Network Theory

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Gyan Ranjan Biswal received his B.E. in Electronics Engineering from the Pt. Ravishankar Shukla University, India in 1999 and M. Tech. (Honors) in Instrumentation & Control Engineering from the Chhattisgarh Swami Vivekananda Technical University, India in 2009 followed by Ph.D. in Electrical Engineering, specialized in the area of Power System Instrumentation (Power Generation Automation) from the Indian Institute of Technology Roorkee, India in 2013.

He is expertise in Design and Development of cooling systems for large size electrical generators, and the C&I of process industries. He has been in academia for about twelve years. Presently, he is with VSS University of Technology, Burla, India at the capacity of Head and Associate Professor, EEE from Dec. 2016. He has more than 65 publications in various Journals and Conferences of Internationally repute to his credit. He also holds a patent as well, and filed one more. He also adapted one international edition book published by Pearson India. He received research grants of US$90,000 (INR 50 lakhs). He has been supervised 09 Masters’ theses, and registered 04 PhD theses. He has also been recognized with many national and international awards by elite bodies. He has been awarded with CICS award under the head of Indian National Science Academy for travel support to USA, MHRD Fellowship by Govt. of India, and Gopabandhu Das Scholarship in his career. His major areas of interests are Power System Instrumentation, Industrial Automation, Robust and Intelligent Control, the Smart Sensors, IoT enabled Smart Sensors, the Smart Grid, Fuel Cell lead Sustainable Sources of Energy, and System Reliability.

Dr. Biswal is a Fellow IE (India), Senior Member of IEEE, USA, and Life Member of ISTE, India. He is actively involved in review panels of different societies of international repute viz. IEEE, IFAC, and the ISA. Currently, he is also actively involved as a Member of IEEE–SA (Standards Association) working groups; IEEE P1876 WG, IEEE P21451-001 WG, and IEEE P1415. He has also been invited for delivering guest lectures at World Congress on Sustainable Technologies (WCST) Conf. 2012, London, UK, INDICON 2015, New Delhi, India, National Power Training Institute (NPTI), Nangal, India, and G.B. Pant Engineering College, Pauri, Gharwal, India, Surendra Sai University of Technology (formerly UCE), Burla, and as a guest expert in 2016 IEEE PES General Meeting Boston, MA, USA.
Network Theory

MODULE-I (9 HOURS) [Online mode: 5 HOURS + 1 Test]

MODULE-II (7 HOURS) [Online mode: 5 HOURS + 1 Test]
Two Port networks: Types of port Network, short circuit admittance parameter, open circuit impedance parameters, Transmission parameters, Condition of Reciprocity and Symmetry in two port network, Inter-relationship between parameters, Input and Output Impedances in terms of two port parameters, Image impedances in terms of ABCD parameters, Ideal two port devices, ideal transformer. Tee and Pie circuit representation, Cascade and Parallel Connections.

MODULE-III (8 HOURS) [Online mode: 5 HOURS + 1 Test]
Network Functions & Responses: Concept of complex frequency, driving point and transfer functions for one port and two port network, poles & zeros of network functions, Restriction on Pole and Zero locations of network function, Time domain behavior and stability from pole-zero plot, Time domain response from pole zero plot.
Three Phase Circuits: Analysis of unbalanced loads, Neutral shift, Symmetrical components, Analysis of unbalanced system, power in terms of symmetrical components.

MODULE-IV (9 HOURS) [Online mode: 5 HOURS + 1 Test]
Network Synthesis: Realizability concept, Hurwitz property, positive realness, properties of positive real functions, Synthesis of R-L, R-C and L-C driving point functions, Foster and Cauer forms.

MODULE-V (6 HOURS) [Online mode: 5 HOURS + 1 Test]
Filters: Classification of filters, Characteristics of ideal filters.
Text and Reference Books

Recommended Text Books:


Reference Books:

* “Basic Circuit Theory, Huelsman, PHI, 3rd ed.,
<table>
<thead>
<tr>
<th>Reference Sites</th>
<th></th>
</tr>
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<tbody>
<tr>
<td>2.  MIT OpenCourseWare:</td>
<td><a href="https://ocw.mit.edu/index.htm">https://ocw.mit.edu/index.htm</a></td>
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</tbody>
</table>
Upon successful completion of this course, you (students) will be able to

<table>
<thead>
<tr>
<th>CO1</th>
<th>Analyze coupled circuits and understand the difference between the steady state and transient response of 1st and 2nd order circuit and understand the concept of time constant.</th>
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<tbody>
<tr>
<td>CO2</td>
<td>Learn the different parameters of two port network.</td>
</tr>
<tr>
<td>CO3</td>
<td>Concept of network function and three phases circuit and know the difference of balanced and unbalanced system and importance of complex power and its components.</td>
</tr>
<tr>
<td>CO4</td>
<td>Synthesis the electrical network.</td>
</tr>
<tr>
<td>CO5</td>
<td>Analyse the network using graph theory and understand the importance of filters in electrical system.</td>
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What do you mean by Network Theory??

- Network Analysis (by means of mathematical modeling): when an electrical/electronic circuit is analyzed for the given/known input(s) to find a desired output/response(s), is called “network analysis”.

- Network Analysis (by means of graph theory): when a complex matrix (electrical/electronic circuit) is analyzed for the multiple given/known input(s) to find desired output/responses, is also called “network analysis”. Typically, this exercise is done by graph theory as the ancestor consumes longer time to reach the solutions.

- Network Synthesis: when an electrical/electronic circuit is supposed to be designed for the given/known input(s) and output/response(s), is called “network synthesis”.

VSSUT, Burla

Network Theory

Dr. Gyan Ranjan Biswal
Introduction

- Filter design (passive type): an electrical circuit which is designed to select a particular band of frequency(ies) to pass through it, using R-L-C elements.

What do you mean by Network Theory???

- Thus, it is very essential to mathematically model and analysis a electrical/electronic circuit to judge the validity/ efficacy of it. It helps to find the effectiveness of the circuit in real life scenario. otherwise a poor selection of types of circuit elements and magnitude of elements / sources/ load may leads to improper findings or even in worst cases uncontrolled responses.

