WITH LOAD DAMPING

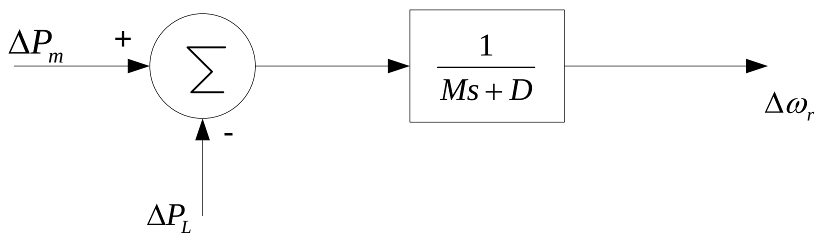
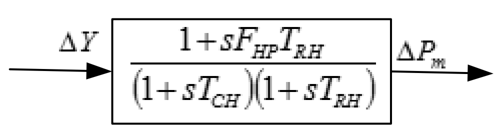


FIG.-7.4 BLOCK DIAGRAM OF GENERATOR WITH LOAD DAMPING

In the absence of speed governor the system response to a load change is determined by the inertia constant and the damping constant.

TURBINE MODEL

Within the speed range of concern, the turbine mechanical power is essentially a function of valve or gate position and independent of frequency. The turbine may be steam-turbine or hydro-turbine. The transfer function of turbine may be represented by simple transfer function having single time constant. The steam turbine may be reheat type or non-reheat type.



Turbine
Reheat type

FIG.-7.5 TURBINE MODEL

ISOCHRONOUS GOVERNOR

Isochronous means constant speed. Isochronous Governor adjusts the turbine valve to bring the frequency to the nominal or scheduled.

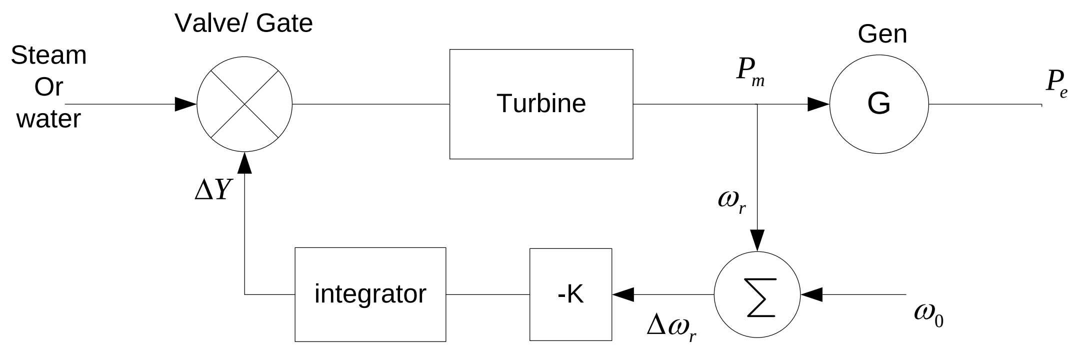


FIG.-7.6 GOVERNOR MODEL

An increase in P_e causes frequency to decay at a rate determined by inertia of rotor. As speed drops the P_m increases. Increase in speed gives $P_m > P_e$ and Speed return to reference value. The steady state turbine power increases by an amount equal to additional load. Isochronous Governor works satisfactorily for a single isolated load or if one generator is required to take change in load.

For a multi generator case (i. e. for load sharing) generator speed regulation or droop characteristics must be provided.

GOVERNOR WITH SPEED DROOP CHARACTERISTICS

The block diagram of governor with speed droop characteristics is provided in Fig.-7.7. Which can be simplified to Fig.-7.8 and then Fig.-7.9. This type of governor is characterized as a proportional controller with a gain of $\frac{1}{R}$.

Where $T_G = \frac{1}{KR}$.

