Network Theory

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Gyan Ranjan Biswal received his B.E. in Electronics Engineering from the Pt. Ravishankar Shukla University, India in 1999 and M. Tech. (Honors) in Instrumentation & Control Engineering from the Chhattisgarh Swami Vivekananda Technical University, India in 2009 followed by Ph.D. in Electrical Engineering, specialized in the area of Power System Instrumentation (Power Generation Automation) from the Indian Institute of Technology Roorkee, India in 2013.

He is expertise in Design and Development of cooling systems for large size electrical generators, and the C&I of process industries. He has been in academia for about twelve years. Presently, he is with VSS University of Technology, Burla, India at the capacity of Head and Associate Professor, EEE from Dec. 2016. He has more than 65 publications in various Journals and Conferences of Internationally repute to his credit. He also holds a patent as well, and filed one more. He also adapted one international edition book published by Pearson India. He received research grants of US$90,000 (INR 50 lakhs). He has been supervised 09 Masters' theses, and registered 04 PhD theses. He has also been recognized with many national and international awards by elite bodies. He has been awarded with CICS award under the head of Indian National Science Academy for travel support to USA, MHRD Fellowship by Govt. of India, and Gopabandhu Das Scholarship in his career. His major areas of interests are Power System Instrumentation, Industrial Automation, Robust and Intelligent Control, the Smart Sensors, IoT enabled Smart Sensors, the Smart Grid, Fuel Cell lead Sustainable Sources of Energy, and System Reliability.

Dr. Biswal is a Fellow IE (India), Senior Member of IEEE, USA, and Life Member of ISTE, India. He is actively involved in review panels of different societies of international repute viz. IEEE, IFAC, and the ISA. Currently, he is also actively involved as a Member of IEEE–SA (Standards Association) working groups; IEEE P1876 WG, IEEE P21451-001 WG, and IEEE P1415. He has also been invited for delivering guest lectures at World Congress on Sustainable Technologies (WCST) Conf. 2012, London, UK, INDICON 2015, New Delhi, India, National Power Training Institute (NPTI), Nangal, India, and G.B. Pant Engineering College, Pauri, Gharwal, India, Surendra Sai University of Technology (formerly UCE), Burla, and as a guest expert in 2016 IEEE PES General Meeting Boston, MA, USA.

Syllabus

Network Theory

MODULE-I (9 HOURS) [Online mode: 5 HOURS + 1 Test]

Analysis of Coupled Circuits: Self-inductance and Mutual inductance, Coefficient of coupling, Series connection of coupled circuits, Dot convention, Ideal Transformer, Analysis of multi-winding coupled circuits, Analysis of single tuned and double tuned coupled circuits.

Transient Response: Transient study in series RL, RC, and RLC networks by time domain and Laplace transform method with DC and AC excitation. Response to step, impulse and ramp inputs of series RL, RC and RLC circuit.

MODULE-II (7 HOURS) [Online mode: 5 HOURS + 1 Test]

Two Port networks: Types of port Network, short circuit admittance parameter, open circuit impedance parameters, Transmission parameters, Condition of Reciprocity and Symmetry in two port network, Inter-relationship between parameters, Input and Output Impedances in terms of two port parameters, Image impedances in terms of ABCD parameters, Ideal two port devices, ideal transformer. Tee and Pie circuit representation, Cascade and Parallel Connections.

MODULE-III (8 HOURS) [Online mode: 5 HOURS + 1 Test]

Network Functions & Responses: Concept of complex frequency, driving point and transfer functions for one port and two port network, poles & zeros of network functions, Restriction on Pole and Zero locations of network function, Time domain behavior and stability from pole-zero plot, Time domain response from pole zero plot.

Three Phase Circuits: Analysis of unbalanced loads, Neutral shift, Symmetrical components, Analysis of unbalanced system, power in terms of symmetrical components.

MODULE-IV (9 HOURS) [Online mode: 5 HOURS + 1 Test]

Network Synthesis: Realizability concept, Hurwitz property, positive realness, properties of positive real functions, Synthesis of R-L, R-C and L-C driving point functions, Foster and Cauer forms.

MODULE-V (6 HOURS) [Online mode: 5 HOURS + 1 Test]

Graph theory: Introduction, Linear graph of a network, Tie-set and cut-set schedule, incidence matrix, Analysis of resistive network using cut-set and tie-set, Dual of a network.

Filters: Classification of filters, Characteristics of ideal filters.

Text and Reference Books

Recommended Text Books:

1. "Introductory Circuit Analysis", Robert L. Boylestad, Pearson, 12th ed., 2012.
2. "Network Analysis", M. E. Van Valkenburg, Pearson, 3rd ed., 2006.
3. "Engineering Circuit Analysis", W. Hayt, TMH, 2006.
4. "Network Analysis & Synthesis", Franklin Fa-Kun. Kuo, John Wiley & Sons.

Reference Books:

* "Basic Circuit Theory, Huelsman, PHI, 3rd ed.,
* "HUGHES Electrical and Electronic Technology", Revised by J. Hiley, K. Brown, and I. M. Smith, Pearson, 10th ed., 2011.
* "Circuits and Networks", Sukhija and Nagsarkar, Oxford Univ. Press, 2012.
* "Fundamentals of Electric Circuits", C. K. Alexander and M. N. O. Sadiku, McGraw-Hill Higher Education, 3rd ed., 2005.
* "Fundamentals of Electrical Engineering", L. S. Bobrow, Oxford University Press, 2nd ed., 2011.
* "Circuit Theory (Analysis and Synthesis)", A. Chakrabarti, Dhanpat Rai pub.

Other Important References

Reference Sites:	
1.	NPTEL, The National Programme on Technology Enhanced Learning (NPTEL): https://nptel.ac.in/
2.	MIT OpenCourseWare : https://ocw.mit.edu/index.htm

Course Outcomes

Upon successful completion of this course, you (students) will be able to

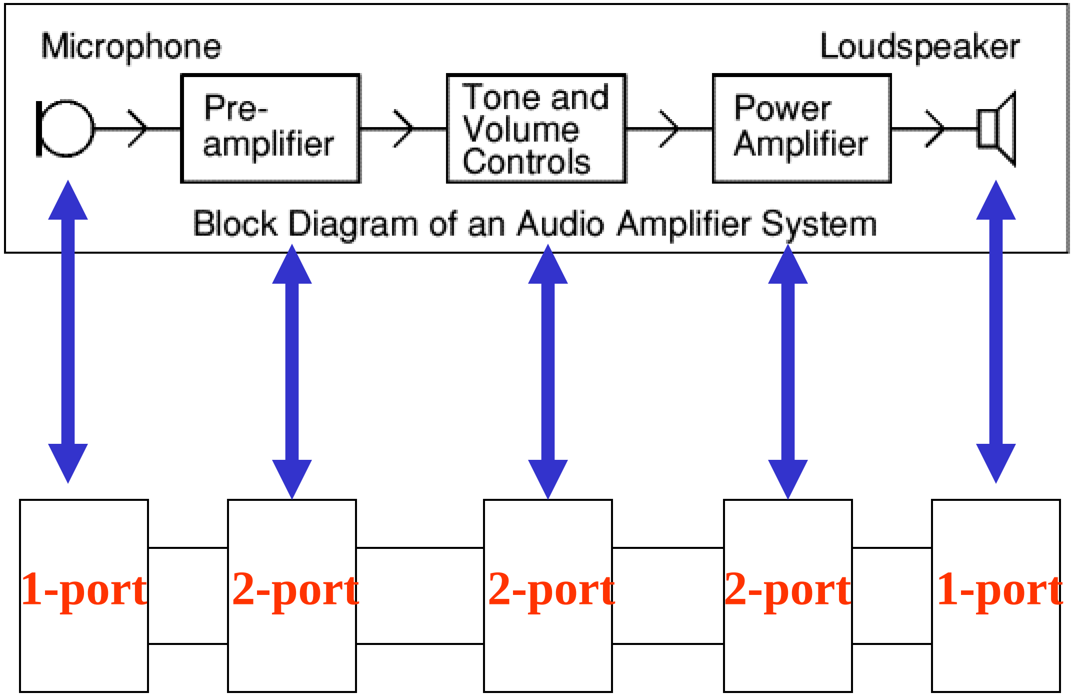
CO1	Analyze coupled circuits and understand the difference between the steady state and transient response of 1st and 2nd order circuit and understand the concept of time constant.
CO2	Learn the different parameters of two port network.
CO3	Concept of network function and three phases circuit and know the difference of balanced and unbalanced system and importance of complex power and its components.
CO4	Synthesis the electrical network.
CO5	Analyse the network using graph theory and understand the importance of filters in electrical system.

Two-Port Analysis

- ❖ **Special thanks to Oxford University Press, India and**
- ❖ **The University of Tennessee, USA**

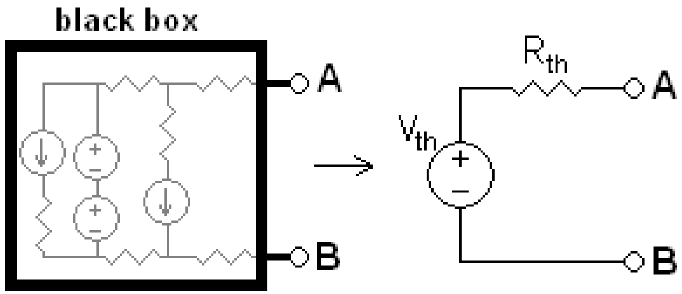
- Review of one ports
- Various two-port descriptions
- Terminated nonlinear two-ports
- Impedance and admittance matrices of two-ports
- Other two-port parameter matrices
- The hybrid matrices
- The transmission matrices
- Interconnection of networks

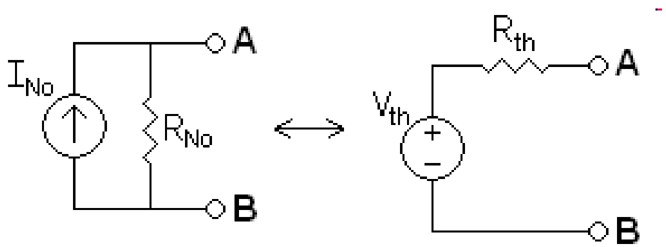
Review of one ports



Review of one ports

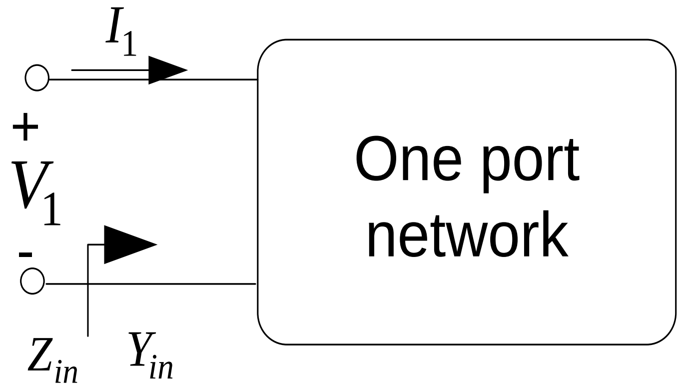
Thevenin's Equivalent Circuit





Norton's Equivalent Circuit

LTI one ports



Input impedance

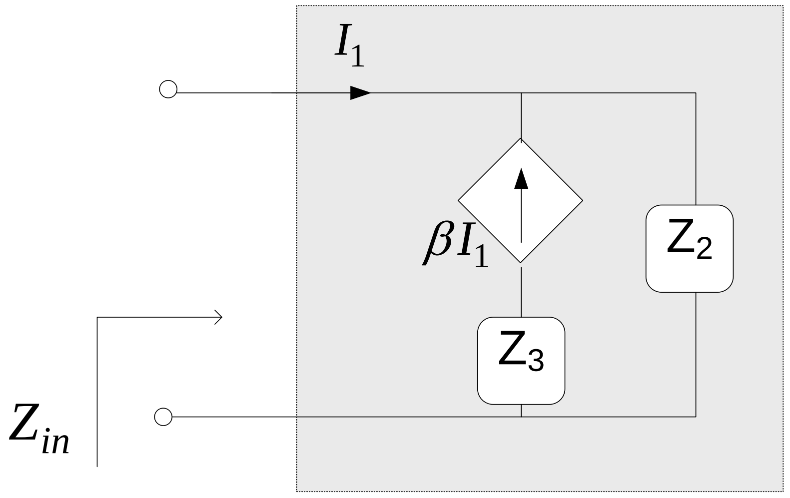
Input admittance

$$Z_{in} = \frac{V_1}{I_1}$$

$$Y_{in} = \frac{I_1}{V_1}$$

Example 1

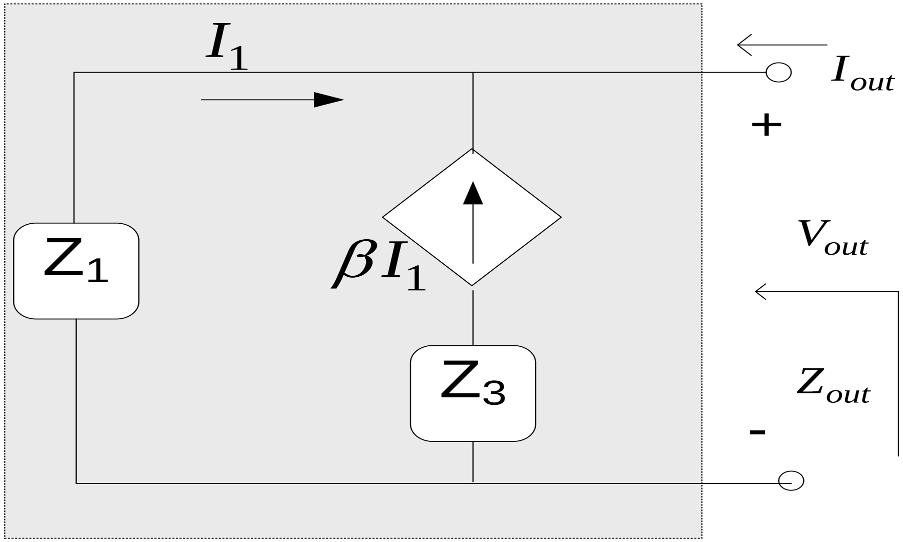
Determine the input impedance of the circuit in Fig.



$$I_{in} = I_1 = -\beta I_1 + \frac{V_{in}}{Z_2} \qquad V_{in} = (1+\beta) Z_2 I_{in} \qquad Z_{in} = (1+\beta) Z_2$$

Example 2

Determine the output impedance of the circuit in Fig.

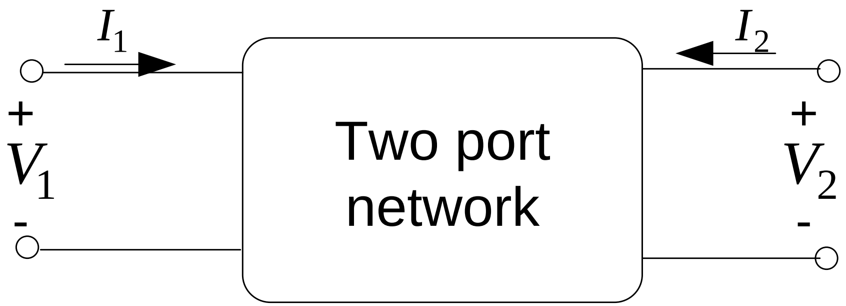


$$I_{out} = -I_1 - \beta I_1 = (1+\beta)\frac{V_{out}}{Z_1}$$

$$Z_{out} = \frac{V_{out}}{I_{out}} = \frac{Z_1}{1+\beta}$$

A two-port network

- Circuits can be considered by theirs terminal variables
- Voltages and currents are terminal's variables
- Complex circuit can be analyzed more easily.
- There are many kinds of two port parameters.



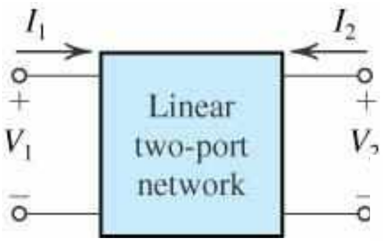
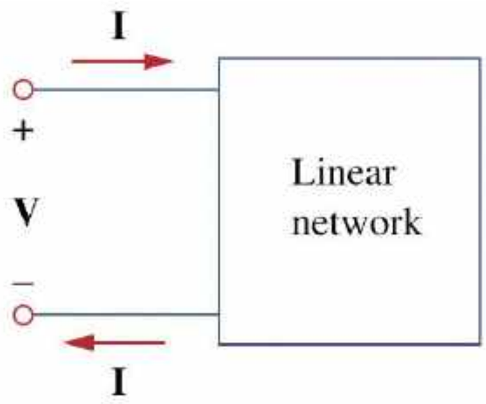
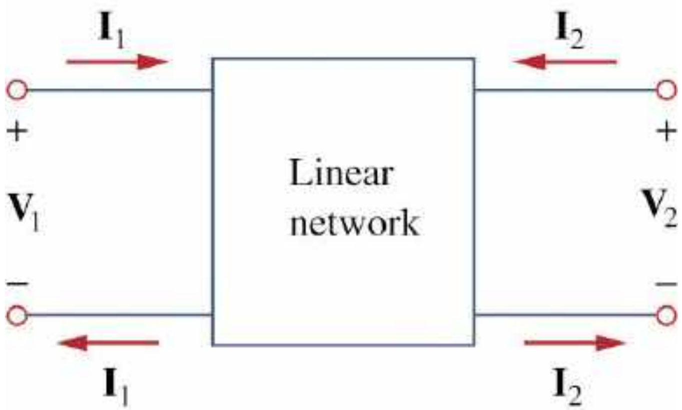


Figure: The reference directions of the four port variables in a linear two-port network.



(a)



(b)

- A **two-port Network** is an electrical network with two separate ports for input and output.

$$\mathbf{i} = g(\mathbf{v})$$ or $$i_1 = g_1(v_1, v_2)$$ $$i_2 = g_2(v_1, v_2)$$

Port current

Port voltage

$$\mathbf{v} = r(\mathbf{i})$$ or $$v_1 = r_1(i_1, i_2)$$ $$v_2 = r_2(i_1, i_2)$$

Or hybrid

$$v_1 = h_1(i_1, v_2)$$

$$i_2 = h_2(i_1, v_2)$$

Impedance Parameters

- Impedance parameters are very useful in designing impedance matching and power distribution system. Two port network can either be voltage or current driven. The input and output terminal voltage can be presented as follows:

$$V_1 = z_{11}I_1 + z_{12}I_2$$

$$V_2 = z_{21}I_1 + z_{22}I_2$$

where impedance parameters of the system is $z = \begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix}$