- Five Majors of Electrical & Electronics Engineering: (founding terminologies)
  - V:
  - I:
  - R:
  - L:
  - C:
Relationship of basic quantities in terms of Circuit Parameters

![Diagram showing relationships between voltage, current, resistance, capacitance, inductance, and energy](image)

**Fig. 1-15. Chart illustrating the relationships of basic quantities in terms of the circuit parameters.** (From M. Kawakami, *EREM Chart*, Kyoritsu Shuppan Co., Tokyo.)

**Table 1-1. SUMMARY OF RELATIONSHIPS FOR THE PARAMETERS**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Basic Relationship</th>
<th>Voltage-Current Relationships</th>
<th>Energy</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R$</td>
<td>$v = Ri$</td>
<td>$v_R = Ri_R$</td>
<td>$w_R = \int_{-\infty}^{t} v_R i_R , dt$</td>
</tr>
<tr>
<td>$G = \frac{1}{R}$</td>
<td>$\psi = Li$</td>
<td>$v_L = L \frac{di_L}{dt}$</td>
<td>$w_L = \frac{1}{2} L i_L^2$</td>
</tr>
<tr>
<td>$L$</td>
<td>$q = Cv$</td>
<td>$v_C = \frac{1}{C} \int_{-\infty}^{t} i_C , dt$</td>
<td>$w_C = \frac{1}{2} C v_C^2$</td>
</tr>
<tr>
<td>$C$</td>
<td>$q = Cv$</td>
<td>$i_C = C \frac{dv_C}{dt}$</td>
<td></td>
</tr>
</tbody>
</table>

*Table adapted from M. Kawakami, *EREM Chart*, Kyoritsu Shuppan Co., Tokyo.*
Introduction

Self Inductance (L) and Mutual Inductance (M):

❖ Unlike Capacitance which drive through *electric field*, behavior of inductance (that drive through *magnetic field*) is a special class.

❖ Network Theorems: Superposition/ Thevenin’s / Norton’s/ MPT....and so on....

❖ Linear and Non-linear elements: is/ isn’t governed by linear differential eqs.
  ▪ R , L, C

❖ Skin Effect: *why so vital???
Skin Effect

Skin effect: Uniform distribution of current throughout the cross-section of a conductor exists only for DC. As the frequency of AC increases, the nonuniformity of current density becomes more pronounced. An increase in frequency causes generation of induced current density (skin effect). This phenomenon is called the skin effect.

Lenz's law
- When the induced voltage is high, high is induced
- When the induced voltage is low, low is induced

No: This effect causes an EMF to be induced at large conductivity, even with zero flux. GMA of GMD.
Introduction

Some other way of Network Classifications:

1) Active and Passive elements
2) Unilateral and Bilateral elements
3) Lumped and Distributed elements
4) Linear and Non-linear elements
In order to completely comprehend mutual inductance, it is important to take a relook at Eq. (1.18). Consider an air cored coil, shown in Fig. 9.1, carrying a time varying current $i(t)$ A. The direction of the flux $\Phi$, shown upward in Fig. 9.1, is obtained by the application of the right-hand rule which states that if the fingers of the right hand are wrapped around the coil in the direction of the current, then the thumb points to the direction of the flux. Assume that the change $di$ in the current produces a change in flux $d\Phi$ weber and completely links all the $N$ turns of the coils. The voltage $e(t)$ V induced in the coil, according to Faraday's law of electromagnetic induction, is proportional to the time rate of change of flux linkages, and is given by

$$
e(t) = N \frac{d\Phi}{dt}$$

$$e(t) = N \frac{d\Phi}{di(t)} \times \frac{di(t)}{dt}$$

(9.1)
So how do we apply Superposition theorems over RL circuits?

\[ e(t) = L \frac{di(t)}{dt} \]

Comparison of Eqs (1.18) and (9.1) gives

\[ L = N \frac{d\Phi}{di} = d\left(\frac{N\Phi}{di}\right) = \frac{d\Psi}{di} H \]

(9.2)

where \( \Psi = N\Phi \) Weber-turns is the flux linkage.

For an air cored coil, \( \Psi \) varies linearly with the variation in \( i \), hence \( (d\Psi/dt) = \) constant, and from Eq. (9.2), inductance \( L \) may be written as

\[ L = \frac{N\Phi}{i} = \frac{\Psi}{i} H \]

(9.3)

When the core of a coil consists of a magnetic material instead of air, Eq. (9.3) does not hold due to the non-linear \( \Psi - i \) characteristic.
In Eq. (9.3), the self-flux linking the coil forms the basis for defining inductance \( L \) of the coil and hence it is termed as the self-inductance of the coil.

Now, \( \Phi = \frac{\text{mmf}}{\text{reluctance}} = \frac{Ni}{R} \)  \( (9.4) \)

where \( R \) is the reluctance of the magnetic path.

Substituting Eq. (9.4) in Eq. (9.3), the expression for \( L \) becomes

\[ L = \frac{N\Phi}{i} = \frac{N^2}{R} \]  \( (9.5) \)
Figure 9.2(a) shows coil 1 having $N_1$ turns is placed on a common magnetic core near coil 2 with $N_2$ turns. The voltage induced in coil 2 is proportional to the rate of change of current $i_1(t)$ in coil 1. If $\frac{di_1}{dt}$ is the increase of current in coil 1 in $dt$, then emf $e_2(t)$ induced in coil 2 may be expressed as

$$e_2(t) \propto \frac{di_1(t)}{dt}$$

or

$$e_2(t) = M_{12} \frac{di_1(t)}{dt}$$

(9.6)

where $M_{12}$ is called mutual inductance between coils 1 and 2. The unit of mutual inductance is the same as for self-inductance, namely henry (H). Two coils have a mutual inductance of 1 H if an emf of 1 V is induced in one coil when the current in the other coil varies uniformly at the rate of 1 A/S.
Fig. 9.2  Mutual inductance between two coils (a) coil 1 excited by current $i_1(t)$ and (b) coil 2 excited by current $i_2(t)$

If $d\Phi_{12}$ weber is the increase of the mutual flux in coil 2 due to the increase of $di_1$ in coil 1, then emf induced in coil 2 is

$$e_2(t) = N_2 \frac{d\Phi_{12}}{dt} \tag{9.7}$$
Mutual Coupling

From expressions (9.6) and (9.7)

\[
M_{12} \frac{di_1}{dt} = N_2 \frac{d\Phi_{12}}{dt} = N_2 \frac{d\Phi_{12}}{di_1} \frac{di_1}{dt}
\]

then

\[
M_{12} = N_2 \frac{d\Phi_{12}}{di_1}
\]  \hspace{1cm} (9.8)

If the reluctance of the magnetic core is constant, then the ratio \(\frac{d\Phi_{12}}{di_1}\) is constant and is equal to flux per ampere, and Eq. (9.8) may be written as

\[
M_{12} = \frac{N_2 \Phi_{12}}{i_1} = \frac{\psi_{12}}{i_1}
\]  \hspace{1cm} (9.9)

where \(\psi_{12}\) is the flux linkages of coil 2 due to a current \(i_1\) in coil 1.

