Mechanics of Materials

Direct Stress :-

1.1 Load : A body/members subjected to external forces, which is called Load on the member.

→ Since the member is in equilibrium the resultant of all the forces acting on it must be zero, but they produce a tendency for the body to be displaced/deformed.

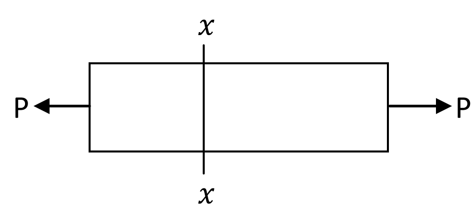
→ The displacement/deformation is resisted by the internal forces of cohesion between particles of material

→ Direct pull/push is the simplest type of Load.

→ Unit of Load. (Kg/Newton)

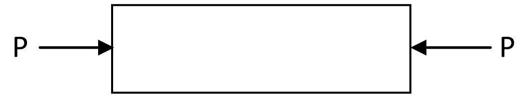
1Kg = 9.81 Newton (N)

Tension



Ex Steel wire of a crane hook
(rope)

Compression



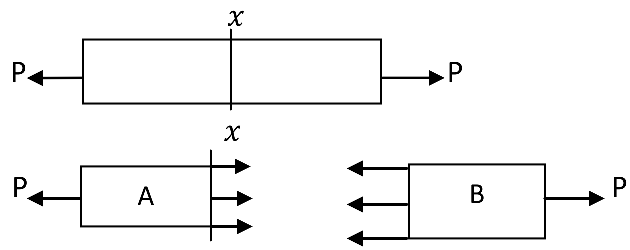
Ex Leg of a table.

→ In both cases, two equal & opposite forces act on a member & tendency to deform/fracture it,

→ Here, the member is in static condition.

→ It the member is in motion, Load may be due to dynamic action/inertia forces.

1.2 Stress :-



→ The force is distributed among the internal forces of cohesion, which is called stresses.

→ Cut the member through section 'XX'.

→ Each portion (A & B) is in equilibrium under the action of external Load 'P' & stresses at 'X'x.

→ Stresses which are normal to the plane on which they act are called direct stresses.

(tensile/compressive)

→ Force transmitted across any section divided by the area of that section is called intensity of stress or stress.

$$\sigma = P/A$$

1.3 Principle of St. Venant :- It states that the actual distribution of Load over the surface of its application will not affect the distribution of stress or strain on sections of body which are at an appreciable distance.

Ex- For points in rod distant more than 3-times its greatest width from the area of Loading

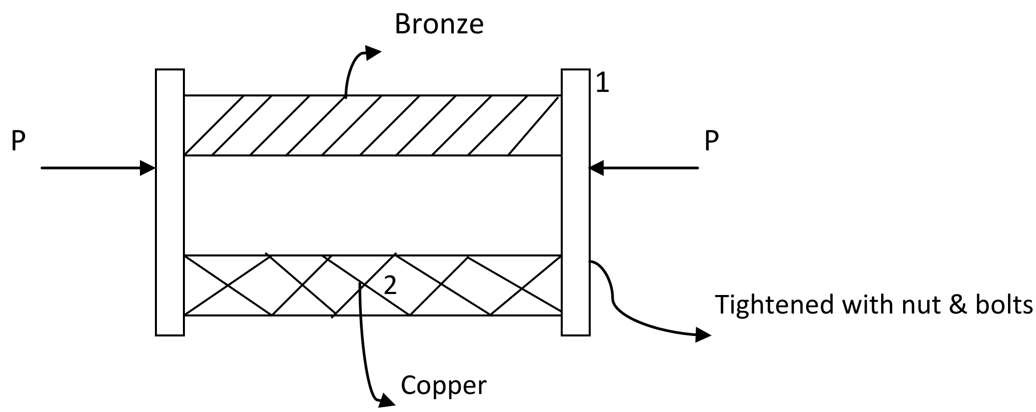
1.4 strain

Longitudinal Strain = $\left(\frac{\sigma l}{l}\right)$

1.5 Modulus of elasticity

$$E = \frac{\left(P/_A\right)}{\left(\frac{\sigma l}{l}\right)} = \left(\frac{Pl}{A\sigma l}\right)$$

1.6 Compound Bars



Both bars have same initial length & after deformation the two bars must remain together.

→ Therefore, strain in each part must be same.

→ Stress developed in each part (bar) will be different. Hence, the Load shared each bar will be different.

→ Suppose Loads are w_1, w_2 & corresponding area of $C/_S$ are A_1 & A_2

(strain)$_1$ = (strain)$_2$

$$\Rightarrow \boxed{\frac{W_1}{A_1E_1} = \frac{W_2}{A_2E_2}} \quad .... (1)$$

→ Total Load ; $\boxed{P = W_1 + W_2}$

$$\Rightarrow P = W_1 + \left(\frac{A_2}{A_1}, \frac{E_2}{E_1}\right) W_1$$

$$\Rightarrow P = W_1 \left(1 + \frac{A_2 E_2}{A_1 E_1}\right)$$

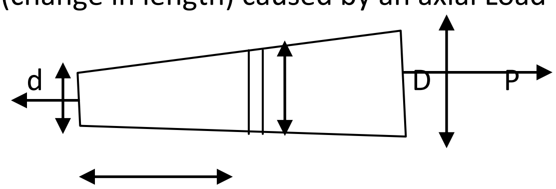
$W_1 = ? = \left(\frac{P A_1 E_1}{A_1 E_1 + A_2 E_2}\right)$

$W_2 = ? =$

*A tensile or compressive member which consists of two or more bars/tubes in parallel, of different materials is called a 'compound bar'.

Q – 1 A rod of length 'l' tapers uniformly from a diameter 'D' at one end to a diameter 'd' at the other. Find the extension (change in length) caused by an axial Load 'P'.

Ans



→ Diameter of rod at a distance of 'x' from smaller end is

$$D_x = \left[d + \frac{(D-d)x}{l}\right]$$

→ The extension/elongation of a short length 'dx' is

$$= \frac{P.dx}{\left(\frac{\pi}{4} D_x^2\right) E} = \left[\frac{4P d_x}{\pi D_x^2 E}\right]$$

Total elongation for the whole rod;

$$\sigma l = \int_o^l \frac{4P.dx}{\pi D_x^2 . E}$$

$$= \int_o^l \frac{4P.dx}{\pi\left[d + \frac{(D-d)x^2}{l}\right].E}$$

*take $\left[d + \frac{(D-d)x^2}{l}\right] = t$

$$= -\left(\frac{l}{D-d}\right)\cdot\frac{4P}{\pi E}\left[\frac{1^l}{d+\frac{(D-d)x}{l}}\right]$$

$$= \frac{4Pl}{\pi E(D-d)}\left(\frac{1}{d}-\frac{1}{D}\right)$$

$$=\left(\frac{4Pl}{\pi DdE}\right)$$

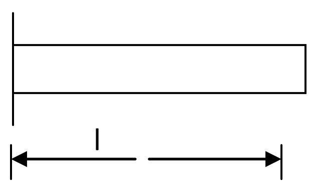
$$Strain = \left(\frac{\sigma l}{l}\right) = \left(\frac{4P}{\pi DdE}\right)$$

*Stress will vary as the cross – section (A) will vary from point to point.

1.7 Temperature stresses in bars :-

Let ‘L’ = Coefficient of thermal expansion for the material/bar.

→ When a bar is subjected to change in temperature, stress is developed in bar. These stresses are known as temperature stresses.



Temprature = t^0 C

Length of bar = l

Co-efficient of thermal expansion = L

→ Elongation/extension in bar = (L+l)

⇒ σl=L+l

*t= change in temperature

→σ = $\sum E$

$= \frac{\sigma l}{l} . E = (L + E)$

→ Consider two bars of different material having same initial length(l). If it will be subjected to change in temperature (l^0C) freely/independently; then their expansion will depend on the soccreponding thermal-expansion-coefficient.

Example.

Steel bar & copper bar

(L_s) → (L_c)

Expansion → $L_s tl$…………………$L_c tl$

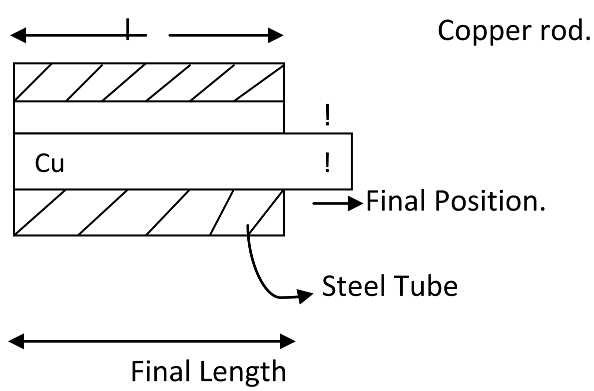
✓ Which bar will expand more, it will depend on coefficient-of-thermal-expansion.