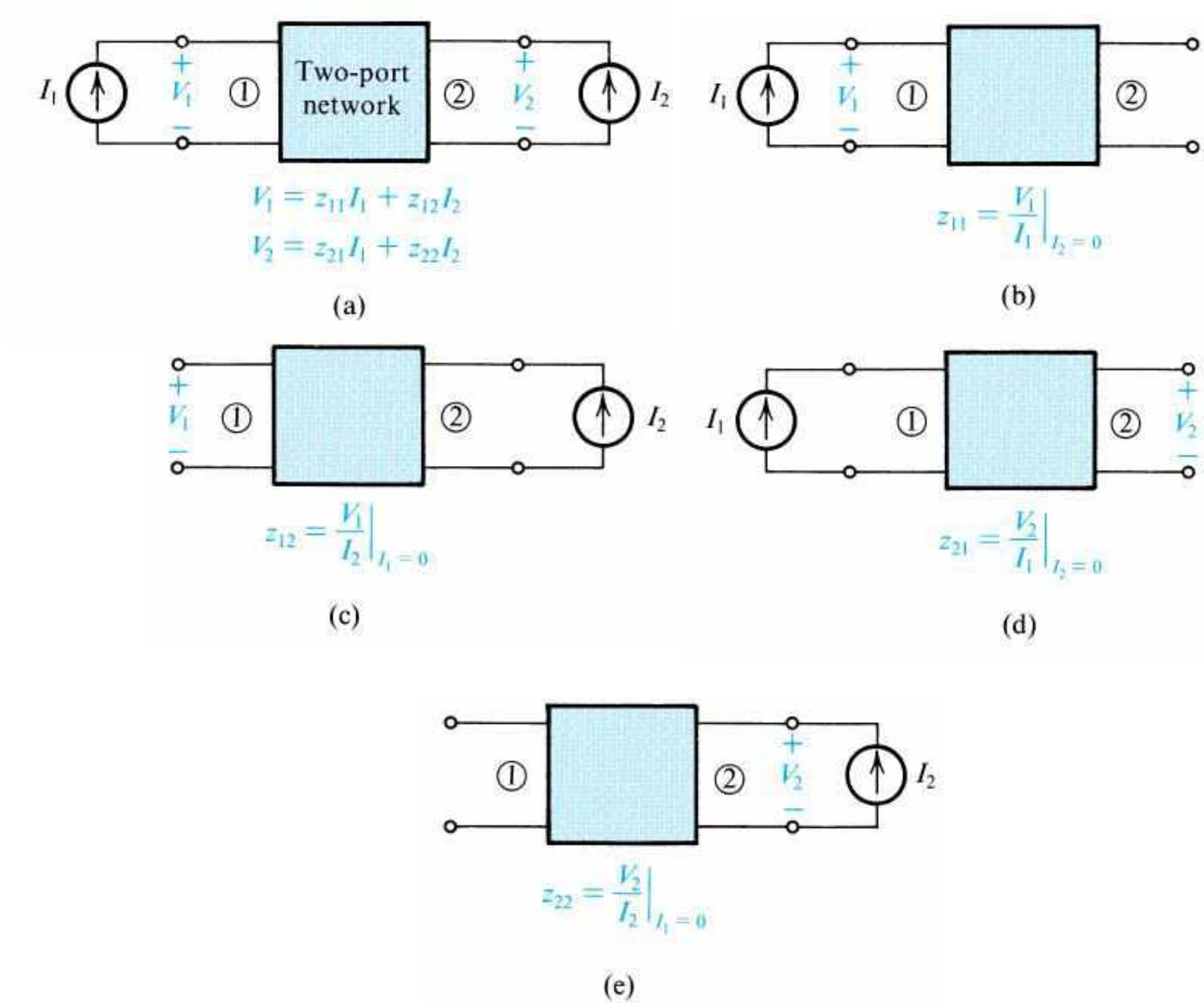
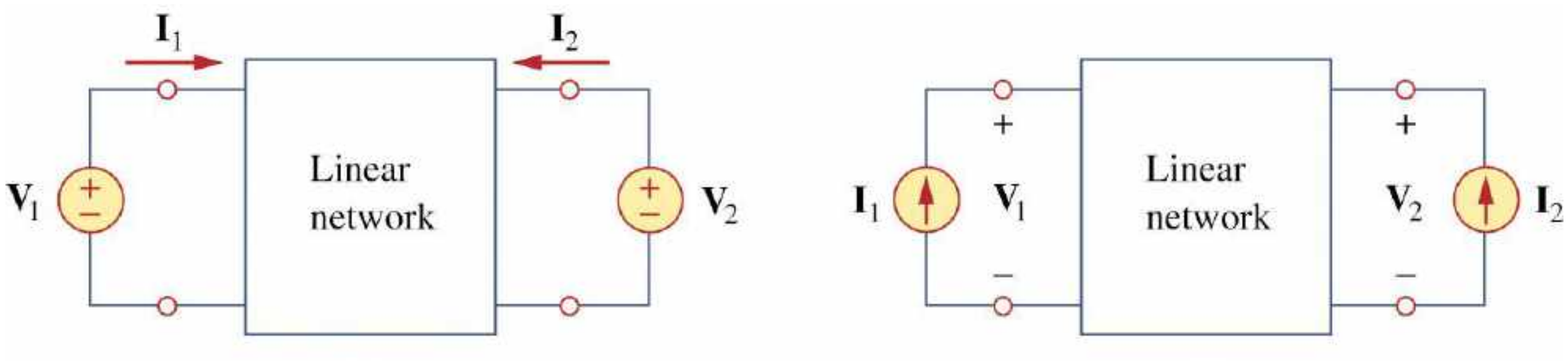


Figure: Definition and conceptual measurement circuits for *z* parameters.

$$\mathbf{V}_1 = \mathbf{z}_{11}\mathbf{I}_1 + \mathbf{z}_{12}\mathbf{I}_2$$
$$\mathbf{V}_2 = \mathbf{z}_{21}\mathbf{I}_1 + \mathbf{z}_{22}\mathbf{I}_2$$

$$\begin{bmatrix} \mathbf{V}_1 \\ \mathbf{V}_2 \end{bmatrix} = \begin{bmatrix} \mathbf{z}_{11} & \mathbf{z}_{12} \\ \mathbf{z}_{21} & \mathbf{z}_{22} \end{bmatrix} \begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \end{bmatrix} = [\mathbf{z}] \begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \end{bmatrix}$$



(a) (b)

$$\mathbf{z}_{11} = \left.\frac{\mathbf{V}_1}{\mathbf{I}_1}\right|_{\mathbf{I}_2=0}, \quad \mathbf{z}_{12} = \left.\frac{\mathbf{V}_1}{\mathbf{I}_2}\right|_{\mathbf{I}_1=0}$$

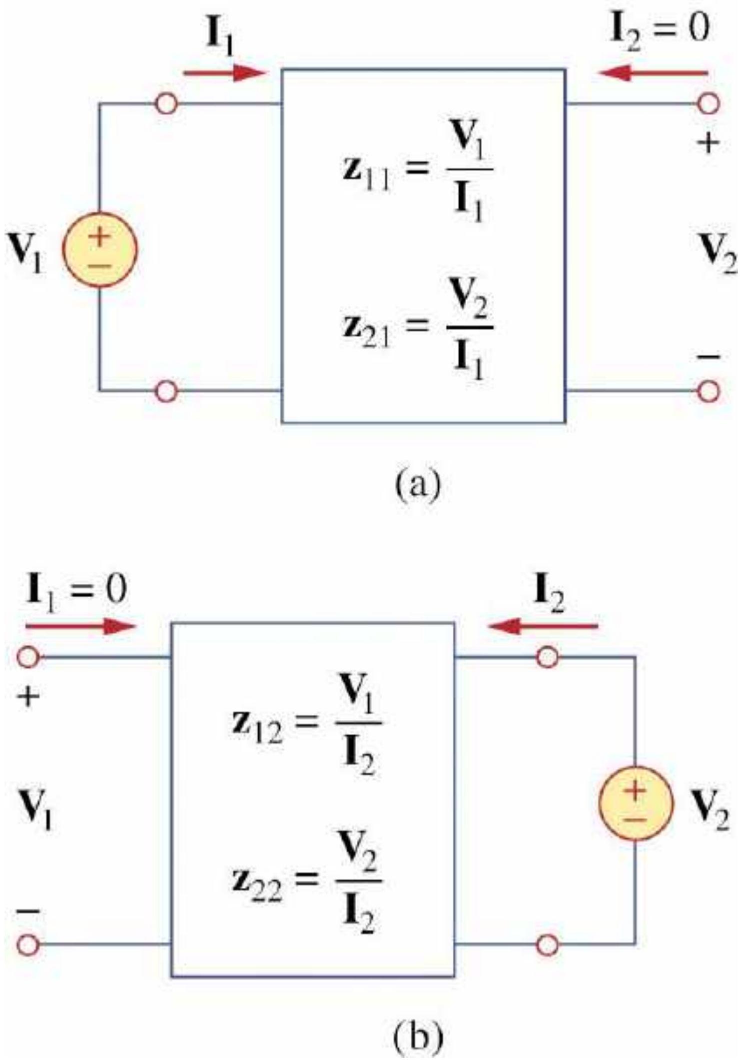
$$\mathbf{z}_{21} = \left.\frac{\mathbf{V}_2}{\mathbf{I}_1}\right|_{\mathbf{I}_2=0}, \quad \mathbf{z}_{22} = \left.\frac{\mathbf{V}_2}{\mathbf{I}_2}\right|_{\mathbf{I}_1=0}$$

$\mathbf{z}_{11}$ = Open-circuit input impedance

$\mathbf{z}_{12}$ = Open-circuit transfer impedance from port 1 to port 2

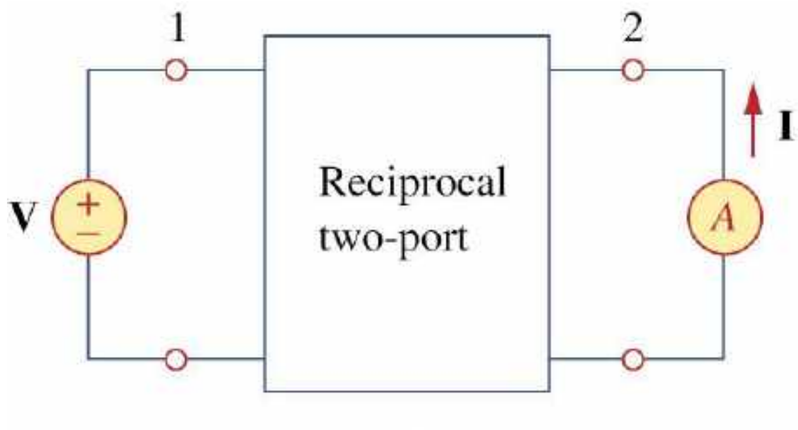
$\mathbf{z}_{21}$ = Open-circuit transfer impedance from port 2 to port 1

$\mathbf{z}_{22}$ = Open-circuit output impedance

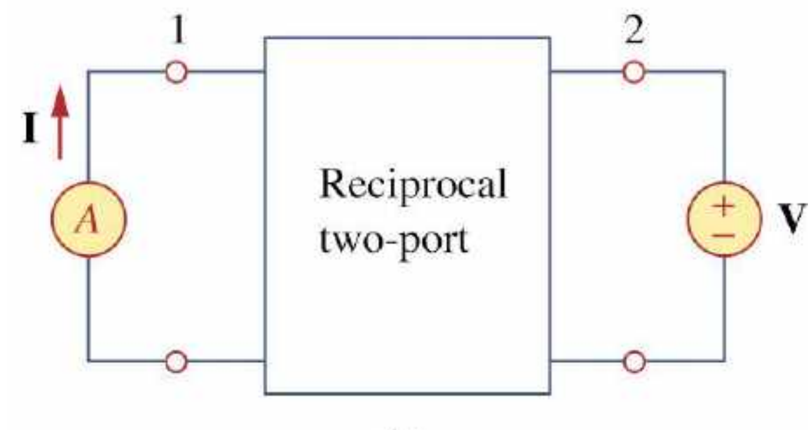


$$z_{11} = \frac{V_1}{I_1}, \quad z_{21} = \frac{V_2}{I_1}$$

$$z_{12} = \frac{V_1}{I_2}, \quad z_{22} = \frac{V_2}{I_2}$$

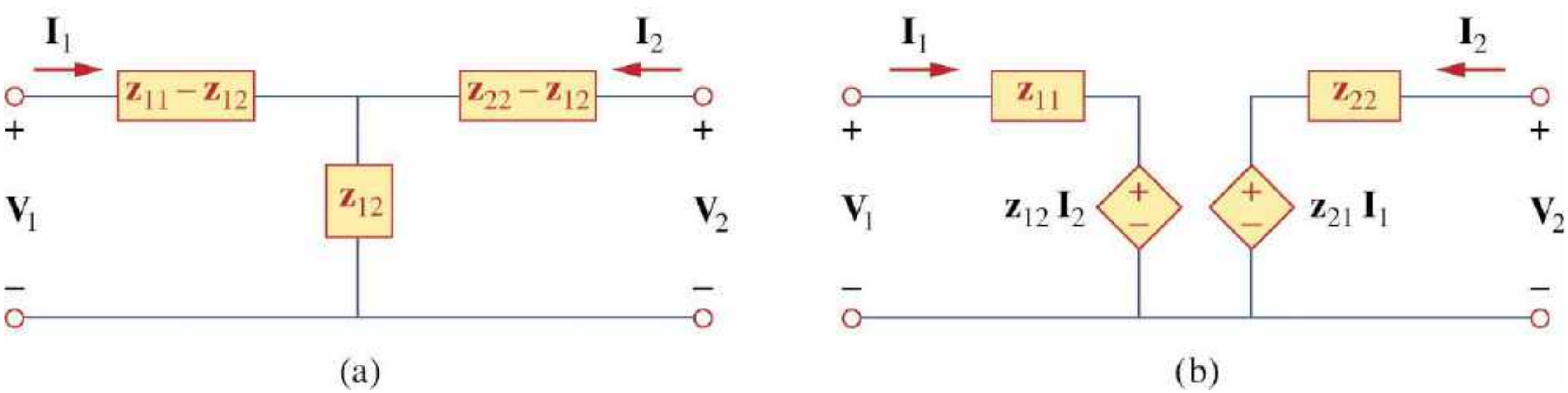


(a)

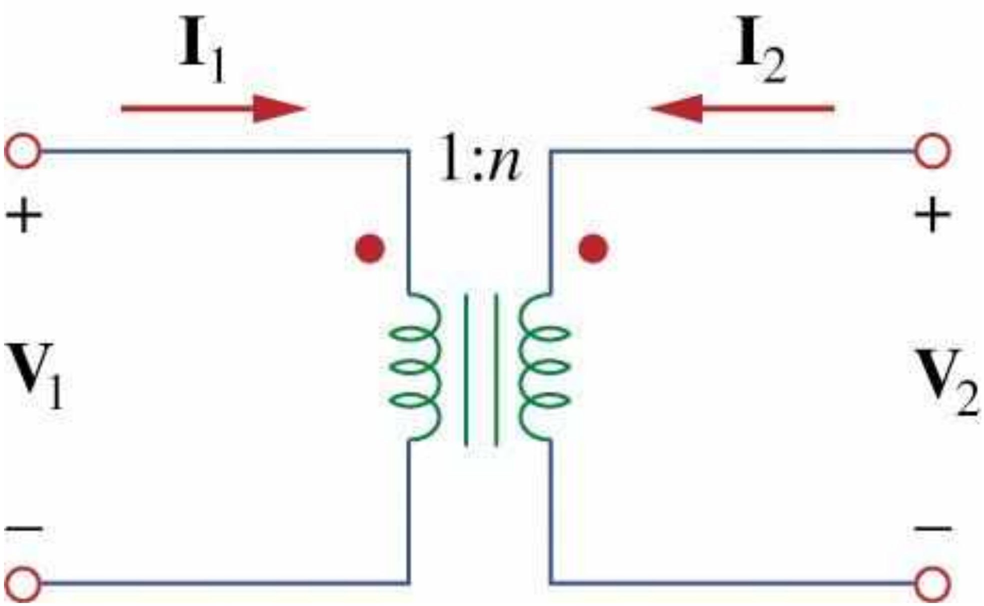


(b)

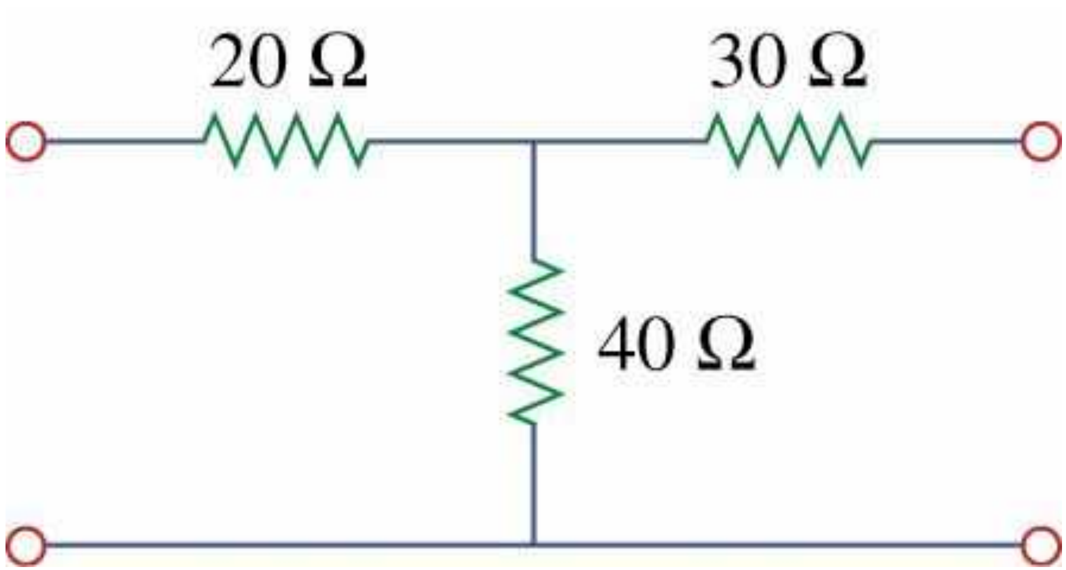
$$\mathbf{z}_{21} = \mathbf{z}_{12}$$


I1
I2
z11 − z12
z22 − z12
z12
V1
V2
+
−
(a)
z11
z22
z12 I2
z21 I1
(b)

$$\mathbf{V}_1 = \frac{1}{n}\mathbf{V}_2, \quad \mathbf{I}_1 = -n\mathbf{I}_2$$



- Determine the *z* parameters for the circuit:

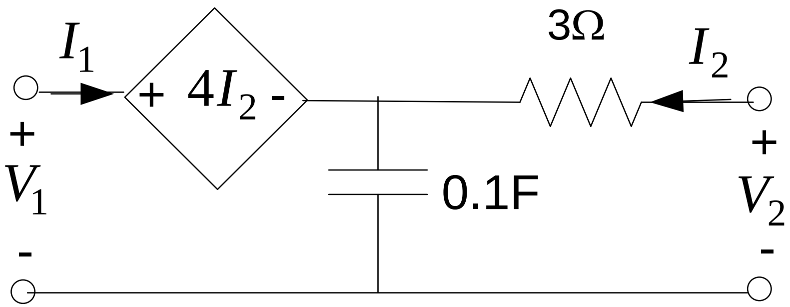


z_{11}	3.000E+01
z_{21}	2.000E+01

z_{12}	2.000E+01
z_{22}	5.000E+01

Example

Determine the impedance parameters from the circuit in Fig.



In frequency domain

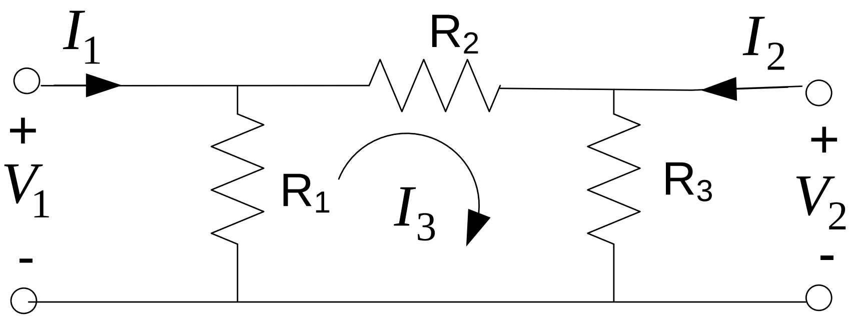
$$V_1 = 4I_2 + \frac{10}{s}(I_1 + I_2) = \frac{10}{s}I_1 + (4 + \frac{10}{s})I_2$$

$$V_2 = 3I_2 + \frac{10}{s}(I_1 + I_2) = \frac{10}{s}I_1 + (3 + \frac{10}{s})I_2$$

$$Z = \begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix} = \begin{bmatrix} \frac{10}{s} & \frac{4s+10}{s} \\ \frac{10}{s} & \frac{3s+10}{s} \end{bmatrix}$$

Example

Compute the z-parameter of the circuit in Fig.