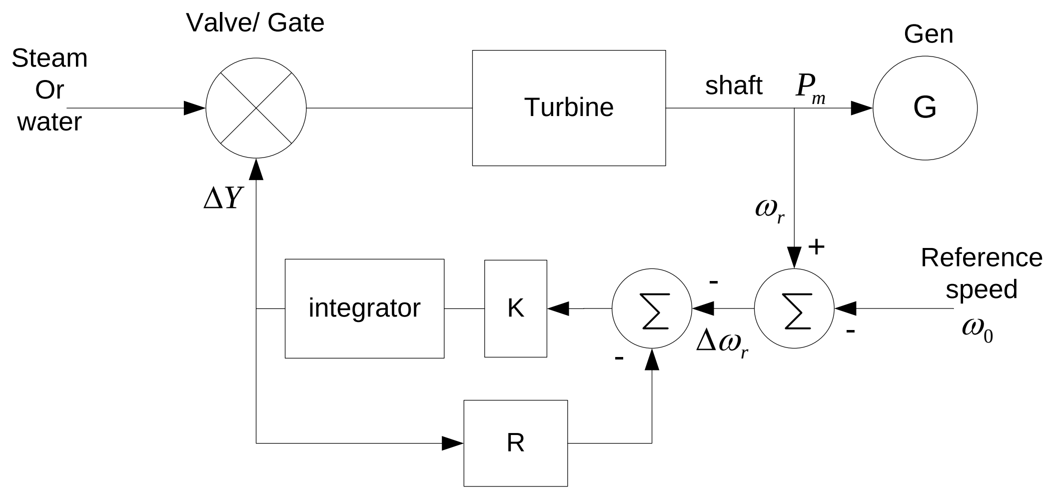


FIG.-7.7 BLOCK DIAGRAM OF GOVERNOR WITH STEADY STATE FEEDBACK:

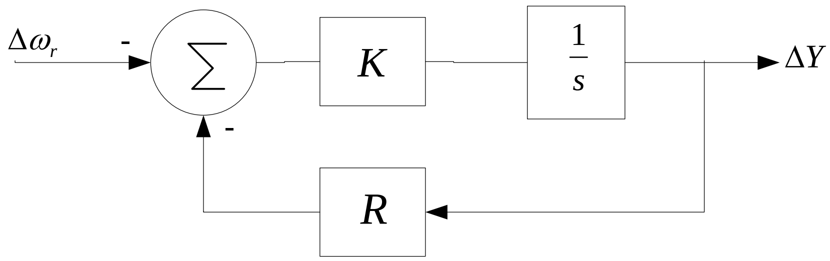


FIG.-7.8 SIMPLIFIED BLOCK DIAGRAM OF GOVERNOR WITH STEADY STATE FEEDBACK:

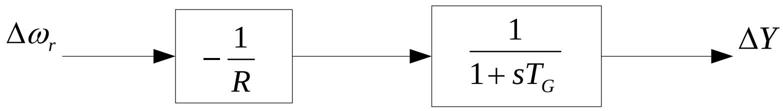


FIG.-7.9 SIMPLIFIED BLOCK DIAGRAM OF GOVERNOR WITH STEADY STATE FEEDBACK:

SPEED REGULATION OR DROOP

The % Speed regulation or droop is given by (7.10). The value of R determines the steady state speed versus load characteristic of generator unit as shown in Fig.-7.10.

$$\%R = (\%\text{ speed or frequency change})/(\%\text{ power output change}) \times 100 \qquad (7.10)$$

$$= \frac{\omega_{NL} - \omega_{FL}}{\omega_0} \times 100$$

ω_{NL} & ω_{FL} - Steady state speed at no load and full load

ω_0 - Nominal or rated speed

But ratio of frequency deviation to power output deviation is known as R.

$$R = \frac{\Delta f}{\Delta P} \qquad (7.11)$$

$$\Delta f = f - f_0 \tag{7.12}$$

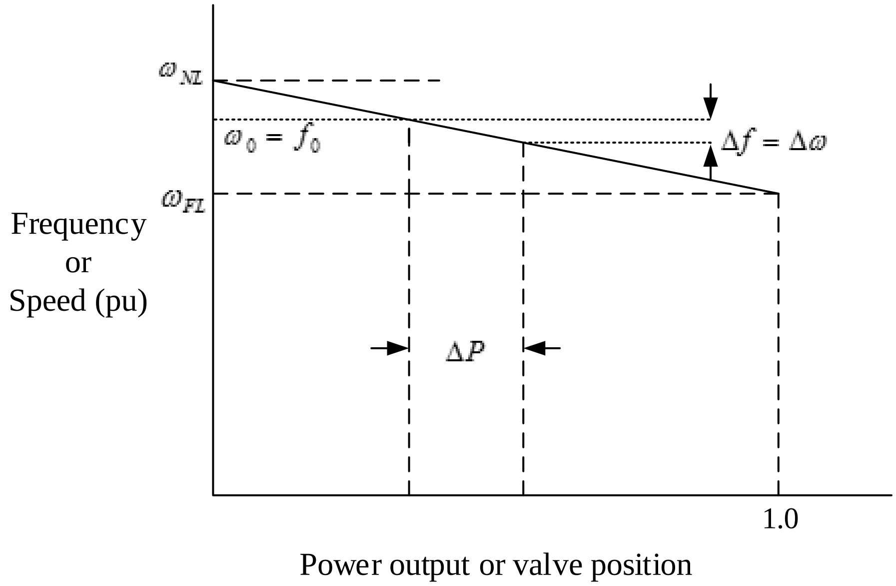


FIG.-7.10 IDEAL STEADY STATE CHARACTERISTICS OF GOVERNOR WITH SPEED DROOP

The 5% droop or regulation means 5% frequency deviation causes 100% change in valve position or power output

