Network Theory

Gyan Ranjan Biswal
PhD (IITR), FIE, SMIEEE, LMISTE
HOD and Associate Professor
Department of Electrical & Electronics Engineering (EEE)
Veer Surendra Sai University of Technology (formerly UCE), Burla
PIN – 768018, Sambalpur (Odisha), India

E-mail: gyanbiswal@vssut.ac.in
URL: http://www.vssut.ac.in
Google Scholar: Gyan Biswal
Home Page: http://in.linkedin.com/pub/gyan-biswal/14/458/a8
ORCID id: https://orcid.org/0000-0001-7730-1985

Contact Hours: Tuesday, 04:30 PM to 05:30 PM at E-106

Gyan Ranjan Biswal received his B.E. in Electronics Engineering from the Pt. Ravishankar Shukla University, India in 1999 and M. Tech. (Honors) in Instrumentation & Control Engineering from the Chhattisgarh Swami Vivekananda Technical University, India in 2009 followed by Ph.D. in Electrical Engineering, specialized in the area of Power System Instrumentation (Power Generation Automation) from the Indian Institute of Technology Roorkee, India in 2013.

He is expertise in Design and Development of cooling systems for large size electrical generators, and the C&I of process industries. He has been in academia for about twelve years. Presently, he is with VSS University of Technology, Burla, India at the capacity of Head and Associate Professor, EEE from Dec. 2016. He has more than 65 publications in various Journals and Conferences of Internationally repute to his credit. He also holds a patent as well, and filed one more. He also adapted one international edition book published by Pearson India. He received research grants of US$90,000 (INR 50 lakhs). He has been supervised 09 Masters' theses, and registered 04 PhD theses. He has also been recognized with many national and international awards by elite bodies. He has been awarded with CICS award under the head of Indian National Science Academy for travel support to USA, MHRD Fellowship by Govt. of India, and Gopabandhu Das Scholarship in his career. His major areas of interests are Power System Instrumentation, Industrial Automation, Robust and Intelligent Control, the Smart Sensors, IoT enabled Smart Sensors, the Smart Grid, Fuel Cell lead Sustainable Sources of Energy, and System Reliability.

Dr. Biswal is a Fellow IE (India), Senior Member of IEEE, USA, and Life Member of ISTE, India. He is actively involved in review panels of different societies of international repute viz. IEEE, IFAC, and the ISA. Currently, he is also actively involved as a Member of IEEE–SA (Standards Association) working groups; IEEE P1876 WG, IEEE P21451-001 WG, and IEEE P1415. He has also been invited for delivering guest lectures at World Congress on Sustainable Technologies (WCST) Conf. 2012, London, UK, INDICON 2015, New Delhi, India, National Power Training Institute (NPTI), Nangal, India, and G.B. Pant Engineering College, Pauri, Gharwal, India, Surendra Sai University of Technology (formerly UCE), Burla, and as a guest expert in 2016 IEEE PES General Meeting Boston, MA, USA.

Syllabus

Network Theory

MODULE-I (9 HOURS) [Online mode: 5 HOURS + 1 Test]

Analysis of Coupled Circuits: Self-inductance and Mutual inductance, Coefficient of coupling, Series connection of coupled circuits, Dot convention, Ideal Transformer, Analysis of multi-winding coupled circuits, Analysis of single tuned and double tuned coupled circuits.

Transient Response: Transient study in series RL, RC, and RLC networks by time domain and Laplace transform method with DC and AC excitation. Response to step, impulse and ramp inputs of series RL, RC and RLC circuit.

MODULE-II (7 HOURS) [Online mode: 5 HOURS + 1 Test]

Two Port networks: Types of port Network, short circuit admittance parameter, open circuit impedance parameters, Transmission parameters, Condition of Reciprocity and Symmetry in two port network, Inter-relationship between parameters, Input and Output Impedances in terms of two port parameters, Image impedances in terms of ABCD parameters, Ideal two port devices, ideal transformer. Tee and Pie circuit representation, Cascade and Parallel Connections.

MODULE-III (8 HOURS) [Online mode: 5 HOURS + 1 Test]

Network Functions & Responses: Concept of complex frequency, driving point and transfer functions for one port and two port network, poles & zeros of network functions, Restriction on Pole and Zero locations of network function, Time domain behavior and stability from pole-zero plot, Time domain response from pole zero plot.