Similarly, if \(\Phi_{21}, \psi_{21}\) are respectively the flux and flux linkages of coil 1 due to a current \(i_2(t)\) in coil 2 as shown in Fig. 9.2(b), the mutual inductance \(M_{21}\) between coil 2 and coil 1 can be expressed as

\[
M_{21} = \frac{N_1 \Phi_{21}}{i_2} = \frac{\psi_{21}}{i_2}
\]  \hspace{1cm} (9.10)
Coefficient of Coupling

From Fig. 9.2, it is seen that only a part of the flux links the two coils. A part of the total flux produced by one coil, called the leakage flux, links only the coil producing the flux, while the balance flux, called the mutual flux, links the other coil. If a fraction $k$ of total flux $\Phi_1$ produced by current $I_1$ in coil 1 links coil 2, then the mutual flux $\Phi_{12} = k\Phi_1$. Similarly, $\Phi_{21} = k\Phi_2$, where $\Phi_2$ is the total flux produced by current $I_2$ in coil 2, and $\Phi_{21}$ is the mutual flux that links coil 1. Then, using Eq. (9.4), gives

$$\Phi_1 = \frac{N_1 I_1}{R}, \quad \text{and} \quad \Phi_2 = \frac{N_2 I_2}{R} \tag{9.11}$$

where $R$ is the reluctance of the magnetic circuit.

Substituting the values of $\Phi_{12}$ and $\Phi_{21}$ in Eqs (9.9) and (9.10) yields

$$M_{12} = k \frac{N_1 N_2}{R} H \quad \text{and} \quad M_{21} = k \frac{N_1 N_2}{R} H \tag{9.12}$$

Hence, for a bilateral circuit,

$$M_{12} = M_{21} = M \tag{9.13}$$

Then

$$M^2 = M_{12} \times M_{21} = k^2 \left(\frac{N_1 N_2}{R} \right)^2 \tag{9.14}$$

Using Eq. (9.5) $L_1$ and $L_2$, the self-inductances of coils 1 and 2, respectively, are expressed as
Coefficient of Coupling

\[ L_1 = \frac{N_1^2}{R}, \quad \text{and} \quad L_2 = \frac{N_2^2}{R} \]

From Eq. (9.15), the product \( L_1L_2 \) may be obtained as

\[ L_1L_2 = \frac{N_1^2 N_2^2}{R^2} \]

Substituting Eq. (9.16) in Eq. (9.14) and simplifying leads to

\[ M = k \sqrt{L_1L_2} \]

The term \( k \) is called the coefficient of coupling. When 100 percent of the flux lines link each coil, then \( k = 1 \). The term coupling coefficient is widely used in radio networks to denote the degree of coupling between two coils. If the two coils are close together, most of the flux produced by the current in one coil passes through the other coil, and the coils are said to be tightly coupled. If the coils are well apart, then a small part of the flux produced by the current in one coil passes through the other coil, and the coils are said to be loosely coupled.
Example 9.1: Two coils having 750 and 1200 turns, respectively, are wound on a common non-magnetic core. The leakage flux and mutual flux, due to a current of 7.5 A in coil 1, is 0.25 mWb and 0.75 mWb, respectively. Calculate the (a) self-inductance of the coils, (b) mutual inductance, and (c) coefficient of coupling.

Solution: From the data, it is seen that $\Phi_{11} = 0.25$ mWb and $\Phi_{12} = 0.75$ mWb. Therefore, the total flux linking coil 1 is $\Phi_1 = \Phi_{11} + \Phi_{12} = 1.0$ mWb.

(a) From Eq. (9.3), the self-inductance of the coil is computed as follows:

$$L_1 = \frac{750 \times 1.0 \times 10^{-3}}{7.5} = 100 \text{ mH}$$

To compute the mutual inductance, Eq. (9.9) is employed in the following manner:

$$M = M_{12} = \frac{1200 \times 0.75 \times 10^{-3}}{7.5} = 120 \text{ mH}$$

The coefficient of coupling is calculated as

$$k = \frac{\Phi_{12}}{\Phi_1} = \frac{0.75}{1.0} = 0.75$$
Using Eq. (9.17) leads to the self-inductance $L_2$. Thus,

$$L_2 = \frac{M^2}{k^2 L_1} = \frac{(120 \times 10^{-3})^2}{(0.75)^2 \times (100 \times 10^{-3})} = 256 \text{ mH}$$
Mutual Coupling

Two coils have 750 and 1200 turns respectively on a non-magnetic core. The leakage flux is mutual flux denoted by \( I = 7.5 \, \text{mH} \). In coil 1, the flux is 0.25 mwb and 0.75 mwb in coil 2. Calculate:

(a) \( L_1 \), \( L_2 \) (self inductance)
(b) Mutual ind. \( M \)
(c) Coupling coefficient \( k \)

For \( \phi_1 = 0.25 \, \text{mwb} \), \( \phi_2 = 0.75 \, \text{mwb} \)

\[ \phi_1 \text{ (total flux)} = \phi_1 + \phi_2 = 1.0 \, \text{mwb} \]

\[ L_1 = \frac{N_1 \phi_1}{I} = \frac{750 \times 1 \times 10^{-3}}{7.5} = 100 \, \text{mH} \]

\[ M = \frac{N_1 \phi_2}{I} = \frac{1200 \times 0.75 \times 10^{-3}}{7.5} = 120 \, \text{mH} \]

Now \( L_2 = \frac{M^2}{k^2 L_1} = \frac{(120 \times 10^{-3})^2}{(0.75)^2} = 256 \, \text{mH} \)

\[ k = \frac{M^2}{L_1 L_2} = \frac{(0.75)^2}{100 \times 256} = 0.075 \]
It is seen in Section 9.4.2 that the sign of the voltage of a mutual inductance can be conveniently established if the direction of the windings of coupled coils is known. If dots are placed at the terminals to indicate currents entering or leaving the respective coils, then these dots provide a symbolic representation of the instantaneous polarity of the coupled coils and the pictorial representation of the core with its windings is not required. Thus, mutually coupled coils of the coupled circuit of Fig. 9.4(a) may then be represented by the equivalent circuit of Fig. 9.4(b).

