CE 15023

FLUID DYNAMICS

LECTURE NOTES

MODULE-III

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Course Contents

Module – III

Drag and Lift: Introduction; Types of Drag, Drag on a sphere, Cylinder, Flat plate & on an air foil, Polar diagram, Profile Drag, Lift on immersed bodies.

Turbulent Flow in pipes: Reynolds observation on pipe flow, Causes and characteristics of turbulence. Reynolds stresses, Prandtl's Mixing length Theory, Velocity distribution in Rough pipes, Karman – Prandtl's resistance equations.

Lecture 1

Turbulent Flow in Pipes

Unlike fully developed laminar flow in pipes, turbulent flow occurs more frequently in many practical situations. However, this phenomenon is more complex to analyze. Hence, many empirical relations are developed to understand the characteristics of common flow problems. Before, going into these solutions and empirical relations, first few concepts and characteristics of turbulent flows are discussed.

Transition from Laminar to Turbulent Flow

Consider a situation in which a water reservoir is connected to a pipe. The water is initially at rest and is allowed to flow through pipe and the flow rate is regulated by a valve. By opening the valve slowly, the flow velocity and hence the Reynolds number increases from zero to the maximum steady state value. For initial time period, the Reynolds number is small enough for laminar flow to occur. At some time, when the Reynolds number reaches 2100, intermittent spots and random fluctuations appear indicating the flow transition to turbulent condition. This process continues till the Reynolds number value reaches 4000 beyond which the flow becomes fully turbulent.

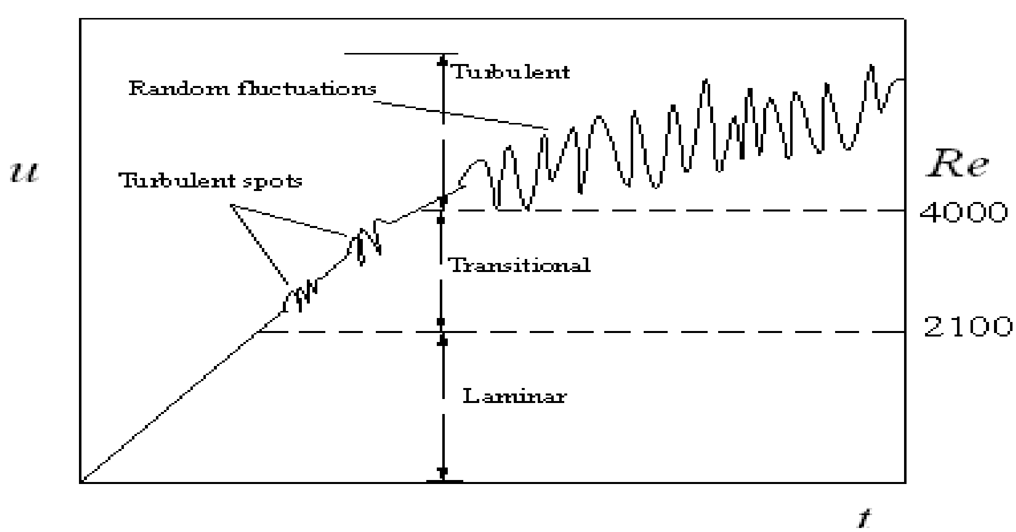

Fig. 1: Transition phenomena in a pipe flow.

This phenomena is typically shown in Fig. 1 where the axial velocity component of flow at given location is given by $u = u(t)$. The flow characteristics such as pressure drop and heat transfer depends strongly on the nature of fluctuations and randomness.

Reynolds Time-Averaging Concept in Turbulent Flow

The fundamental difference between laminar and turbulent flow is the chaotic and random behavior of flow properties the chaotic and random behavior of flow properties such as velocity, pressure, shear stress, temperature etc. One way to handle such high Reynolds number flow is to standardize in terms of mean/average value of flow parameters. Such a technique is known as "Reynolds Time-Averaging Concept". In this method, the flow parameters are expressed in terms of two quantities; one is the time-average value and the other is the fluctuating part with respect to average value.