LOAD SHARING BY THE UNITS

Let us consider two generators having with droop chaaracteristics share a load change in a power system. The frequency of both can be taken as equal as they belong to same system. Let the load increases by ΔP_L . It shall cause slow down of the generators and the governor has to increase the output till they reach a new common frequency. It can be seen that change in frequency for both the generators has to be same as shown in Fig.-7.11.

$$\Delta P_1 = P_1^{'} - P_1 = \frac{\Delta f}{R_1} \tag{7.13}$$

$$\Delta P_2 = P_2^{'} - P_2 = \frac{\Delta f}{R_2} \tag{7.14}$$

$$\frac{\Delta P_1}{\Delta P_2} = \frac{R_2}{R_1} \tag{7.15}$$

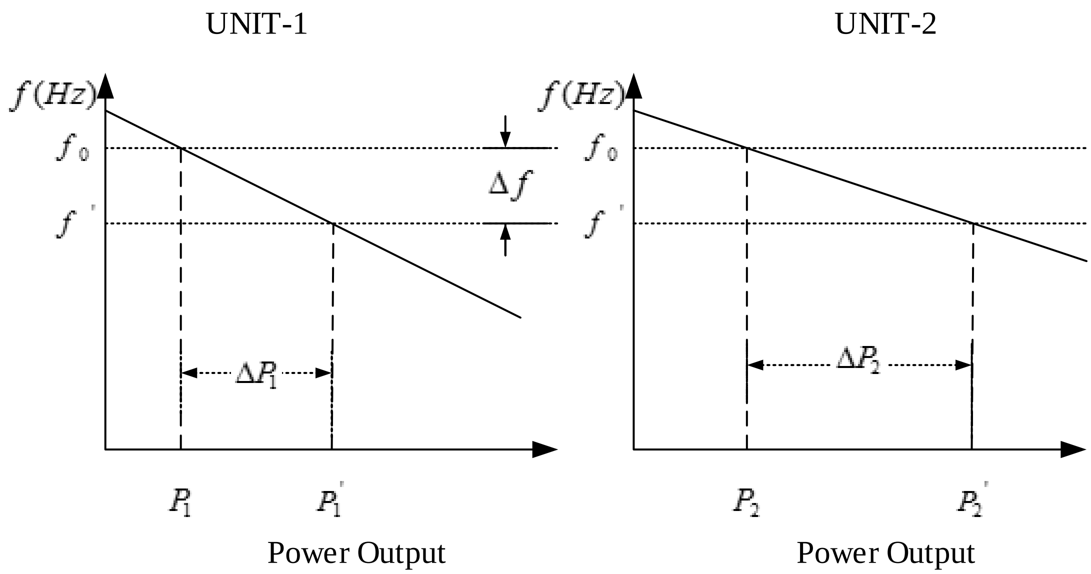


FIG.-7.11 LOAD SHARING BETWEEN TWO GENERATORS HAVING DIFFERENT REGULATION

If the % of regulation of the units are equal, the change in the outputs will be nearly in proportion to its rating. Because of the droop characteristics the increase in power output is accompanied by a steady state speed or frequency deviation $(\Delta\omega_{ss})$

CONTROL OF GENERATOR UNIT POWER OUTPUT

The relationship between speed and load can be adjusted by changing an input shown as load reference set point Fig.-7.12.

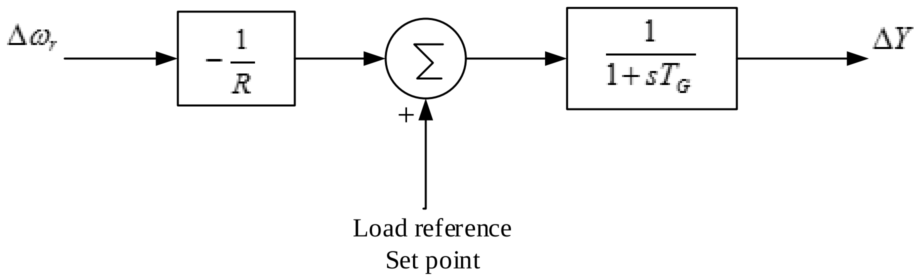


FIG.-7.12 BLOACK DIAGRAM OF GOVERNOR WITH LOAD REFERENCE CONTROL

When two or more generating units are operating in parallel the speed droop characteristic (corresponding to a load reference setting) of each generator unit merely establishes the proportion of the load picked up by the unit when a sudden change in system load occurs. The output of each unit at any given system frequency can be varied only by changing its load reference, which in effect moves the speed droop characteristic up and down.

ACTUAL SPEED DROOP CHARACTERISTIC

In actual practice, the characteristic departs from the straight line relationship as shown in Fig.-7.13. Steam turbines have a number of valves each having non-linear flow area versus position characteristic. Hence, they have the speed droop characteristic of general nature of curves. The hydraulic turbines, which have a single gate, tend to have the characteristic similar to curve. The actual speed droop characteristic may exhibit incremental regulation ranging from 2% to 12%. Depending on the unit output modern EHG systems minimize these variations in incremental regulation by using linearizing circuits or first stage pressure feedback.