Three Phase Circuits: Analysis of unbalanced loads, Neutral shift, Symmetrical components, Analysis of unbalanced system, power in terms of symmetrical components.

MODULE-IV (9 HOURS) [Online mode: 5 HOURS + 1 Test]

Network Synthesis: Realizability concept, Hurwitz property, positive realness, properties of positive real functions, Synthesis of R-L, R-C and L-C driving point functions, Foster and Cauer forms.

MODULE-V (6 HOURS) [Online mode: 5 HOURS + 1 Test]

Graph theory: Introduction, Linear graph of a network, Tie-set and cut-set schedule, incidence matrix, Analysis of resistive network using cut-set and tie-set, Dual of a network.

Filters: Classification of filters, Characteristics of ideal filters.

Text and Reference Books

Recommended Text Books:

1. "Introductory Circuit Analysis", Robert L. Boylestad, Pearson, 12th ed., 2012.
2. "Network Analysis", M. E. Van Valkenburg, Pearson, 3rd ed., 2006.
3. "Engineering Circuit Analysis", W. Hayt, TMH, 2006.
4. "Network Analysis & Synthesis", Franklin Fa-Kun. Kuo, John Wiley & Sons.

Reference Books:

* "Basic Circuit Theory, Huelsman, PHI, 3rd ed.,
* "HUGHES Electrical and Electronic Technology", Revised by J. Hiley, K. Brown, and I. M. Smith, Pearson, 10th ed., 2011.
* "Circuits and Networks", Sukhija and Nagsarkar, Oxford Univ. Press, 2012.
* "Fundamentals of Electric Circuits", C. K. Alexander and M. N. O. Sadiku, McGraw-Hill Higher Education, 3rd ed., 2005.
* "Fundamentals of Electrical Engineering", L. S. Bobrow, Oxford University Press, 2nd ed., 2011.
* "Circuit Theory (Analysis and Synthesis)", A. Chakrabarti, Dhanpat Rai pub.

Other Important References

Reference Sites:	
1.	NPTEL, The National Programme on Technology Enhanced Learning (NPTEL): https://nptel.ac.in/
2.	MIT OpenCourseWare : https://ocw.mit.edu/index.htm

Course Outcomes

Upon successful completion of this course, you (students) will be able to

CO1	Analyze coupled circuits and understand the difference between the steady state and transient response of 1st and 2nd order circuit and understand the concept of time constant.
CO2	Learn the different parameters of two port network.
CO3	Concept of network function and three phases circuit and know the difference of balanced and unbalanced system and importance of complex power and its components.
CO4	Synthesis the electrical network.
CO5	Analyse the network using graph theory and understand the importance of filters in electrical system.

What do you mean by Network Theory???

- Network Analysis (by means of mathematical modeling): when an electrical/electronic circuit is analyzed for the given/ known input(s) to find a desired output/ response(s), is called “network analysis”.

- Network Analysis (by means of graph theory): when a complex matrix (electrical/electronic circuit) is analyzed for the multiple given/ known input(s) to find desired output/ responses, is also called “network analysis”. Typically, this exercise is done by graph theory *as the ancestor consumes longer time to reach the solutions*.

- Network Synthesis: when an electrical/electronic circuit is supposed to be designed for the given/ known input(s) and output/ response(s), is called “network synthesis”.

- Filter design (passive type): an electrical circuit which is designed to select a particular band of frequency(ies) to pass through it, using R-L-C elements.

What do you mean by Network Theory???

- Thus, it is very essential to mathematically model and analysis a electrical/electronic circuit to judge the validity/ efficacy of it. It helps to find the effectiveness of the circuit in real life scenario. otherwise a poor selection of types of circuit elements and magnitude of elements / sources/ load may leads to *improper findings* or even in worst cases *uncontrolled responses*.
- Five Majors of Electrical & Electronics Engineering: (founding terminologies)

- V :
- I :
- R :
- L :
- C :