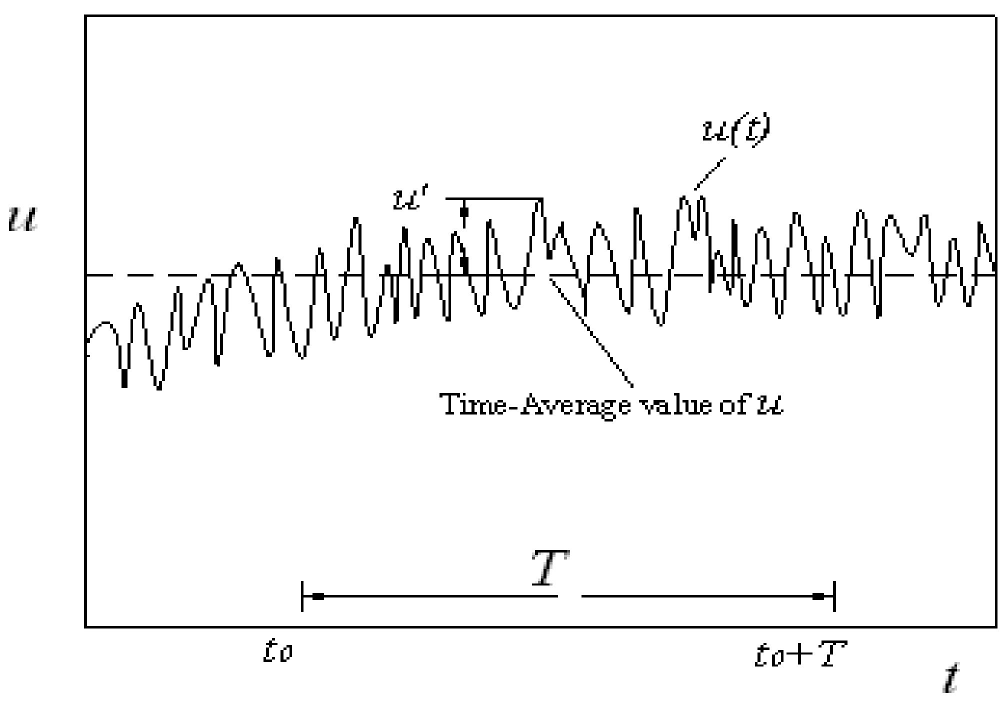

Fig. 2: Time averaging concept in turbulent flows.

For example, referring to Fig. 2, if $u = u(x,y,z,t)$, then mean value of turbulent function is defined by,

$$\bar{u} = \frac{1}{T}\int_0^T u(x,y,z,t)\,dt \qquad (1)$$

where T is the averaging time period . The fluctuation part (or the time varying part) is then defined by,

$$u' = u - \bar{u} \qquad (2)$$

Turbulent Intensity (φ)

It is clear from Eqs. (1) and (2) that the fluctuation has zero mean value. However, the mean square value of fluctuating is not zero and is the measure of "turbulent intensity". It is often defined as the ratio of square root of mean square of fluctuating velocity to the time average velocity. Mathematically, it may be written as:

$$\varphi = \frac{\sqrt{\left[(u')^2\right]_{avg}}}{\bar{u}} \qquad (3)$$

where the mean square value of fluctuating velocity is,

$$\left[(u')^2\right]_{avg} = \frac{1}{T}\int_{t_0}^{t_0+T} (u')^2\,dt \qquad (4)$$

The larger the turbulent intensity, the more will be fluctuation in parameters. Typical values of φ range form 0.0001 to 0.1.

Turbulent Stresses

In order to define stresses in turbulent flows, let us write the *x*- momentum equation with time averaging and fluctuation terms; i.e.

$$\rho\frac{d\bar{u}}{dt} = -\frac{\partial\bar{p}}{\partial x} + \rho g + \frac{\partial}{\partial x}\left(\mu\frac{\partial\bar{u}}{\partial x} - \rho\left[(u')^2\right]_{avg}\right) + \frac{\partial}{\partial y}\left(\mu\frac{\partial\bar{u}}{\partial y} - \rho\left[u'v'\right]_{avg}\right) + \frac{\partial}{\partial z}\left(\mu\frac{\partial\bar{u}}{\partial z} - \rho\left[u'w'\right]_{avg}\right) \qquad (5)$$

The three correlation terms $-\rho\left[\left(u'\right)^2\right]_{avg}, -\rho\left[u'v'\right]_{avg}$ and $-\rho\left[u'w'\right]_{avg}$ are called "turbulent stresses" because they have same dimensions as that of laminar shear stress terms i.e. $\mu\frac{\partial\bar{u}}{\partial x}, \mu\frac{\partial\bar{u}}{\partial y}$ and $\mu\frac{\partial\bar{u}}{\partial z}$. The turbulent stresses are unknown and must be related to experimental flow conditions and geometry. However, experiments in pipe flows reveal that the stress associated with $-\rho\left[u'v'\right]_{avg}$ in *y*- direction is dominant.

Hence, with reasonable accuracy, the momentum equation

is reduced to,

$$\rho\frac{d\bar{u}}{dt} = -\frac{\partial\bar{p}}{\partial x} + \rho g + \frac{\partial\tau}{\partial x} \qquad (6)$$

where

$$\tau = \mu\frac{\partial\bar{u}}{\partial y} + \left\{\rho\left[u'v'\right]_{avg}\right\} = \tau_{lam} + \tau_{turb} \qquad (7)$$

The typical trend of a turbulent-shear layer for a pipe flow is shown in Fig. 3. It is seen that "laminar shear" is dominant near the wall whereas turbulent shear dominates in the "outer layer". There is an intermediate region called "overlap layer" where both laminar and turbulent shear are important.

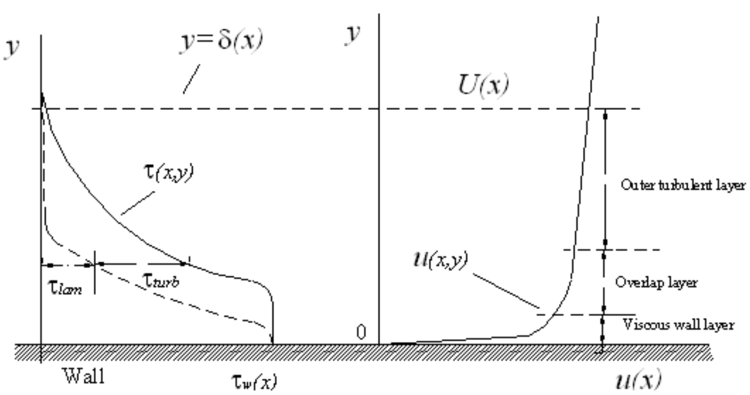

Fig. 3: Typical velocity and shear stress distribution in turbulent flow.