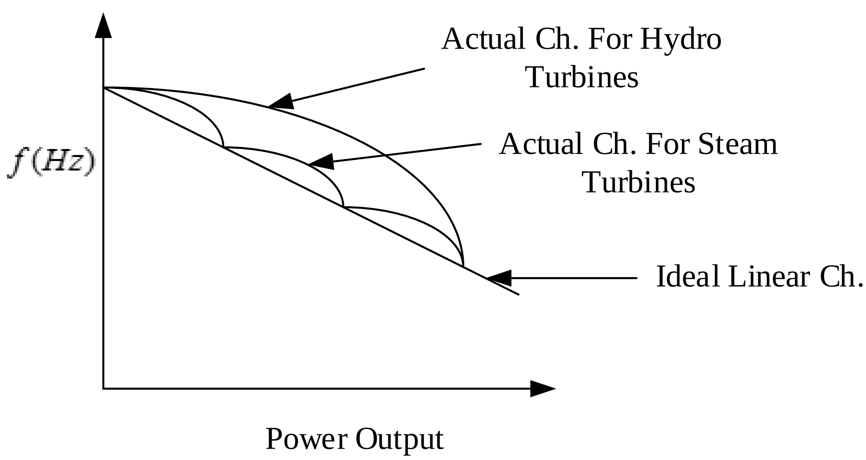


FIG.-7.13 SPEED FROOP CHARACTERISTICS OF DIFFERENT GOVERNOR

COMPOSITE REGULATING CHARACTERISTIC OF POWER SYSTEM

In load frequency control we are interested in the collective performance of all generators in power system. The inter machine oscillations and transmission system performance are therefore not considered. All the generators as well as loads can be represented by a single equivalent as shown in Fig.-7.14 as M_{eq}, D respectively.

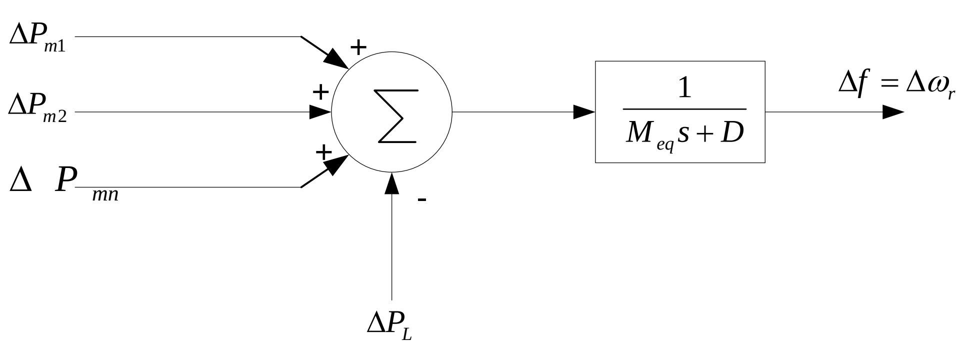


FIG.-7.14 EQUIVALENT SYSTEM GENERATORS AND LOAD

The composite power or frequency characteristic of a power system thus depends upon the combined effect of droops of all generator speed governors. It also depends on the frequency characteristic of all the loads in the system. The steady state frequency deviation following a load change ΔP_L for the equivalent system can be given by

$$\Delta f_{ss} = \frac{-\Delta P_L}{\left(\frac{1}{R_1} + \frac{1}{R_2} + \cdots\cdots\cdots + \frac{1}{R_n}\right) + D} \tag{7.16}$$

$$\Delta f_{ss} = \frac{-\Delta P_L}{\frac{1}{R_{eq}} + D} \tag{7.17}$$

Where

$$R_{eq} = \left(\frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \cdots\cdots\cdots + \frac{1}{R_n}}\right) \tag{7.18}$$

Thus the composite frequency response characteristic of the system is represented by

$$\beta = \frac{-\Delta P_L}{\Delta f_{ss}} = \frac{1}{R_{eq}} + D \ \text{(MW/Hz)} \tag{7.19}$$

It is also known as stiffness of the system and $\frac{1}{\beta}$ as the composite regulating characteristic of system.