Relationship of basic quantities in terms of Circuit Parameters

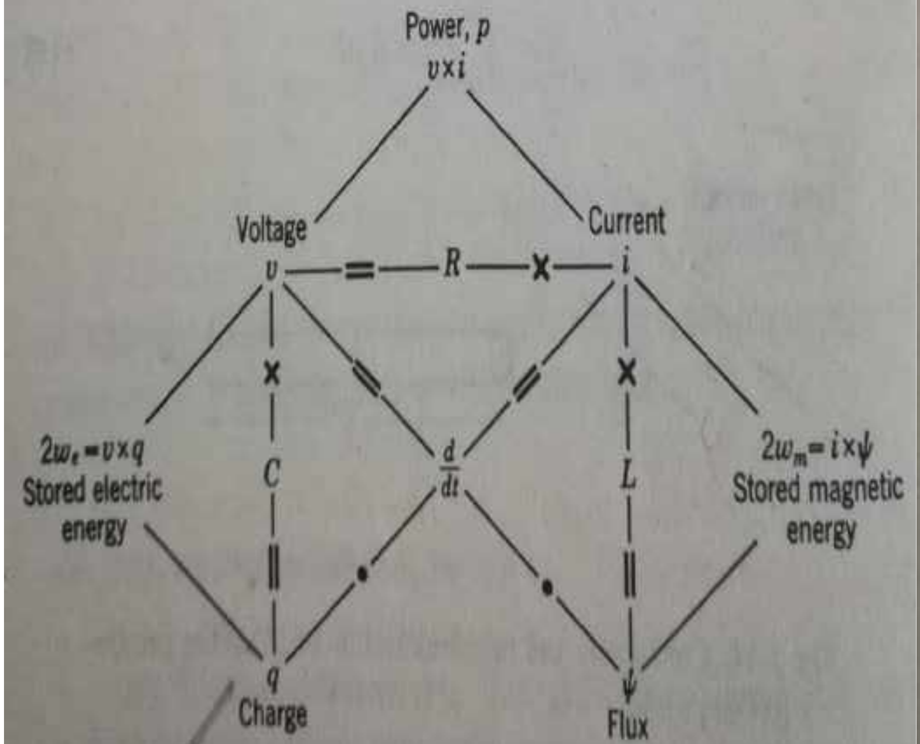


Fig. 1-15. Chart illustrating the relationships of basic quantities in terms of the circuit parameters. (From M. Kawakami, *EREM Chart*, Kyoritsu Shuppan Co., Tokyo.)

Table 1-1. SUMMARY OF RELATIONSHIPS FOR THE PARAMETERS

Parameter	*Basic Relationship*	*Voltage-Current Relationships*		*Energy*
R $G = \frac{1}{R}$	$v = Ri$	$v_R = Ri_R$	$i_R = Gv_R$	$w_R = \int_{-\infty}^{t} v_R i_R\, dt$
L (or M)	$\psi = Li$	$v_L = L\frac{di_L}{dt}$	$i_L = \frac{1}{L}\int_{-\infty}^{t} v_L\, dt$	$w_L = \frac{1}{2}Li^2$
C $D = \frac{1}{C}$	$q = Cv$	$v_C = \frac{1}{C}\int_{-\infty}^{t} i_C\, dt$	$i_C = C\frac{dv_C}{dt}$	$w_C = \frac{1}{2}Cv^2$

Self Inductance (L) and Mutual Inductance (M):

- Unlike Capacitance which drive through *electric field*, behavior of inductance (that drive through *magnetic field*) is a special class.

- Network Theorems: Superposition/ Thevenin's / Norton's/ MPT....and so on....

- Linear and Non-linear elements: is/ isn't governed by linear differential eqs.
- R , L, C

- Skin Effect: *why so vital???*

Skin effect: Uniform distribution of current throughout the cross section of a conductor exists only for DC. As the frequency of AC ↑es, the nonuniformity of distribution becomes more pronounced.

An increase in frequency causes nonuniform current density (J). This phenomenon is called skin effect.

$$I = \int J \cdot ds \Rightarrow J = \frac{I}{\pi r^2} \ (A/m^2)$$

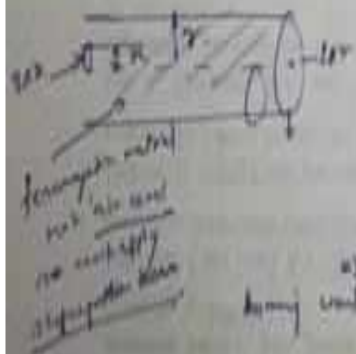
Lenz law

φ in high ... → ∝ less induced voltage ∝ high current density (AC)

φ flux linkage less → ∝ high induced voltage ∝ less current density

Note: As skin effect is voltage in ... due to larger conductor size even at lower freq. GMR & GMD