Or the simplest approach is apply Delta-Star conversion, and then solve through Z-parameter

$$V_1 = R_1 I_1 - R_1 I_3$$

$$V_2 = R_3 I_2 + R_3 I_3$$

$$0 = -R_1 I_1 + R_3 I_2 + (R_1 + R_2 + R_3) I_3$$

$$I_3 = \frac{R_1}{R_1 + R_2 + R_3} I_1 - \frac{R_3}{R_1 + R_2 + R_3} I_2$$

$$V_1 = (R_1 - \frac{R_1^2}{R_1 + R_2 + R_3})I_1 + \frac{R_1R_3}{R_1 + R_2 + R_3} I_2$$

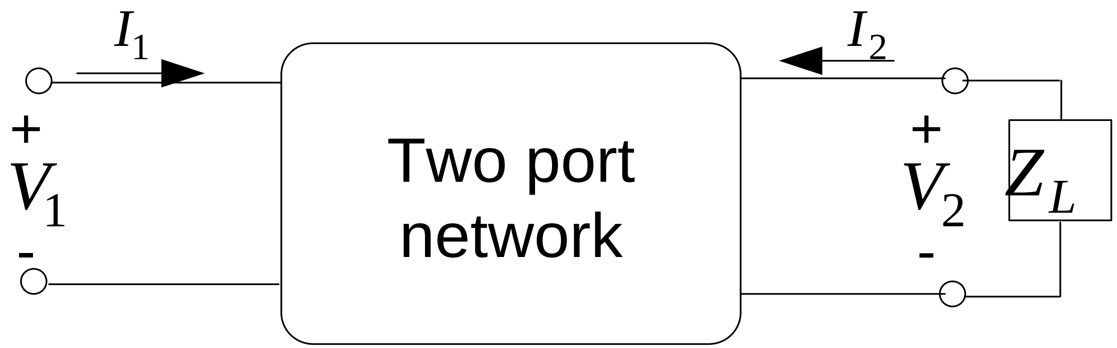
$$= \frac{R_1(R_2 + R_3)}{R_1 + R_2 + R_3} I_1 + \frac{R_1R_3}{R_1 + R_2 + R_3} I_2$$

$$V_2 = \frac{R_1R_3}{R_1 + R_2 + R_3} I_1 + (R_3 - \frac{R_3^2}{R_1 + R_2 + R_3})I_2$$

$$= \frac{R_1R_3}{R_1 + R_2 + R_3} I_1 + \frac{R_3(R_1 + R_2)}{R_1 + R_2 + R_3} I_2$$

$$\begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{21} \end{bmatrix} = \begin{bmatrix} \frac{R_1(R_2 + R_3)}{R_1 + R_2 + R_3} & \frac{R_1R_3)}{R_1 + R_2 + R_3} \\ \frac{R_1R_3)}{R_1 + R_2 + R_3} & \frac{R_3(R_1 + R_2)}{R_1 + R_2 + R_3} \end{bmatrix}$$

Z parameter analysis of terminated two-port



Z-parameter equations

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} \qquad V_2 = -Z_L I_2$$

$$\begin{bmatrix} V_1 \\ 0 \end{bmatrix} = \begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} + Z_L \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

From Crammer's rules

$$I_1 = \frac{\begin{vmatrix} V_1 & z_{12} \\ 0 & z_{22}+Z_L \end{vmatrix}}{\begin{vmatrix} z_{11} & z_{12} \\ z_{21} & z_{22}+Z_L \end{vmatrix}} = \frac{(z_{22}+Z_L)V_1}{z_{11}(z_{22}+Z_L)-z_{12}z_{21}}$$

The input impedance Z_{in}

$$Z_{in} = z_{11} - \frac{z_{12}z_{21}}{z_{22}+Z_L}$$

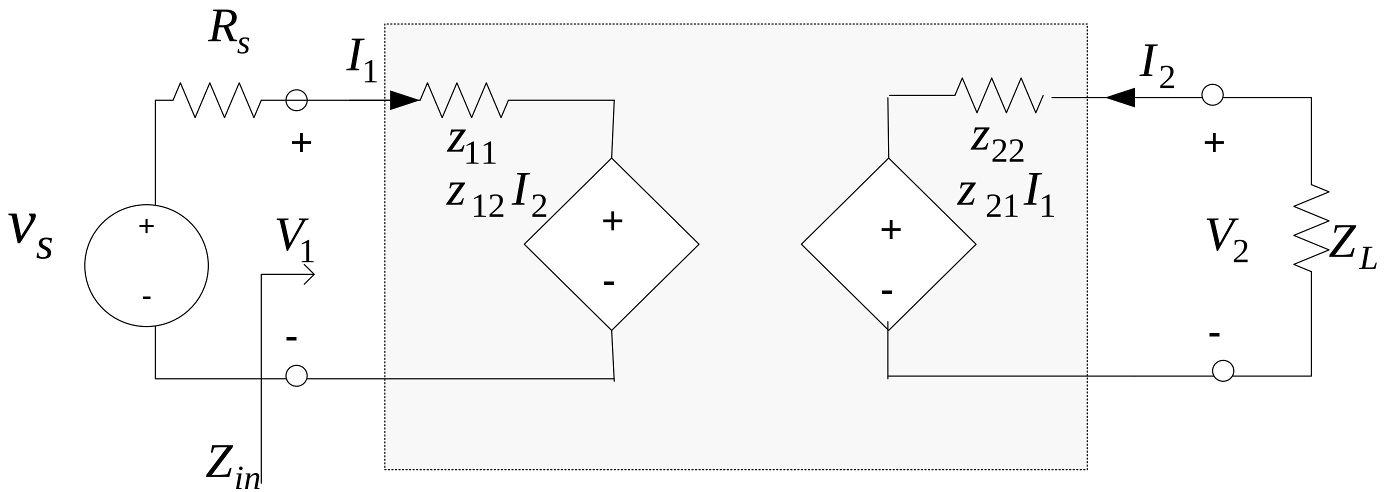
and

$$z_{21}I_1 = -(z_{22}+Z_L)I_2$$

$$I_2 = -\frac{z_{21}}{z_{22}+Z_L}I_1$$

$$V_1 = z_{11}I_1 + z_{12}I_2 = \left(z_{11} - \frac{z_{12}z_{21}}{z_{22} + Z_L} \right) I_1$$

Gain: $$\frac{V_2}{V_s} = \frac{V_1}{V_s} \cdot \frac{V_2}{V_1} = \frac{Z_{in}}{Z_{in} + Z_s} \cdot \frac{Z_L}{z_{22} + Z_L} \frac{z_{21}}{Z_{in}} = \frac{Z_L}{z_{22} + Z_L} \cdot \frac{z_{21}}{Z_{in} + Z_s}$$

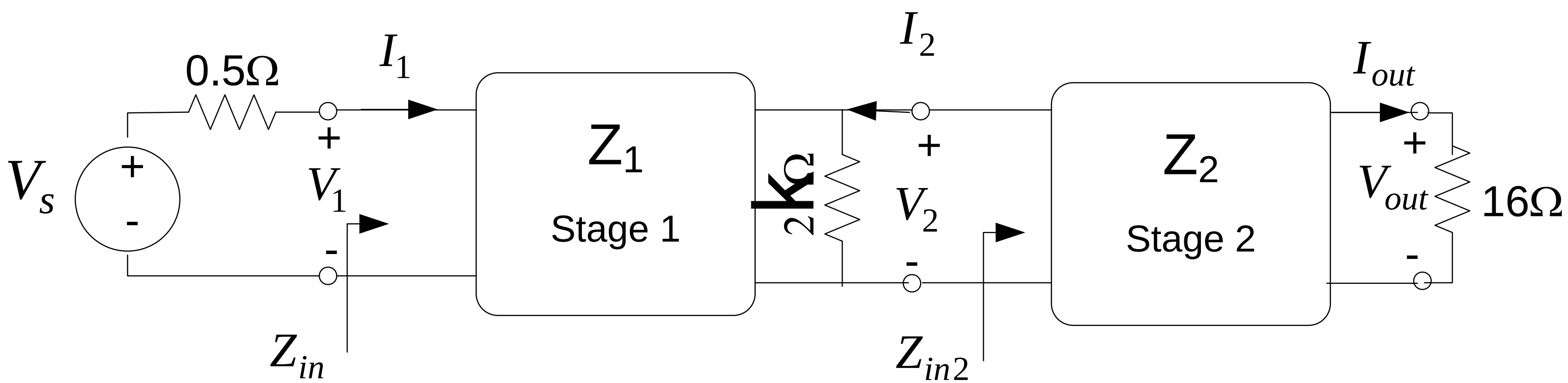


Terminated two-port Z-parameter model

Example:

The circuit in Figure is a two-stage transistor amplifier. The Z-parameters for each stage are

$$Z_1 = \begin{bmatrix} 350 & 2.667 \\ -10^6 & 6,667 \end{bmatrix} \qquad Z_2 = \begin{bmatrix} 1.0262\times10^6 & 6,790.8 \\ 1.0258\times10^6 & 6,793.5 \end{bmatrix}$$



Determine a) The input impedance Z_{in}

b) The overall voltage gain

c) Check the matching of the load and output impedance

Solution

$$Z_{in2} = z_{11} - \frac{z_{12} z_{21}}{z_{22} + Z_L}$$

$$= 1.0262 \cdot 10^6 - \frac{6790.8 \times 1.0258 \cdot 10^6}{6793.5 + 16}$$

$$= 3{,}159 \quad \Omega$$

$$\frac{V_{out}}{V_2} = \frac{Z_L}{z_{22} + Z_L} \frac{z_{21}}{Z_{in2}}$$

$$= \frac{16(1.0258 \cdot 10^6)}{(16 + 6793.5)3{,}159}$$

$$= 0.7629$$

$$Z_{L1} = 2k \,//\, Z_{in2} = 2000 \,//\, 3159 = 1224.7\Omega$$

$$Z_{in} = z_{11} - \frac{z_{12} z_{21}}{z_{22} + Z_{L1}}$$

$$= 350 + \frac{2.667 \times 10^{6}}{6667 + 1224.7}$$

$$= 687.9 \quad \Omega$$

$$\frac{V_2}{V_s} = \frac{Z_{L1}}{Z_{L1} + z_{22}} \frac{z_{21}}{Z_s + Z_{in}}$$

$$= \left(\frac{1224.7}{1224.7 + 6667} \right) \left(\frac{-10^{6}}{75 + 687.9} \right)$$

$$= -203.4$$

0.902 225.6

$$\frac{V_2}{V_s} = \frac{V_1}{V_s} \cdot \frac{V_2}{V_1} = \frac{Z_{in}}{Z_{in} + Z_s} \cdot \frac{Z_L}{z_{22} + Z_L} \frac{z_{21}}{Z_{in}} = \frac{Z_L}{z_{22} + Z_L} \cdot \frac{z_{21}}{Z_{in} + Z_s}$$

The overall voltage gain

$$A_{VS} = \frac{V_{out}}{V_s} = \frac{V_{out}}{V_2}\frac{V_2}{V_s}$$

$$= 0.7629 \times (-203.4)$$

$$= -155.2\ V/V$$

Out put impedance

$$Z_{out} = \left.\frac{V_2}{I_2}\right|_{V_s=0}$$

Job for you… To show that?

$$Z_{out} = z_{22} - \frac{z_{12}z_{21}}{R_s + z_{11}}$$

$$Z_{out1} = z_{22} - \frac{z_{12} z_{21}}{R_s + z_{11}}$$

$$= 6667 + \frac{2.667 \times 10^6}{0.5 + 350}$$

$$= 14.276\,\mathrm{k}\,\Omega$$

$$R_{s2} = Z_{out1} \,//\, 2k = 1.7542\,\mathrm{k}\,\Omega$$

$$Z_{out} = 6793.5 - \frac{6790.8 \cdot 1.0258 \times 10^6}{1754.24 + 1.0262 \times 10^6}$$

$$= 16.93\,\Omega$$

Thus, the load is closely matched to the output impedance.

Admittance Parameters

- Admittance parameters are very useful for describing the network when impedance parameters may not be existed. This is solved by finding the second set of parameters by expressing the terminal current in term of the voltage. The input and output terminal current can be presented as follows:

$$I_1 = y_{11}V_1 + y_{12}V_2$$
$$I_2 = y_{21}V_1 + y_{22}V_2$$

where admittance parameters of the system is y = $\begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix}$

These parameters are call short-circuited admittance parameters.

$$y_{11} = \left.\frac{I_1}{V_1}\right|_{V_2=0} \qquad y_{12} = \left.\frac{I_1}{V_2}\right|_{V_1=0}$$

$$y_{21} = \left.\frac{I_2}{V_1}\right|_{V_2=0} \qquad y_{22} = \left.\frac{I_2}{V_2}\right|_{V_1=0}$$

where y_{11} short-circuit input impedance

y_{12} short-circuit transfer impedance from port 2 to 1

y_{21} short-circuit transfer impedance from port 1 to 2

y_{22} short-circuit output impedance

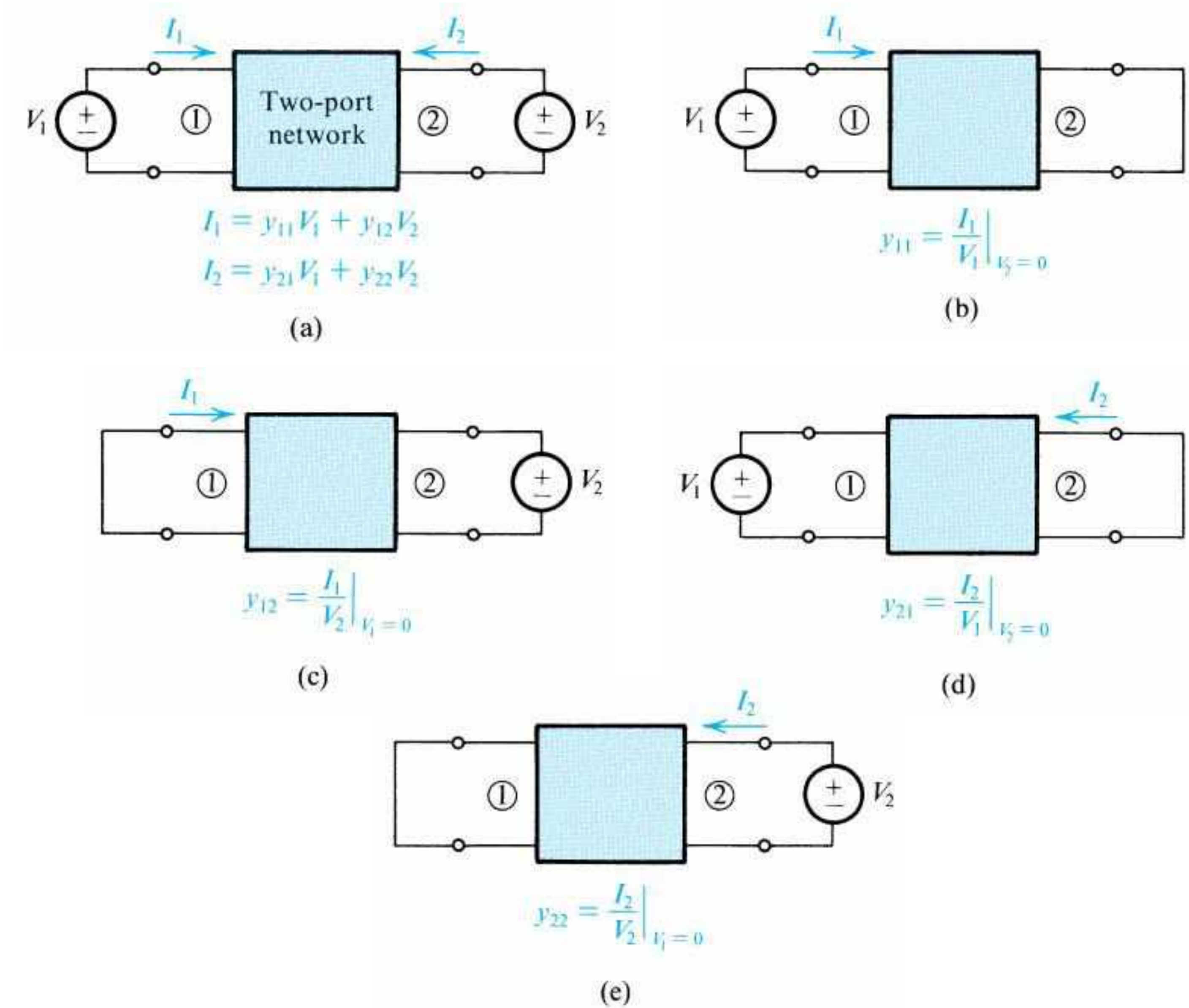
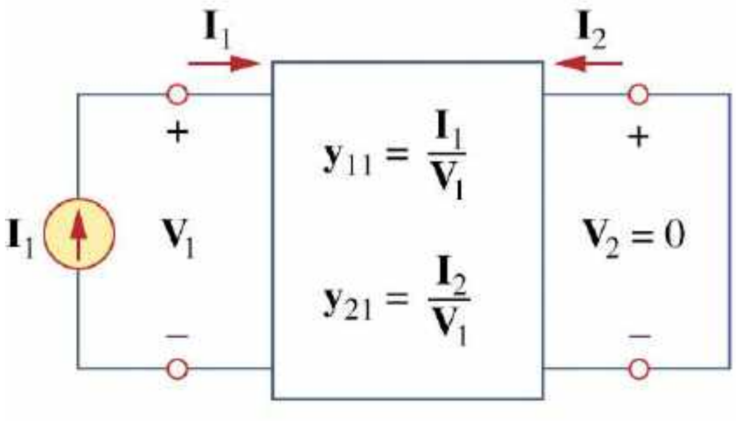
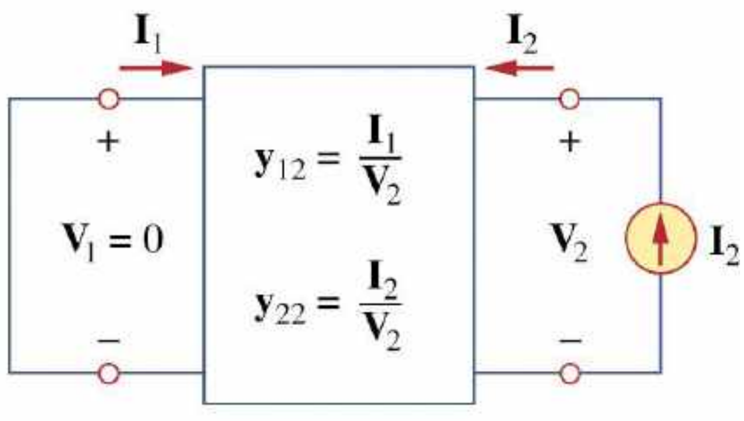


Figure: Definition and conceptual measurement circuits for *y* parameters.



(a)



(b)

$$\mathbf{I}_1 = \mathbf{y}_{11}\mathbf{V}_1 + \mathbf{y}_{12}\mathbf{V}_2$$
$$\mathbf{I}_2 = \mathbf{y}_{21}\mathbf{V}_1 + \mathbf{y}_{22}\mathbf{V}_2$$

$$\begin{bmatrix}\mathbf{I}_1\\ \mathbf{I}_2\end{bmatrix} = \begin{bmatrix}\mathbf{y}_{11} & \mathbf{y}_{12}\\ \mathbf{y}_{21} & \mathbf{y}_{22}\end{bmatrix}\begin{bmatrix}\mathbf{V}_1\\ \mathbf{V}_2\end{bmatrix} = [\mathbf{y}]\begin{bmatrix}\mathbf{V}_1\\ \mathbf{V}_2\end{bmatrix}$$

$$\mathbf{y}_{11} = \left.\frac{\mathbf{I}_1}{\mathbf{V}_1}\right|_{\mathbf{V}_2=0}, \quad \mathbf{y}_{12} = \left.\frac{\mathbf{I}_1}{\mathbf{V}_2}\right|_{\mathbf{V}_1=0}$$

$$\mathbf{y}_{21} = \left.\frac{\mathbf{I}_2}{\mathbf{V}_1}\right|_{\mathbf{V}_2=0}, \quad \mathbf{y}_{22} = \left.\frac{\mathbf{I}_2}{\mathbf{V}_2}\right|_{\mathbf{V}_1=0}$$

$\mathbf{y}_{11}$ = Short-circuit input admittance

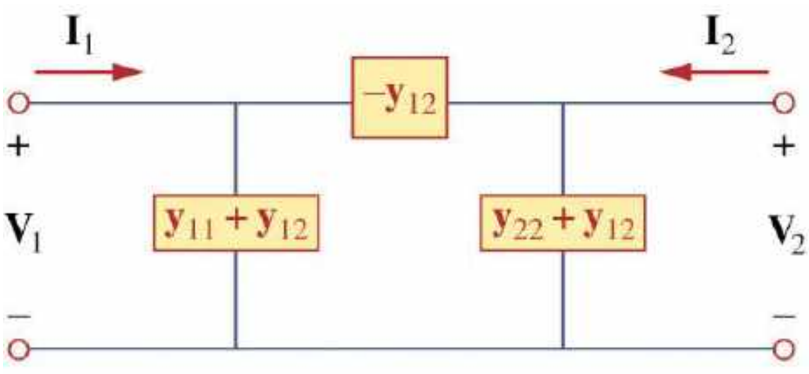
$\mathbf{y}_{12}$ = Short-circuit transfer admittance from port 1 to port 2

$\mathbf{y}_{21}$ = Short-circuit transfer admittance from port 2 to port 1

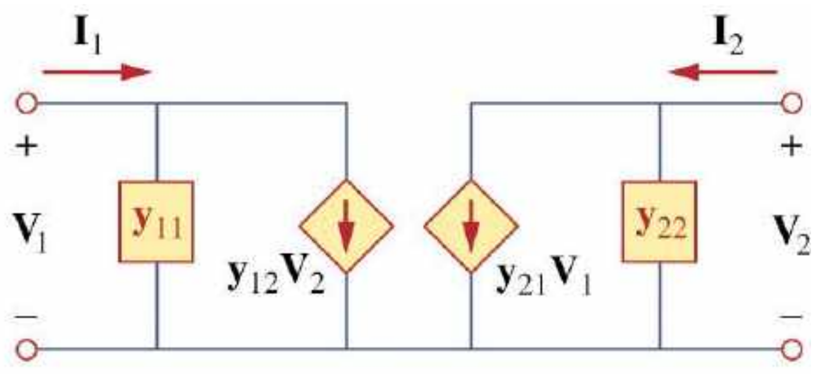
$\mathbf{y}_{22}$ = Short-circuit output admittance

$$\mathbf{y}_{11} = \frac{\mathbf{V}_1}{\mathbf{I}_1}, \quad \mathbf{y}_{21} = \frac{\mathbf{V}_2}{\mathbf{I}_1}$$

$$\mathbf{y}_{12} = \frac{\mathbf{V}_1}{\mathbf{I}_2}, \quad \mathbf{y}_{22} = \frac{\mathbf{V}_2}{\mathbf{I}_2}$$