→ Assume L_c = 18 x $10^{-6}/_{0C}$

L_s = 11 x $10^{-6}/_{0C}$

It shows that capper bar will expand more than steel bar it were free.

*For the case of compound bar, both will expand simultaneously so that final lengths will be same.



Equilibrium equation :-

Copper will be prevented from expanding its full amount & it is in compression & steel tube is in tension. Finally the compound bar takes an intermediate position. Therefore,

Compression in copper rod = Tension in steel tube

$\Rightarrow$ $\boxed{\sigma_{cu} A_{cu} = \sigma_s A_s}$

$$\begin{cases} \sigma_{cu} = Compressive\ stress\ in\ copper \\ \sigma_s = Tensile\ stress\ in\ steel \end{cases}$$

Compatibility equation :-

Initial & final lengths are same.

$\Rightarrow$ (Temperature strain of rod – compressive strain)

$(L_{cu} t)$

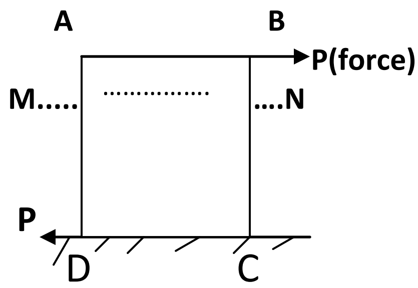
= (Temperature strain of tube + tensile strain)

$(L_s . t)$

$$\begin{cases} \text{Compressive strain} = \left(\frac{\sigma_{cu}}{E_{cu}}\right) \\ \text{Tensile strain} = \left(\frac{\sigma_s}{E_s}\right) \end{cases}$$

Shear Stress :-

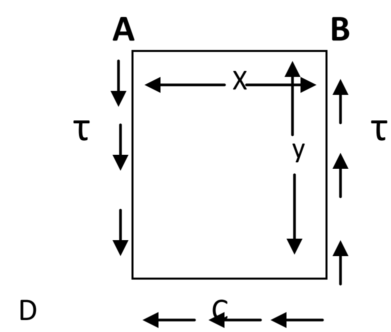
2.1 Shear Stress :-



→ Apply a force (P) tangentially on the face of 'AB'. Then, there is a tendency for one part of body to slide over/shear from other part across any section 'MN'.

→ If the C/S 'MN' is parallel to the Load (P) & say it 'A'; then τ (Shear stress) = $\frac{P}{A}$

2.2 Complementary Sheer Stress :-



→ Let there be a shear stress acting on planes AB & CD.

These stresses will form a couple equal to (τ.xz)y

A = (xz) = Area of 'AB' plane.

Force = τ x A

= τ x (xz)

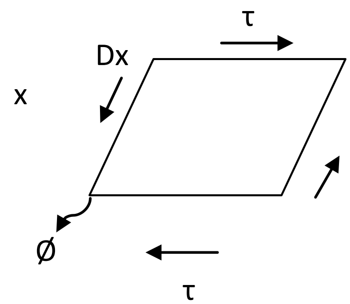
→ The couple can be balanced by tangential forces on plane 'AD' & 'BC' The stresses on that plane are called complementary shear stresses (τ')

$$(\tau.xz)\, y = (\tau'.yz)\, x$$

$$\Rightarrow \boxed{\tau' = \tau}$$

*Every shear stress is accompanied by an equal complementary shear stress on planes at right angles.

2.3 Shear strain :-



Ø = Change in right angles

$= \left(\frac{dx}{x}\right)$

For small value of 'Ø'

$\tan Ø = Ø = \frac{dx}{x}$

2.4 Modules of rigidity :-

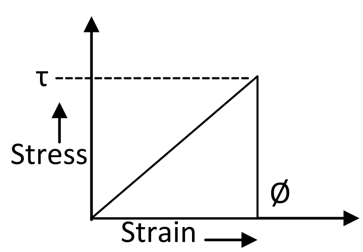
→ Shear strain is proportional to shear stress within certain Limit.

$$\frac{Shear\ Stress}{Sheer\ Strain} = G$$

$$\Rightarrow \boxed{\frac{\tau}{Ø} = G} \rightarrow N/_{mm^2}$$

Shear modules/modules of rigidity

2.5 Strain energy :-



Strain energy = Work done in straining

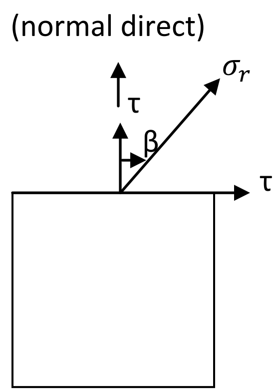
$= \frac{1}{2}$ (Final couple) x

(Angle turned through.)

Complex Stress / Compound Stress :-

3.1 Introduction –

→ If both direct & shear stress will act into a section, the resultant of both will be neither normal nor tangential to the plane.



angle = β(say)

Suppose ‘$б_r$’ is the resultant of normal stress (σ) & shear (τ); & it makes an angle ‘θ’ with the normal to the plane.

$$Б_r = \sqrt{\sigma^2 + \tau^2}$$

$$\text{Tan}\beta = \left(\frac{\tau}{\sigma}\right)$$

$$\Rightarrow \beta = tan^{-1}\left(\frac{\tau}{\sigma}\right)$$

3.2 Stress acting on a plane inclined θ^0 to the direction of the transverse section & $(90+\theta)^2$ to the Force :-

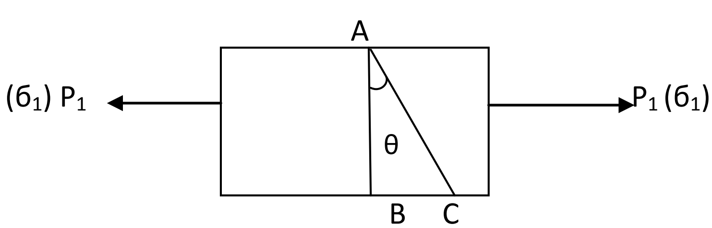
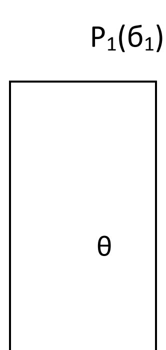
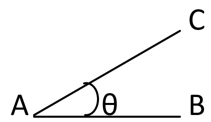
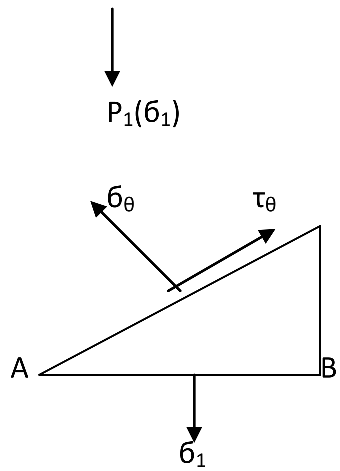


Fig (2)





Fig(3)



Fig(4)

→ Take a thickness of 't' ⊥ar to the figure-4.

→ Resolve the forces in the direction of 'б$_\theta$'

Б$_\theta$.AC.t = б$_1$.AB.t.cosθ

⇒ б$_\theta$ = б$_1$.$\left(\frac{AB}{AC}\right)$.cosθ

= б$_1 . \cos^2\theta$

→ Resolve the forces in the direction of 'τ_θ'

τ_θ.AC.t = б$_1$.AB.t.sinθ

↓ cos(90-θ)