TURBINE-GOVERNOR SYSTEM

The complete turbine governing system is now shown in Fig.-7.15

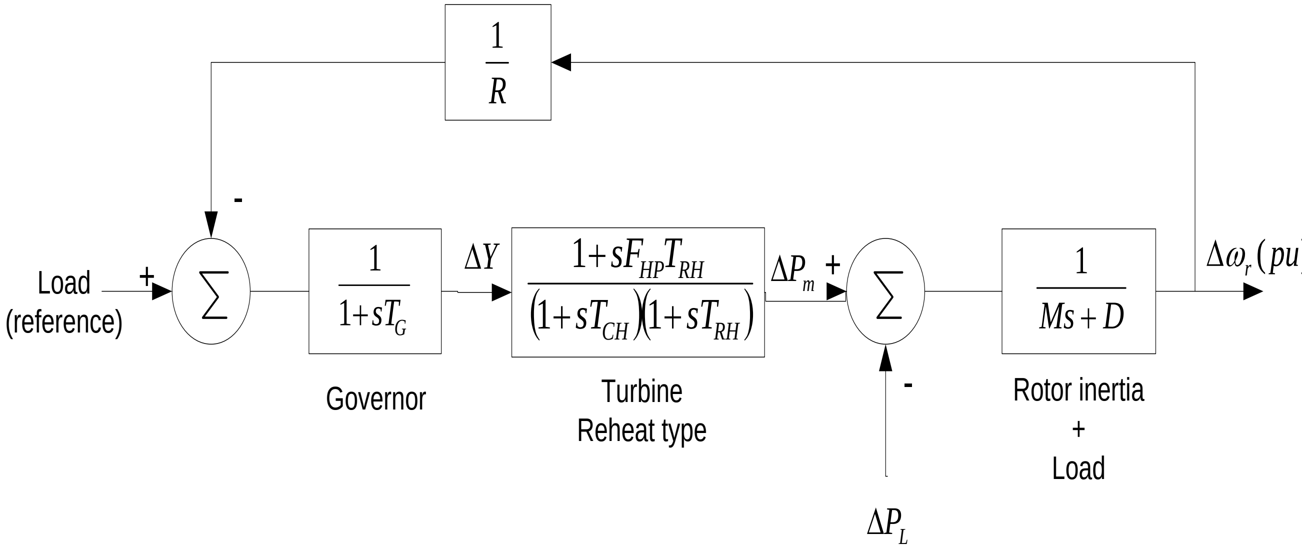


FIG.-7.15 COMPLETE BLOCK DIAGRAM OF GENERATOR WITH TURBINE GOVERNING SYSTEM

For reheat type turbine $T_{RH} = 0$

Governor of hydraulic units require transient droop compensation shown in Fig.-7.16 for stable speed control performance.

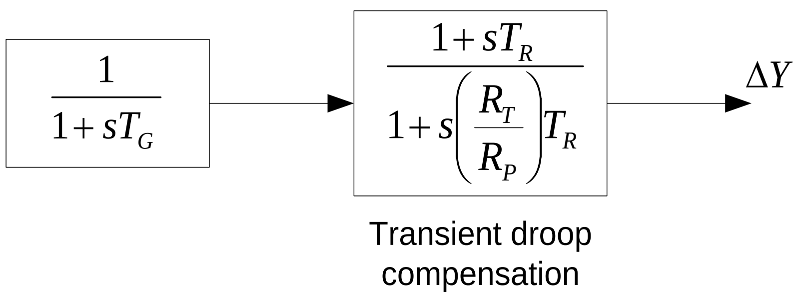


FIG.-7.16 TRANSIENT DROOP COMPENSATION OF HYDRAULIC GOVERNOR

AUTOMATIC GENERATION CONTROL

The primary objectives of AGC are to regulate frequency to the specified nominal values and to maintain the interchange power between control areas at the scheduled values by adjusting the output of selected generator. This function is referred as (LFC) A secondary objective is to distribute the required change in generation among units to minimize the operating cost.

INTERCONNECTED POWER SYSTEM OR MULTI AREA CONTROL

Let us consider two area of power system connected by means of tie line. The electrical equivalent of the system is shown in Fig.-7.17 and control block diagram is Fig.-7.18