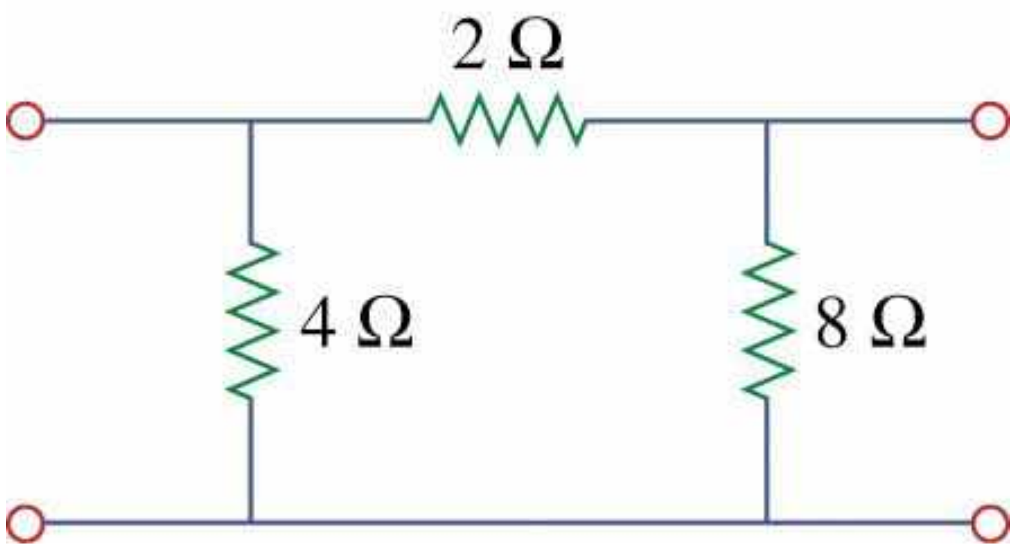


(a)



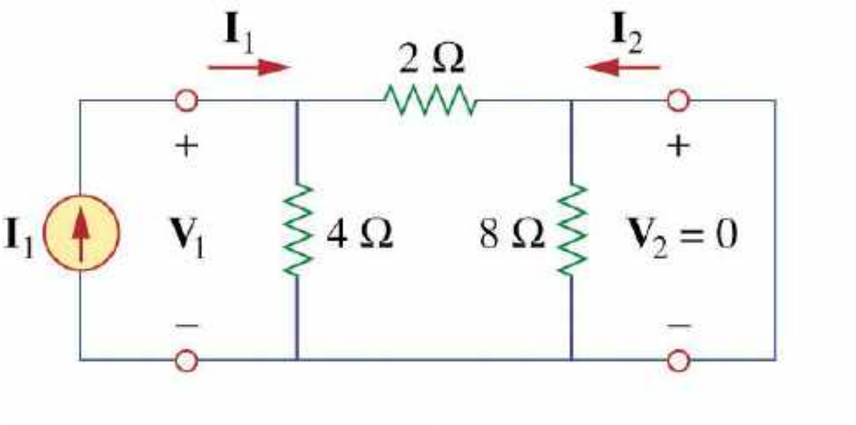
(b)

- Obtain the y parameters for the Π network shown:

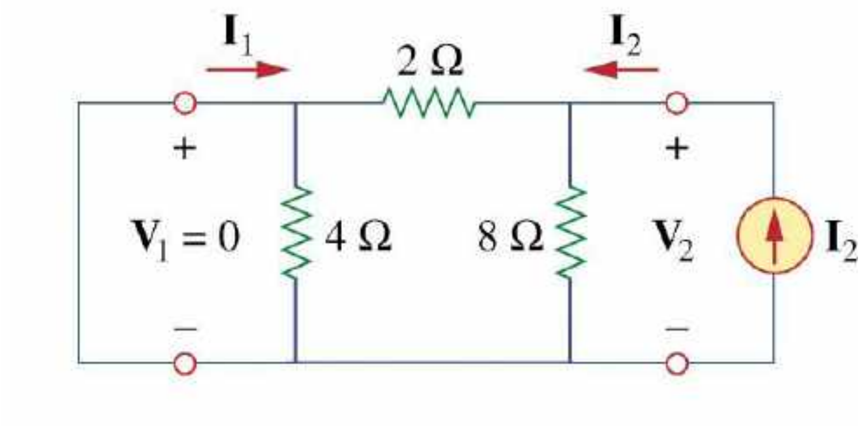


y_{11}	7.500E-01
y_{21}	-5.000E-01

y_{12}	-5.000E-01
y_{22}	6.250E-01



(a)



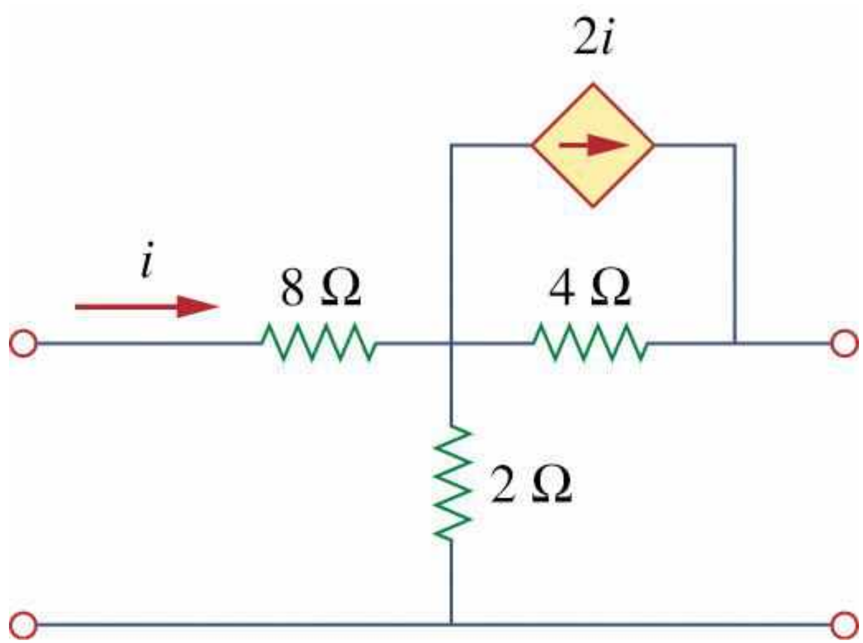
(b)

$$\mathbf{y}_{12} = -\frac{1}{2}\,\mathrm{S} = \mathbf{y}_{21}$$

$$\mathbf{y}_{11} + \mathbf{y}_{12} = \frac{1}{4} \Rightarrow \mathbf{y}_{11} = \frac{1}{4} - \mathbf{y}_{12} = 0.75\,\mathrm{S}$$

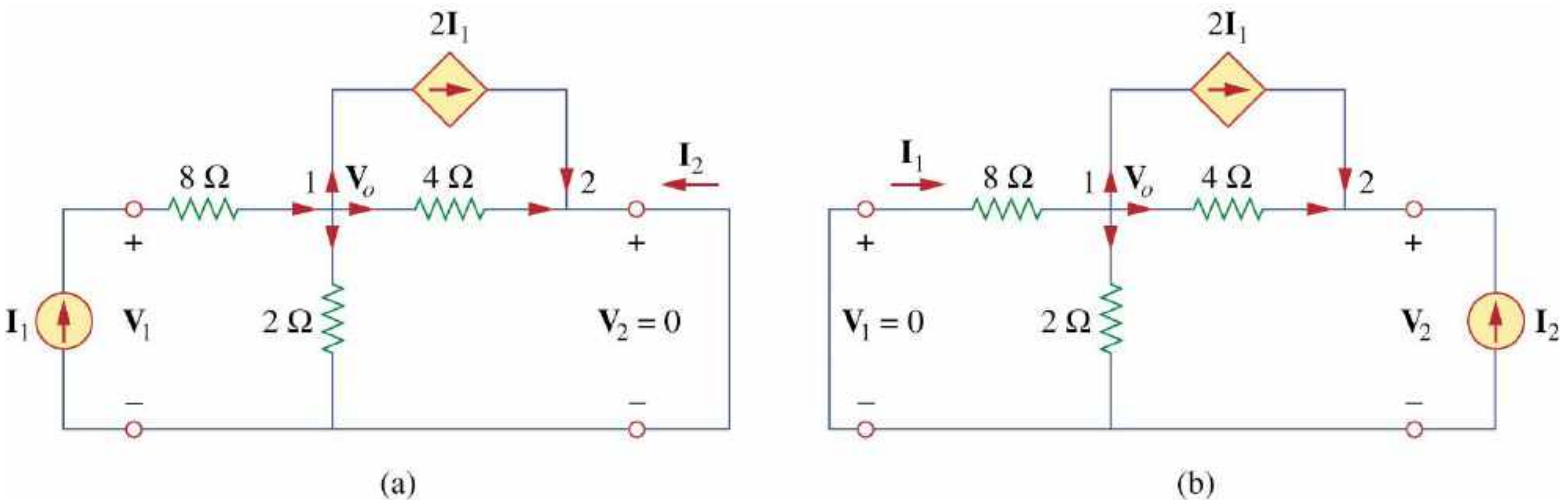
$$\mathbf{y}_{22} + \mathbf{y}_{12} = \frac{1}{8} \Rightarrow \mathbf{y}_{22} = \frac{1}{8} - \mathbf{y}_{12} = 0.625\,\mathrm{S}$$

- Determine the *y* parameters for the *T* network shown:



y_{11}	1.500E-01
y_{21}	-2.500E-01

y_{12}	-5.000E-02
y_{22}	2.500E-01



2I1
8 Ω
1
Vo
4 Ω
2
I2
I1
V1
2 Ω
V2 = 0
+
–
(a)
2I1
I1
8 Ω
1
Vo
4 Ω
2
V1 = 0
2 Ω
V2
I2
+
–
(b)

$$\text{At node 1, } \frac{\mathbf{V}_1 - \mathbf{V}_o}{8} = 2\mathbf{I}_1 + \frac{\mathbf{V}_o}{2} + \frac{\mathbf{V}_o - 0}{4}$$

$$\text{But } \mathbf{I}_1 = \frac{\mathbf{V}_1 - \mathbf{V}_o}{8}, \text{ therefore, } 0 = \frac{\mathbf{V}_1 - \mathbf{V}_o}{8} + \frac{3\mathbf{V}_o}{4}$$

$$0 = \mathbf{V}_1 - \mathbf{V}_o + 6\mathbf{V}_o \Rightarrow \mathbf{V}_1 = -5\mathbf{V}_o$$

$$\text{Hence, } \mathbf{I}_1 = \frac{-5\mathbf{V}_o - \mathbf{V}_o}{8} = -0.75\,\mathbf{V}_o$$

$$\text{and } \mathbf{y}_{11} = \frac{\mathbf{I}_1}{\mathbf{V}_1} = \frac{-0.75\mathbf{V}_o}{-5\mathbf{V}_o} = 0.15\,\text{S}$$

At node 2, $\dfrac{\mathbf{V}_o - 0}{4} + 2\mathbf{I}_1 + \mathbf{I}_2 = 0$

or $-\mathbf{I}_2 = 0.25\mathbf{V}_o - 1.5\mathbf{V}_o = -1.25\mathbf{V}_o$

Hence, $\mathbf{y}_{21} = \dfrac{\mathbf{I}_2}{\mathbf{V}_1} = \dfrac{1.25\mathbf{V}_o}{-5\mathbf{V}_o} = -0.25 \text{ S}$

Similarly, we get $\mathbf{y}_{12}$ and $\mathbf{y}_{21}$ using Fig. (b). At node 1,

$$\frac{0 - \mathbf{V}_o}{8} = 2\mathbf{I}_\mathbf{1} + \frac{\mathbf{V}_o}{2} + \frac{\mathbf{V}_o - \mathbf{V}_2}{4}$$

But $\mathbf{I}_1 = \dfrac{0 - \mathbf{V}_o}{8}$, therefore, $0 = -\dfrac{\mathbf{V}_o}{8} + \dfrac{\mathbf{V}_o}{2} + \dfrac{\mathbf{V}_o - \mathbf{V}_2}{4}$

or $0 = -\mathbf{V}_o + 4\mathbf{V}_o + 2\mathbf{V}_o - 2\mathbf{V}_2 \Rightarrow \mathbf{V}_2 = 2.5\mathbf{V}_o$

Hence, $\mathbf{y}_{12} = \dfrac{\mathbf{I}_1}{\mathbf{V}_2} = \dfrac{-\mathbf{V}_o/8}{2.5\mathbf{V}_o} = -0.05\,\text{S}$

At node 2, $\dfrac{\mathbf{V}_o - \mathbf{V}_2}{4} + 2\mathbf{I}_1 + \mathbf{I}_2 = 0$

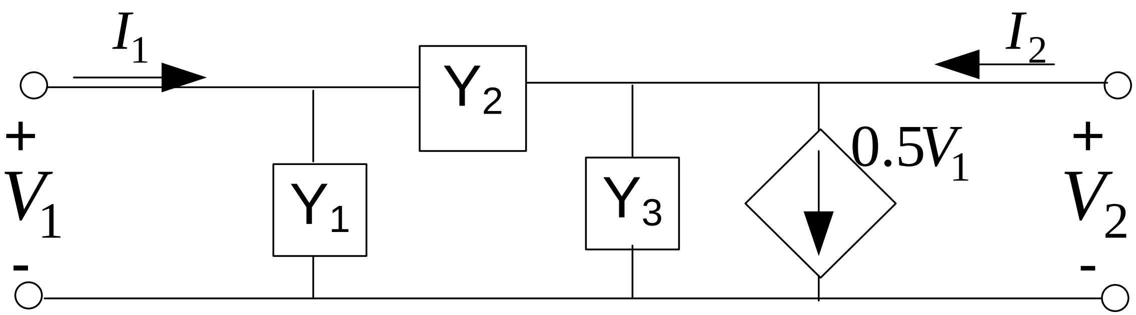
or $-\mathbf{I}_2 = 0.25\mathbf{V}_o - \dfrac{1}{4}(2.5)\mathbf{V}_o - \dfrac{2\mathbf{V}_o}{8} = -0.625\mathbf{V}_o$

Thus, $\mathbf{y}_{22} = \dfrac{\mathbf{I}_2}{\mathbf{V}_2} = \dfrac{0.625\mathbf{V}_o}{2.5\mathbf{V}_o} = 0.25\,\text{S}$

Notice that $\mathbf{y}_{12} \neq \mathbf{y}_{21}$ in this case, since the network is not reciprocal.

Example

Determine the admittance parameters from the circuit in Fig.



$$I_1 = Y_1V_1 + Y_2(V_1 - V_2) = (Y_1 + Y_2)V_1 - Y_2V_2$$

$$I_2 = 0.5V_1 + Y_3V_2 + Y_2(V_2 - V_1) = (0.5 - Y_2)V_1 + (Y_2 + Y_3)V_2$$

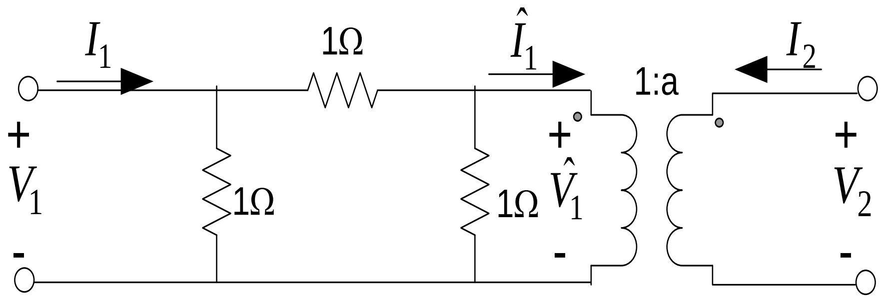
$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} Y_1 + Y_2 & -Y_2 \\ 0.5 - Y_2 & Y_2 + Y_3 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

$$y_{11} = Y_1 + Y_2 \; , \; y_{12} = -Y_2$$

$$y_{21} = 0.5 - Y_2 \; , \; y_{22} = Y_2 + Y_3$$

Example

Compute the y-parameter of the circuit in Fig.



$$I_1 = V_1 + (V_1 - \hat{V}_1) = 2V_1 - \hat{V}_1 = 2V_1 - \frac{1}{a}V_2$$

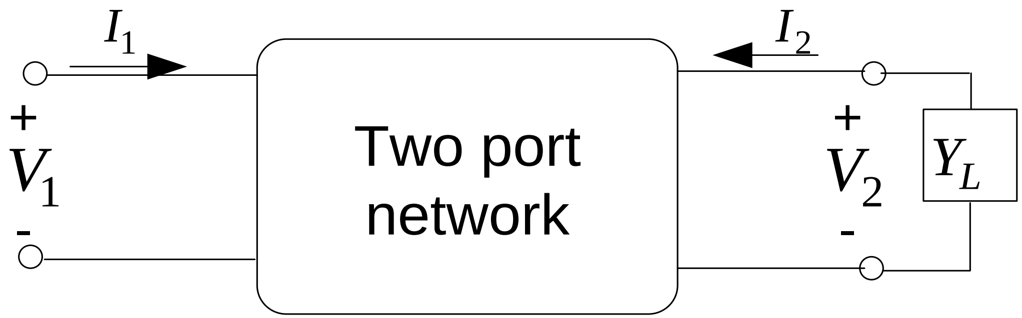
$$I_2 = -\frac{1}{a}\hat{I}_1 = -\frac{1}{a}\left[-\hat{V}_1 + (V_1 - \hat{V}_1)\right] = -\frac{1}{a}V_1 + \frac{2}{a^2}V_2$$

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 2 & -\frac{1}{a} \\ -\frac{1}{a} & \frac{2}{a^2} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

$$y_{11} = 2, \quad y_{12} = -1/a$$

$$y_{21} = -1/a, \quad y_{22} = 2/a^2$$

Y parameter analysis of terminated two-port



Y-parameter equations

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} \qquad I_2 = -Y_L V_2$$

$$\begin{bmatrix} I_1 \\ 0 \end{bmatrix} = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} + Y_L \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

From Crammer's rules

$$V_1 = \frac{\begin{vmatrix} I_1 & y_{12} \\ 0 & y_{22} + Y_L \end{vmatrix}}{\begin{vmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} + Y_L \end{vmatrix}} = \frac{(y_{22} + Y_L)I_1}{y_{11}(y_{22} + Y_L) - y_{12}y_{21}}$$

The input admittance Y_{in}

$$Y_{in} = y_{11} - \frac{y_{12}y_{21}}{y_{11}(y_{22} + Y_L)}$$

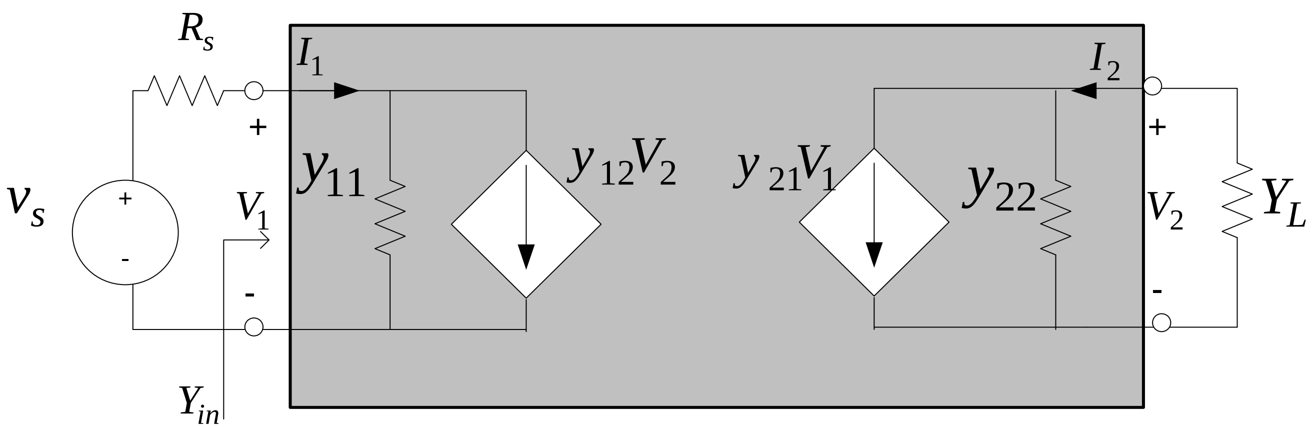
and

$$y_{21}V_1 = -(y_{22} + Y_L)V_2$$

$$V_2 = -\frac{y_{21}}{y_{22} + Y_L}V_1$$

$$I_1 = y_{11}V_1 + y_{12}V_2 = \left(y_{11} - \frac{y_{12}y_{21}}{y_{22} + Y_L} \right) V_1$$

Gain: $$\frac{V_2}{V_1} = -\frac{y_{21}}{y_{22} + Y_L}$$



Terminated two-port Y-parameter model

Two-port Devices and the Hybrid Model

❖H-parameter is the combination of Z and Y parameter defined by

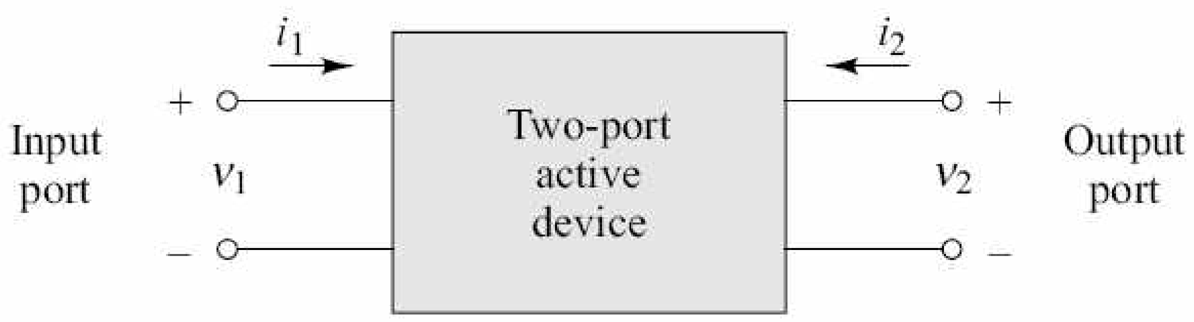


Fig. 8.4 *A two-port network.*

- If the current i_1 and the voltage v_2 are independent and if the two-port is linear.

$$v_1 = h_{11}i_1 + h_{12}v_2$$

$$i_2 = h_{21}i_1 + h_{22}v_2$$

- The quantities h_{11}, h_{12}, h_{21}, and h_{22} are called the *h, or hybrid, parameters.*
- H-parameter is commonly used in transistor modeling.

$$h_{11} \equiv \left.\frac{v_1}{i_1}\right|_{v_2=0}$$ = input resistance with output short-circuit (ohms).

$$h_{12} \equiv \left.\frac{v_1}{v_2}\right|_{i_1=0}$$ = fraction of output voltage at input with input open-circuited, or more simply, reverse-open-circuit voltage amplification (dimensionless).

$$h_{21} \equiv \left.\frac{i_2}{i_1}\right|_{v_2=0}$$ = negative of current transfer ratio (or current gain) with output short-circuited. (Note that the current into a load across the output port would be the negative of i_2.) This parameter is usually referred to, simply, as the *short-circuit current gain* (dimensionless).

$$h_{22} \equiv \left.\frac{i_2}{v_2}\right|_{i_1=0}$$ = output conductance with input open-circuited (mhos).