Dot convention may be employed in the following manner:

(i) When the assumed currents in a mutually coupled pair of coils enter or leave the dotted terminals, the $M$ terms will have the same sign as the $L$ terms.

(ii) When one current enters at the dotted terminal and the other current leaves the other dotted terminal, the $M$ terms will have the opposite sign as the $L$ terms.
Rules of Mutual Coupling and Dot Conversion

\[ \psi_1 = \frac{N_1}{N_1} \Phi_{22} \]

\[ \phi_1 = \frac{1}{\phi_{21}} \frac{\psi_{12}}{\phi_{21}} \]

\[ \phi_{21} = \frac{N_1}{N_1} \]

\[ N_1 = L_1 \]

\[ 1 \times 2 = L_2 \]

1. Current flowing into the dot on one winding induces a voltage in the other winding, making the dotted end (+) and out of dot (−) wire.

2. Current simultaneously flowing into/out of dots on winding induce flux in the core which are additive.
Figure 9.5 shows two mutually coupled coils of self-inductances $L_1\ H$, $L_2\ H$, and a mutual coupling of $M\ H$. The dot marks are provided by the manufacturer as shown in the figure. For the assumed directions of currents in the coils, it is seen that current $i_1(t)$ is entering the dotted terminal at A, while current $i_2(t)$ is leaving the dotted terminal at B. Hence, as per rule (ii) $M$ will have a negative sign. The voltage equations are written as

$$R_1i_1(t) + L_1\frac{di_1(t)}{dt} - M\frac{di_2(t)}{dt} = v_S(t)$$  \hspace{1cm} (9.18)$$

$$R_2i_2(t) + L_2\frac{di_2(t)}{dt} - M\frac{di_1(t)}{dt} = 0$$  \hspace{1cm} (9.19)$$

**Fig. 9.5** Application of the dot convention to write voltage equations
If the voltage source in Fig. 9.5 is a sinusoidal voltage source whose operating frequency is \( f \) Hz, Eqs (9.18) and (9.19) take the following form:

\[
\begin{align*}
R_1 I_1 + j\omega L_1 I_1 - j\omega M I_2 &= V_S \\
R_2 I_2 + j\omega L_2 I_2 - j\omega M I_1 &= 0
\end{align*}
\]  

(9.20)

where \( \omega = 2\pi f \) rad/s and \( V_S, I_1, \) and \( I_2 \) are the effective values of the voltage source and currents in the coils, respectively. In the matrix form Eq. (9.20) can be expressed as

\[
\begin{bmatrix}
(R_1 + j\omega L_1) & -j\omega M \\
-j\omega M & (R_2 + j\omega L_2)
\end{bmatrix}
\begin{bmatrix}
I_1 \\
I_2
\end{bmatrix} = \begin{bmatrix}
V_S \\
0
\end{bmatrix}
\]  

(9.21)

or

\[
\begin{bmatrix}
Z_{11} & Z_{12} \\
Z_{21} & Z_{22}
\end{bmatrix}
\begin{bmatrix}
I_1 \\
I_2
\end{bmatrix} = \begin{bmatrix}
V_S \\
0
\end{bmatrix}
\]  

(9.22)

where \( Z_{11} = (R_1 + j\omega L_1), Z_{12} = Z_{21} = -j\omega M, \) and \( Z_{22} = (R_2 + j\omega L_2). \) The mesh currents \( I_1 \) and \( I_2 \) can be computed by solving Eq. (9.22).

The equivalent network representation of Eq. (9.20) is shown in Fig. 9.6. The veracity of the representation can be verified by developing mesh equations for the network.
Mutual Coupling and Dot Conversion

Fig. 9.6  $T$-equivalent circuit of a mutually coupled circuit
Mutual Coupling and Dot Conversion

\[ V = L_1 \frac{di}{dt} + L_2 \frac{di}{dt} + M \frac{di}{dt} + M \frac{di}{dt} \]

\[ V = (L_1 + L_2 + 2M) \frac{di}{dt} \]

\[ L_{eq} = L_1 + L_2 + 2M \]

\[ V = L_1 \frac{di}{dt} + L_2 \frac{di}{dt} - 2M \frac{di}{dt} \]

\[ L_{eq} = L_1 + L_2 - 2M \]
Mutual Coupling and Dot Conversion

\[ v_1 = L_1 \frac{di_1}{dt} + M \frac{di_2}{dt} \quad (1) \]

\[ v_1 = L_2 \frac{di_2}{dt} + M \frac{di_1}{dt} \quad (2) \]

\[ \text{eq. } (1) = \text{eq. } (2) \]

\[ (L_1 - M) \frac{di_1}{dt} = (L_2 - M) \frac{di_2}{dt} \]

\[ \frac{di_2}{dt} = \left( \frac{L_2 - M}{L_2 - L_1} \right) \frac{di_1}{dt} \quad (3) \]

\[ \frac{d}{dt} (\bar{x} - x_1) = -1 \]

\[ \frac{di}{dt} = \left( \frac{L_2 - M + L_2 - M}{L_2 - M} \right) \frac{di}{dt} \]
Mutual Coupling and Dot Conversion

\[ i_1 \times L = \frac{1}{i_2} \]

\[ i_2 = \frac{1}{i_3} \]

\[ \text{total} \]

\[ \psi_1 \]
Coupling and Energy Transformation

\[ I_{vs} = \frac{V}{Z_a + Z_L} \]