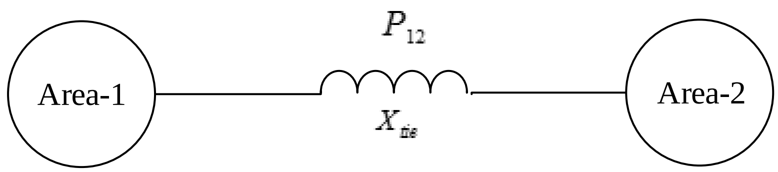


FIG.-7.17 ELECTRICAL EQUIVALENT OF TWO INTERCONNECTED POWER SYSTEM

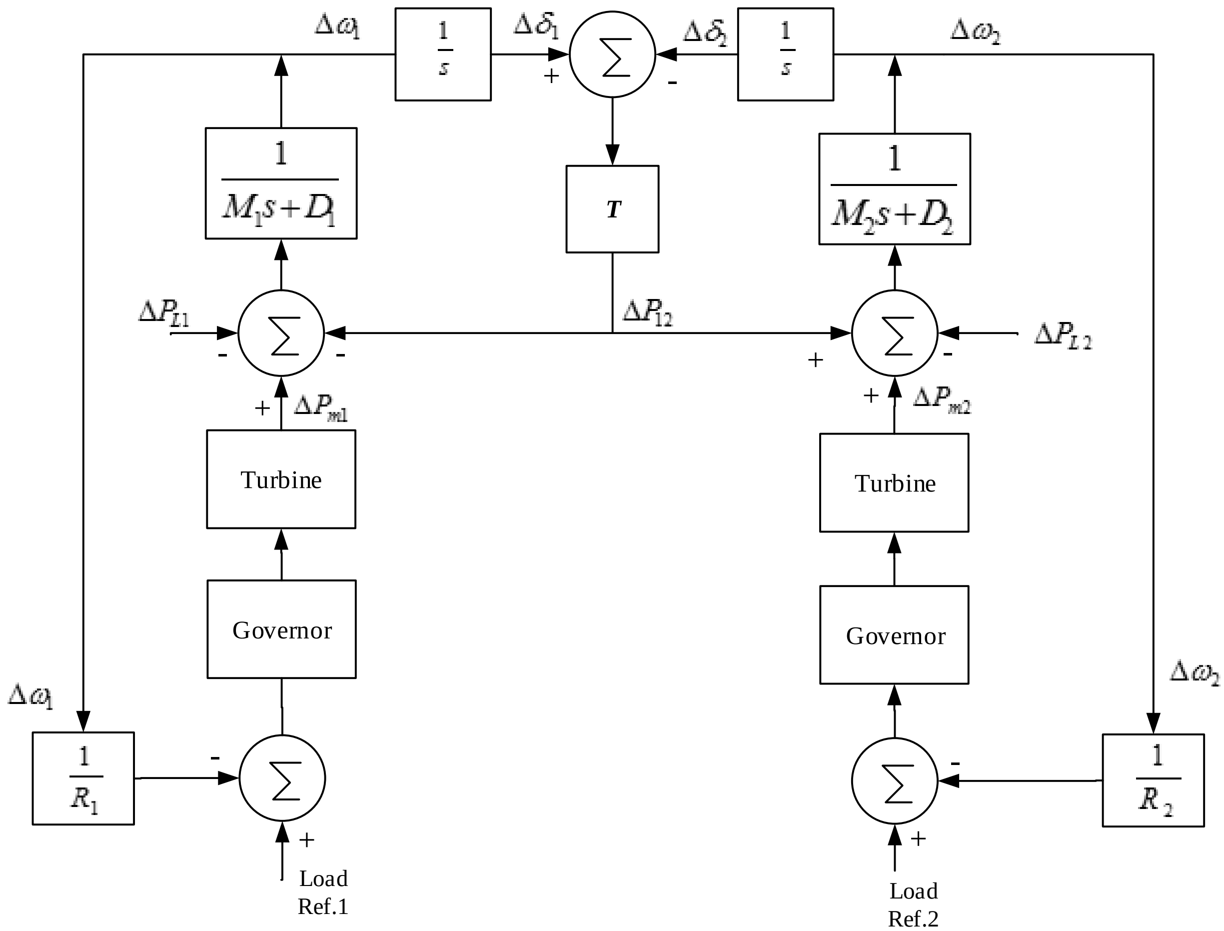


FIG.-7.17 BLOCK DIAGRAM FOR CONTROL OF TWO INTERCONNECTED POWER SYSTEM FOR PRIMARY SPEED CONTROL

The tie line power flow can be given by (7.20)

$$P_{12} = \frac{E_1 E_2}{X_T} \sin(\delta_1 - \delta_2) \tag{7.20}$$

Linearizing about an initial operating point on the power angle curve i.e.

$\delta_1 = \delta_{10}$, $\delta_2 = \delta_{20}$,

We have

$$\Delta P_{12} = T\Delta\delta_{12} \tag{7.21}$$

$$\Delta\delta_{12} = \Delta\delta_1 - \Delta\delta_2 \tag{7.22}$$

Where *T*- Synchronous torque coefficient is given by

$$T = \frac{E_1 E_2}{X_T}\cos(\delta_{10} - \delta_{20}) \tag{7.23}$$

A positive ΔP_{12} represents an increase in power transfer from area-1 to area-2 means increasing the load of area-1 and decreasing the area-2. Therefore, the feedback for ΔP_{12} is negative for area-1 and positive for area-2. The steady state frequency deviation $(f - f_0)$ is the same for two areas for a total load change of ΔP_L and given by (7.24)

$$\Delta f = \Delta\omega_1 - \Delta\omega_2 = \frac{-\Delta P_L}{\left(\frac{1}{R_1} + \frac{1}{R_2}\right) + (D_1 + D_2)} \tag{7.24}$$

Consider the steady state values following an increase in area-1 load by ΔP_{L1}.