An alternate form of shear stress for turbulent flow is given in terms of "eddy viscosity" (η) which is analogous to dynamic viscosity in case of laminar flow. It may be written as;

$$\tau = \eta \frac{d\bar{u}}{dy} \qquad (8)$$

In order to determine the Reynolds stresses in turbulent flows $\left(\rho[u'v']_{avg}\right)$, several empirical theories have been attempted. The most common one is Prandtl's concept of "mixing length". He proposed that the turbulent process could be viewed as the random transport of bundles of fluid particles over a certain distance (l_m) from a region of one velocity to another region of different velocity. This distance is called "mixing length". In this mixing length, the eddy viscosity may be defined as,

$$\eta = \rho l_m^2 \left|\frac{d\bar{u}}{dy}\right| \qquad (9)$$

Thus, turbulent shear stress becomes,

$$\tau_{turb} = \rho l_m^2 \left(\frac{d\bar{u}}{dy}\right)^2 \qquad (10)$$

<u>Turbulent Velocity Profile</u>

A fully developed turbulent flow in a pipe can be divided into three regions which are characterized by their distances from the wall: the viscous sub-layer very near to the pipe wall, the overlap region and the outer turbulent layer throughout the center portion of the flow. Within the viscous sub-layer the viscous shear stress is dominant compared to that of turbulent (or Reynolds) stress i.e. fluid viscosity plays a major role compared to fluid density. In the outer turbulent layer, Reynolds stresses (i.e. fluid density) are dominant and there is a considerable mixing and randomness to the flow.

The character of flow within these two regions is entirely different. Considerable efforts have been made to determine the actual velocity profiles in pipe flows. Some of them are discussed here.

In viscous sub-layer, the velocity profile is written as,

$$\frac{\bar{u}}{u^*} = \frac{yu^*}{\upsilon} \qquad (11)$$

where $y = R - r$ is the distance measured from the wall, υ is the kinematics viscosity of the flow, $\bar{u}$ is the time-averaged x -component of velocity and u^* is called “friction velocity defined by,

$$u^* = \left(\frac{\tau_w}{\rho}\right)^{0.5} \quad (12)$$

The "friction velocity" is not the actual fluid velocity rather it has same dimension as that of velocity. The Eq. (11) sometimes called as the 'law of wall". For smooth wall, Eq. (11) is valid very near to the wall for which $0 \le \frac{yu^*}{\upsilon} \le 5$.

In case of "overlap layer" the following expression commonly known as "Logarithmic Overlap Law" has been proposed;

$$\frac{\bar{u}}{u^*} = 2.5\ln\left(\frac{yu^*}{\upsilon}\right) + 5 \quad (13)$$

The most often used correlation is the "empirical power law velocity profile" defined by,

$$\frac{\bar{u}}{V_c} = \left(1 - \frac{r}{R}\right)^{\frac{1}{n}} \quad (14)$$

where V_c is the centerline velocity and $n = 7$ holds good for many practical flow problems

Moody Chart

The fundamental difference between laminar and turbulent flow is that the shear stress for laminar flow depends on the viscosity of the fluid whereas in case of turbulent flow, it is the function of density of the fluid. In general, the pressure drop Δp, for steady, incompressible turbulent flow in a horizontal round pipe of diameter D can be written in the functional form as,

$$\Delta P_{turb} = f\left(V, D, l, \varepsilon, \mu, \rho\right) \quad (15)$$

where V is the average velocity, l is the length of the pipe and ε is a measure of the roughness of the pipe wall. Similar expression can also be written for the case of laminar flow in which the ε term will be absent because the pressure drop in laminar flow is found to be independent of pipe roughness i.e.

$$\Delta P_{lam} = F\left(V, D, l, \mu, \rho\right) \quad (16)$$

By dimensional analysis treatment, we can found that

$$\left(\frac{\Delta P_l}{\frac{1}{2}\rho V^2}\right)_{lam} = \phi\left(\frac{\rho . V . D}{\mu}, \frac{l}{D}\right)$$

$$\left(\frac{\Delta P_l}{\frac{1}{2}\rho V^2}\right)_{turb} = \tilde{\phi}\left(\frac{\rho . V . D}{\mu}, \frac{l}{D}, \frac{\varepsilon}{D}\right) \qquad (17)$$

The only difference between two expressions in Eq. (17) is that the term (ε/D), which is known as the "relative roughness". In commercially available pipes, the roughness is not uniform; so it is correlated with pipe diameter and the contribution (ε/D) forms a significant value in friction factor calculation. From tests with commercial pipes, Moody gave the values for average pipe roughness listed in Table 1.

Table 1: Average values of roughness for commercial pipes (Table 8.1; Ref. 1)

Material	ε (mm)
Riveted steel	0.9 – 9
Riveted steel	0.3 – 3
wood	0.18 – 0.9
cast iron	0.26
Galvanized iron	0.15
Commercial steel	0.045
Plastic and glass	0(smooth pipes)

Now Eq. (17) can be simplified with reasonable assumption that the pressure drop is proportional to pipe length. It can be done only when,

$$\frac{\Delta P}{\frac{1}{2}\rho V^2} = \frac{l}{D}\tilde{\phi}\left(R_e, \frac{\varepsilon}{D}\right) \qquad (18)$$