\[ I_{cy} = \frac{Z_b - I}{Z_L + Z_b} \]

\[ V = \frac{Z_b - I}{Z_L + Z_b} \]

\[ V = \frac{Z_a - Z_b}{Z_a + Z_b} \]

\[ Z_a = Z_b + Z_L \]
Coupling and Energy Transformation

Fig. P2-8

Fig. 3-22. An example illustrating the procedure by which a current source is shifted in a network.
Consider a pair of mutually coupled coils as shown in Fig. 9.10. Let it be assumed that initially currents \( i_1(t) \) and \( i_2(t) \) in the coils are zero at \( t = 0 \), and the initial energy stored in the circuit is zero. The voltage equations for the circuit are

\[
\begin{align*}
  v_1(t) &= L_1 \frac{di_1(t)}{dt} + M \frac{di_2(t)}{dt} \\
  v_2(t) &= L_2 \frac{di_2(t)}{dt} + M \frac{di_1(t)}{dt}
\end{align*}
\]

(9.23) (9.24)

**Fig. 9.10** Stored energy relationship with two mutually coupled coils

With \( M_{12} = M_{21} = M \) and \( k = \sqrt{L_1L_2} \)
The net energy input to the coupled circuit at any instant of time $t$ is given by

$$w(t) = \int_0^t \left[ v_1(t)i_1(t) + v_2(t)i_2(t) \right] dt$$

(9.25)

Substituting for $v_1(t)$ and $v_2(t)$ from Eqs (9.23) and (9.24), respectively, yields

$$w(t) = \int_0^t \left[ \left\{ L_1 \frac{di_1(t)}{dt} + M \frac{di_2(t)}{dt} \right\} i_1(t) + \left\{ L_2 \frac{di_2(t)}{dt} + M \frac{di_1(t)}{dt} \right\} i_2(t) \right] dt$$

(9.26)

from which

$$w_r(t) = \left( \frac{1}{2} L_1 \left[ i_1(t) \right]^2 + \frac{1}{2} L_2 \left[ i_2(t) \right]^2 + M \left[ i_1(t) \right] \left[ i_2(t) \right] \right) J$$

(9.27)

In case current enters a coil at the dot-marked terminal and leaves at the other dot-marked terminal, then the energy input is given by

$$w_r(t) = \left( \frac{1}{2} L_1 \left[ i_1(t) \right]^2 + \frac{1}{2} L_2 \left[ i_2(t) \right]^2 - M \left[ i_1(t) \right] \left[ i_2(t) \right] \right) J$$

(9.28)
Since the mutually coupled coils are passive elements, the energy stored \( w_\tau(t) \) can never be negative for any values of \( i_1(t) \), \( i_2(t) \), \( L_1 \), \( L_2 \), or \( M \). At best, the stored energy in the circuit can be greater than or equal to zero. If both currents \( i_1(t) \) and \( i_2(t) \) are both positive or both negative, their product will always be positive. It may be inferred from Eqs (9.27) and (9.28) that \( w_\tau(t) \) could possibly be negative only in the case of Eq. (9.28).

Adding and subtracting \( \sqrt{L_1 L_2} i_1(t) i_2(t) \) to Eq. (9.28) and rearranging give

\[
w_\tau(t) = \frac{1}{2} \left[ \sqrt{L_1} i_1(t) - \sqrt{L_2} i_2(t) \right]^2 + \left[ \sqrt{L_1 L_2} - M \right] i_1(t) i_2(t) \quad (9.29)
\]

Equation (9.29) indicates that in order that the stored energy \( w_\tau(t) \geq 0 \), the second term should never be negative since the squared term can only have a minimum value of zero. Hence the necessary condition for \( w_\tau(t) \geq 0 \) is obtained when

\[
\sqrt{L_1 L_2} \geq M \quad (9.30)
\]

Equation (9.30) implies that

\[
M = k \sqrt{L_1 L_2}, \quad \text{for } 0 \leq k \leq 1
\]
Maximum Value of ‘M’

Equation (9.30) implies that

$$M = k \sqrt{L_1 L_2}, \quad \text{for } 0 \leq k \leq 1$$

The inequality in Eq. (9.30) provides a basis for a physical interpretation of the magnetic coupling. If coil 2 is on open circuit $i_2(t) = 0$, the leakage flux which links both the coils is established by current $i_1(t)$ alone. Therefore, it is evident that the leakage flux within coil 2 cannot be greater than the flux contained in coil 1 which signifies the total flux. As such, there has to be an upper limit to the mutual coupling and hence the mutual inductance between the two coils.
Example 9.7  Figure 9.11 shows a mutually coupled linear circuit in which coils 1 and 2 have self-inductances of 0.5 H and 3.0 H, and the coefficient of coupling between the coils is 0.65. If coil 1 is excited by a current \( i_1 = 2.5 \cos(8t) \) A, calculate the energy stored in the circuit at \( t = 0 \) s when terminals AB are (a) open-circuited, and (b) short-circuited.
Example 9.7  Figure 9.11 shows a mutually coupled linear circuit in which coils 1 and 2 have self-inductances of 0.5 H and 3.0 H, and the coefficient of coupling between the coils is 0.65. If coil 1 is excited by a current $i_1 = 2.5 \cos (8t)$ A, calculate the energy stored in the circuit at $t = 0$ s when terminals AB are (a) open-circuited, and (b) short-circuited.