"Inductance of a Conductor due to Internal Flux"



$L = \frac{\lambda}{I}$ → flux (linkage)

$\mathcal{F} = mmf = \oint H \cdot dl = I$ AT

$\oint H_x \cdot ds = I_x$

$\Rightarrow 2\pi x H_x = I_x$

Assuming uniform current density,

$$I_x = \frac{\pi x^2}{\pi r^2} \cdot I$$

$$\therefore H_x = \frac{I_x}{2\pi x} = \frac{\pi x^2}{\pi r^2} \cdot I \cdot \frac{1}{2\pi x}$$

$\Rightarrow H_x = \frac{x}{2\pi r^2} I$ AT/m

$\therefore B_x = \mu H_x = \frac{\mu x I}{2\pi r^2}$ Wb/m²

now; $d\phi = \frac{\mu x I}{2\pi r^2} dx$ Wb/m

$[\because \phi = L \frac{di}{dt}]$

now $d\lambda = \frac{\pi x^2}{\pi r^2} \cdot d\phi$

$\Rightarrow d\lambda = \frac{\pi x^2}{\pi r^2} \cdot \frac{\mu x I}{2\pi r^2}$ Wb-t/m

$\Rightarrow d\lambda = \frac{\mu I x^3}{2\pi r^4}$ Wb-t/m

[so $d\lambda \propto x^3 \propto \frac{1}{r^4}$]

Some other way of Network Classifications:

1) Active and Passive elements
2) Unilateral and Bilateral elements
3) Lumped and Distributed elements
4) Linear and Non-linear elements

Self Inductance

In order to completely comprehend mutual inductance, it is important to take a relook at Eq. (1.18). Consider an air cored coil, shown in Fig. 9.1, carrying a time varying current $i(t)$ A. The direction of the flux Φ, shown upward in Fig. 9.1, is obtained by the application of the right-hand rule which states that if the fingers of the right hand are wrapped around the coil in the direction of the current, then the thumb points to the direction of the flux. Assume that the change di in the current produces a change in flux $d\Phi$ weber and completely links all the N turns of the coils. The voltage $e(t)$ V induced in the coil, according to Faraday's law of electromagnetic induction, is proportional to the time rate of change of flux linkages, and is given by

$$e(t) = N\frac{d\Phi}{dt}$$

$$e(t) = N\frac{d\Phi}{di(t)} \times \frac{di(t)}{dt} \tag{9.1}$$

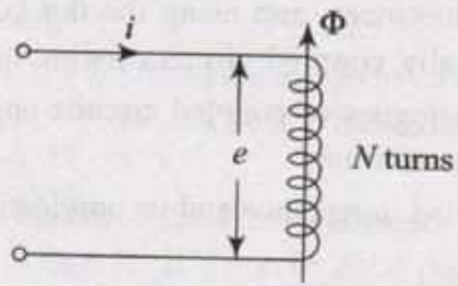


Fig. 9.1 *Self-inductance of a coil*

Self Inductance

$$e(t) = L\frac{di(t)}{dt}$$

Comparison of Eqs (1.18) and (9.1) gives

$$L = N\frac{d\Phi}{di} = \frac{d(N\Phi)}{di} = \frac{d\psi}{di}\ \text{H} \tag{9.2}$$

where $\Psi = N\Phi$ Weber-turns is the flux linkage.

For an air cored coil, Ψ varies linearly with the variation in i, hence $(d\Psi/dt)$ = constant, and from Eq. (9.2), inductance L may be written as

$$L = \frac{N\Phi}{i} = \frac{\psi}{i}\ \text{H} \tag{9.3}$$

When the core of a coil consists of a magnetic material instead of air, Eq. (9.3) does not hold due to the non-linear Ψ–i characteristic.

So how do we apply Superposition theorems over RL circuits?

Self Inductance

In Eq. (9.3), the self-flux linking the coil forms the basis for defining inductance L of the coil and hence it is termed as the self-inductance of the coil.

Now, $$\Phi = \frac{\text{mmf}}{\text{reluctance}} = \frac{Ni}{\mathcal{R}} \quad (9.4)$$

where $\mathcal{R}$ is the reluctance of the magnetic path.