It can be rewritten as,

$$\Delta P = f\left(\frac{l}{D}\right)\left(\frac{\rho V^2}{2}\right) \qquad (19)$$

where f is known as "friction factor" and is defined by,

$$f = \phi\left(R_e, \frac{\varepsilon}{D}\right) \qquad (20)$$

Now, recalling the energy equation for a steady incompressible flow,

$$\frac{p_1}{\gamma}+\frac{V_1^2}{2g}+z_1=\frac{p_2}{\gamma}+\frac{V_2^2}{2g}+z_2+h_l \qquad (21)$$

where h_l is the head loss between two sections. With assumption of horizontal $(z_1=z_2)$ constant diameter pipe $(D_1=D_2 \text{ or } V_1=V_2)$ with fully developed flow,

$$\Delta p=p_1-p_2=\gamma h_l=\rho g h_l \qquad (22)$$

From Eqs. (19) and (22), we can determine head loss as,

$$h_l=f\left(\frac{l}{D}\right)\left(\frac{V^2}{2g}\right) \qquad (23)$$

This is known as *Darcy-Weisbach equation* and is valid for fully developed, steady, incompressible horizontal pipe flow. If the flow is laminar, the friction factor will be independent on $\left(\frac{\varepsilon}{D}\right)$ and simply,

$$f=\frac{64}{R_e} \qquad (24)$$

The functional dependence of friction factor on the Reynolds number and relative roughness is rather complex. It is found from exhaustive set of experiments and is usually presented in the form of curve-fitting formula/data. The most common graphical representation of friction factor dependence on surface roughness and Reynolds number is shown in "Moody Chart" (Fig. 4). This chart is valid universally for all steady, fully developed, incompressible flows.

The following inferences may be made from Moody chart (Fig. 4).

- For laminar flows ($R_e<2100$), $f=(64/R_e)$ and is independent of surface roughness
- At very high Reynolds number $(R_e>4000)$, the flow becomes completely turbulent (wholly turbulent flow) and is independent of Reynolds number. In this case, the laminar sub-layer is so thin that the surface roughness completely dominates the character of flow near the wall. The pressure drop responsible for turbulent shear stress is inertia dominated rather than viscous dominated as found in case of laminar viscous sub-layer. Hence, the friction factor is given by, $f=\phi(\varepsilon/D)$
- The friction factor at moderate Reynolds number $(2100<R_e<4000)$ is indeed dependent on both Reynolds number and relative roughness.
- Even for smooth pipes $(\varepsilon=0)$, the friction factor is not zero i.e. there is always head loss, no matter how smooth the pipe surface is. There is always some microscopic

surface roughness that produces no-slip behavior $(\text{thus } f \neq 0)$ on the molecular level. Such pipes are called "hydraulically smooth".

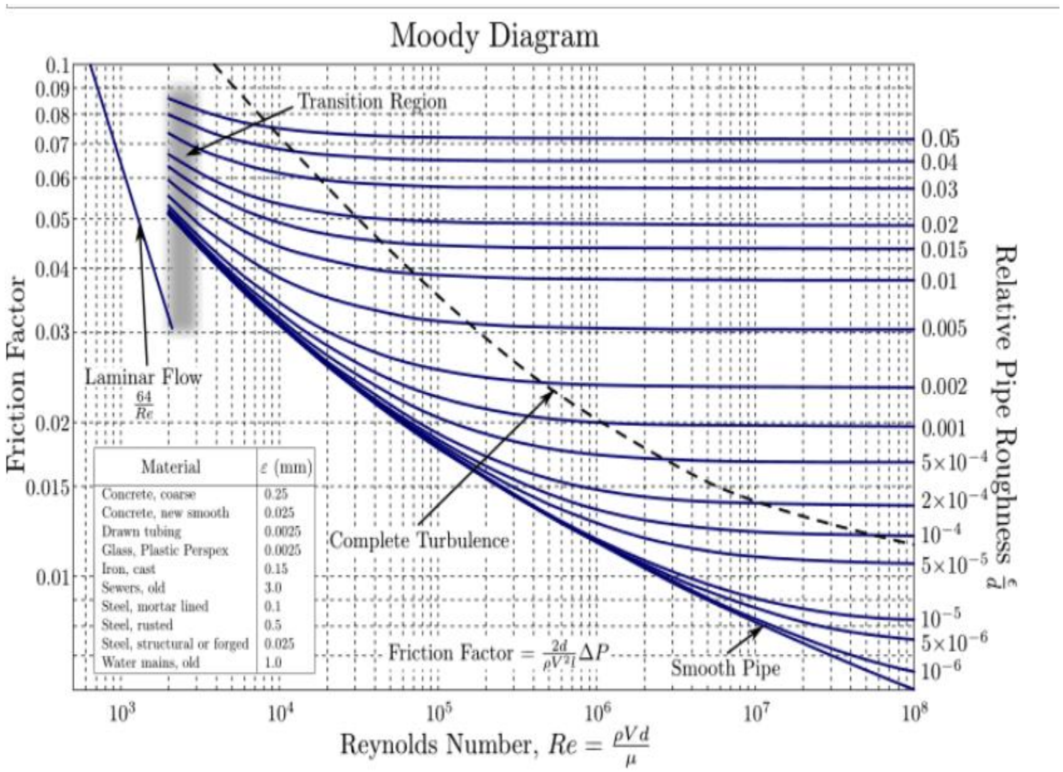

Fig (4) Moody's Chart

It must be noted that Moody chart covers extremely wide range of flow parameters i.e. diameter of the pipes (D), fluid density (ρ), viscosity (μ) and velocities (V) in non-laminar regions of the flow $(4000 < R_e < 10^8)$ that almost accommodates all applications of pipe flows. In the non-laminar regions of fluid flow, Moody chart can be represented by the empirical equation i.e.

$$\frac{1}{\sqrt{f}} = -2\log\left(\frac{\varepsilon/D}{3.7} + \frac{2.51}{R_e\sqrt{f}}\right) \quad (25)$$

This equation is called "Colebrook formula" and is valid with 10% accuracy with the graphical data.