![Figure 9.11](image)
Solution Using Eq. (9.17), the magnitude of $M$ is computed as

$$M = 0.65 \sqrt{0.5 \times 3.0} = 0.7961 \text{ H}$$

(a) Since terminals AB are open-circuited, no current flows through coil 2. All energy is stored in coil 1 is due to $i_1(0) = 2.5 \text{ A}$. Hence,

The energy stored in the circuit $\frac{1}{2} L_1 i_1(0)^2 = \frac{1}{2} \times 0.5 (2.5)^2 = 1.5625 \text{ J}$

(b) The voltage across terminals AB of the mutually coupled circuit is given by

$$v_{AB}(t) = -M \frac{di_1(t)}{dt} + L_2 \frac{di_2(t)}{dt}$$

(9.7.1)

Since current $i_1(t)$ is entering at the dotted terminal and current $i_2(t)$ is leaving at the dotted terminal, $M$ has been assigned a negative sign in Eq. (9.7.1). Further, when the terminals AB are short-circuited, the voltage across the terminals is zero. Therefore, Eq. (9.7.1) is equated to zero and the values are substituted as given below.

$$\frac{3}{i_2(t)} = \frac{d}{dt} \left[ \frac{2.5 \cos(8t)}{3} \right] = -\frac{0.7961 \times 20 \sin(8t)}{3}$$

or

$$i_2(t) = -\frac{0.7961 \times 20}{3} \int_0^t \sin(8t) \, dt = \frac{0.7961 \times 20}{3} \frac{\cos(8t)}{8} = 0.6634 \cos(8t) \text{ A}$$

(9.7.2)

From Eq. (9.7.2), $i_2(0) = 0.6634 \text{ A}$. The total energy stored in the circuit from Eq. (9.28) is

$$w_T(0) = \left( \frac{1}{2} \times 0.5 \times (2.5)^2 + \frac{1}{2} \times 3 \times (0.6634)^2 - 0.7961 \times (2.5) \times (0.6634) \right)$$

$$= 0.9023 \text{ J}$$
Transformer Applications

**Ideal Transformer**

Assume that lines have small, uniform elements.

01. Transformer windings are uniform and that its primary and secondary windings are not mismatched.

02. Core materials are chosen to minimize losses.

03. It requires a minimum of core loss.

Flow in the core

 leakage flow is neglected (i.e., no leakage flux)

**Transformation Ratio**

\[
\frac{V_1}{V_2} = \frac{E_1}{E_2} = \frac{N_1}{N_2} = \frac{I_2}{I_1} \\
V_1 = \sqrt{E_1^2 + 4 \pi f_1 N_1^2}
\]

\[
V_2 = E_2 = \sqrt{E_2^2 + 4 \pi f_2 N_2^2}
\]

(Transformer losses)
Transformer Applications

Coupled coil as Transformer

Linear Transformer

\[ (v_1 + a_j + jw_1)z_1 - jw_1 z_1 = v_0 \]

Ideal Transformer

\[ \frac{V_1}{V_2} = \frac{N_1}{N_2} = \frac{m_i}{m_o} \]

Transformer Core:

\[ \frac{V_1}{V_2} = \frac{N_1}{N_2} = \frac{m_i}{m_o} \]
Transformer Applications

\[ V_s = 2,000 \ \text{lo}^n \ \text{V} \]

\[ V_1 = \begin{bmatrix} 9 \cdots \ \text{Lo}^n \ - \ I_1 \ (0.6 + j 0.675) \end{bmatrix} \ \text{V} \]

\[ V_E = L_2 (0.2 + j 0.675) \ \text{V} \]

\[ \frac{V_1}{n} = 0.01 (0.6 + j 0.675) \ \text{V} \]

\[ V_1 = \begin{bmatrix} 0.01 \ (0.6 + j 0.675) \end{bmatrix} \]

\[ = 164.1 \ (0.6 + j 0.675) \]

\[ = 1 (230 + j 219) \ \text{V} \]

\[ \Rightarrow \frac{V_1}{n} = 0.01 (230 + j 219) = 2.000 \ \text{lo}^n \ \text{V} \]

\[ I_1 = 54.90 \ \text{A} \]

\[ I_1 = 54.90 \ \text{A} \]

\[ V_1 = 54.90 \ \text{A} \times (230 + j 219) \]

\[ = 15.04 \ \text{A} \times 235.50 = 3.525 \ \text{V} \]

\[ V_2 = \frac{V_1}{n} = 162.0 \ \text{lo}^n \ \text{V} \]

\[ I_1 = 65.00 \ \text{A} \times (230 + j 219) \]
Transformer Applications

Open-Circuit (OC) Test / No-Load Test

I.e., armature is excited at rated \( V_1 \) & \( I_1 \) on primary side
(on side of side of load open at (other side) secondary side

\[
Y_0 = \frac{I_1}{V_1} \\
\text{A:} = \frac{P_A}{V_1^2} \\
B_m = \sqrt{Y_0 - A} \\
(\text{G} = \frac{1}{\text{A}})
\]

The O-C test yields:

1. Core loss.
2. Loss in the leakage branch.
3. Loss in the primary branch.
4. Loss in the shunt branch.

A 50/110 kV transformer is connected to 110 supply. If the \( 110 \text{kV} \) side is open-circuited, the meter readings are 110 kVA, 50 kVA. Calculate the parameters of the shunt branch for equilibrium.

LV side:

\[
Y_0 = \frac{I_1}{V_1} = \frac{50}{110} = 0.4545 \text{ A}
\]

HV side:

\[
G_{hv} = G_{hv} \left( \frac{V_1}{110} \right)^2 \\
= 0.073 \times \left( \frac{110}{110} \right)^2 \\
= 0.073 \text{ m}\text{A}^{-1}
\]

Hence, \( G_{hv} = 0.073 \text{ m}\text{A}^{-1} \) and \( B_m = 0.073 \text{ m}\text{A}^{-1} \).
Transformer Applications

**Short Circuit (SC) Test**

The test is used to determine the series parameter of the transformer.

The primary is short-circuited on one side, and a current is excited from a fixed voltage source from the other side.

\[ V_{sc} = I_{sc} \]

\[ Z = \frac{V_{sc}}{I_{sc}} = \sqrt{R^2 + X^2} \]

\[ R_{eq} = R = \frac{P_{eq}}{I_{sc}^2} \]

\[ X_{eq} = X = \sqrt{2R^2 - R^2} \]

The test yields:

1. **full load copper loss**
2. **equivalent \( R \) and \( X \) of the transformer**
Duality Principle

If an electrical network is governed by the same type of equation as the network under consideration, then the network is known as the dual of the network.