For area-1, we have

$$\Delta P_{m1} - \Delta P_{12} - \Delta P_{L1} = \Delta f D_1 \tag{7.25}$$

And for area-2,

$$\Delta P_{m2} + \Delta P_{12} = \Delta f D_2 \tag{7.26}$$

The change in mechanical power depends on regulation. Hence

$$\Delta P_{m1} = -\frac{\Delta f}{R_1} \tag{7.27}$$

$$\Delta P_{m2} = -\frac{\Delta f}{R_2} \tag{7.28}$$

$$\Delta f\left(\frac{1}{R_1} + D_1\right) = -\Delta P_{12} - \Delta P_{L1} \tag{7.29}$$

And $$\Delta f\left(\frac{1}{R_2} + D_2\right) = \Delta P_{12} \tag{7.30}$$

Solving above equations,

$$\Delta f = \frac{-\Delta P_{L1}}{\left(\frac{1}{R_1}+D_1\right)\left(\frac{1}{R_2}+D_2\right)} = -\frac{\Delta P_{L1}}{\beta_1+\beta_2} \tag{7.31}$$

$$\Delta P_{12} = \frac{-\Delta P_{L1}\left(\frac{1}{R_2}+D_2\right)}{\left(\frac{1}{R_1}+D_1\right)\left(\frac{1}{R_2}+D_2\right)} = -\frac{\Delta P_{L1}\beta_2}{\beta_1+\beta_2} \tag{7.32}$$

An increase in area-1 load by ΔP_{L1} results in a frequency reduction in both areas and a tie line flow of ΔP_{12}. Similarly, for a change in area-2 load by ΔP_{L2}, We have

$$\Delta f = -\frac{\Delta P_{L2}}{\beta_1+\beta_2} \tag{7.33}$$

$$\Delta P_{12} = -\Delta P_{21} = -\frac{\Delta P_{L2}\beta_1}{\beta_1+\beta_2} \tag{7.34}$$

FREQUENCY BIAS TIE LINE CONTROL

The basic objective of supplementary control is to restore balance between each area load and generations.

Frequency at schedule value

Net interchanging power with neighbouring area at scheduled value.

The supplementary control in a given area should ideally correct only for change in that area. Thus, A control signal made up of tie line flow deviation added to frequency deviation weighted by a bias factor would accomplish the desired objectives. Thus the control signal is known as area control error (ACE).

A suitable bias factor is its composite frequency response characteristic β.

Thus ACE for area-2 is

$$ACE_2 = \Delta P_{21} + B_2\Delta f \tag{7.35}$$

Where,

$$B_2 = \beta_2 = \frac{1}{R_2} + D_2 \tag{7.36}$$

Similarly, for area-1,

$$ACE_1 = \Delta P_{12} + B_1 \Delta f \tag{7.37}$$

Where,

$$B_1 = \beta_1 = \frac{1}{R_1} + D_1 \tag{7.38}$$

ACE represents the required change in area generation and its unit is MW.