$i = 11$ = input $\qquad o = 22$ = output

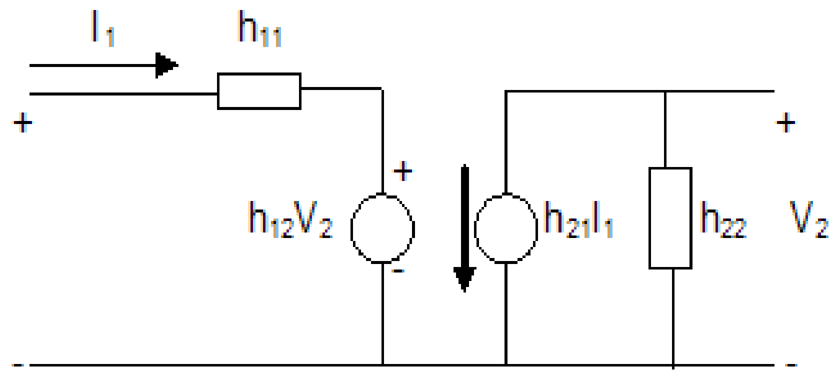
$f = 21$ = forward transfer $\qquad r = 12$ = reverse transfer

In the case of transistors, another subscript (b, e, or c) is added to designate the type of configuration. For example,

$h_{ib} = h_{11b}$ = input resistance in common-base configuration

$h_{fe} = h_{21e}$ = short-circuit forward current gain in common-emitter circuit

- The h-parameters are named specifically as follows:
  - h_{11} = short circuit input impedance
  - h_{12} = open circuit reverse voltage gain
  - h_{21} = short circuit forward current gain
  - h_{22} = open circuit output admittance



$$\mathbf{V}_1 = \mathbf{h}_{11}\mathbf{I}_1 + \mathbf{h}_{12}\mathbf{V}_2$$
$$\mathbf{I}_2 = \mathbf{h}_{21}\mathbf{I}_1 + \mathbf{h}_{22}\mathbf{V}_2$$

$$\begin{bmatrix} \mathbf{V}_1 \\ \mathbf{I}_2 \end{bmatrix} = \begin{bmatrix} \mathbf{h}_{11} & \mathbf{h}_{12} \\ \mathbf{h}_{21} & \mathbf{h}_{22} \end{bmatrix} \begin{bmatrix} \mathbf{I}_1 \\ \mathbf{V}_2 \end{bmatrix} = [\mathbf{h}] \begin{bmatrix} \mathbf{I}_1 \\ \mathbf{V}_2 \end{bmatrix}$$

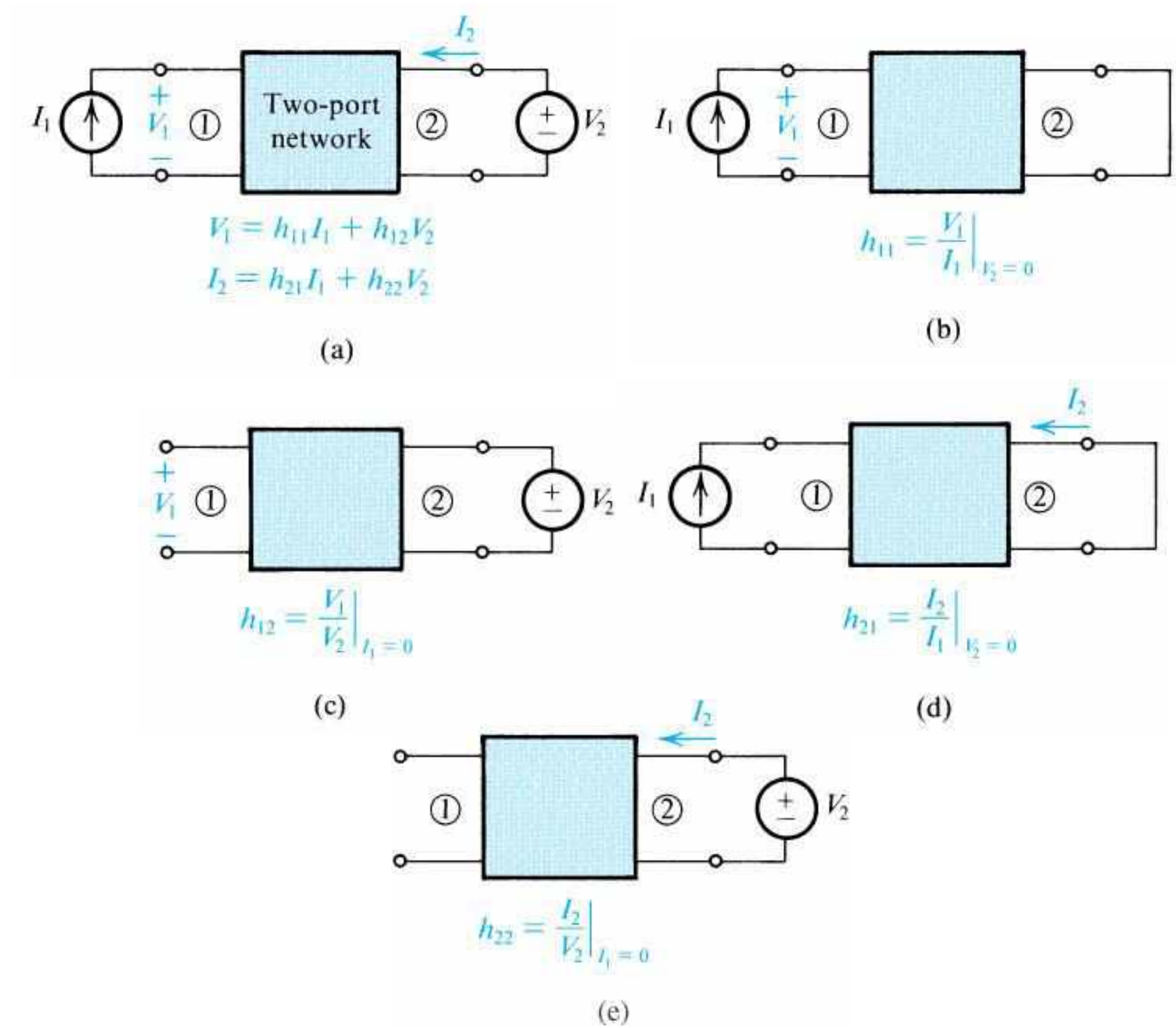


Figure : Definition and conceptual measurement circuits for *h* parameters.

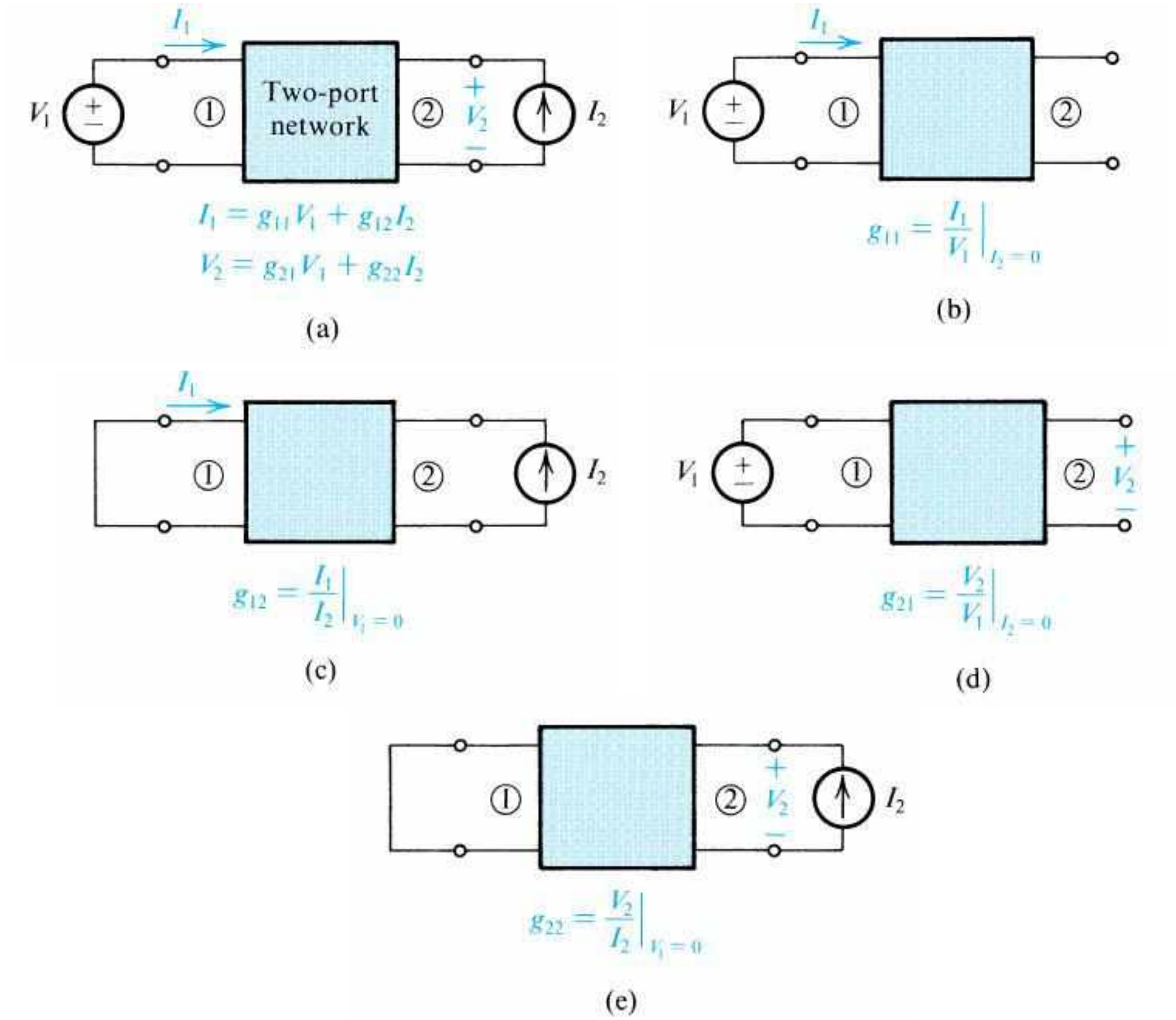
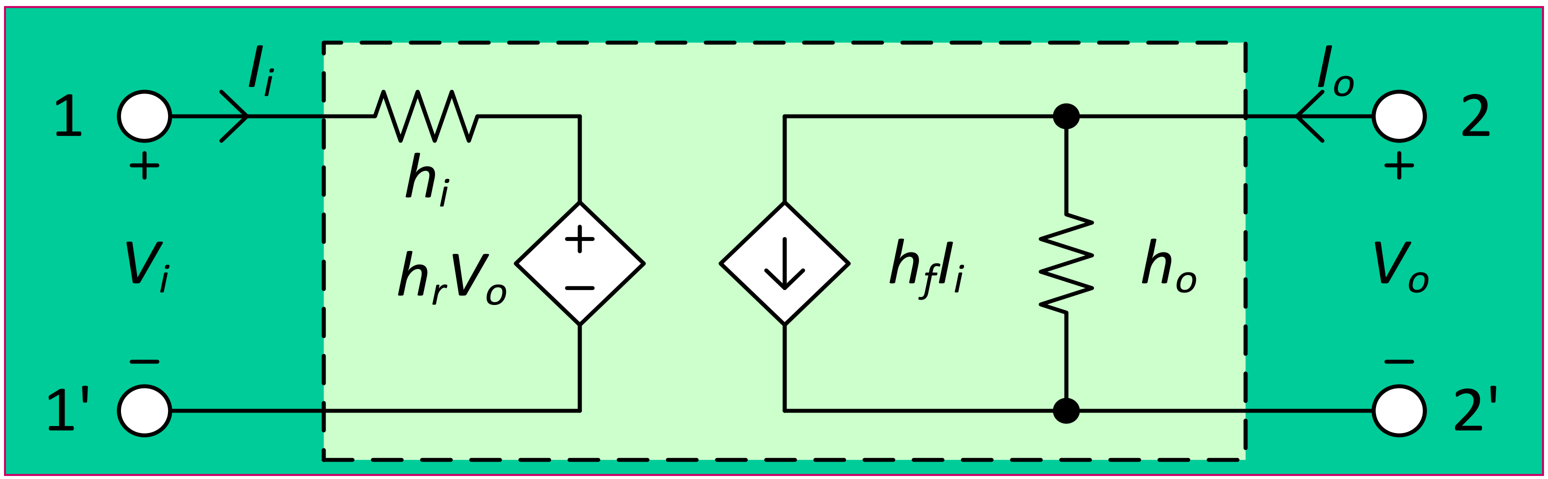


Figure: Definition and conceptual measurement circuits for g parameters.

Hybrid Parameter Model



$$V_i = h_{11} I_i + h_{12} V_o = h_i I_i + h_r V_o$$

$$I_o = h_{21} I_i + h_{22} V_o = h_f I_i + h_o V_o$$

h-Parameters

$$h_{11} = \frac{V_i}{I_i} \Bigg|_{V_o = 0} \qquad h_{12} = \frac{V_i}{V_o} \Bigg|_{I_i = 0}$$

$$h_{21} = \frac{I_o}{I_i} \Bigg|_{V_o = 0} \qquad h_{22} = \frac{I_o}{V_o} \Bigg|_{I_i = 0}$$

$h_{11} = h_i$ = Input Resistance
$h_{12} = h_r$ = Reverse Transfer Voltage Ratio
$h_{21} = h_f$ = Forward Transfer Current Ratio
$h_{22} = h_o$ = Output Admittance

The Model

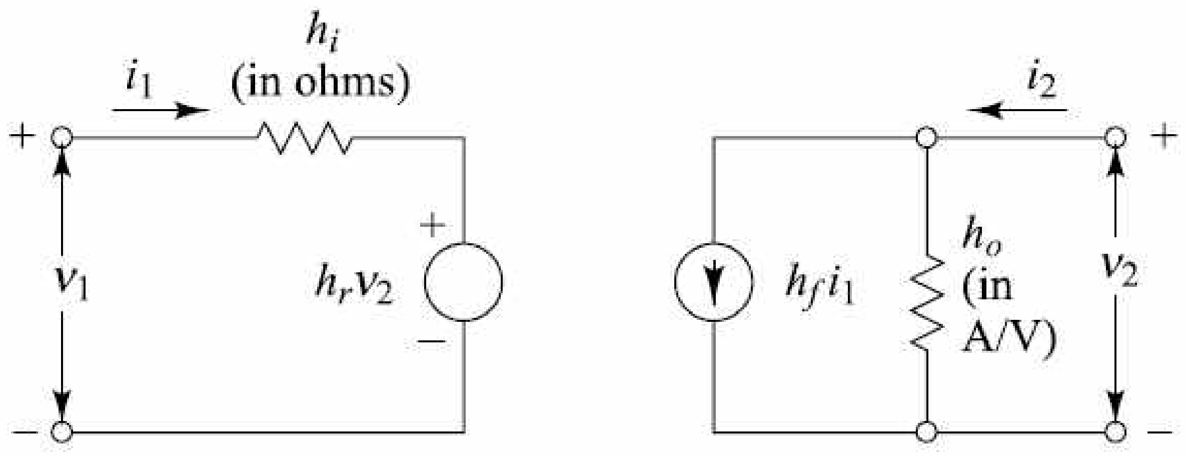


Fig. 8.5 *The hybrid model for the two-port network of Fig. 8.4. The parameters h_r and h_f are dimensionless.*

Transistor Hybrid Model

- *CE- configuration*

$$v_B = f_1(i_B, v_C)$$
$$i_C = f_2(i_B, v_C)$$

- Using Taylor's series expansion

$$\Delta v_B = \left.\frac{\partial f_1}{\partial i_B}\right|_{V_C} \Delta i_B + \left.\frac{\partial f_1}{\partial v_C}\right|_{I_B} \Delta v_C$$

$$\Delta i_C = \left.\frac{\partial f_2}{\partial i_B}\right|_{V_C} \Delta i_B + \left.\frac{\partial f_2}{\partial v_C}\right|_{I_B} \Delta v_C$$

- For small signal (incremental analysis)

$$v_b = h_{ie} i_b + h_{re} v_c$$
$$i_c = h_{fe} i_b + h_{oe} v_c$$

Where

$$h_{ie} = \frac{\partial f_1}{\partial i_B} = \left.\frac{\partial v_B}{\partial i_B}\right|_{V_C} \qquad h_{re} = \frac{\partial f_1}{\partial v_C} = \left.\frac{\partial v_B}{\partial v_C}\right|_{I_B}$$

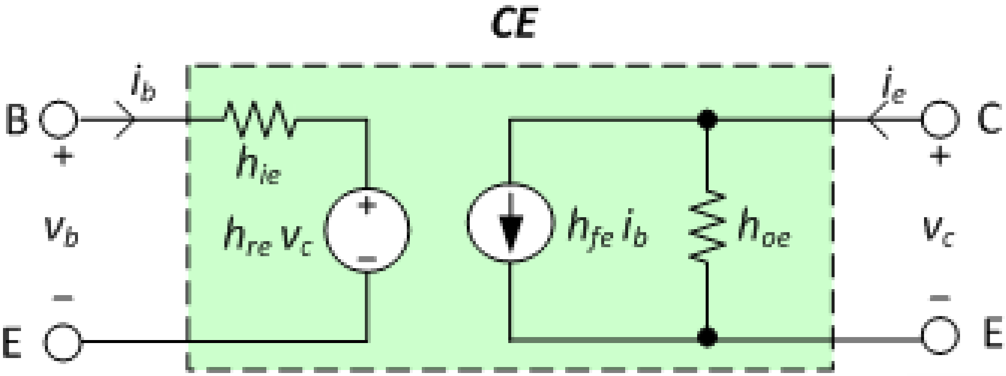
$$h_{fe} = \frac{\partial f_2}{\partial i_B} = \left.\frac{\partial i_C}{\partial i_B}\right|_{V_C} \qquad h_{oe} = \frac{\partial f_2}{\partial v_C} = \left.\frac{\partial i_C}{\partial v_C}\right|_{I_B}$$

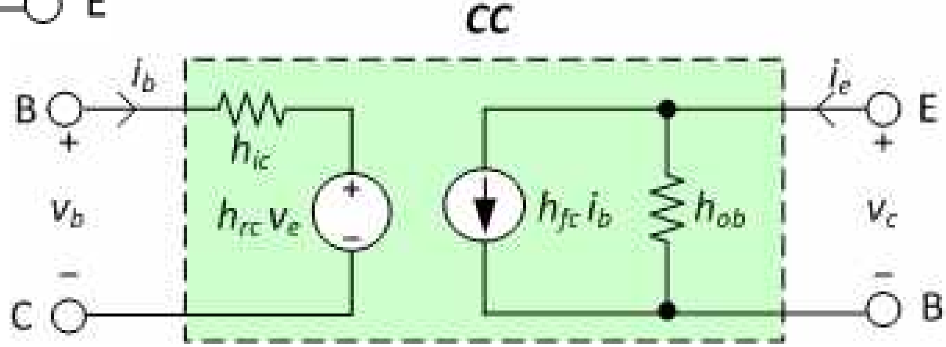
- If a parameter is constant, its incremental change is zero.

For example: if V_C is constant, then it is equivalent to v_c=0.

if I_B is constant, then it is equivalent to i_b=0.

$$h_{re} = \left.\frac{\partial v_B}{\partial v_C}\right|_{I_B} = \left.\frac{v_b}{v_c}\right|_{i_b=0} \quad or \quad h_{re} = \left.\frac{V_b}{V_c}\right|_{I_b=0}$$





Graphical analysis of CE configuration

- The large-signal response of transistors are obtained graphically. For small signals the transistor operates with reasonable linearity, and we inquire into small-signal linear models which represent the operation of the transistor in the active region.