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Dual</th>
</tr>
</thead>
<tbody>
<tr>
<td>i, e</td>
<td>V, i</td>
</tr>
<tr>
<td>R</td>
<td>G, R</td>
</tr>
<tr>
<td>L</td>
<td>C</td>
</tr>
<tr>
<td>Loop current (E)</td>
<td>Node pair voltage (V)</td>
</tr>
<tr>
<td>No. of loops</td>
<td>No. of node pair</td>
</tr>
<tr>
<td>Mesh current</td>
<td>Node potential</td>
</tr>
<tr>
<td>Link / cord</td>
<td>Tree branch</td>
</tr>
<tr>
<td>s, e</td>
<td>O, C</td>
</tr>
<tr>
<td>Source path</td>
<td>Shunt (1st) path</td>
</tr>
<tr>
<td>1</td>
<td>Source</td>
</tr>
</tbody>
</table>
Dual Circuit

1. Place 1 node in each loop.
2. Draw lines from one node to another through elements in original network, traversing one node at a time.
3. Continue the process till the no. of possible paths through single elements is exhausted.
Dependent Sources

1. VCVS (Voltage Controlled Voltage Source)
   \[ V_i = A \cdot V_i \]

2. VCES (Voltage Controlled Current Source)
   \[ \begin{align*}
   i_s &= A \cdot V_i \\
   i_s &= 0 \\
   V_i &= 0
   \end{align*} \]

3. CCCS
   \[ \begin{align*}
   i &= A \cdot i_s \\
   V_i &= 0
   \end{align*} \]

4. CCVS
   \[ \begin{align*}
   V_i &= A \cdot i_s \\
   V_i &= 0
   \end{align*} \]

A → Power Source
0 → Ground
A → RMS Power
- = DC Power
- = DC Power
- = DC Power
If \( V_a - V_b = 6 \text{V} \) then
\[ V_c - V_d = 9. \]

Unfortunately, the rest of the notes are not legible. However, it seems to involve a network diagram with labeled voltages and currents, which are not clearly visible due to the handwriting style and quality of the scan.
Dependent Sources

\[ I = 9 \]

\[ V = 18 \text{V} \]

\[ V_A = 10 \text{A} \]

\[ V + 100 \text{V} + \frac{V_A}{10} = 0 \]

\[ V_A = 10 \text{V} \]

\[ V = -\frac{90}{10} = -9 \text{V} \]

\[ I = \frac{V}{10} = \frac{-90}{10} = -9 \text{A} \]

If all the elements of the \( \Delta \) currents are scaled by a factor \( k \), the elements of company \( \lambda \) currents will be scaled by a factor \( k^2 \).

\[ R_1 = k R_c \]

\[ R_2 = k R_3 \]

\[ R_3 = k R_3 \]
Dependent Sources

\[ I = 9 \]

\[ \begin{align*}
10 \Omega & \quad & 10 \Omega & \quad & V = 10 \text{V} \\
\end{align*} \]

\[ \begin{align*}
10 \text{A} & \quad & 10 \Omega & \quad & I = 1 \text{A} \\
\end{align*} \]

Apply source transformation.
Supplementary Sheets/ Problems
Mutual Coupling and Dot Conversion

The analysis starts with:

\[ \Phi_2 = \frac{1}{M} \left( E_2 - \frac{E_1}{N} \right) \]

\[ \Phi_2 = \frac{1}{N} \left( E_2 - \frac{E_1}{M} \right) \]

\[ R_{g1} = \frac{1}{M} \]

\[ R_{g2} = \frac{1}{N} \]

\[ \Phi_1 \times \Phi_2 = 0.325 \times 10^{-6} \text{ A.m} \]

\[ \text{Current} = \text{Voltage} / \text{Resistance} \]

\[ F = 1264 \text{ V} \]
Mutual Coupling and Dot Conversion

2. A coil is made of 40 turns. The coil carries a current of 2 A. Find the magnetic flux density in the coil.

(a) Given: $I = 2$ A, $N = 40$ turns.

(b) Write the expression for magnetic flux density and find the energy stored in the coil.

(c) For $N = 40$, find the flux density.

\[ \phi = N \times I \times B \]

\[ B = \frac{\phi}{N \times I} \]

Notes:
- Flux density:
  \[ B = \frac{10 \times 10^{-2} \times 10 \times 10^{-3}}{2} = 10^{-2} \text{ mT} \]
- Magnetic flux:
  \[ \phi = 4 \times 10^{-5} \text{ Wb} \]
- Magnetic field:
  \[ H = \frac{B}{\mu_0} = \frac{10^{-2}}{4\pi \times 10^{-7}} = 7955 \frac{A}{m} \]

Notes:
- Flux density from Ampere's law: $\phi$ is quadratic time dependent hence $d\phi/dt$ will give $I$.

Notes:
- From Ampere's rule, $\phi = \frac{1}{2} N I^2 t$.

Notes:
- For $I$, use $\phi = \frac{1}{2} N I^2 t$.

Notes:
- From Ampere's law, $\phi = \frac{1}{2} N I^2 t$.

Notes:
- From Ampere's law, $\phi = \frac{1}{2} N I^2 t$.
Mutual Coupling and Dot Conversion
Example 9.3 In Fig. 9.5, the source voltage $V_S = 15 \angle 0^\circ$ V, at $\omega = 12$ rad/s, $R_1 = 1.5 \, \Omega$, $L_1 = 1$ H, $R_2 = 450 \, \Omega$, $L_2 = 100$ H, and $M = 10$ H. (a) Develop the voltage equations and compute the currents in the two coils. (b) Calculate the output voltage $V_2$ across the resistor $R_2$ and determine the voltage gain ratio $|V_2/V_S|$. (c) What is the minimum and maximum values of the voltage gain and the conditions at which these occur?

Solution

(a) Dot convention (ii) is applicable since the current in coil 1 is entering at the dotted terminal and current in coil 2 is leaving at the dotted terminal. Using Eq. (9.20), the voltage equations are

\[
1.5I_1 + j12 \times 1I_1 - j12 \times 10I_2 = 15 \angle 0^\circ
\]

\[
450I_2 + j12 \times 100I_2 - j12 \times 10I_1 = 0
\]

or

\[
(1.5 + j12)I_1 - j120I_2 = 15 \angle 0^\circ
\]

\[-j120I_1 + (450 + j1200)I_2 = 0
\]

Solving the above equations simultaneously yields

$I_1 = 2.6583 \angle -15.20^\circ$ A and $I_2 = 0.2489 \angle 5.36^\circ$ A
Mutual Coupling and Dot Conversion

(b) Output voltage across the resistor $R_2$,

$$V_2 = 450 \times 0.2489 \angle 5.36^\circ = 112.0089 \angle 5.36^\circ \text{ V}$$

and

$$\left| \frac{V_2}{V_s} \right| = \frac{112.0089}{15} = 7.4673$$

It may be noted that the output voltage is greater than the input voltage. However, a phase shift also occurs in the output voltage.