Example 9.1. *A test for determining the equivalent sand grain roughness of a certain pipe gave the following data :*

diameter of pipe	= 30 cm
discharge	= 0.47 m^3/sec
head loss in 10 metres	= 1.9 metres
kinematic viscosity of fluid	= 10^{-6} m^2/sec.

Determine the equivalent sand grain roughness of the pipe. What is the maximum roughness in order that the pipe may act as hydrodynamically smooth at the given discharge ?

Solution. The average velocity of flow,

$$V = \frac{Q}{A} = \frac{0.47}{\frac{\pi}{4} \times (0.3)^2} = \frac{0.47}{0.071} = 6.65 \text{ m/s}$$

the Reynolds number of flow, $\mathbf{R} = \dfrac{V \cdot d}{\nu} = \dfrac{6.65 \times 0.3}{10^{-6}} = 1.995 \times 10^6$.

From the Darcy-Weisbach equation, the friction factor is obtained as

$$h_L = f \frac{L}{d} \frac{V^2}{2g}$$

$$\therefore \quad f = \frac{h_L . d . 2g}{LV^2}$$

$$= \frac{1.9 \times 0.3 \times 2 \times 9.81}{10 \times (6.65)^2} = 0.0253.$$

From the resistance formula for rough pipes, Eq. (9.62)

$$\frac{1}{\sqrt{f}} = 2 \log_{10} \frac{r_0}{k_s} + 1.74$$

or $$\frac{1}{\sqrt{0.0253}} - 1.74 = 2 \log_{10} \frac{r_0}{k_s}$$

or $$2 \log_{10} (r_0/k_s) = \frac{1}{0.159} - 1.74 = 4.56$$

$$r_0/k_s = 190.5$$

or $$k_s = \frac{r_0}{190.5} = \frac{0.15}{190.5} \text{ m} = 0.785 \times 10^{-3} \text{ m}.$$

If δ' = nominal thickness of laminar sub-layer, then for hydrodynamically smooth pipe,

$$\frac{k_s}{\delta'} < \frac{1}{4}$$

The nominal thickness of laminar sub-layer is defined by

$$\frac{u_* \delta'}{\nu} = 11.6$$

where $$u_* = \sqrt{\frac{\tau_0}{\rho}} \text{ and } \tau_0 = \frac{f}{8} \rho V^2$$

$$\therefore \quad u_* = V \sqrt{\frac{f}{8}} \quad \text{or} \quad u_* = 0.65 \times \sqrt{\frac{0.02}{8}} = 0.37 \text{ m/s}$$

$$\therefore \quad \delta' = 11.6 \, \nu/u_* = \frac{11.6 \times 10^{-6}}{0.37} = 31.35 \times 10^{-6} \text{ m}.$$

Source:A.K Jain

For the pipe to behave as hydrodynamically smooth at the given discharge, the maximum roughness,

$$k_s = \frac{1}{4}\delta' = \frac{31.35 \times 10^{-6}}{4} = 7.84 \times 10^{-6} \text{ m.}$$

Example 9.2. *A pipeline carrying water has surface protusions of average height 0.10 mm. If the shear stress developed is 7.85 N/m² determine whether the pipe surface acts as smooth, rough or in transition. For water take ρ = 1000 kg/m³ and kinematic viscosity ν = 0.93 × 10⁻² stokes.*

Solution. $k_s = 0.10$ mm, $\tau = 7.85$ N/m²

$$u_* = \sqrt{\frac{\tau_0}{\rho}} = \sqrt{\frac{7.85}{1000}} = 0.0877 \text{ m/s}$$

$$\frac{u_* k_s}{\nu} = \frac{0.0877 \times 0.10/1000}{0.93 \times 10^{-2} \times 10^{-4}} = 9.42$$

Since the roughness Reynolds number $\frac{u_* k_s}{\nu}$ lies between 4 and 100, the pipe surface acts is in transition.

Source: A.K Jain

Lecture 2

Drag and Lift

Introduction

In aerodynamics, the **lift-to-drag ratio**, or **L/D ratio**, is the amount of lift generated by a wing or vehicle, divided by the aerodynamic drag it creates by moving through the air. A higher or more favorable L/D ratio is typically one of the major goals in aircraft design; since a particular aircraft's required lift is set by its weight, delivering that lift with lower drag leads directly to better fuel economy in aircraft, climb performance, and glide ratio.

The term is calculated for any particular airspeed by measuring the lift generated, then dividing by the drag at that speed. These vary with speed, so the results are typically plotted on a 2D graph. In almost all cases the graph forms a U-shape, due to the two main components of drag.

Lift and Drag for Flow About a Rotating Cylinder

The pressure at large distances from the cylinder is uniform and given by p_0.