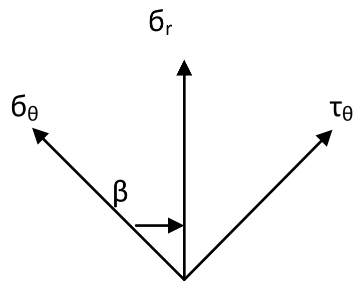
⇒ τ_θ = $\frac{б_1}{2}$ sin2θ

→ б$_r$ = Resultant stress

$= \sqrt{б_\theta^2 + \tau_\theta^2}$

$= б_1\sqrt{(cos^4\theta + cos^2\theta . sin^2\theta .)}$

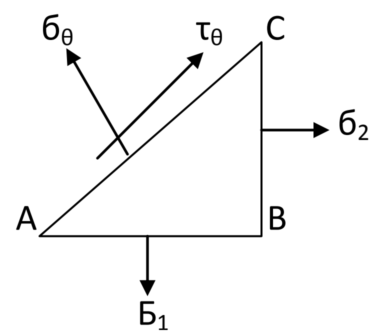
$= б_1 . cos\theta$



$tan_\beta = \left(\frac{\tau_\theta}{\sigma_\theta}\right)$

$\Rightarrow \beta = tan^{-1}\left(\frac{\tau_\theta}{\sigma_\theta}\right)$

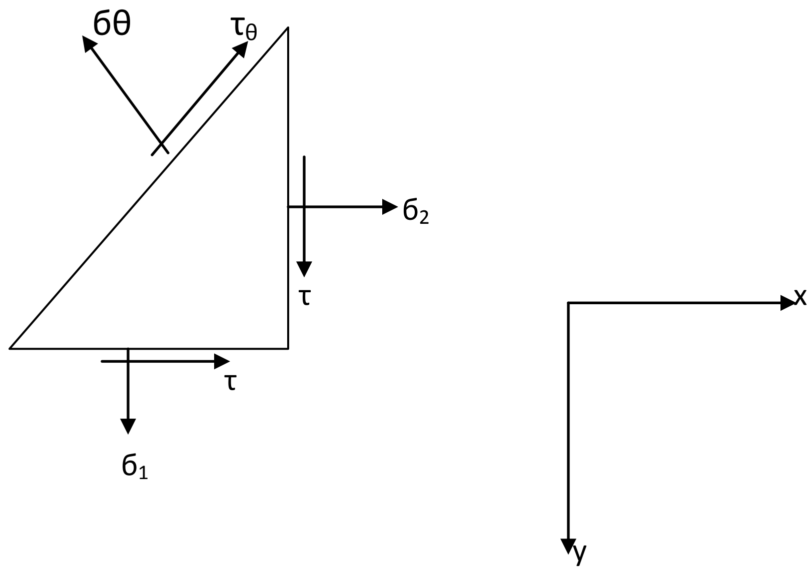
3.3 Stresses on an inclined plane due to two normal stresses (Pure normal stresses)



$(б_1 x AB x t) \cos\theta + б_2(BC) \times t \times \sin\theta$

=

3.5 Two dimensional stress system :-



$\sigma_\theta = \frac{1}{2}(\sigma_1 + \sigma_2) + \frac{1}{2}(\sigma_1 - \sigma_2)\cos 2\theta + \tau.\sin 2\theta$

$\tau_\theta = \frac{1}{2}(\sigma_1 - \sigma_2)\sin 2\theta - \tau\cos 2\theta$

3.6 Principal Planes & Principal Stresses :-

From para (3.5), we observed that $\sigma_\theta = 0$, for some values of 'θ'.

*The planes on which the shear component is zero $(\tau_\theta = 0)$are called principal planes.

$\tau_\theta = 0$ { equ^n of 'τ_θ' is from (3.5) para.

$\Rightarrow \tan 2\theta = \left(\frac{2\tau}{\sigma_y - \sigma_x}\right)$

→ The stresses on the principal planes are called principal stresses.

It is pure normal (tension/compression)

$$\text{Sin}2\theta = \pm \frac{2\tau}{\sqrt{(\sigma_y - \sigma_x)^2 + 4\tau^2}}$$

$$\text{Cos}2\theta = \pm \frac{(\sigma_y - \sigma_x)}{\sqrt{(\sigma_y - \sigma_x)^2 + 4\tau^2}}$$

Principal stresses $(б_\theta) = \frac{1}{2}\left(б_y + б_x\right) \pm$

$$\frac{\frac{1}{2}(б_y - б_x)^2}{\sqrt{(б_y - б_x)^2 + 4\tau^2}} \pm \frac{\tau.2\tau}{\sqrt{(б_y - б_x)^2 + 4\tau^2}}$$

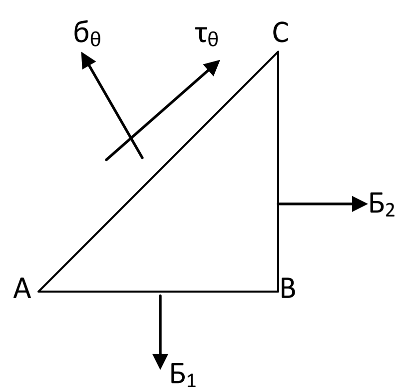
*Principal stresses are the maximum & minimum of normal stresses

- ✓ They may be maximum tensile & maximum compressive stress.

→ Simplified from

$$(б_\theta) = \frac{(б_y + б_x)}{2} \pm \frac{1}{2}\sqrt{(б_y - б_x)^2 + 4\tau^2}$$

3.8 Maximum Shear Stresses :-



$б_\theta$ x (ACxt) = {($б_1$xACxt) x sinθ

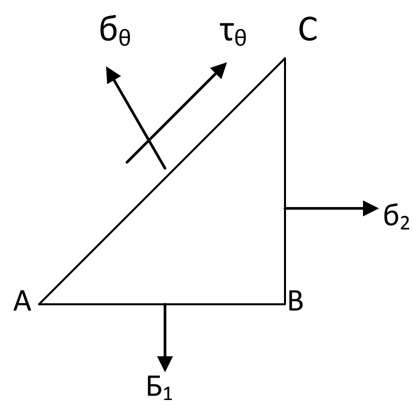
-($б_2$xBCxt)cosθ }

$\Rightarrow \tau_\theta = \frac{1}{2}(б_2 - б_1)\sin 2\theta$

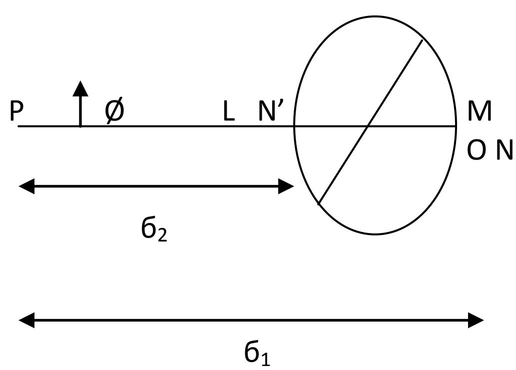
*If $2\theta = 90^0$, $\sin 2\theta = 1$

$$(\tau_\theta)_{max} = \left(\frac{б_2 - б_1}{2}\right)$$

3.9 Mohr’s stress circle :-



Section of + he speciman



Mohr’s stress circle

$PL = б_2$ & $PM = б_1$

OL → ‘BC’ Plane $(б_2)$

OM → ‘AB’ Plane $(б_1)$

→ Plane ‘AC’ is obtained by rotating the ‘AB’ through ‘θ’ anticlockwise.

→ In stress circle, OM (AB-Plane) is rotated through ‘2θ’ in anticlockwise direction.

PN = PO + ON

$= \frac{1}{2}(б_1 + б_2) + \frac{1}{2}(б_1 - б_2)cos2\theta$

$= б_1\left(\frac{1-cos2\theta}{2}\right) + б_1\left(\frac{1+cos2\theta}{2}\right)$

$= б_1 \sin^2\theta + б_2 cos^2\theta$

$= б_\theta$ (Normal stress component on ‘AC’)

RN $= \frac{1}{2}(б_1 - б_2)sin2\theta$

$= \tau_\theta$ (shear stress component on ‘AC’)

$$б_r = \sqrt{б_\theta^2 + C_\theta^2}$$

= PR

*$б_\theta = \left(\frac{б_1+б_2}{2}\right) + \frac{1}{2}(б_1 - б_2)cos2\theta$

↑

Dexvation

$\tau_\theta = \left(\frac{б_1-б_2}{2}\right) sin2\theta$

Let $\left(\frac{б_1+б_2}{2}\right) = б_{av}$

$\left(\frac{б_1-б_2}{2}\right) = б_{max}$

Hence, $б_\theta = б_{av} + б_{max}\cos 2\theta$

& $\tau = \tau_{ma}x.\sin 2\theta$

$\Rightarrow (б_\theta - б_{av}) = \tau_{max}\sqrt{1 - sin^2 2\theta}$

$\Rightarrow (б_\theta - б_{av}) = \tau_{max}\sqrt{1 - \frac{\tau^2}{\tau_{max}^2}}$