(c) If $R_2$ is made equal to zero, i.e., a short-circuit condition, $V_2 = 0$, hence the minimum ratio $\left| \frac{V_2}{V_s} \right| = 0$ results.

If $R_2$ is made equal to $\infty$, i.e., an open-circuit condition which results in $I_2 = 0$. Thus, the two voltage equations which are obtained from Eq. (9.20) are

$$(1.5 + j12) I_1 = 15 \angle 0^\circ, \text{ or } I_1 = \frac{15 \angle 0^\circ}{(1.5 + j12)} = 1.2403 \angle -82.88^\circ \text{ A}$$

$$V_2 = -j120 I_1 = -j120 \times 1.2403 \angle -82.88^\circ$$

$$= 148.8417 \angle -172.88^\circ \text{ V}$$

Maximum gain

$$\left| \frac{V_2}{V_s} \right| = \frac{148.8417 \angle -172.88^\circ}{15 \angle 0^\circ} = 9.9228$$
Transformer Applications

1. \( V_x = \frac{V_{in}}{V_{max}} \times 100 \)
   
   \( V_x = \frac{X_{max}}{X_{in}} \times 100 \)
   
   \( V_x = \frac{X_{max}}{X_{in}} \times 100 \)

2. \( V_x = \frac{X_{max}}{X_{in}} \times 100 \)
   
   \( V_x = \frac{X_{max}}{X_{in}} \times 100 \)
   
   \( V_x = \frac{X_{max}}{X_{in}} \times 100 \)

3. \( T = \frac{V_{in}}{V_{max}} \times 100 \)
   
   \( T = \frac{X_{max}}{X_{in}} \times 100 \)
   
   \( T = \frac{X_{max}}{X_{in}} \times 100 \)

4. \( V_x = \frac{X_{max}}{X_{in}} \times 100 \)
   
   \( V_x = \frac{X_{max}}{X_{in}} \times 100 \)
   
   \( V_x = \frac{X_{max}}{X_{in}} \times 100 \)

5. \( T = \frac{V_{in}}{V_{max}} \times 100 \)
   
   \( T = \frac{X_{max}}{X_{in}} \times 100 \)
   
   \( T = \frac{X_{max}}{X_{in}} \times 100 \)
Dependent Sources

Mesh Analysis with a Dependent Source

(a) Output of dependent source
(b) Power dissipated by the 4 Ω resistor
(c) Power supplied by the 6V source
Dependent Sources

Node Analysis with Dependent Sources

Calculate:

1. Node voltages
2. $\frac{V}{P}$ of the dependent current source.
Dependent Sources

Fig. 3-29. Magnetically coupled network which is analyzed in Example 6.

Fig. 3-30. A network representation which is equivalent to that shown in Fig. 3-29.
3-26. A network with magnetic coupling is shown in the figure. For the network, $M_{12} = 0$. Formulate the loop equations for this network using the Kirchhoff voltage law.
Fig. 3-15. Two examples of extraneous elements so far as terminal behavior is concerned.

Fig. 3-21. Two equivalent networks illustrating the manner in which one source is replaced by two such that the Kirchhoff current law is still satisfied at each node.
1-21. The current in a 1-H inductor follows the variation shown in the accompanying figure. The current increases from 0 at \( t = 0 \) at the rate of 1 amp/sec (for several seconds, at least). Find: (a) the flux linkages in the system after 1 sec, (b) the time rate of change of flux linkages in the system after 2 sec, and (c) the quantity of charge having passed through the inductor after 1 sec.

\[ i(t) = (1 - e^{-t}) \text{ amp, } t > 0. \]

At a certain time the current has a value of 0.63 amp. (a) At what rate is the current changing? (b) What is the value of the total flux linkages? (c) What is the rate of change of flux linkages? (d) What is the voltage across the inductor? (e) How much energy is stored in the magnetic field of the inductor? (f) What is the voltage across the resistor? (g) At what rate is energy being stored in the magnetic field of the inductor? (h) At what rate is energy being dissipated as heat? (i) At what rate is the battery supplying energy?
25. In the circuit shown the capacitor is charged to a voltage of 1 V, and at \( t = 0 \) the switch \( K \) is closed. The current in the circuit is known to be of the form \( i(t) = e^{-t} \) amp, \( t > 0 \). At a certain time the current has a value of 0.37 amp. (a) At what rate is the voltage across the capacitor changing? (b) What is the value of the charge on the capacitor?

(c) What is the time rate of change of the product \( CV \)? (d) What is the voltage across the capacitor? (e) How much energy is stored in the electric field of the capacitor? (f) What is the voltage across the resistor? (g) At what rate is energy being taken from the electric field of the capacitor? (h) At what rate is energy being dissipated as heat?

Fig. P1-25.

1-39. The figure shows a piecewise linear characteristic. Let \( x = q_c \) and \( y = v_c \) so that the characteristic represents a nonlinear capacitor. If the voltage applied to the capacitor is that shown in Fig. P1-32, plot the corresponding \( i_c(t) \).

1-40. Repeat Prob. 1-39 if the voltage waveform is that shown in Fig. P1-30.
Source Transformations

Fig. 3-17. Source transformation for a network with a single inductor.

Fig. 3-18. Source transformation involving one capacitor.
3-36. For the network shown in the figure, determine the numerical value of the branch current $i_1$. All sources in the network are time invariant.

![Network Diagram](image)

**Fig. P3-36.**
3.37. In the network of the figure, all sources are time invariant. Determine the numerical value of \( i_2 \).

3.38. In the given network, all sources are time invariant. Determine the branch current in the 2-\( \Omega \) resistor.

3.39. In the network of the figure, all voltage sources and current source are time invariant, and all resistors have the value \( R = \frac{1}{2}\Omega \). Solve for the four node-to-datum voltages.
Thank you