Deploying Bernoulli's equation between the points at infinity and on the boundary of the cylinder,

$$p_b = \rho g\left[\frac{U_0^2}{2g} + \frac{p_0}{\rho_g} - \frac{U_b^2}{2g}\right] \quad (23.9)$$

Hence,

$$U_b = \frac{1}{r}\frac{\partial\phi}{\partial\theta} = -2U_0\sin\theta - \frac{r}{2\pi}\left[\frac{U_0}{\chi}\right]^{1/2} \quad (23.10)$$

From Eqs (23.9) and (23.10) we can write

$$p_b = \rho g\left[\frac{U_0^2}{2g} + \frac{p_0}{\rho g}\right] - \left[\frac{-2U_0\sin\theta - \frac{\tau}{2\pi}\left(\frac{U_0}{\chi}\right)^{1/2}}{2g}\right]^2 \quad (23.11)$$

The lift may calculated as

$$L = -\int_0^{2\pi} p_b \sin\theta \left[\frac{\chi}{U_0}\right]^{1/2} d\theta$$

$$L = -\int_0^{2\pi}\left\{\frac{\rho U_0^2}{2} + p_0 - \frac{\rho\left[-2U_0\sin\theta - \frac{\Gamma}{2\pi}\left(\frac{U_0}{\chi}\right)^{1/2}\right]^2}{2}\right\}\left[\frac{x}{U_0}\right]^{1/2}(\sin\theta)d\theta$$

or,

$$L = -\int_0^{2\pi}\left[\frac{\rho U_0^2}{2}\left(\frac{\chi}{U_0}\right)^{1/2}\sin\theta + p_0\left(\frac{\chi}{U_0}\right)^{1/2}\sin\theta - \frac{\rho}{2}\left\{4U_0^2\sin^2\theta + \frac{4U_0 r\sin\theta}{2\pi}\left(\frac{U_0}{\chi}\right)^{1/2} + \frac{\Gamma^2}{4\pi^2}\left[\frac{U_0}{\chi}\right.\right.\right.$$

$$L = -\int_0^{2\pi}\left[\frac{\rho U_0^2}{2}\left(\frac{\chi}{U_0}\right)^{1/2}\sin\theta + p_0\left(\frac{\chi}{U_0}\right)^{1/2}\sin\theta - 2\rho U_0^2\sin^3\theta\left(\frac{\chi}{U_0}\right)^{1/2} - \frac{\rho U_0\Gamma}{\pi}\sin^2\theta - \frac{\rho\Gamma^2}{8\pi^2}\left(\frac{\chi}{U_0}\right.\right.$$

$$L = \rho U_0 \Gamma \tag{23.12}$$

The drag force , which includes the multiplication by cosθ (and integration over 2π) is zero.

- Thus the inviscid flow also demonstrates lift.
- lift becomes a simple formula involving only the density of the medium, free stream velocity and circulation.
- in two dimensional incompressible steady flow about a boundary of any shape, the lift is always a product of these three quantities.----- **Kutta- Joukowski theorem**

Aerofoil Theory

Aerofoils are streamline shaped wings which are used in airplanes and turbo machinery. These shapes are such that the drag force is a very small fraction of the lift. The following nomenclatures are used for defining an aerofoil

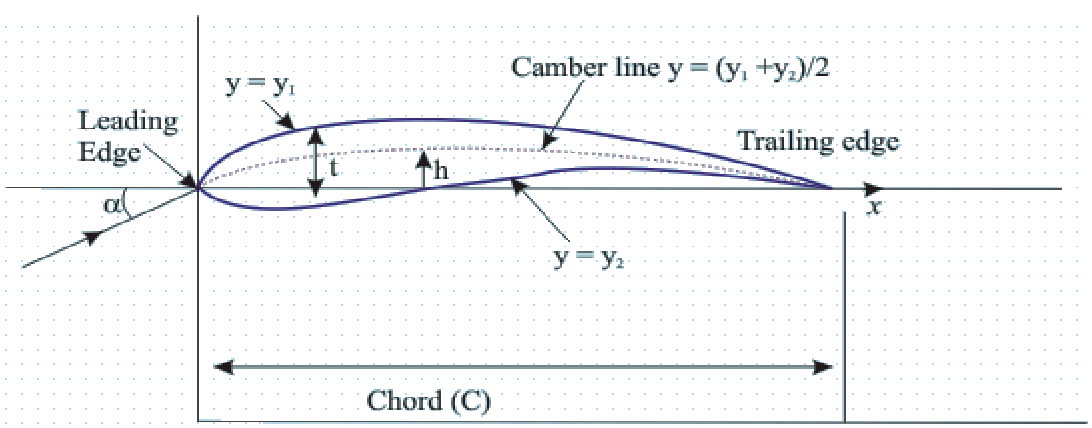

Fig 23.4 Aerofoil Section

- The **chord** (C) is the distance between the leading edge and trailing edge.
- The length of an aerofoil, normal to the cross-section (i.e., normal to the plane of a paper) is called the **span** of a aerofoil.
- The **camber line** represents the mean profile of the aerofoil. Some important geometrical parameters for an aerofoil are the ratio of maximum thickness to chord (t/C) and the ratio of maximum camber to chord (h/C). When these ratios are small, an aerofoil can be considered to be thin. For the analysis of flow, a thin aerofoil is represented by its camber.

The theory of thick cambered aerofoils uses a complex-variable mapping which transforms the inviscid flow across a rotating cylinder into the flow about an aerofoil shape with circulation.