$\Rightarrow (б_\theta - б_{av})^2 + \tau^2 = \tau_{max}^2$

This is an equation of a circle having radius = τ_{max}

$\left(\frac{б_1 - б_2}{2}\right)$

& centre is on x-axis at

$\left(б_{av} = \frac{б_1 + б_2}{2}\right)$

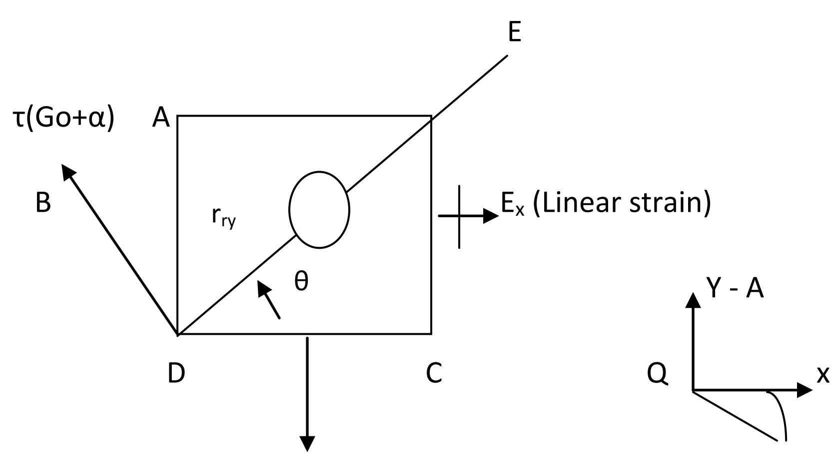
→ Plane Stress

→ Plane Strain

→ Bending of Beam :- (N-A), Fibre Layer

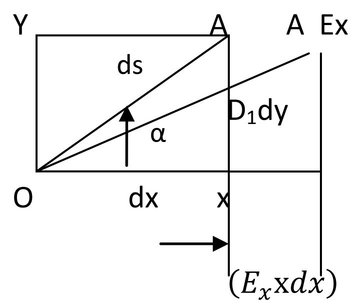
→ Stresses in Beam $(б_x, б_y, \tau_{xy}), б_z = 0$

Plane stress problem (thickness :- z-direc^n)



Ey

r_{xy}



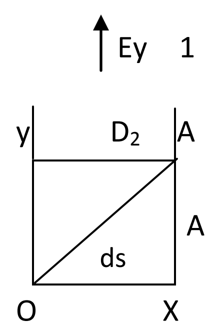
$E_x, E_y, r_{xy}.$

Ex

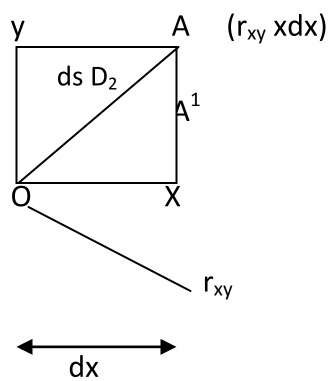
$$E_x = \left(\frac{D_1 A\prime}{ds}\right)$$

Ey

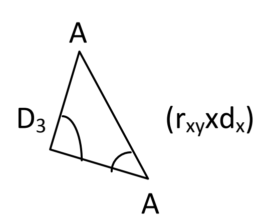
$$E_{y\alpha} = \left(\frac{D_1 A1}{ds}\right)$$



r_{xy}



$AA^1 = XX^1$



$XX^1 = (r_{xy} x dx)$

$$r_{xy\alpha} = \frac{(r_{xy} \times d_x) cos\alpha . D_3 A}{ds}$$

$$E_\alpha = E_{x\alpha} + E_{y\alpha} + r_{xy\alpha}$$

$$\Rightarrow E_a = E_x cos^2\alpha + Eysin^2\alpha - r_{xy} sin\alpha . cos\alpha.$$

$$E_b = E(90+\alpha) = E_x cos^2\alpha + Eysin^2\alpha \frac{+r_{xy} sin^2\alpha}{2}$$

3.10 Analysis of strain

$$E_y = E_x cos^2\alpha$$

$$+E_y sin^2\alpha + \left(\frac{r_{xy}}{2}\right) sin2\alpha$$

$$\Rightarrow E_\alpha = E_x\left(\frac{1+cos2\alpha}{2}\right) + E_y\left(\frac{1-cos2\alpha}{2}\right) + \left(\frac{r_{xy}}{2}\right) sin2\alpha$$

$$\Rightarrow E_\alpha = \left(\frac{E_x+E_y}{2}\right) + \left(\frac{E_x-E_y}{2}\right) cos2\alpha + \left(\frac{r_{xy}}{2}\right) sin2\alpha$$

→ To get principal strain, $(E_{1,} E_2)$are the maximum & minimum value of 'Eα', hence, put $\left(\frac{dE_\alpha}{d\alpha}\right) = 0$

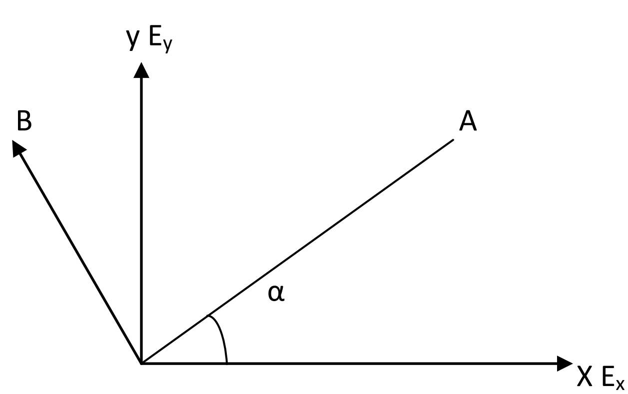
⇒ tan2α = $\left(\frac{r_{xy}}{E_x - E_y}\right)$

(* Principal stresses are the maximum & minimum values of normal stresses in the two dimensional geometry.

E_x, E_y → Linear strain

r_{xy} → Shear strain in 'XOY' plane.

E_a, E_b → Linear strain in 'OA' & 'OB' direc^n



$$E_a = (E_\alpha) = E_x.\cos^2\alpha + E_y \sin^2\alpha - \left(\frac{r_{xy}}{2}\right)\sin 2\alpha$$

$$E_b = E_{(90+\alpha)} = E_x.\sin^2\alpha + E_y \cos^2\alpha + \left(\frac{r_{xy}}{2}\right).\sin 2\alpha$$

& $r_{ab} \rightarrow$ *Shear strain corresponds to*

'E_a' & E_b'

↓ ↓

E_α $E(OP+\alpha^n)$

→ simplifying the equ^n for (E_a, E_b, & r_{ab})

$$E_a = \left(\frac{E_x+E_y}{2}\right) + \left(\frac{E_x-E_y}{2}\right)\cos 2\alpha - \left(\frac{r_{xy}}{2}\right).\sin 2\alpha$$

$$E_b = \left(\frac{E_x+E_y}{2}\right) + \left(\frac{E_x-E_y}{2}\right)\cos 2\alpha - \left(\frac{r_{xy}}{2}\right).\sin 2\alpha$$

$$y_{ab} = (E_x - E_y).\sin 2\alpha + r_{xy}.\cos 2\alpha$$

or $\left(\frac{r_{ab}}{2}\right) = \left(\frac{E_x-E_y}{2}\right).\sin 2\alpha + \left(\frac{r_{xy}}{2}\right)\cos 2\alpha$

→ A state of plane strain is defined by E_x, E_y, r_{xy}.

→ There are two mutually $\perp^{ar}$ directions (Lines) O1 & O2 which remain right angle to each other after defromation.

*If we will find Linear strains in that directions (01 & 02), it will give the maximum & minimum values.

→ Therefore, $Y_{12} = 0$

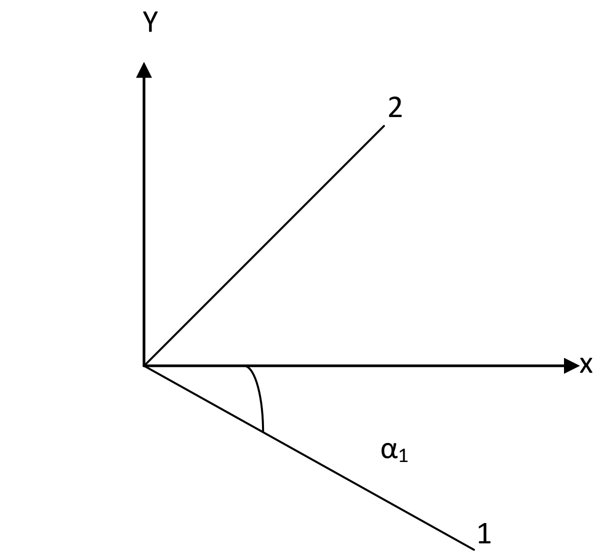
If we will put $Y_{ab} = 0$, we aill get the necessay condition for Principal strains.