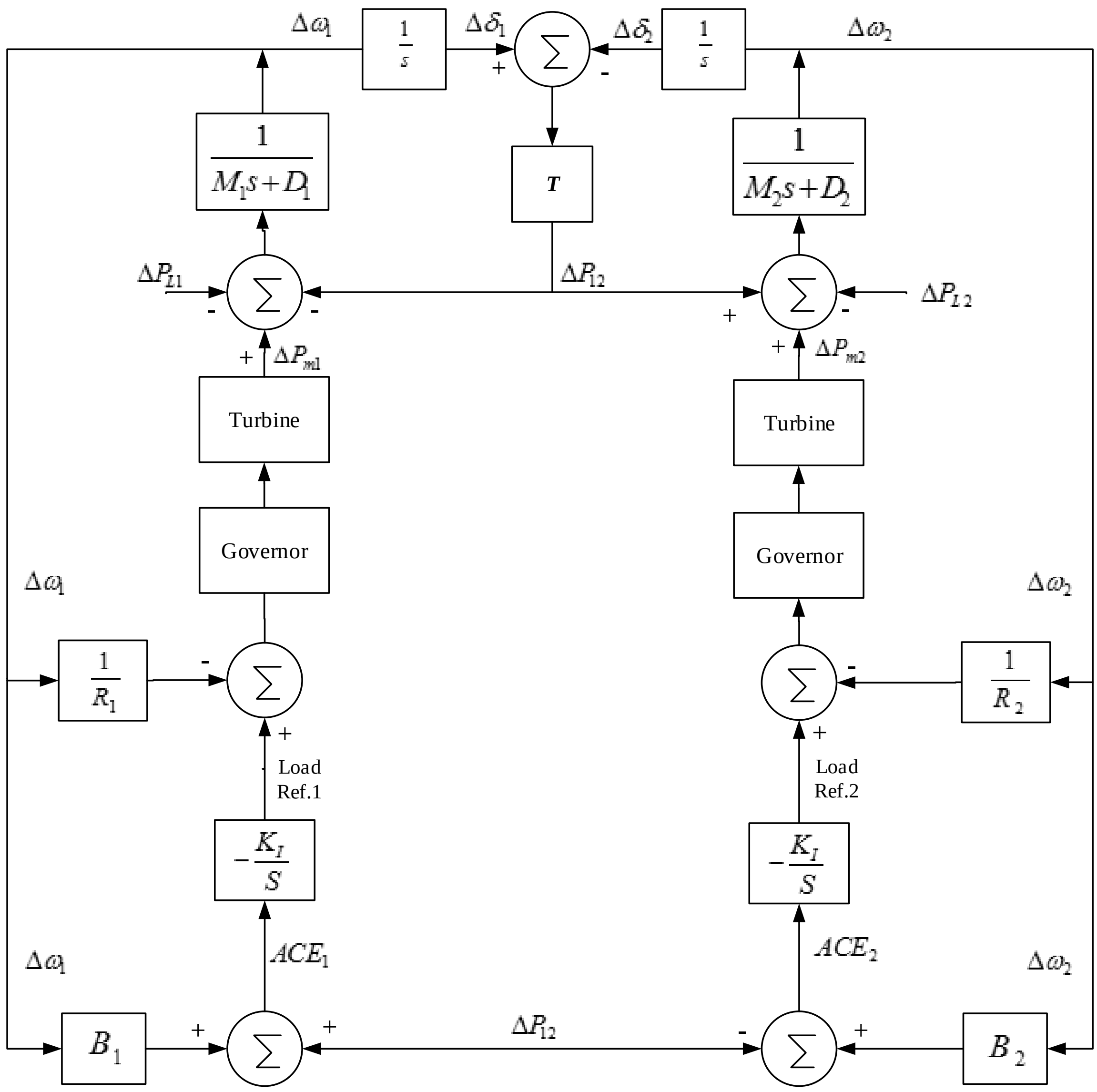


FIG.-7.18 BLOCK DIAGRAM FOR CONTROL OF TWO INTERCONNECTED POWER SYSTEM WITH SUPPLEMENTARY CONTROL

Basis for selection of bias factor

$$ACE_1 = A_1 \Delta P_{12} + B_1 \Delta f = 0 \tag{7.39}$$

$$ACE_2 = A_2 \Delta P_{21} + B_2 \Delta f = 0 \tag{7.40}$$

The above equations results in $\Delta P_{12} = 0$ and $\Delta f = 0$

If the load sudden increases $\Rightarrow$ Frequency decreases $\Rightarrow$ Generator respond $\Rightarrow$ Frequency deviation determined by the regulation characteristics of both systems

$$\Delta f_R = -\frac{\Delta P_{L1}}{\beta_1 + \beta_2} \tag{7.41}$$

The supplementary control which is slower than the primary speed control will now commence responding.

With $B_1 = \beta_1$ and $B_2 = \beta_2$

$$ACE_1 = \Delta P_{12} + B_1 \Delta f_R \tag{7.42}$$

$$ACE_1 = -\frac{\Delta P_{L1}}{\beta_1 + \beta_2}\left(\beta_1 + B_2\right) \tag{7.43}$$

$$ACE_1 = -\Delta P_{L1} \tag{7.44}$$

$$ACE_2 = \Delta P_{21} + B_2 \Delta f_R \tag{7.45}$$

$$ACE_2 = -\frac{\Delta P_{L1}}{\beta_1 + \beta_2}\left(-\beta_2 + B_2\right) \tag{7.46}$$

$$ACE_2 = 0 \tag{7.47}$$

Only supplementary control in area-1 will respond to ΔP_{L1} and change generation so as to bring ACE to zero. The load change in area-1 is thus unobservable to supplementary control in area-2.

If B_1 and B_2 were set to double their respective area frequency response characteristic.

$$ACE_1 = \Delta P_{12} + B_1 \Delta f_R = -\frac{\Delta P_L}{\beta_1 + \beta_2}\left(2\beta_1 + \beta_2\right) \tag{7.48}$$

$$ACE_1 = -\Delta P_{L1}\left(1 - \frac{1}{\beta_2}\right) \tag{7.49}$$

$$ACE_2 = -\Delta P_{12} + 2B_2 \Delta f_R = -\frac{\Delta P_{L1}}{\beta_2} \tag{7.50}$$

Thus, both area supplementary control would respond and correct the frequency deviation twice as fast.

CONCLUSION

This class note has been prepared for the use by the students as a supplement to this course. The students are advised to through this notes along with the progress of the subject in the classes. In the mean time the students must refer the prescribed text books and reference books. The author wishes best of luck to the students.