Flow Around a Thin Aerofoil

- Thin aerofoil theory is based upon the superposition of uniform flow at infinity and a continuous distribution of clockwise free vortex on the camber line having circulation density $\gamma(s)$ per unit length .
- The circulation density $\gamma(s)$ should be such that the resultant flow is tangent to the camber line at every point.
- Since the slope of the camber line is assumed to be small, $\gamma(s)ds = \gamma(\eta)d\eta$. The total circulation around the profile is given by

$$\Gamma = \int_0^C \gamma(\eta)\, d\eta \qquad (23.13)$$

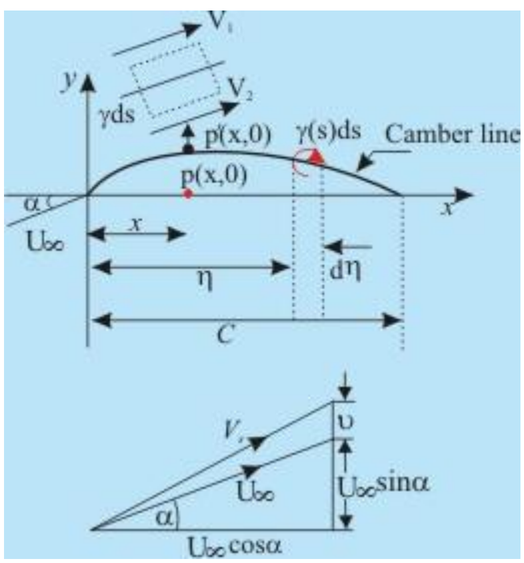

Fig 23.5 Flow Around Thin Aerofoil

A vortical motion of strength $\gamma d\eta$ at x= η develops a velocity at the **point p** which may be expressed as

$$d\upsilon = \frac{\gamma(\eta)d\eta}{2\pi(\eta - x)} \text{ acting upwards}$$

The total induced velocity in the upward direction at **point p** due to the entire vortex distribution along the camber line is

$$\upsilon(x) = \frac{1}{2\pi}\int_0^C \frac{\gamma(\eta)d\eta}{(\eta - x)} \qquad (23.14)$$

For a small camber (having small α), this expression is identically valid for the induced velocity at **point p'** due to the vortex sheet of variable strength $\gamma(s)$ on the camber line. The resultant velocity due to U_∞ and v(x) must be tangential to the camber line so that the slope of a camber line may be expressed as

$$\frac{dy}{dx} = \frac{U_\infty \sin\alpha + \upsilon}{U_\infty \cos\alpha} = \tan\alpha + \frac{\upsilon}{U_\infty \cos\alpha}$$

$$\frac{dy}{dx} = \alpha + \frac{\nu}{U_\infty}\left[\text{since } \alpha \text{ is very small}\right] \qquad (23.15)$$

From Eqs (23.14) and (23.15) we can write

$$\frac{dy}{dx} = \alpha + \frac{1}{2\pi U_\infty}\int_0^c \frac{\gamma(\eta)d\eta}{\eta - x}$$

Consider an element ds on the camber line. Consider a small rectangle (drawn with dotted line) around ds. The upper and lower sides of the rectangle are very close to each other and these are parallel to the camber line. The other two sides are normal to the camber line. The circulation along the rectangle is measured in clockwise direction as

$V_1 ds - V_2 ds = \gamma ds$ [normal component of velocity at the camber line should be zero]

or $V_1 - V_2 = \gamma$

If the mean velocity in the tangential direction at the camber line is given by $Vs = (V_1 + V_2)/2$, it can be rewritten as

$V_1 = Vs + \frac{\gamma}{2}$ and $V_2 = V_s - \frac{\gamma}{2}$

if v is very small $[v << U_\infty], V_s$ becomes equal to $U_\infty \gamma(s)$. The difference in velocity across the camber line brought about by the vortex sheet of variable strength causes pressure difference and generates lift force.

Generation of Vortices Around a Wing

- **The lift around an aerofoil is generated following Kutta-Joukowski theorem** . Lift is a product of ρ , U_∞ and the circulation Γ.

$$Lift = \rho U_\infty \Gamma$$

- When the motion of a wing starts from rest, vortices are formed at the trailing edge.
- At the start, there is a velocity discontinuity at the trailing edge. This is eventual because near the trailing edge, the velocity at the bottom surface is higher than that at the top surface. This discrepancy in velocity culminates in the formation of vortices at the trailing edge.
- Figure 23.6(a) depicts the formation of starting vortex by impulsively moving aerofoil. However, the starting vortices induce a counter circulation as shown in Figure 23.6(b). The circulation around a path (ABCD) enclosing the wing and just shed (starting) vortex must be zero. Here we refer to **Kelvin's theorem** once again.

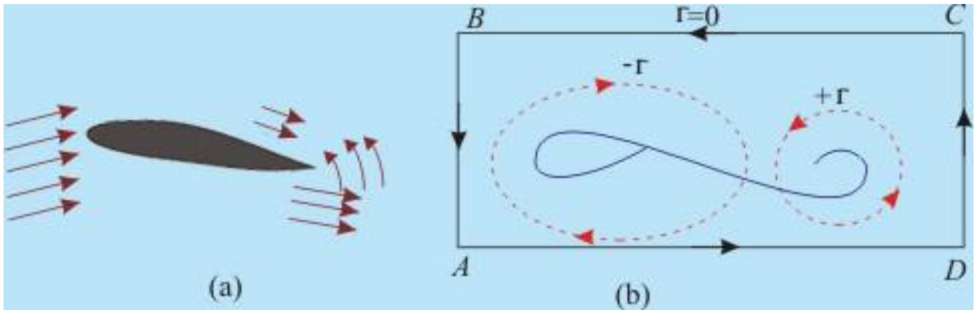

Fig 23.6 Vortices Generated when an Aerofoil Just Begins to Move

- Initially, the flow starts with the zero circulation around the closed path. Thereafter, due to the change in angle of attack or flow velocity, if a fresh starting vortex is shed, the circulation around the wing will adjust itself so that a net zero vorticity is set around the closed path.
- Real wings have finite span or finite aspect ratio (AR) λ , defined as

$$\lambda = \frac{b^2}{A_s} \qquad (23.16)$$

where b is the span length, A_s is the plan form area as seen from the top..