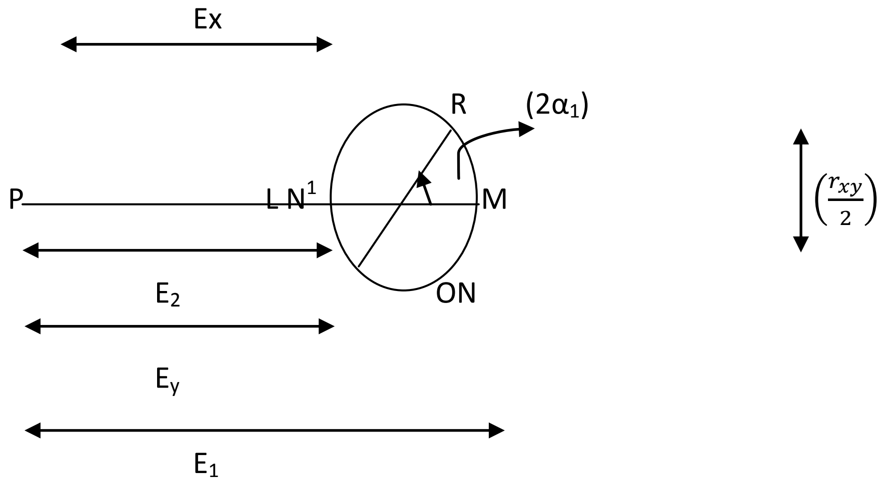
$Y_{ab} = 0$

$\Rightarrow \tan 2\alpha_1 = \left(\frac{r_{xy}}{E_y - E_x}\right)$

Principal $$E_1 = \left(\frac{E_x+E_y}{2}\right) + \sqrt{\left(\frac{E_x-E_y^2}{2}\right) + \left(\frac{r_{xy}}{2}\right)^2}$$

Strains $$E_2 = \left(\frac{E_x+E_y}{2}\right) + \sqrt{\left(\frac{E_x-E_y^2}{2}\right) + \left(\frac{r_{xy}}{2}\right)^2}$$





$(E_x - E_y)$ =N'N

RN = $\left(\frac{r_{xy}}{2}\right)$

E_1 = PM = PO + OM = (PO+OR)

E_2 = PL = (PO – LO)

= (PO – OR)

:- Mohr's Strain Circle :-

1) Analysis of strain :-

→ Ex→ Linear strain in x-direction.

Ey → Linear strain in y-direction.

Ø→ Shear strain.

Eθ → linear strain in a direction inclined 'θ' to x-axis.

Max[m] velocity of strain. E_1 = Principal strain on principal plane.

Min[m] velocity of strain. E_2 = Principal strain on principal plane.

→ tan 2θ = $\frac{Ø}{(E_x+E_y)}$

→ $E_1, E_2 = \frac{1}{2}(E_x + E_y) \pm \frac{1}{2}\sqrt{(E_x - E_y)^2 + Ø^2}$

2- Mohr's Strain Circle :-

→ Horizontal axis for Linear strain (E_x, E_y)

→ Vertical axis for <u>half shear strain.</u>

(1/2Ø)

Shearing Force & Bending Moment:-

4.1 Beam (definition) –

Beams are usually a straight horizontal member to carry vertical Load & they are suitably supported to resist the vertical (transverse) Load & induced bending in an axial plane.

- ✓ S/S Beam
- ✓ Cantilever Beam
- ✓ Fixed Beam
- ✓ Contineous Beam

*Frames are often used in building & are composed of beams & columns that are either pin or fixed connected.

4.2 Types of Load –

1) Conuntrated Load (Paint Load)
2) Distributed Load

{ Uniforming distributed
  Varying from point to point

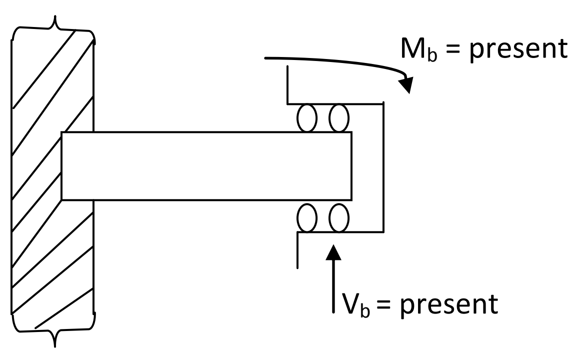
4.3 Types of support –

1) Simple or free support (Roller)

Beam restes freely on it. The reaction will be normal to the support.

2) Hinge/pinned

3) Fixed

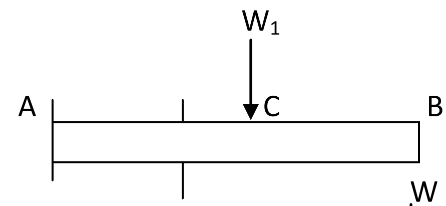


Shearing force :- of any section of a beam represents the tendency of the beam to one side of section to slide or shear Laturaly relative to other portion.

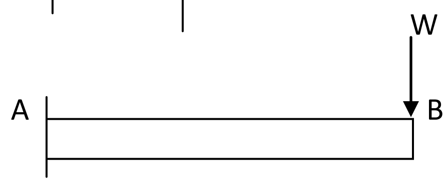
Or

→ The shear force at any section of a beam is the algebraic sum of the Lateral components of the forces scting on either side of the section.

Ex (1) -



Ex (2) -



Sign Convention :- From Left ⟶ Upward (+ve)

From Right ⟶ Downward (+ve)

*When a force is neither nor lateral direction it must be resolved in the two cesual direction (Horizontal component & vertical components). The vertical component will be taken into account in the shearing sorce.

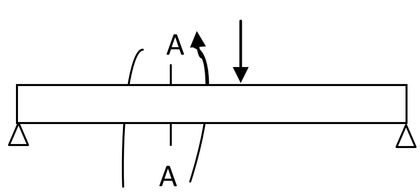
→The shear-force-diagram shows the variation of shear force along the length of beam.

Bending moment :- is the algebraic sum of moments about the section (x-x) of all the forces acting on either left or right side of the section.

Sign convention

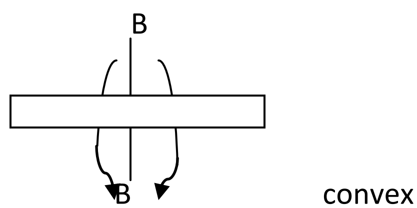
From Left - → Clockwise (+ve)

From Right → anticloskwise (+ve)

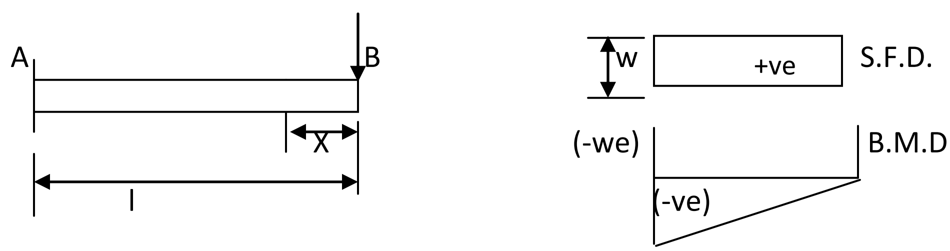


*This refers to sagging bending moment because it tends to make the beam concave upward at section A-A.

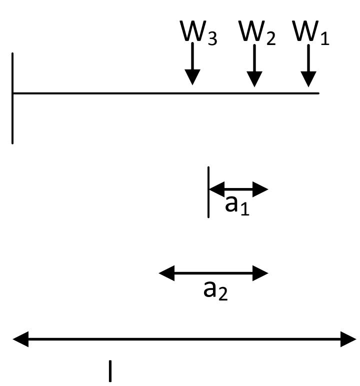
→ The negative bending moment is termed as hogging.