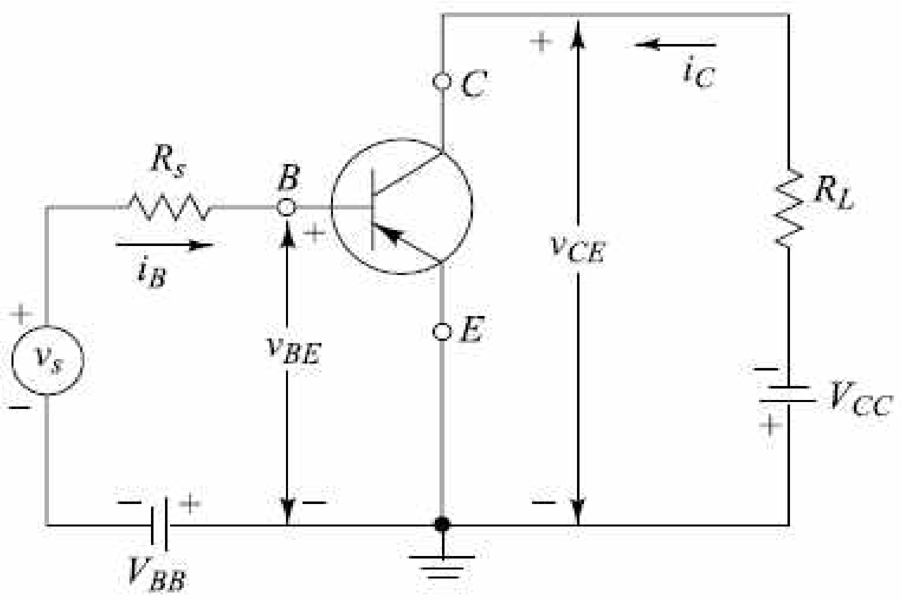


Fig. 8.1 *The CE transistor configuration.*

Graphical Analysis of the CE Configuration

- **Notation** instantaneous values are represented by lowercase letters (*i* for current, *v* for voltage, and *p* for power). Maximum, average (dc), and effective, or root-mean-square (rms), values are represented by the uppercase letter (dc) values instantaneous total values are indicated by the uppercase subscript of the proper electrode varying components from some quiescent value are indicated by the lowercase subscript.

$$I_C = f\left(I_B, V_{CE}\right)$$

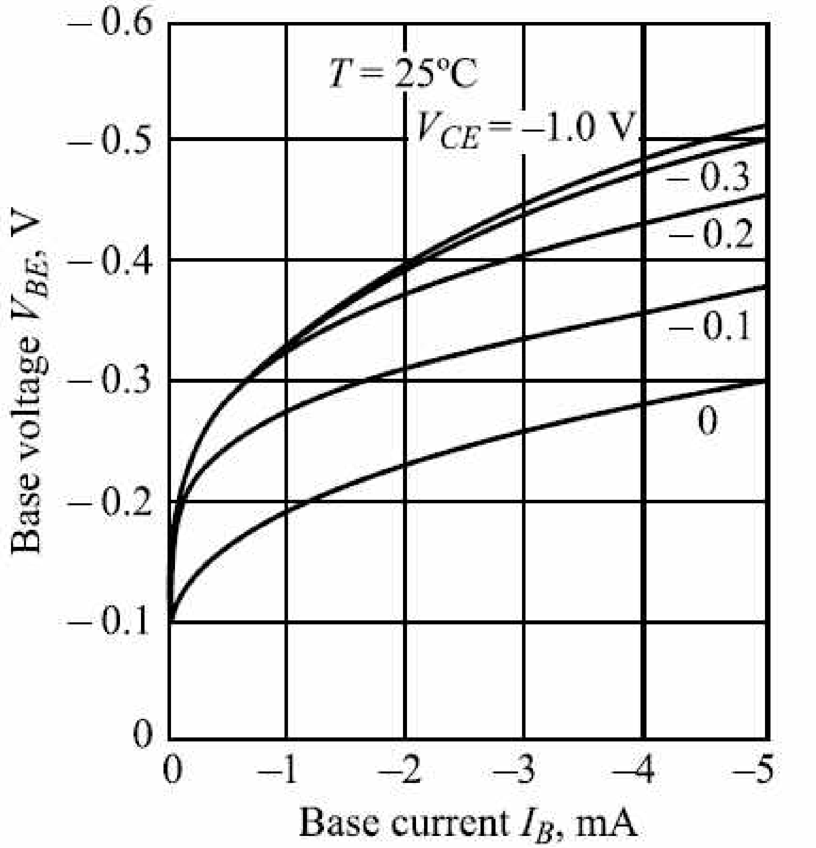
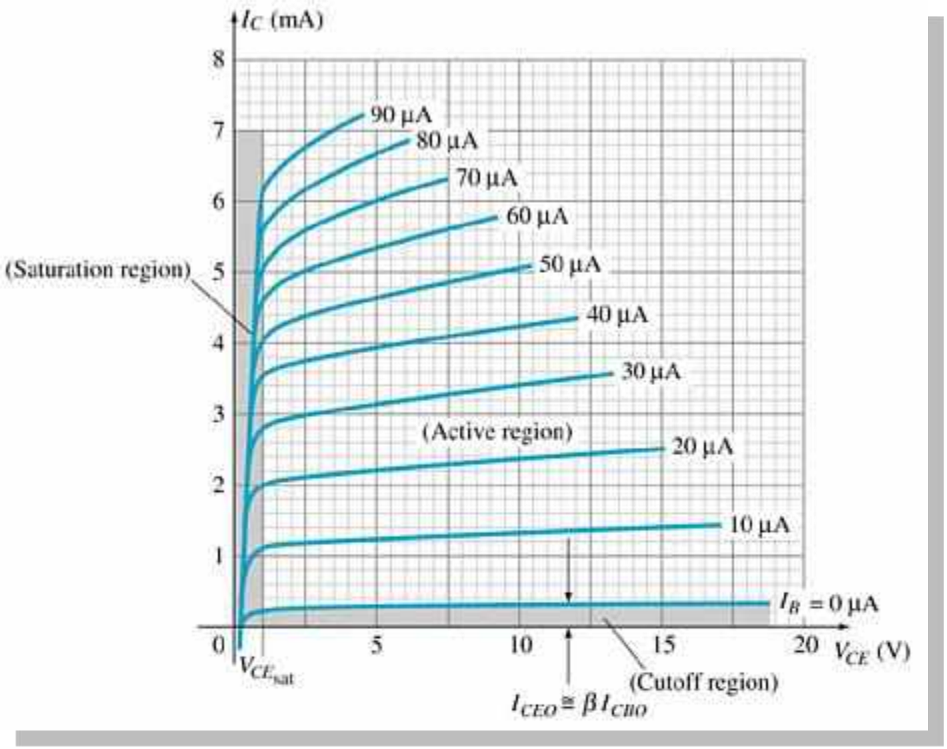
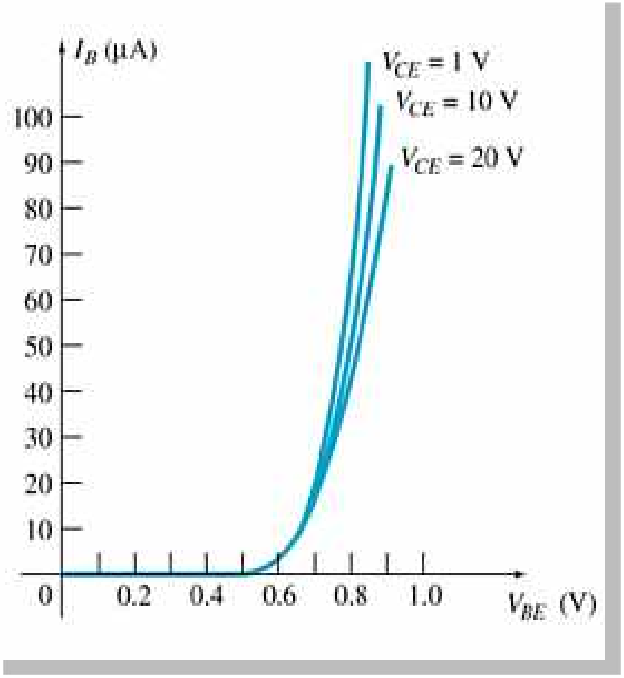


Fig. 5.11 *Typical common-emitter input characteristics of the p-n-p germanium junction transistor of Fig. 5.10.*

I/P and O/P Characteristics



O/P or Collector Characteristics (*n-p-n*)

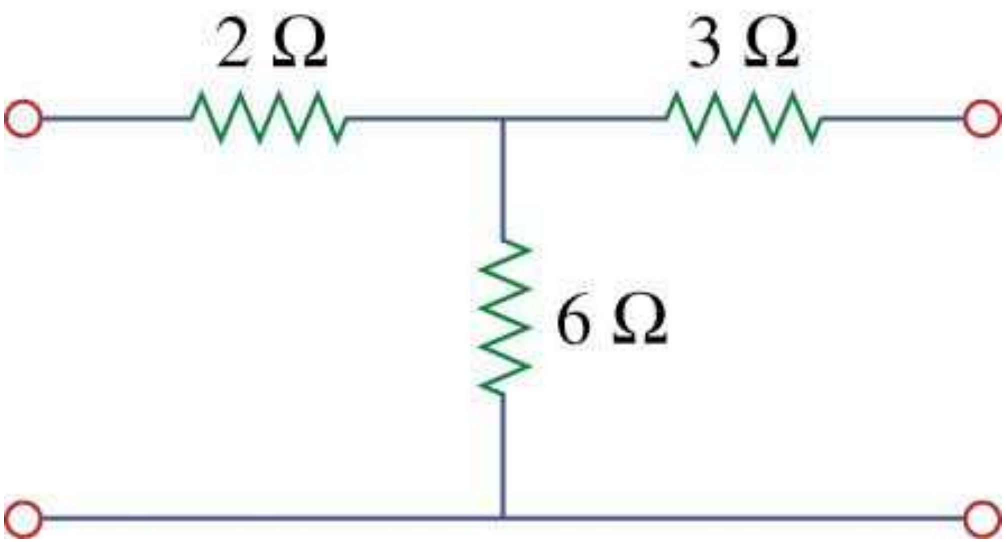


I/P or Base Characteristics (*n-p-n*)

$$I_C = f\left(I_B, V_{CE}\right)$$

$$V_{BE} = f\left(I_B, V_{CE}\right)$$

- Find the hybrid parameters for the two-port network:



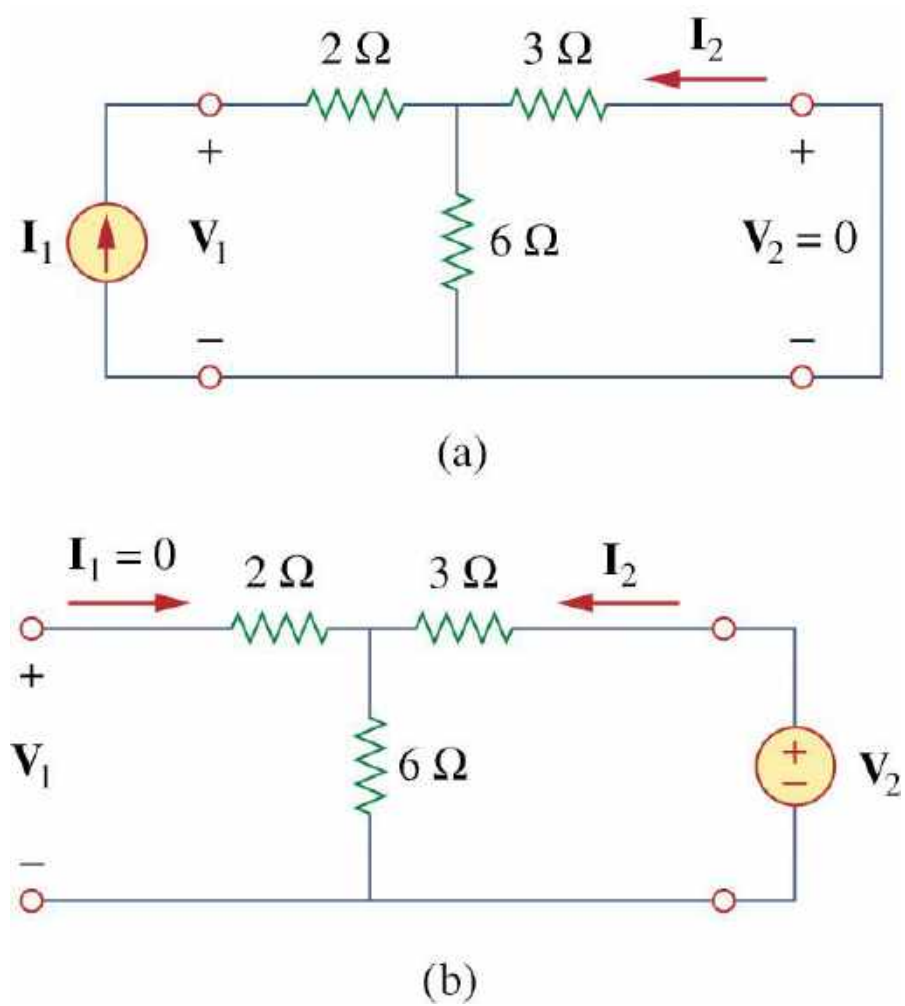
From Fig. (a),

$$\mathbf{V}_1 = \mathbf{I}_1(2 + 3\|6) = 4\mathbf{I}_1$$

Hence, $\mathbf{h}_{11} = \dfrac{\mathbf{V}_1}{\mathbf{I}_1} = 4\ \Omega$

$$-\mathbf{I}_2 = \frac{6}{6+3}\mathbf{I}_1 = \frac{2}{3}\mathbf{I}_1$$

Hence, $h_{21} = \dfrac{\mathbf{I}_2}{\mathbf{I}_1} = -\dfrac{2}{3}$



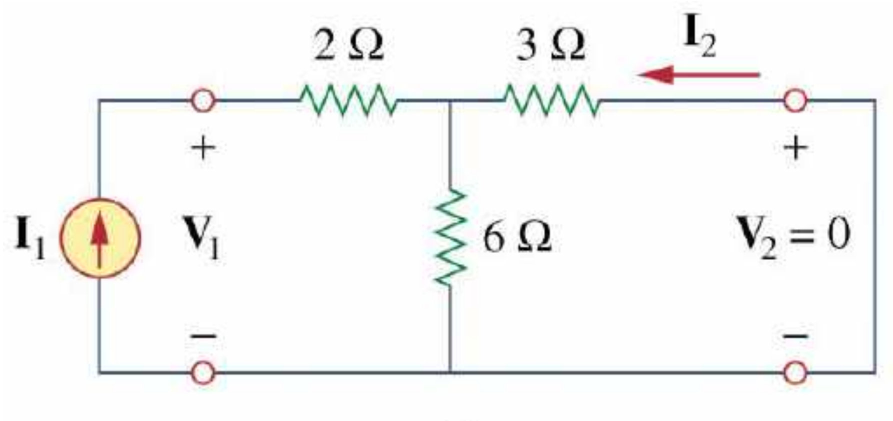
From Fig.(b),

$$\mathbf{V}_1 = \frac{6}{6+3}\mathbf{V}_2 = \frac{2}{3}\mathbf{V}_2$$

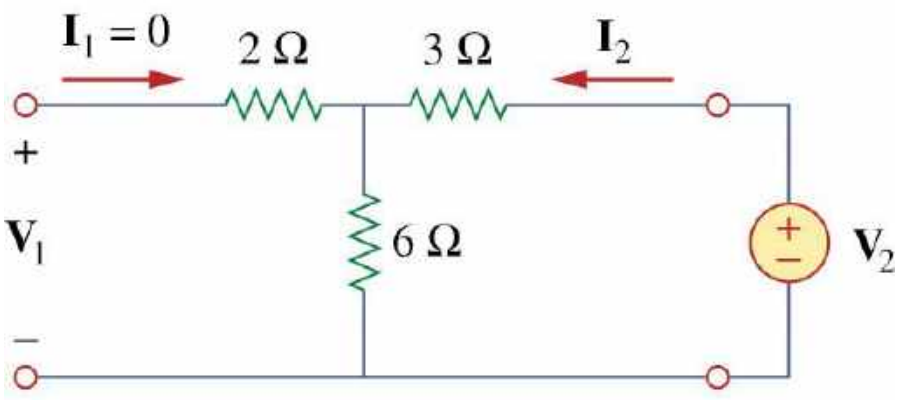
Hence, $\mathbf{h}_{12} = \frac{\mathbf{V}_1}{\mathbf{V}_2} = \frac{2}{3}$

Also, $\mathbf{V}_2 = (3+6)\mathbf{I}_2 = 9\mathbf{I}_2$

Thus, $\mathbf{h}_{22} = \frac{\mathbf{I}_2}{\mathbf{V}_2} = \frac{1}{9}$ S



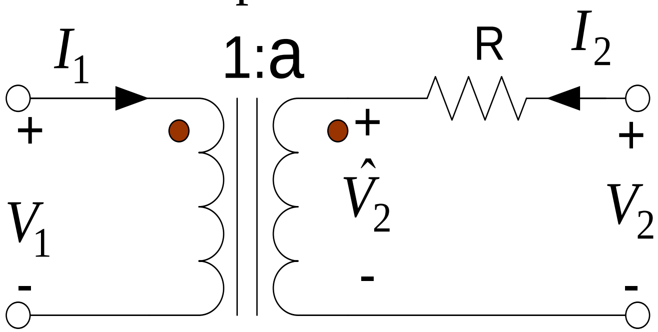
(a)



(b)

Example

Determine the h-parameter of the two-port circuit shown in Fig.



$$V_1 = \frac{1}{a}\hat{V}_2 \qquad I_1 = -aI_2$$

$$\hat{V}_2 = V_2 - RI_2 = \frac{R}{a}I_1 + V_2$$

$$V_1 = \frac{R}{a^2}I_1 + \frac{1}{a}V_2$$

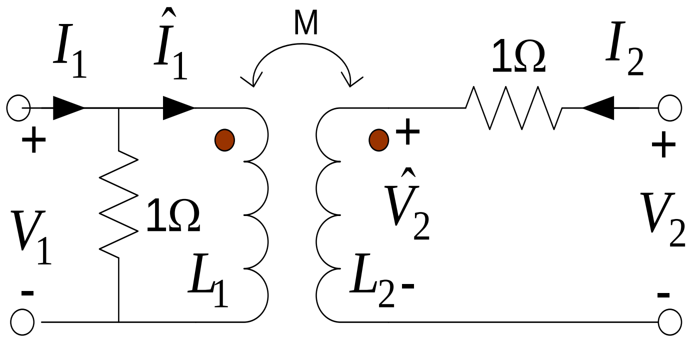
$$I_2 = \frac{V_2 - \hat{V}_2}{R} = -\frac{\hat{V}_2}{R} + \frac{V_2}{R}$$

$$= -\frac{1}{a}I_1 + 0V_2$$

$$\begin{bmatrix} V_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} \frac{R}{a^2} & \frac{1}{a} \\ -\frac{1}{a} & 0 \end{bmatrix} \begin{bmatrix} I_1 \\ V_2 \end{bmatrix}$$

Example

Find the h-parameter of the circuit in Fig. assuming L_1=L_2=M=1H



In frequency domain

$$V_1 = sL_1\hat{I}_1 + sMI_2$$

$$\hat{I}_1 = I_1 - V_1$$

$$(1+sL_1)V_1 - sMI_2 = sL_1I_1$$

$$V_2 = \hat{V}_2 + I_2$$

$$\hat{V}_2 = sL_2 I_2 + sM\hat{I}_1 = sL_2 I_2 + sM(I_1 - V_1)$$

$$V_2 = (1 + sL_2)I_2 + sM(I_1 - V_1)$$

$$sMV_1 - (1 + sL_2)I_2 = sMI_1 - V_2$$

In matrix form

$$\begin{bmatrix} 1+sL_1 & -sM \\ sM & -(1+sL_2) \end{bmatrix} \begin{bmatrix} V_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} sL_1 & 0 \\ sM & -1 \end{bmatrix} \begin{bmatrix} I_1 \\ V_2 \end{bmatrix}$$

$$\begin{bmatrix} V_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 1+sL_1 & -sM \\ sM & -(1+sL_2) \end{bmatrix}^{-1} \begin{bmatrix} sL_1 & 0 \\ sM & -1 \end{bmatrix} \begin{bmatrix} I_1 \\ V_2 \end{bmatrix}$$

With $L_1 = L_2 = M = 1$ H

$$\begin{bmatrix} V_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 1+s & -s \\ s & -(1+s) \end{bmatrix}^{-1} \begin{bmatrix} s & 0 \\ s & -1 \end{bmatrix} \begin{bmatrix} I_1 \\ V_2 \end{bmatrix}$$

$$= \frac{1}{2s+1} \begin{bmatrix} s & s \\ -s & s+1 \end{bmatrix} \begin{bmatrix} I_1 \\ V_2 \end{bmatrix}$$

Transmission parameter

The T-parameter or transmission parameters are used in power system and it is called **ABCD parameter**. The transmission parameter is defined by

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} t_{11} & t_{12} \\ t_{21} & t_{22} \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix} \quad \text{or} \quad \begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$$

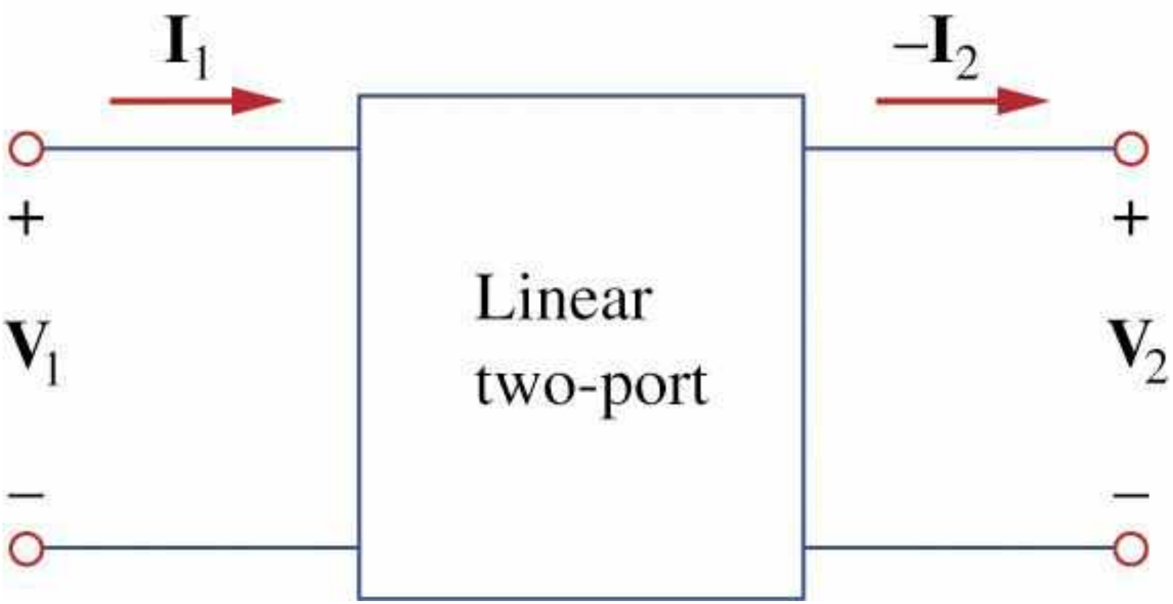
This means that the power flows into the input port and flow out to the load from the output port.