Substituting Eq. (9.4) in Eq. (9.3), the expression for L becomes

$$L = \frac{N\Phi}{i} = \frac{N^2}{\mathcal{R}} \quad (9.5)$$

Mutual Inductance

Figure 9.2(a) shows coil 1 having N_1 turns is placed on a common magnetic core near coil 2 with N_2 turns. The voltage induced in coil 2 is proportional to the rate of change of current $i_1(t)$ in coil 1. If di_1 is the increase of current in coil 1 in dt s, then emf $e_2(t)$ induced in coil 2 may be expressed as

$$e_2(t) \propto \frac{di_1(t)}{dt}$$

or

$$e_2(t) = M_{12} \frac{di_1(t)}{dt} \qquad (9.6)$$

where M_{12} is called mutual inductance between coils 1 and 2. The unit of mutual inductance is the same as for self-inductance, namely henry (H). Two coils have a mutual inductance of 1 H if an emf of 1 V is induced in one coil when the current in the other coil varies uniformly at the rate of 1 A/S.

Mutual Coupling

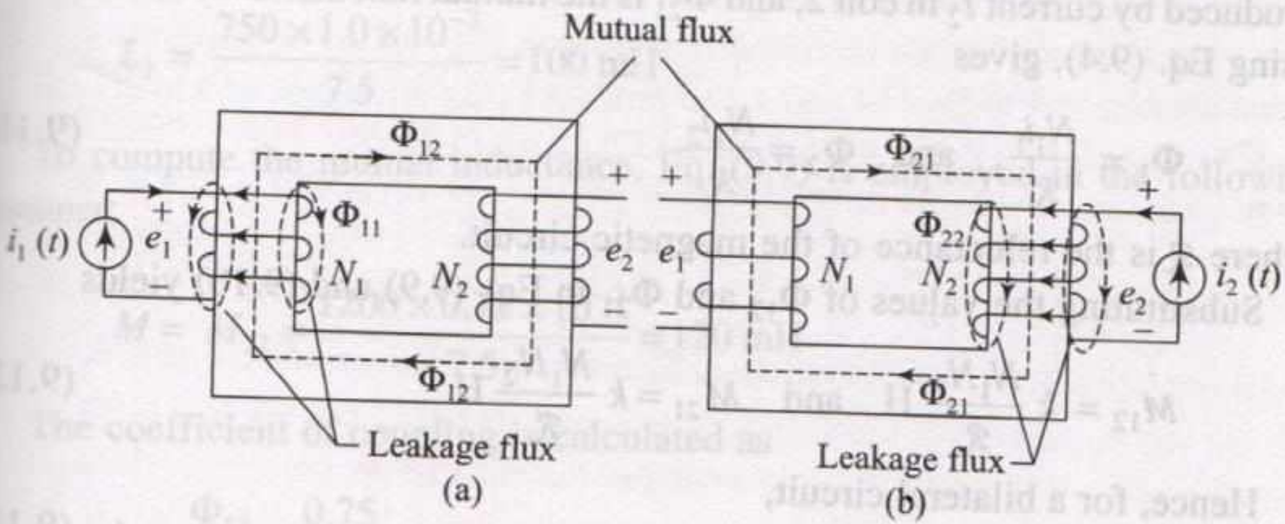


Fig. 9.2 *Mutual inductance between two coils (a) coil 1 excited by current $i_1(t)$ and (b) coil 2 excited by current $i_2(t)$*

If $d\Phi_{12}$ weber is the increase of the mutual flux in coil 2 due to the increase of di_1 A in coil 1, then emf induced in coil 2 is

$$e_2(t) = N_2 \frac{d\Phi_{12}}{dt} \tag{9.7}$$

Mutual Coupling

From expressions (9.6) and (9.7)

$$M_{12}\frac{di_1}{dt} = N_2\frac{d\Phi_{12}}{dt} = N_2\frac{d\Phi_{12}}{di_1}\frac{di_1}{dt}$$

then

$$M_{12} = N_2\frac{d\Phi_{12}}{di_1} \tag{9.8}$$

If the reluctance of the magnetic core is constant, then the ratio $d\Phi_{12}/di_1$ is constant and is equal to flux per ampere, and Eq. (9.8) may be written as

$$M_{12} = \frac{N_2\Phi_{12}}{i_1} = \frac{\psi_{12}}{i_2} \tag{9.9}$$

where ψ_{12} is the flux linkages of coil 2 due to a current i_1 in coil 1.