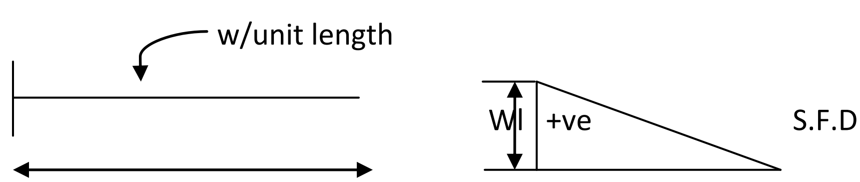
Q – (1)

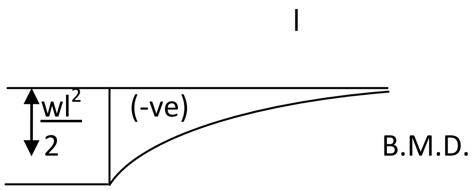


Q – (2)

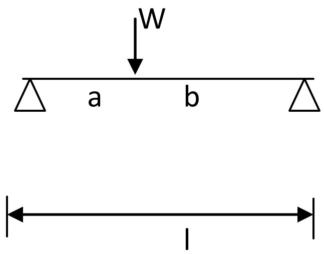


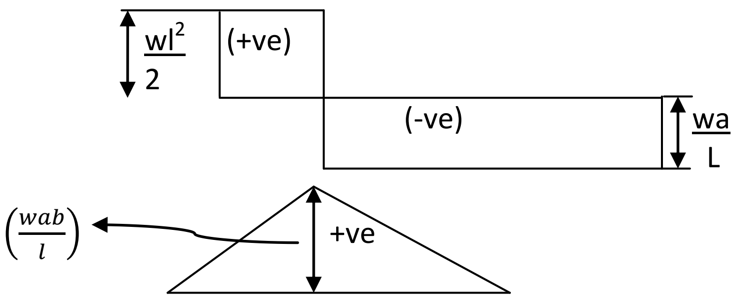
Q – (3)





Q – (4)





Thin Cylinders & Spheres :-

1) Thin cylinder under internal pressure :-

L=Length of Cylinders

D= inside/internal dia

P = applied internal pressure

σ_1 = Circumferential stress.

σ_2 = Longitudinal stress.

P = Applied internal pressure in all direct^n

→ Three principal stresses act on the cylinder

(Circumferential, Longitudinal & radial stresses)

→If t/d < 1/20; the circumferential stress (U1) & longitudinal stress (2) are consist over the thickness.

$\left(\frac{t}{d}\right) < \frac{1}{20}$ { The radial stress is equal to internal pressure (P)

At the inside surface & zero at the outside surface.

→ Consider a half cylinder of length 'l'

→σ_1 (Circumferential stress) acts on an area 2tl

{ C/s area = 9txl)

For both open end of half cylinder, C/s area = 2x(txl) }

{ C/s area = (tx1)

For both the open end of half cylinder, C/s area = 2 x (txl) }

→ Equating the vertical pressure forces in diametral plant, w have

σ_1 x (2tl) = pdl   dl = Projected horizontal Area

$$\Rightarrow \boxed{\sigma_1 = \frac{pd}{2t}}$$

→ Considering forces in <u>horizontal direction</u>.

(Length/Longitudinal direction.)

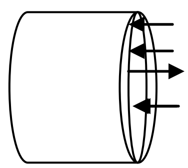
- ✓ σ_2 acts on the area $(2\pi r)xt$.

  Perimeter.

- ✓ Equating forces

$$\sigma_2 (2\pi r x t) = P x (\pi r^2)$$

$$\Rightarrow \boxed{\sigma_2 = \left(\frac{pr}{2t}\right)}$$



2- Thin spherical shell under internal pressure :-

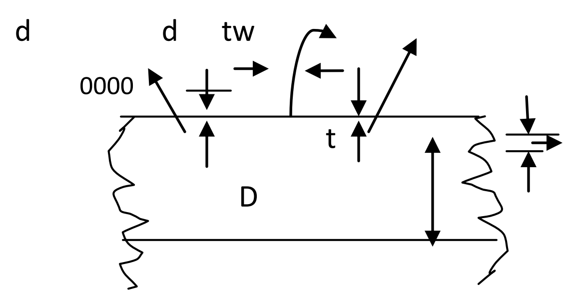
→ By Symmetry, both the principal stresses are equal ($\sigma_1 = \sigma_2$).

$$\sigma (2\pi r t) = P(\pi r^2)$$

$$\Rightarrow \boxed{\sigma = \left(\frac{pr}{2t}\right)}$$

3- Wire winding of thin cylinders :-

→ To strengthen the tube against the applied internal pressure; It may be wound with wire under tension.



→ Replace the wire by an equivalent cylindrical shell of thickness 'tw'

$$t_w \times d = \frac{\pi d^2}{4}$$ } C/s is same in longitudinal plane.

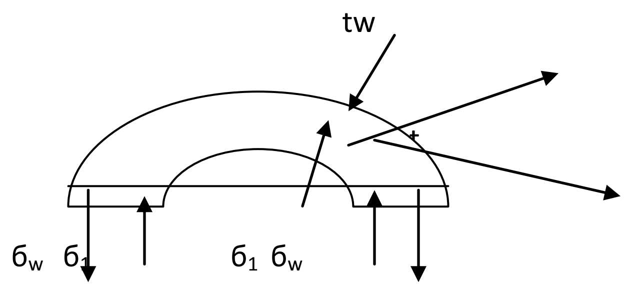
→ Initial tensite stress in wire ($б_w$)

Suppose initial tension in wire = 'T'

$$\boxed{б_w = \frac{T}{\pi r^2}}$$

⇒

→ If there is no internal pressure ‘P’,



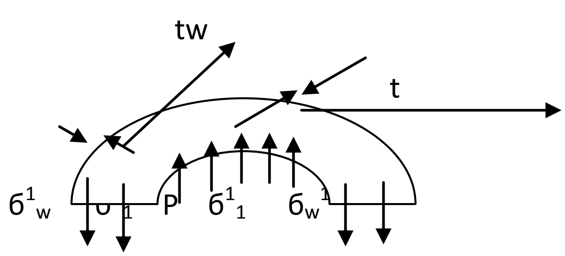
then ‘$б_1$’ will become compressive in nature, due to pressure of wire.

- ✓ $б_1$ = compressive circumferential stress-
- ✓ $б_1 \times t \times l = б_w \times t_w \times l$

→ When internal pressure (P) is applied; the stresses in cylinder. ($б^1{}_1$) & stresses in wire ($б^1{}_w$).

Equating $б^1{}_1 \times (2tl) + б_w{}^1 \times (2t_wl) = P(2r)$ dia

Final circumferential stress.



→ Final longitudinal stress.

$$б_2 = \left(\frac{pr}{2t}\right)$$

*Wire is under stress in only one direct

→ Since wire & cylinder is in contact, the change in circumferential (hwp) strain must be same. Hence,

$$\frac{[б_1 + (б_1^1 - v_2^{\sigma})]}{E} = \left(\frac{б_w^1 - б_w}{E_w}\right)$$

Strain in cylinder.

*→E1 = $(\frac{б_1 - \nu\sigma_2 - \nu\sigma_3}{E})$ →Hooks low for 3-D stress system.

-: Bucking of Column :-

→ Column is a compressive member. It is in transverse direction to the principal Loading direction. That is called bucking of columns.

→ The resistance towards bucking/(bending) is determined by EI.

E = elastic modulus

I = moment of inertia

→ Short column (strut) :- Effective length of compression member exceeds three times the least leteral dimensions.

- ✓ Long column.

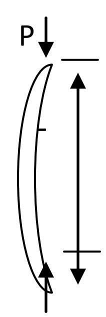
→ Slenderness ratio = $(\frac{leH,x}{i_x})$or $(\frac{leH,y}{iyy})$

→ For short column $(\frac{leH,x}{i_{xx}})$or $(\frac{leH,y}{i_{yy}})$ < 40

I_{xx} = radius of gyration w.r.t x-was.

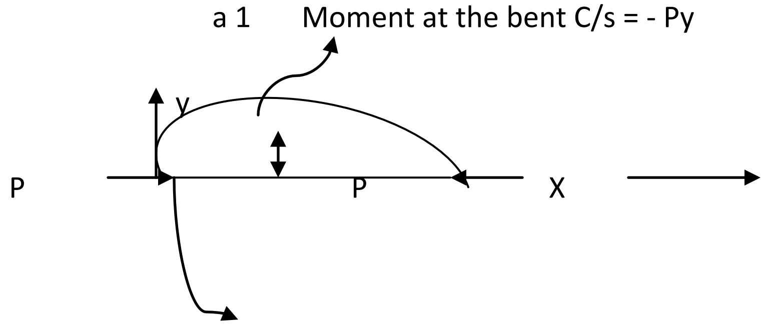
I_{yy} = radius of gyration w.r.t y- axis

radius of gyration w.r.tHinged strut Axially Loaded:-



→ Suppose the strut is detlected under an end load ‘P’.

a 1 Moment at the bent C/s = - Py



O(origin)

From Deflection of beam, we have

$EI.\frac{d^2y}{dx^2} = M$ $\frac{Signconvention}{Left :-Clockwise.}$ (+ve)

$= -Py$ Right :- Anticlockwise (+ve)