T-parameter can be calculated from

$$t_{11} = \left.\frac{V_1}{V_2}\right|_{I_2=0} \qquad t_{12} = -\left.\frac{V_1}{I_2}\right|_{V_2=0}$$

$$t_{21} = \left.\frac{I_1}{V_2}\right|_{I_2=0} \qquad t_{22} = -\left.\frac{I_1}{I_2}\right|_{V_2=0}$$

} Open or short circuit at the output port


$\mathbf{I}_1$
$-\mathbf{I}_2$
+
+
$\mathbf{V}_1$
Linear
two-port
$\mathbf{V}_2$
−
−

a = Open-circuit voltage gain

b = Negative short-circuit transfer impedance

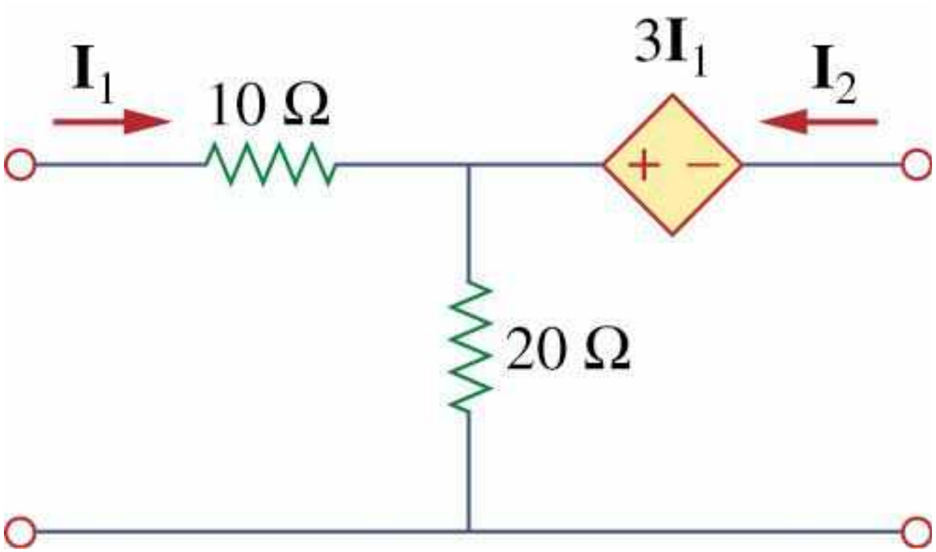
c = Open-circuit transfer admittance

d = Negative short-circuit current gain

$$\mathbf{AD} - \mathbf{BC} = 1, \quad \mathbf{ad} - \mathbf{bc} = 1$$

A = D

- Find the transmission parameters for the two-port network:

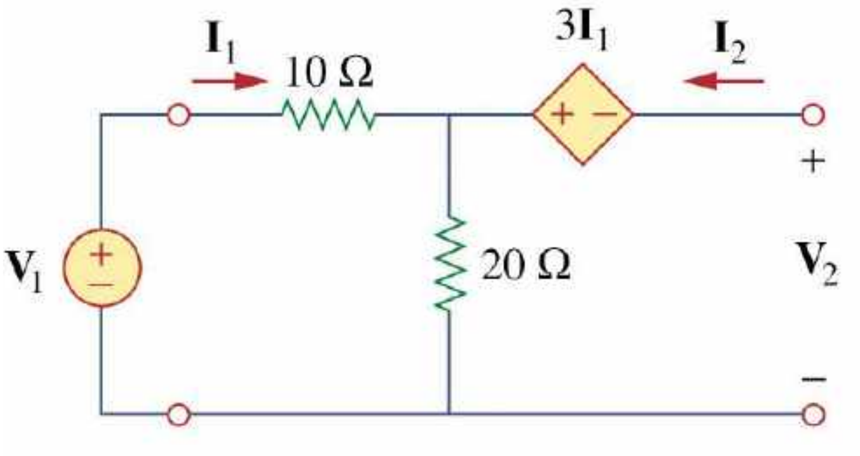


From Fig.(a),

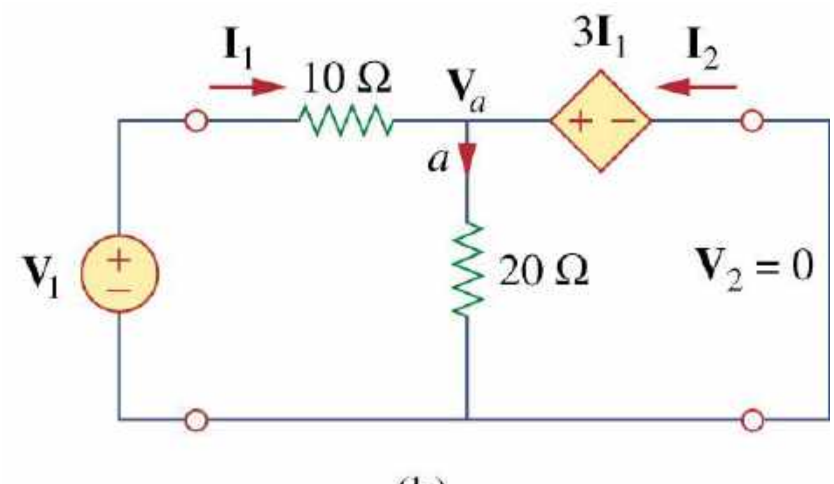
$\mathbf{V}_1 = (10+20)\mathbf{I}_1 = 30\mathbf{I}_1$ and $\mathbf{V}_2 = 20\mathbf{I}_1 - 3\mathbf{I}_1 = 17\mathbf{I}_1$

Thus

$$\mathbf{A} = \frac{\mathbf{V}_1}{\mathbf{V}_2} = \frac{30\mathbf{I}_1}{17\mathbf{I}_1} = 1.765, \quad \mathbf{C} = \frac{\mathbf{I}_1}{\mathbf{V}_2} = \frac{\mathbf{I}_1}{17\mathbf{I}_1} = 0.0588 \text{ S}$$



(a)



(b)

From Fig.(b),

$$\frac{\mathbf{V}_1 - \mathbf{V}_a}{10} - \frac{\mathbf{V}_a}{20} + \mathbf{I}_2 = 0$$

But $\mathbf{V}_a = 3\mathbf{I}_1$ and $\mathbf{I}_1 = (\mathbf{V}_1 - \mathbf{V}_a)/10$,

$$\Rightarrow \mathbf{V}_a = 3\mathbf{I}_1, \quad \mathbf{V}_1 = 13\mathbf{I}_1$$

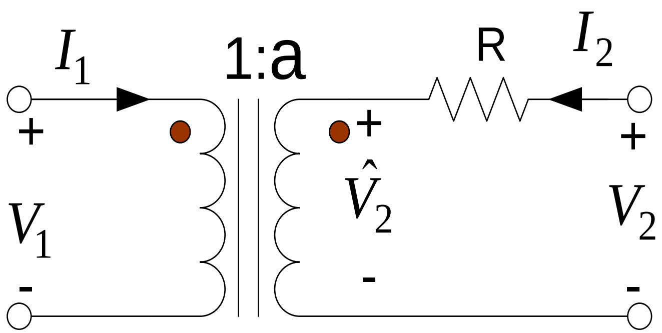
$$\Rightarrow \mathbf{I}_1 - \frac{3\mathbf{I}_1}{20} + \mathbf{I}_2 = 0 \Rightarrow \frac{17}{20}\mathbf{I}_1 = -\mathbf{I}_2$$

Thus,

$$\mathbf{D} = -\frac{\mathbf{I}_1}{\mathbf{I}_2} = \frac{20}{17} = 1.176, \quad \mathbf{B} = -\frac{\mathbf{V}_1}{\mathbf{V}_2} = \frac{-13\mathbf{I}_1}{(17/20)\mathbf{I}_1} = 15.29\ \Omega$$

Example

Determine the t-parameter of the circuit shown in Fig .



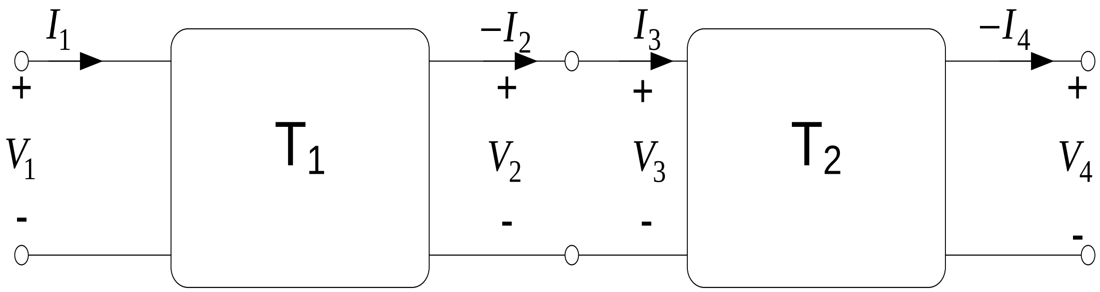
$$V_1 = \frac{1}{a}\hat{V}_2 = \frac{1}{a}(V_2 - RI_2)$$

$$I_1 = -aI_2$$

$$\therefore \begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} \frac{1}{a} & \frac{R}{a} \\ 0 & a \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$$

Example

One of the most importance characteristics of the two-port circuit with T-parameter is to determine the overall cascade parameter.



$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = T_1 \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix} \qquad \begin{bmatrix} V_3 \\ I_3 \end{bmatrix} = T_2 \begin{bmatrix} V_4 \\ -I_4 \end{bmatrix} \qquad V_2 = V_3 \,, \quad -I_2 = I_3$$

Thus

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = T_1 T_2 \begin{bmatrix} V_4 \\ -I_4 \end{bmatrix}$$

Inverse Transmission parameter

$$\begin{bmatrix} V_2 \\ I_2 \end{bmatrix} = \begin{bmatrix} A' & B' \\ C' & D' \end{bmatrix} \begin{bmatrix} V_1 \\ -I_1 \end{bmatrix}$$

$$A' = \left.\frac{V_2}{V_1}\right|_{I_1=0} \qquad B' = -\left.\frac{V_2}{I_1}\right|_{V_1=0}$$

$$C' = \left.\frac{I_2}{V_1}\right|_{I_1=0} \qquad D' = -\left.\frac{I_2}{I_1}\right|_{V_1=0}$$

TABLE 19.1 Conversion of two-port parameters.

	z		y		h		g		T		t	
z	z_{11}	z_{12}	$\frac{y_{22}}{\Delta_y}$	$-\frac{y_{12}}{\Delta_y}$	$\frac{\Delta_h}{h_{22}}$	$\frac{h_{12}}{h_{22}}$	$\frac{1}{g_{11}}$	$-\frac{g_{12}}{g_{11}}$	$\frac{A}{C}$	$\frac{\Delta_T}{C}$	$\frac{d}{c}$	$\frac{1}{c}$
	z_{21}	z_{22}	$-\frac{y_{21}}{\Delta_y}$	$\frac{y_{11}}{\Delta_y}$	$-\frac{h_{21}}{h_{22}}$	$\frac{1}{h_{22}}$	$\frac{g_{21}}{g_{11}}$	$\frac{\Delta_g}{g_{11}}$	$\frac{1}{C}$	$\frac{D}{C}$	$\frac{\Delta_t}{c}$	$\frac{a}{c}$
y	$\frac{z_{22}}{\Delta_z}$	$-\frac{z_{12}}{\Delta_z}$	y_{11}	y_{12}	$\frac{1}{h_{11}}$	$-\frac{h_{12}}{h_{11}}$	$\frac{\Delta_g}{g_{22}}$	$\frac{g_{12}}{g_{22}}$	$\frac{D}{B}$	$-\frac{\Delta_T}{B}$	$\frac{a}{b}$	$-\frac{1}{b}$
	$-\frac{z_{21}}{\Delta_z}$	$\frac{z_{11}}{\Delta_z}$	y_{21}	y_{22}	$\frac{h_{21}}{h_{11}}$	$\frac{\Delta_h}{h_{11}}$	$-\frac{g_{21}}{g_{22}}$	$\frac{1}{g_{22}}$	$-\frac{1}{B}$	$\frac{A}{B}$	$-\frac{\Delta_t}{b}$	$\frac{d}{b}$
h	$\frac{\Delta_z}{z_{22}}$	$\frac{z_{12}}{z_{22}}$	$\frac{1}{y_{11}}$	$-\frac{y_{12}}{y_{11}}$	h_{11}	h_{12}	$\frac{g_{22}}{\Delta_g}$	$-\frac{g_{12}}{\Delta_g}$	$\frac{B}{D}$	$\frac{\Delta_T}{D}$	$\frac{b}{a}$	$\frac{1}{a}$
	$-\frac{z_{21}}{z_{22}}$	$\frac{1}{z_{22}}$	$\frac{y_{21}}{y_{11}}$	$\frac{\Delta_y}{y_{11}}$	h_{21}	h_{22}	$-\frac{g_{21}}{\Delta_g}$	$\frac{g_{11}}{\Delta_g}$	$-\frac{1}{D}$	$\frac{C}{D}$	$\frac{\Delta_t}{a}$	$\frac{c}{a}$
g	$\frac{1}{z_{11}}$	$-\frac{z_{12}}{z_{11}}$	$\frac{\Delta_y}{y_{22}}$	$\frac{y_{12}}{y_{22}}$	$\frac{h_{22}}{\Delta_h}$	$-\frac{h_{12}}{\Delta_h}$	g_{11}	g_{12}	$\frac{C}{A}$	$-\frac{\Delta_T}{A}$	$\frac{c}{d}$	$-\frac{1}{d}$
	$\frac{z_{21}}{z_{11}}$	$\frac{\Delta_z}{z_{11}}$	$-\frac{y_{21}}{y_{22}}$	$\frac{1}{y_{22}}$	$-\frac{h_{21}}{\Delta_h}$	$\frac{h_{11}}{\Delta_h}$	g_{21}	g_{22}	$\frac{1}{A}$	$\frac{B}{A}$	$\frac{\Delta_t}{d}$	$-\frac{b}{d}$
T	$\frac{z_{11}}{z_{21}}$	$\frac{\Delta_z}{z_{21}}$	$-\frac{y_{22}}{y_{21}}$	$-\frac{1}{y_{21}}$	$-\frac{\Delta_h}{h_{21}}$	$-\frac{h_{11}}{h_{21}}$	$\frac{1}{g_{21}}$	$\frac{g_{22}}{g_{21}}$	A	B	$\frac{d}{\Delta_t}$	$\frac{b}{\Delta_t}$
	$\frac{1}{z_{21}}$	$\frac{z_{22}}{z_{21}}$	$-\frac{\Delta_y}{y_{21}}$	$-\frac{y_{11}}{y_{21}}$	$-\frac{h_{22}}{h_{21}}$	$-\frac{1}{h_{21}}$	$\frac{g_{11}}{g_{21}}$	$\frac{\Delta_g}{g_{21}}$	C	D	$\frac{c}{\Delta_t}$	$\frac{a}{\Delta_t}$
t	$\frac{z_{22}}{z_{12}}$	$\frac{\Delta_z}{z_{12}}$	$-\frac{y_{11}}{y_{12}}$	$-\frac{1}{y_{12}}$	$\frac{1}{h_{12}}$	$\frac{h_{11}}{h_{12}}$	$-\frac{\Delta_g}{g_{12}}$	$-\frac{g_{22}}{g_{12}}$	$\frac{D}{\Delta_T}$	$\frac{B}{\Delta_T}$	a	b
	$\frac{1}{z_{12}}$	$\frac{z_{11}}{z_{12}}$	$-\frac{\Delta_y}{y_{12}}$	$-\frac{y_{22}}{y_{12}}$	$\frac{h_{22}}{h_{12}}$	$\frac{\Delta_h}{h_{12}}$	$-\frac{g_{11}}{g_{12}}$	$-\frac{1}{g_{12}}$	$\frac{C}{\Delta_T}$	$\frac{A}{\Delta_T}$	c	d

$\Delta_z = z_{11}z_{22} - z_{12}z_{21}$, $\Delta_h = h_{11}h_{22} - h_{12}h_{21}$, $\Delta_T = AD - BC$

$\Delta_y = y_{11}y_{22} - y_{12}y_{21}$, $\Delta_g = g_{11}g_{22} - g_{12}g_{21}$, $\Delta_t = ad - bc$

Some Extra Problems

- Find [**z**] and [**g**] of a two-port network if

$$[\mathbf{T}] = \begin{bmatrix} 10 & 1.5\,\Omega \\ 2\,\mathrm{S} & 4 \end{bmatrix}$$

- Solution:

If $\mathbf{A} = 10,\ \mathbf{B} = 1.5,\ \mathbf{C} = 2,\ \mathbf{D} = 4,$ the determinant of the matrix is

$$\Delta_T = \mathbf{AD} - \mathbf{BC} = 40 - 3 = 37$$

$$\mathbf{z}_{11} = \frac{\mathbf{A}}{\mathbf{C}} = \frac{10}{2} = 5, \quad \mathbf{z}_{12} = \frac{\Delta_T}{\mathbf{C}} = \frac{37}{2} = 18.5$$

$$\mathbf{z}_{21} = \frac{1}{\mathbf{C}} = \frac{1}{2} = 0.5, \quad \mathbf{z}_{22} = \frac{\mathbf{D}}{\mathbf{C}} = \frac{4}{2} = 2$$

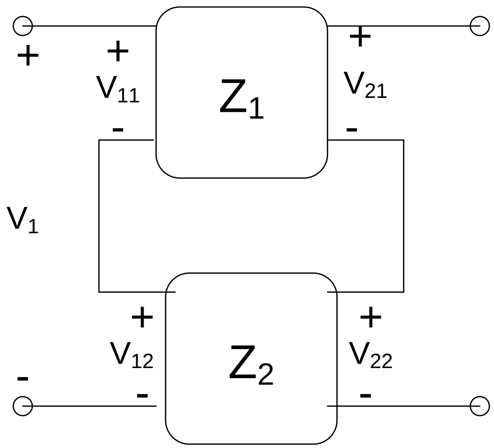
$$\mathbf{g}_{11} = \frac{\mathbf{C}}{\mathbf{A}} = \frac{2}{10} = 0.2, \quad \mathbf{g}_{12} = -\frac{\Delta_T}{\mathbf{A}} = -\frac{37}{10} = -3.7$$

$$\mathbf{g}_{21} = \frac{1}{\mathbf{A}} = \frac{1}{10} = 0.1, \quad \mathbf{g}_{22} = \frac{\mathbf{B}}{\mathbf{A}} = \frac{1.5}{10} = 0.15$$

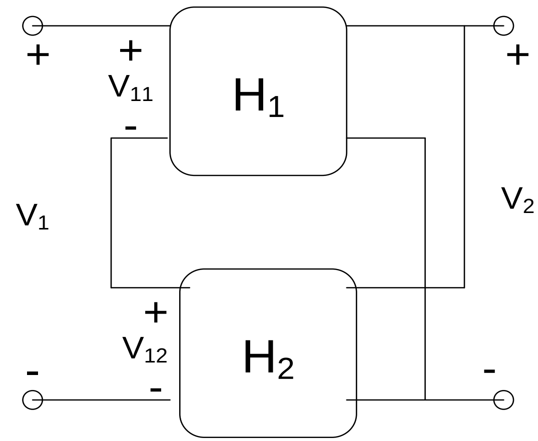
$$\text{Thus, } [\mathbf{z}] = \begin{bmatrix} 5 & 18.5 \\ 0.5 & 2 \end{bmatrix}\Omega, \qquad [\mathbf{g}] = \begin{bmatrix} 0.2\,\text{S} & -3.7 \\ 0.1 & 0.15\,\Omega \end{bmatrix}$$

Interconnection of two-port network

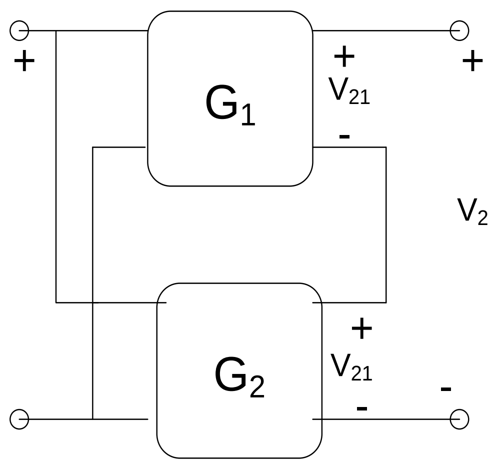
- Two port networks can be connected in series parallel or cascaded
- Series and parallel of two-port have 4 configurations
  - Series input-series output (Z-parameter)
  - Series input-parallel output (h-parameter)
  - Parallel in put-series output (g or h^{-1}-parameter)
  - Parallel input-parallel output (Y-parameter)
- With proper choice of parameters the combined parameters can be added together.