Similarly, if Φ_{21}, ψ_{21} are respectively the flux and flux linkages of coil 1 due to a current $i_2(t)$ in coil 2 as shown in Fig. 9.2(b), the mutual inductance M_{21} between coil 2 and coil 1 can be expressed as

$$M_{21} = \frac{N_1\Phi_{21}}{i_2} = \frac{\psi_{21}}{i_2} \tag{9.10}$$

Coefficient of Coupling

From Fig. 9.2, it is seen that only a part of the flux links the two coils. A part of the total flux produced by one coil, called the leakage flux, links only the coil producing the flux, while the balance flux, called the mutual flux, links the other coil. If a fraction k of total flux Φ_1 produced by current I_1 in coil 1 links coil 2, then the mutual flux $\Phi_{12} = k\Phi_1$. Similarly, $\Phi_{21} = k\Phi_2$, where Φ_2 is the total flux produced by current I_2 in coil 2, and Φ_{21} is the mutual flux that links coil 1. Then, using Eq. (9.4), gives

$$\Phi_1 = \frac{N_1 i_1}{\mathcal{R}}, \quad \text{and} \quad \Phi_2 = \frac{N_2 i_2}{\mathcal{R}} \tag{9.11}$$

where $\mathcal{R}$ is the reluctance of the magnetic circuit.

Substituting the values of Φ_{12} and Φ_{21} in Eqs (9.9) and (9.10) yields

$$M_{12} = k\frac{N_1 N_2}{\mathcal{R}}\,\text{H} \quad \text{and} \quad M_{21} = k\frac{N_1 N_2}{\mathcal{R}}\,\text{H} \tag{9.12}$$

Hence, for a bilateral circuit,

$$M_{12} = M_{21} = M \tag{9.13}$$

Then

$$M^2 = M_{12} \times M_{21} = k^2\left(\frac{N_1 N_2}{\mathcal{R}}\right)^2 \tag{9.14}$$

Using Eq. (9.5) L_1 and L_2, the self-inductances of coils 1 and 2, respectively, are expressed as

Coefficient of Coupling

$$L_1 = \frac{N_1^2}{\mathcal{R}}, \text{H} \quad \text{and} \quad L_2 = \frac{N_2^2}{\mathcal{R}}, \text{H} \tag{9.15}$$

From Eq. (9.15), the product $L_1 L_2$ may be obtained as

$$L_1 L_2 = \frac{N_1^2 N_2^2}{\mathcal{R}^2} \tag{9.16}$$

Substituting Eq. (9.16) in Eq. (9.14) and simplifying leads to

$$M = k\sqrt{L_1 L_2} \tag{9.17}$$

The term k is called the coefficient of coupling. When 100 percent of the flux lines link each coil, then $k = 1$. The term coupling coefficient is widely used in radio networks to denote the degree of coupling between two coils. If the two coils are close together, most of the flux produced by the current in one coil passes through the other coil, and the coils are said to be tightly coupled. If the coils are well apart, then a small part of the flux produced by the current in one coil passes through the other coil, and the coils are said to be loosely coupled.

Q.1:

Example 9.1 Two coils having 750 and 1200 turns, respectively, are wound on a common non-magnetic core. The leakage flux and mutual flux, due to a current of 7.5 A in coil 1, is 0.25 mWb and 0.75 mWb, respectively. Calculate the (a) self-inductance of the coils, (b) mutual inductance, and (c) coefficient of coupling.

Solution From the data, it is seen that $\Phi_{11} = 0.25$ mWb and $\Phi_{12} = 0.75$ mWb. Therefore, the total flux linking coil 1 is $\Phi_1 = \Phi_{11} + \Phi_{12} = 1.0$ mWb.

(a) From Eq. (9.3), the self-inductance of the coil is computed as follows:

$$L_1 = \frac{750 \times 1.0 \times 10^{-3}}{7.5} = 100 \text{ mH}$$

To compute the mutual inductance, Eq. (9.9) is employed in the following manner:

$$M = M_{12} = \frac{1200 \times 0.75 \times 10^{-3}}{7.5} = 120 \text{ mH}$$

The coefficient of coupling is calculated as

$$k = \frac{\Phi_{12}}{\Phi_1} = \frac{0.75}{1.0} = 0.75$$

Mutual Coupled Coils

Using Eq. (9.17) leads to the self-inductance L_2. Thus,

$$L_2 = \frac{M^2}{k^2 L_1} = \frac{\left(120\times10^{-3}\right)^2}{(0.75)^2\times\left(100\times10^{-3}\right)} = 256\ \text{mH}$$