$\Rightarrow$ $\boxed{\frac{d^2y}{dx^2} + \alpha^2 y = o}$ -------1

… Where $\alpha^2 = \frac{\quad}{EI}$

The solution of equ[n] 1

$Y = A\sin\alpha x + B\cos\alpha x$

(i) At x = o, y = o ; B = o

(ii) At x = l, y = o ; $A\sin\alpha l = o$

If 'A' = 'O' ; then y=o for all values of 'x' & strut will not Bucket.

or $\sin\alpha l = O$

$\Rightarrow$ $\boxed{\alpha l = \pi}$

$\Rightarrow \alpha^2 = \frac{\pi^2}{l^2}$

$\Rightarrow$ $\boxed{\frac{P}{EI} = \frac{\pi^2}{l^2}}$ (2)

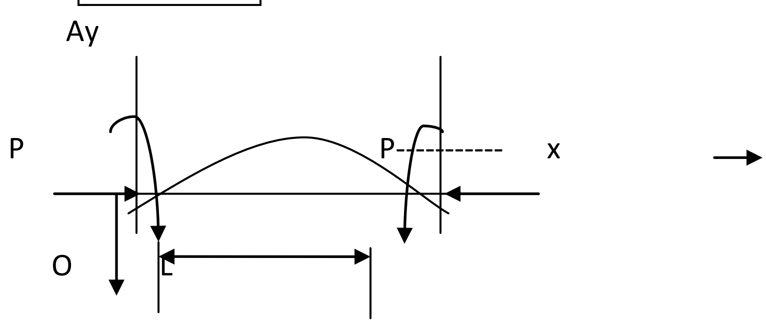
This will determine the least value of 'P' which will cause the strut to bucket.

→ This is called <u>'Euler crippling load'.</u>

(P_e)

$\boxed{P_2 = \frac{\pi^2 EI}{l^2}}$ -------------(3)

2- Strut nds:-



The equation for deflection of beam is

$$EI.\frac{d^2y}{dx^2} = -Py + M$$

$\Rightarrow \frac{d^2y}{dx^2}+(\frac{P}{EI})y = (\frac{M}{EI})$

$\Rightarrow$ $$\frac{d^2y}{dx^2} + \alpha^2 y = (\frac{M}{EI})$$ $(\alpha^2 = \frac{P}{EI})$

--------------(4)

The solution of equ$^\sigma$ 4 is

$Y = A\sin\alpha x + B\cos\alpha x + (\frac{M}{EI\alpha^2})$

(i) At x=o, y=o; $B + \left(\frac{M}{EI\alpha^2}\right) = o$

$\Rightarrow$ $B = \frac{-M}{P}$

(ii) $\frac{dy}{dx}$ = O (at x = o, y = o);

Slope.

$\frac{dy}{dx} = \alpha A.\cos\alpha x - \alpha B\sin\alpha x$

$\Rightarrow O = \alpha A$

$\Rightarrow$ A = O ---------------(6) Because e. $\alpha^2 = \frac{P}{EI}$

Hence, $Y = \frac{M}{P}(1-\cos\alpha x)$ (7)

(iii) At x = l, y = O;

$\cos\alpha l = 1$

Put in

Equ$^{\sigma}$(7) ⇒ $\boxed{A1 = 2\pi}$

→ Therefore, the least value of 'P' is

$\alpha^2 l^2 = 4\pi^2$

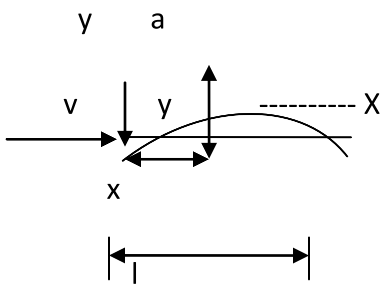
$\Rightarrow \frac{P_e}{EI} = \frac{4\pi^2}{l^2}$

$$\Rightarrow \boxed{P_e = \left(\frac{4\pi^2 EI}{l^2}\right)} \quad (8)$$

→ It we wil[l] ... equal to '2l' then,

the least value of load is ($\pi^2 EI/l^2$).

3- <u>Direction fixed at one end & position fixed at the other end:-</u>



→ Suppose 'V' is the literal force which applied to maintain the position of pinned-end.

→ Then, the deflection equation will become

$$EI \frac{d^2y}{dx^2} = -Py - Vx$$

$$\Rightarrow \frac{d^2y}{dx^2} = -\frac{P}{EI} y - \frac{V}{EI} x$$

$$\Rightarrow \boxed{\frac{d^2y}{dx^2} + \alpha^2 y = \frac{-Vx}{EI}} \quad 9)$$

The solution to equ$^{\sigma}$ 9 is:

$$Y = A \sin \alpha x + B \cos \alpha x \frac{-Vx}{P} \text{ -------- } (10)$$

(i) At x = o, y = o

$\therefore$ **B = O** ------ (a)

(ii) At x = l, y = O

$$A \sin \alpha l = \frac{Vl}{P}$$)

(iii) At x = l, y = O

$$\frac{dy}{dx} = O$$

$$\Rightarrow ((\alpha A.\cos \alpha x - \alpha B \sin \alpha x - \frac{V}{P}) = O$$

$$\Rightarrow (\alpha A.\cos \alpha l = \frac{V}{P}) \text{ -------- (C)}$$

→ We have,

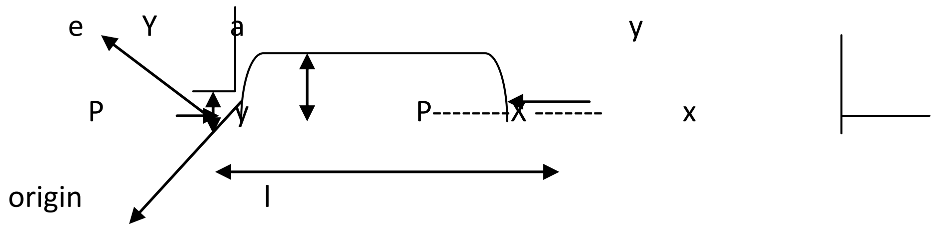
tan αl = αl

⇒ **Al = 4.5** ion

→ The crippling load 'P_e' is

$$P_e = \left[\frac{2.05\pi^2 EI}{l^2}\right] \text{ \_\_\_\_\_\_\_\_ } (10)$$

4- Columns with eccentric loading :-



y = the distance from the line of action of load.

→ The deflection equ$^{\sigma}$ is

$$EI.\frac{d^2y}{dx^2} = -Py$$

⇒ $$\boxed{\frac{d^2y}{dx^2} + \alpha^2 y = 0}$$

→ The solution to the above differential equation is

Y = Asinαx + Bcosαx

(i) At x=o, y=e; $\boxed{B = e}$

(ii) At x = l/2, $\frac{dy}{dx}$ = o;

⇒ $(\frac{dy}{dx})_{x=M_2} = \alpha A \cos\frac{\alpha l}{2} - \alpha B \sin\frac{\alpha l}{2}$

⇒ Acos $\frac{\alpha l}{2}$ - Bsin $\frac{\alpha l}{2}$ = o

⇒ tan $\frac{\alpha l}{2}$ = $(\frac{A}{e})$

*→ Therefore, the column/start will deflect for all values of 'P'.

- ✓ For the case of tan $\frac{\alpha l}{2}$ = 8

⇒ $\frac{\alpha l}{2} = \frac{\pi}{2}$ (smallest –angle)

⇒ $\boxed{\alpha l = \pi}$

- ✓ The corresponding crippling load is

$$P_e = \left(\frac{\pi^2 EI}{l^2}\right)$$

→ Then, $y = e\left[\left(\tan\frac{\alpha l}{2}\right)\sin\alpha + \cos\alpha x\right]$----(11)

→ Due to additional B.M. set up by deflection here, the stunt will fail by compressive stress before enter load is reached.

→ Eqn$^{\alpha}$ II (x=l/2) = $e\left[\frac{\sin^2\frac{\alpha l}{2}+\cos^2\frac{\alpha l}{2}}{\cos\frac{\alpha l}{2}}\right]$

$= e\sec\frac{\alpha l}{2}$ x = l/2

$(M)_{max} = (p \times y_{max})$

$= P.e.\sec\frac{\alpha l}{2}$

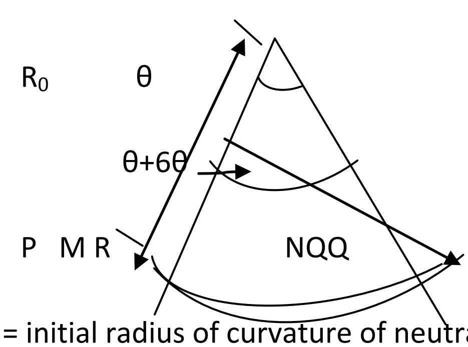
→ Maximum stress developed is due to combined bending & direct stress

$$\sigma = \frac{P}{A} + \frac{M}{Z}$$

Section modulus= $\left(\frac{I}{y}\right)$

$$= \frac{Pe\ (P_{cr}+0.26P)}{(P_{cr}-P)}$$

5- Column/strut with initial curvature:-



R_o = initial radius of curvature of neutral surface.