- For a wing of finite span, the end conditions affect both the lift and the drag. In the leading edge region, pressure at the bottom surface of a wing is higher than that at the top surface. The longitudinal vortices are generated at the edges of finite wing owing to pressure differences between the bottom surface directly facing the flow and the top surface.

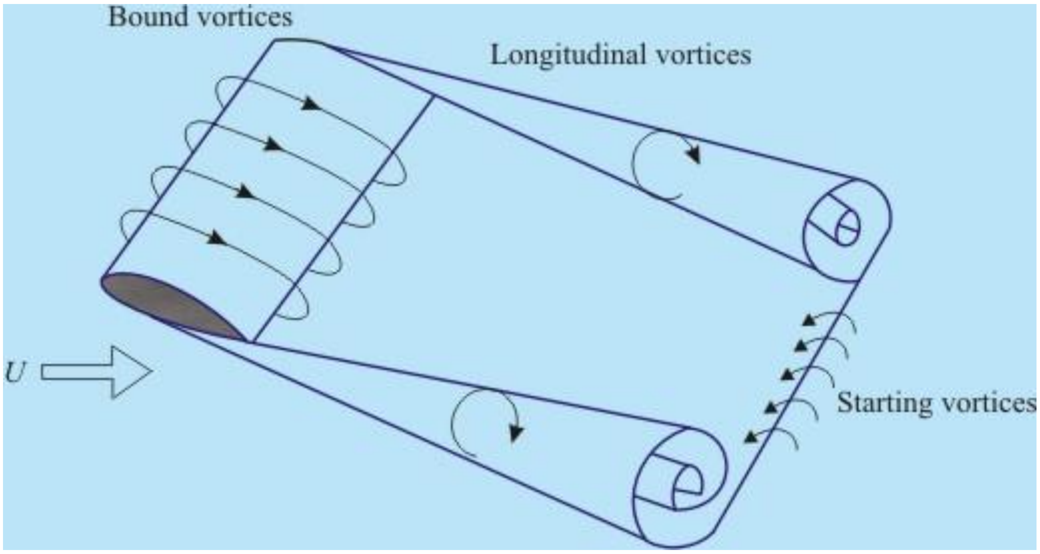

Fig 23.7 Vortices Around a Finite Wing

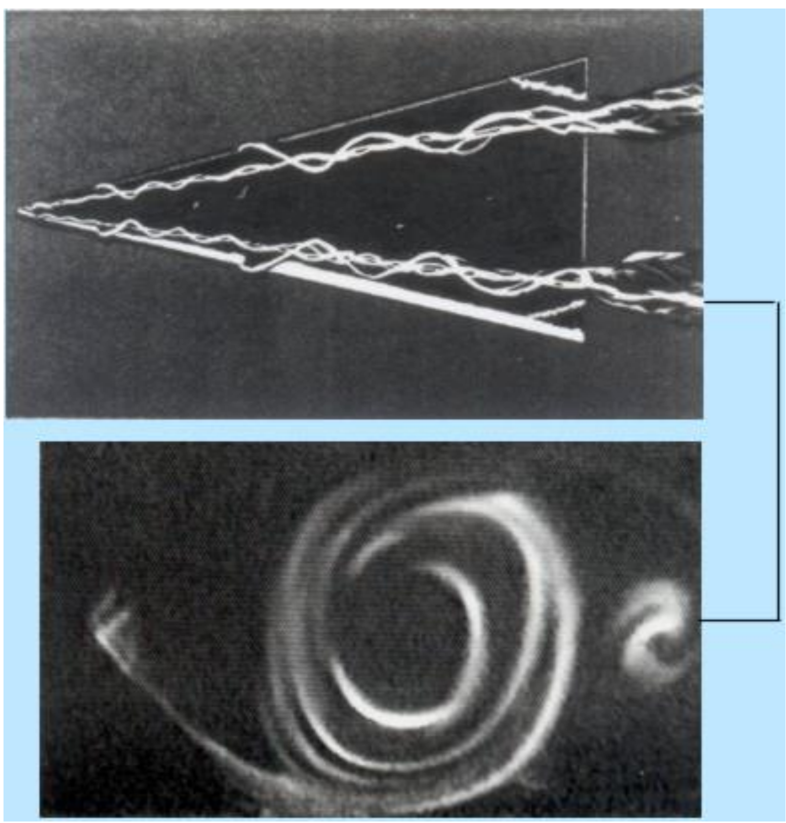

Fig 23.8 Generation of Longitudinal Vortices

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1. Fluid Mechanics by A.K. Jain, Khanna Publishers

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1. Fluid Mechanics and Hydraulic Machines, Modi & Seth, Standard Pulishers

2. Introduction to Fluid Mechanics and Fluid Machines, S.K. Som & G. Biswas,