Max. value of M

$$W_T(t) \ge 0$$

$$\therefore \sqrt{L_1 L_2} \ge M$$

$$\therefore M = k\sqrt{L_1 L_2} \quad \text{for } 0 \le k \le 1$$

Ex (4). Two coils having 750 & 1200 turns resp. on a non-magnetic core. The leakage flux & mutual flux due to I = 7.5 A in coil 1 is 0.25 mWb & 0.75 mWb resp.
Calculate (a) L_1, L_2 (self inductance) (b) Mutual ind (M)
(c) co-efficient of coupling (k)

Soln

(a) $\phi_{11} = 0.25$ mWb, $\phi_{12} = 0.75$ mWb

$\therefore \phi_1$ (total flux) $= \phi_{11} + \phi_{12} = 1.0$ mWb

$$L_1 = \frac{N_1 \phi_1}{I} = \frac{750 \times 1 \times 10^{-3}}{7.5} = 100 \text{ mH}$$

$$M = M_{12} = \frac{1200 \times 0.75 \times 10^{-3}}{7.5} = \frac{N_2 \phi_{21}}{I} = 120 \text{ mH}$$

$$\therefore k = \frac{\phi_{12}}{\phi_1} = \frac{0.75}{1.0} = 0.75$$

$$\text{now } L_2 = \frac{M^2}{k^2 L_1} = \frac{(120 \times 10^{-3})^2}{(0.75)^2 \times 100 \times 10^{-3}} = 256 \text{ mH}$$

$\mathcal{F} = N I = \phi \mathcal{R}$

$\phi_1 = \frac{N_1 I_1}{\mathcal{R}} = \frac{L_1 I_1}{N_1}$

$\phi_{12} = \frac{N_1 I_1}{\mathcal{R}}$

$M_{12} = k \frac{N_1 N_2}{\mathcal{R}}$ H

$M_{21} = k \frac{N_1 N_2}{\mathcal{R}}$ H

$M_{12} = M_{21}$ (bilateral)