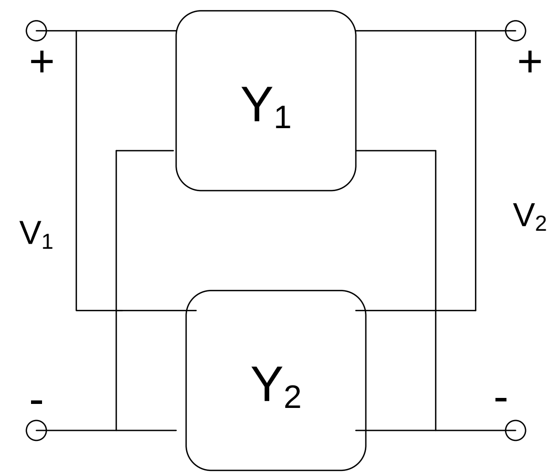
$Z=Z_1+Z_2$



$H=H_1+H_2$



$G=G_1+G_2$

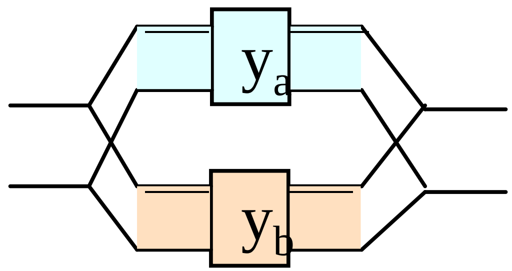


$Y=Y_1+Y_2$

Interconnection of Two- Port Networks

Three ways that two ports are interconnected:

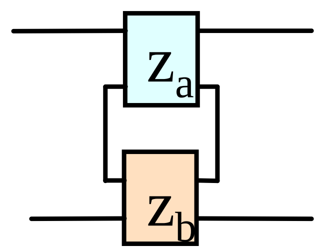
* Parallel



Y parameters

$$[y] = [y_a] + [y_b]$$

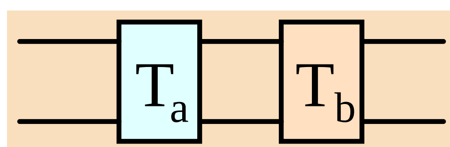
* Series



Z parameters

$$[z] = [z_a] + [z_b]$$

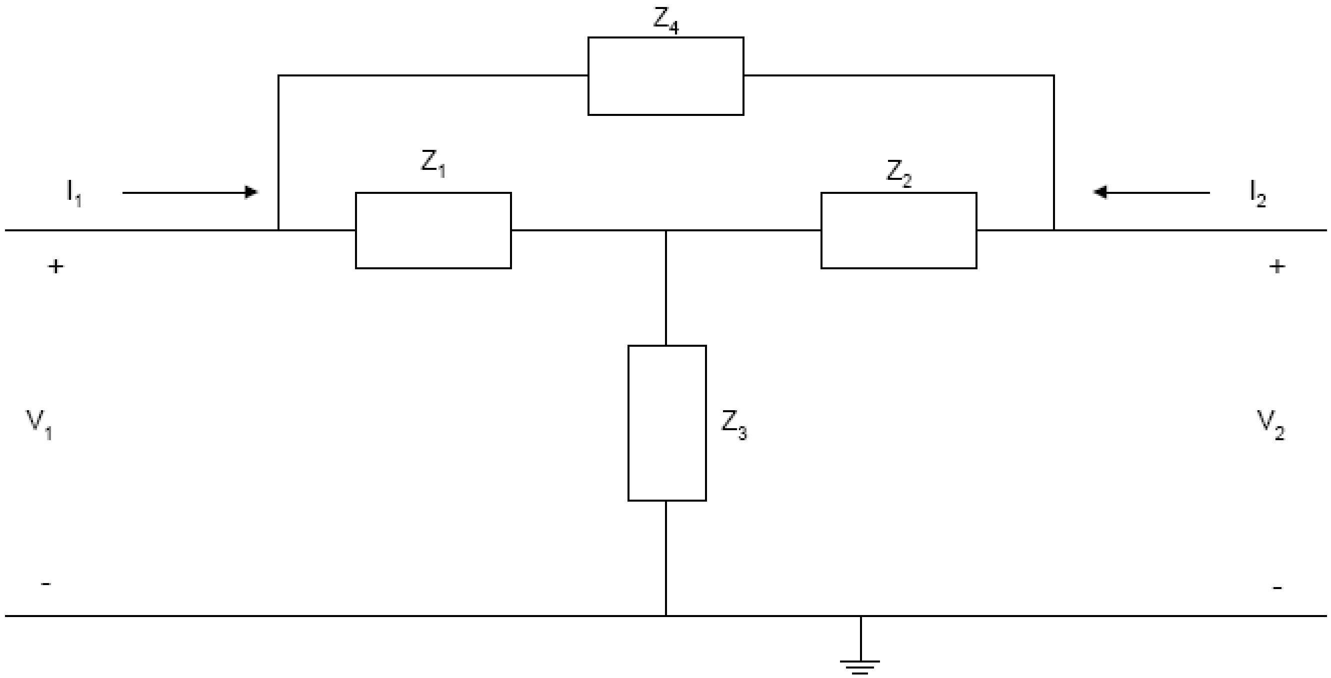
* Cascade

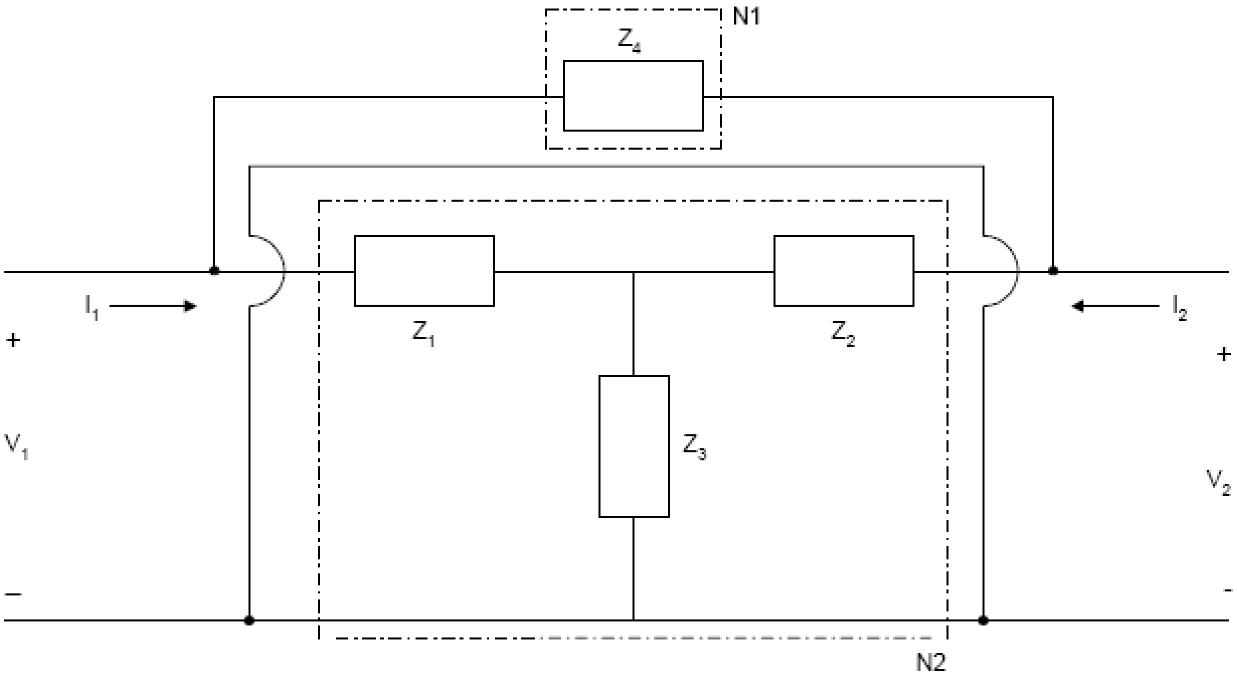


ABCD parameters

$$[T] = [T_a]\,[T_b]$$

Example: Bridge-T network





N1 // N2

For network N2

$$[Z] = \begin{bmatrix} Z_1 + Z_3 & Z_3 \\ Z_3 & Z_2 + Z_3 \end{bmatrix} \longrightarrow$$

$$y_{11} = \frac{Z_2 + Z_3}{Z_1 Z_2 + Z_1 Z_3 + Z_2 Z_3}$$

$$y_{12} = \frac{-Z_3}{Z_1 Z_2 + Z_1 Z_3 + Z_2 Z_3}$$

$$y_{21} = \frac{-Z_3}{Z_1 Z_2 + Z_1 Z_3 + Z_2 Z_3}$$

$$y_{22} = -\frac{Z_1 + Z_3}{Z_1 Z_2 + Z_1 Z_3 + Z_2 Z_3}$$

For network N1

$$[T] = \begin{bmatrix} 1 & Z_4 \\ 0 & 1 \end{bmatrix} \longrightarrow$$

$$y_{11} = \frac{1}{Z_4}$$

$$y_{12} = -\frac{1}{Z_4}$$

$$y_{21} = -\frac{1}{Z_4}$$

$$y_{22} = \frac{1}{Z_4}$$

Y-parameters of the bridge-T network are

$$y_{11eq} = \frac{1}{Z_4} + \frac{Z_2 + Z_3}{Z_1 Z_2 + Z_1 Z_3 + Z_2 Z_3}$$

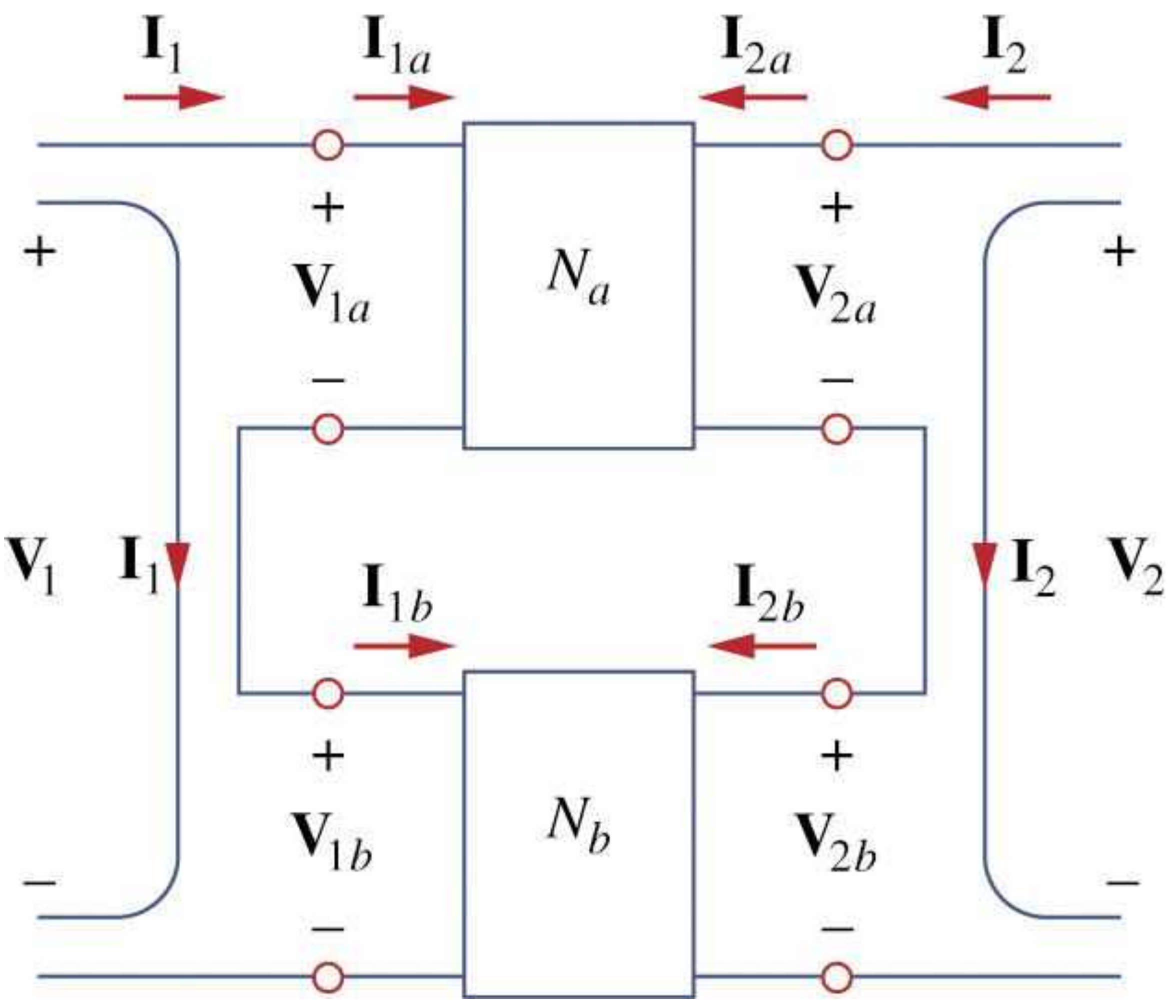
$$y_{12eq} = -\frac{1}{Z_4} - \frac{Z_3}{Z_1 Z_2 + Z_1 Z_3 + Z_2 Z_3}$$

$$y_{21eq} = -\frac{1}{Z_4} - \frac{Z_3}{Z_1 Z_2 + Z_1 Z_3 + Z_2 Z_3}$$

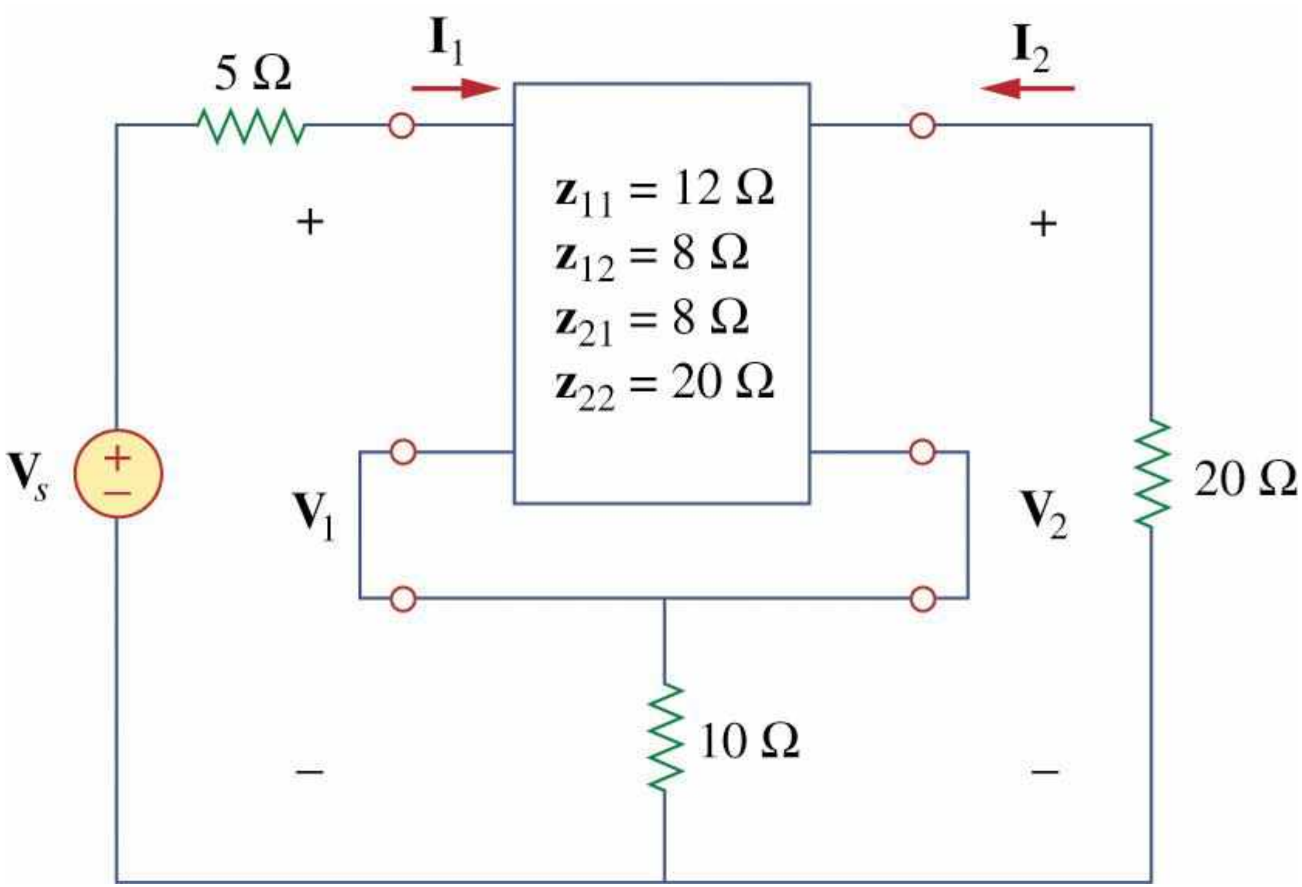
$$y_{22eq} = \frac{1}{Z_4} + \frac{Z_1 + Z_3}{Z_1 Z_2 + Z_1 Z_3 + Z_2 Z_3}$$

Interconnection of Networks

$$[\mathbf{z}] = [\mathbf{z}_a] + [\mathbf{z}_b]$$



- Evaluate $\mathbf{V}_2/\mathbf{V}_1$ in the circuit in Fig.:



This may be regarded as two - ports in series.

For N_b,

$$\mathbf{z}_{12b} = \mathbf{z}_{21b} = 10 = \mathbf{z}_{11b} = \mathbf{z}_{22b}$$

Thus,

$$[\mathbf{z}] = [\mathbf{z}_a] + [\mathbf{z}_b] = \begin{bmatrix} 12 & 8 \\ 8 & 20 \end{bmatrix} + \begin{bmatrix} 10 & 10 \\ 10 & 10 \end{bmatrix} = \begin{bmatrix} 22 & 18 \\ 18 & 30 \end{bmatrix}$$

But

$$\mathbf{V}_1 = \mathbf{z}_{11}\mathbf{I}_1 + \mathbf{z}_{12}\mathbf{I}_2 = 22\mathbf{I}_1 + 18\mathbf{I}_2$$

$$\mathbf{V}_2 = \mathbf{z}_{32}\mathbf{I}_1 + \mathbf{z}_{22}\mathbf{I}_2 = 18\mathbf{I}_1 + 30\mathbf{I}_2$$

Also, at the input port $\mathbf{V}_1 = \mathbf{V}_s - 5\mathbf{I}_1$

and at the output port $\mathbf{V}_2 = -20\mathbf{I}_2 \Rightarrow \mathbf{I}_2 = -\dfrac{\mathbf{V}_2}{20}$

$$\Rightarrow \mathbf{V}_s - 5\mathbf{I}_1 = 22\mathbf{I}_1 - \frac{18}{20}\mathbf{V}_2 \Rightarrow \mathbf{V}_s = 27\mathbf{I}_1 - 0.9\mathbf{V}_2$$

$$\Rightarrow \mathbf{V}_2 = 18\mathbf{I}_1 - \frac{30}{20}\mathbf{V}_2 \Rightarrow \mathbf{I}_1 = \frac{2.5}{18}\mathbf{V}_2$$

$$\Rightarrow \mathbf{V}_s = 27 \times \frac{2.5}{18}\mathbf{V}_2 - 0.9\mathbf{V}_2 = 2.85\mathbf{V}_2$$

And also, $\dfrac{\mathbf{V}_2}{\mathbf{V}_s} = \dfrac{1}{2.85} = 0.3509$

Thank you