R = radius of curvature under the action of pure bending.

→ Strain in the layer of fibre (y-distance from neutral axis)

$$= \left[\frac{PQ^1PQ}{PQ}\right] = \left[\frac{(R+y)(\theta+\sigma\theta)-(R_0+y)\theta}{(R_0+y)\theta}\right]$$

$$= \frac{[R(\theta+\sigma\theta)-R_0\theta-y\sigma\theta]}{(R_0+y)\theta} = \frac{y\sigma\theta}{(R_0+y)\theta}$$

… $R((\theta + \sigma\theta)$= $(R_0 + \theta)$= Length

Of neutral surface (MN)

*If y << R_o(y is neglected in compression with ('R_o')

Hence,

$$R(\theta+\sigma\theta) = R_o\theta$$

$$\Rightarrow \sigma\theta = (\frac{R_o - R}{R})\theta$$

→ stain = $\frac{y\sigma\theta}{(R_o+y)\theta}$

R_o

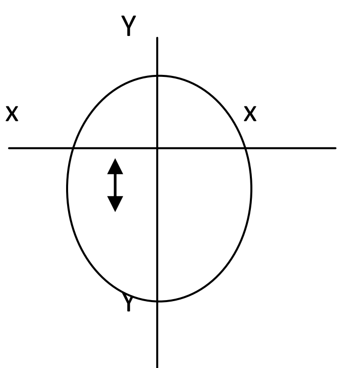
$$= \left[\frac{y(\frac{R_o - R}{R})\theta}{R_{o.\theta}}\right]$$

$$= \frac{y}{R_o}\left(\frac{R_o - R}{R}\right)$$

$$= y\left(\frac{1}{R} - \frac{1}{R_o}\right)$$

→ Normal stress = σ = x strain

$$\begin{pmatrix} Lateral\ stress \\ is\ negelectd \end{pmatrix} = Ey = \left(\frac{1}{R} - \frac{1}{R_o}\right)$$



x – x = Neutral axis.

σA = Elemental cross-setting

'y' from the $\frac{N-A}{(xx)}$.

*For pure bending case, the net normal force on the C/s must be zero.

$$\int б.\ dA = \int E\left(\frac{1}{R} - \frac{1}{R_o}\right)y.dA = O$$

Normal force

→ Bending moment is balanced by the moment of normal force about (x-x).

$$M = \int бy\ dA$$

$\Rightarrow$ M = E $\left(\frac{1}{R} - \frac{1}{R_0}\right) \int y^2 . d^A$

$\Rightarrow$

$$M = EI\left(\frac{1}{R} - \frac{1}{R_0}\right) \quad \text{-- (12)}$$

→ Initial radius of curvature (R_o)

$$R_o = \frac{1}{\left(\frac{d^2y}{dx^2}\right)} \quad (13)$$

Column with initial curvature :-

→ From equ$^{\sigma}$ 12 & 13, we have

$$\begin{cases} EI\left(\frac{1}{R} - \frac{1}{R_0}\right) = M \\ R_0 = \frac{1}{\left(\frac{d^2y}{dx^2}\right)} \end{cases}$$

→ The differential equ$^{\sigma}$ for the deflection

$$EI = \left(\frac{1}{R}\right) = M + EI\left(\frac{1}{R_0}\right)$$

$\Rightarrow$

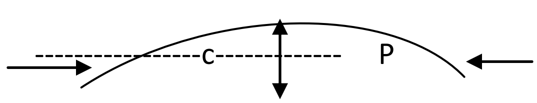
$$EI\,\frac{d^2y}{dx^2} = M + EI.\frac{d^2y_0}{dx^2}$$

→ Assume the cu cular/portablic)

$$y_0 = C.\sin\frac{\pi^2}{l}$$

→ Equ$^{\sigma}$ 14 can be written as

$$\frac{d^2y}{dx^2} + \alpha^2 y = \frac{d^2y0}{dx^2} \quad (14)$$



$$\Rightarrow \quad \boxed{\frac{d^2y}{dx^2} + \alpha^2 y = -\left(\frac{C\pi^2}{l^2}\right)\left(\sin\frac{\pi x}{l}\right)}$$

-------- (15)

→ The complete 801^{n} to eqn$^{\sigma}$ 15 is:

y = A sinαx + Bcosαx –

$$\left(\frac{(\frac{C\pi^2}{l^2})}{(\frac{-\pi^2}{l^2}+l^2)}\right) sin\frac{\pi x}{l}$$

(i)At x = o, y = O; $\boxed{B = O}$

(ii) At x = $\frac{l}{2}$, $\frac{dy}{dx}$ = O; $\boxed{A = O}$

→ Therefore, y = $\left[\frac{(\frac{C\pi^2}{l^2})}{\frac{\pi^2}{l^2}-l^2}\right] sin\frac{\pi x}{l}$

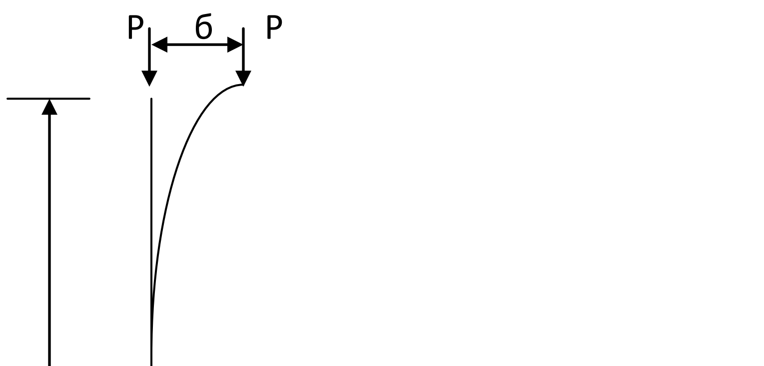
= $\left(\frac{CxP_{cr}}{P_{cr}-P}\right) sin\frac{\pi x}{l}$

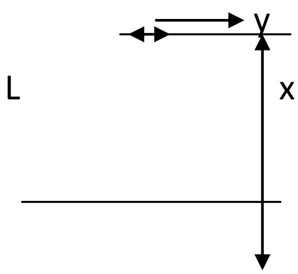
+++++++++++++++++

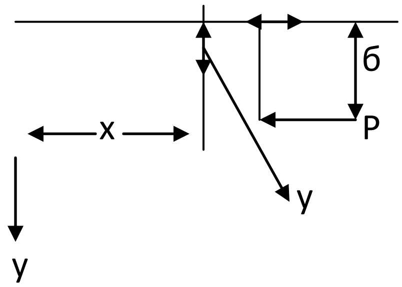
-: Bucking of Columns :-

1) Short column & Long column :-

Long column (Euler's formula) :- (Base-fixed & other end free)







→ We Like to find out the minimum value of Load (P) for which the column isbuckled.

→ The minimum Load (P) is called critical Load/crippingLoad/Euler Load.

$$M_x = -P(б - y)$$

$$\Rightarrow EI\left(\frac{d^2y}{dx^2}\right) = -P(б - y)$$

$$\Rightarrow \frac{d^2y}{dx^2} - \left(\frac{P}{EI}\right)y = -Pб$$

Assume $\alpha^2 = \frac{P}{EI}$

$$y = Asin\alpha x + Bcos\alpha x + б$$

Boundary condition

(i) at x = o, y = o

(ii) at x = l, y = (б)

(iii) at x = 0, $\frac{dy}{dx} = 0$

A = o, B =

y = б (1-cosαx)

put x = l, y = б

→ б = б (1cosαl)

⇒ бcos(αl) = o

→ If $б = o$,the column stands straight in vertical direction

& No Limitation is imposed to get the magnitude of Load 'P'.

Cosαl = o

⇒ αl = $\frac{n\pi}{2}$ (where n = 1,3,5…….

→ To find smallest value of 'P', take n = 1.

α1 = $\frac{\pi}{2}$

$$\Rightarrow \left(\frac{P_{cr}}{EI}.l^2\right) = \frac{\pi^2}{4}$$

$$\Rightarrow P_{cr} = \left(\frac{\pi^2 EI}{4l^2}\right)$$

Note

(a)$P<P_cr$; б = O & column is in stable position.