$L_1 = \frac{N_1^2}{\mathcal{R}}$ H

$L_2 = \frac{N_2^2}{\mathcal{R}}$

$L_1 L_2 = \frac{N_1^2 N_2^2}{\mathcal{R}^2} = \frac{M^2}{k^2}$

Mutual Coupling and Dot Conversion

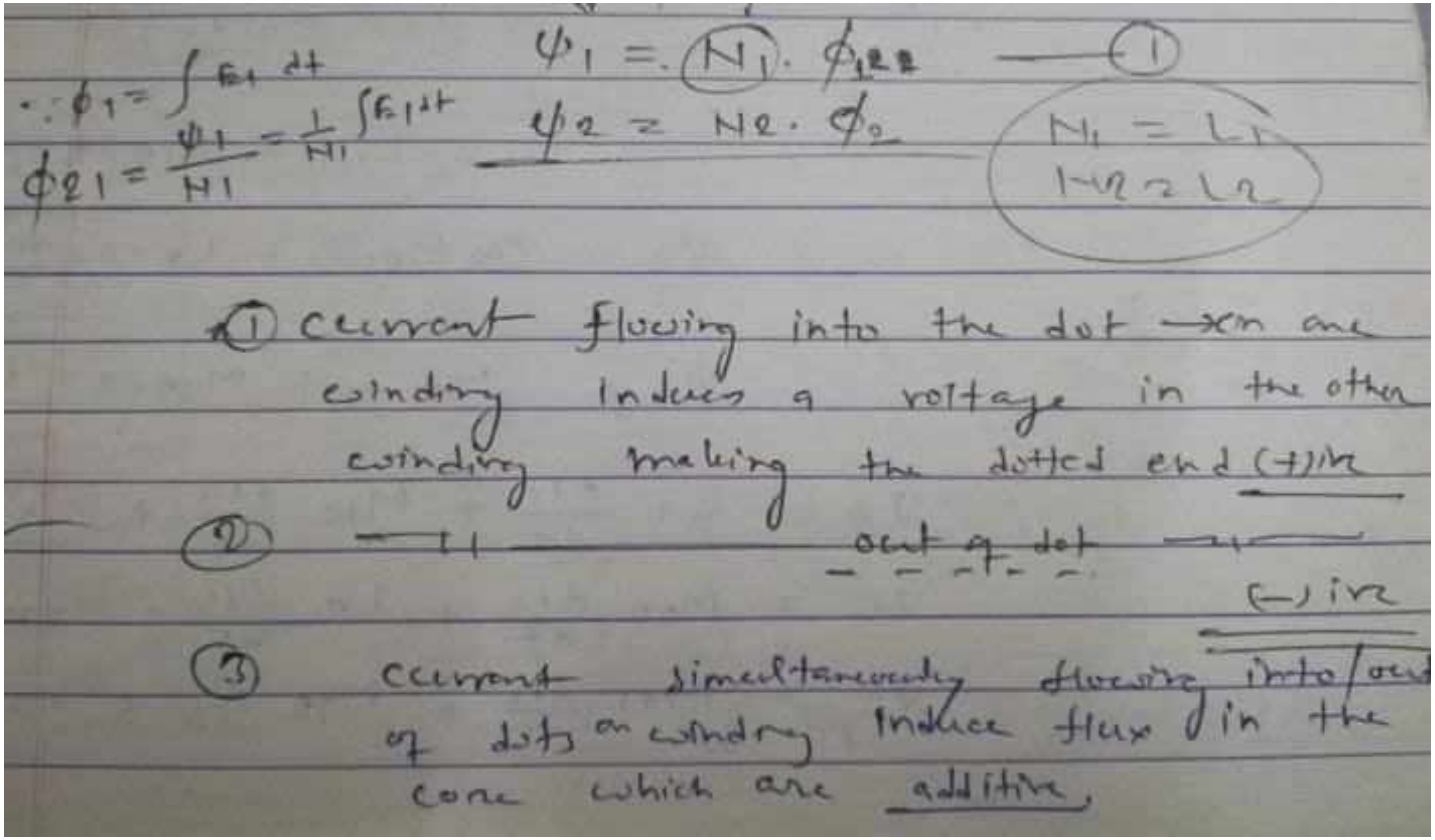
It is seen in Section 9.4.2 that the sign of the voltage of a mutual inductance can be conveniently established if the direction of the windings of coupled coils is known. If dots are placed at the terminals to indicate currents entering or leaving the respective coils, then these dots provide a symbolic representation of the instantaneous polarity of the coupled coils and the pictorial representation of the core with its windings is not required. Thus, mutually coupled coils of the coupled circuit of Fig. 9.4(a) may then be represented by the equivalent circuit of Fig. 9.4 (b).

Dot convention may be employed in the following manner:

(i) When the assumed currents in a mutually coupled pair of coils enter or leave the dotted terminals, the M terms will have the same sign as the L terms.

(ii) When one current enters at the dotted terminal and the other current leaves the other dotted terminal, the M terms will have the opposite sign as the L terms.

Rules of Mutual Coupling and Dot Conversion



$\because \phi_1 = \int E_1 \, dt$

$\phi_{21} = \frac{\psi_1}{N_1} = \frac{1}{N_1} \int E_1 \, dt$

$\psi_1 = N_1 \cdot \phi_{12}$ —— (1)

$\psi_2 = N_2 \cdot \phi_2$

$N_1 = L_1$

$N_2 = L_2$

(1) Current flowing into the dot in one winding induces a voltage in the other winding making the dotted end (+)ve

(2) —"— out of dot —"— (−)ve

(3) Current simultaneously flowing into/out of dots on winding induce flux in the core which are additive.

Mutual Coupling and Dot Conversion

Figure 9.5 shows two mutually coupled coils of self-inductances L_1 H, L_2 H, and a mutual coupling of M H. The dot marks are provided by the manufacturer as shown in the figure. For the assumed directions of currents in the coils, it is seen that current $i_1(t)$ is entering the dotted terminal at A, while current $i_2(t)$ is leaving the dotted terminal at B. Hence, as per rule (ii) M will have a negative sign. The voltage equations are written as

$$R_1 i_1(t) + L_1 \frac{di_1(t)}{dt} - M\frac{di_2(t)}{dt} = v_S(t) \tag{9.18}$$

$$R_2 i_2(t) + L_2 \frac{di_2(t)}{dt} - M\frac{di_1(t)}{dt} = 0 \tag{9.19}$$

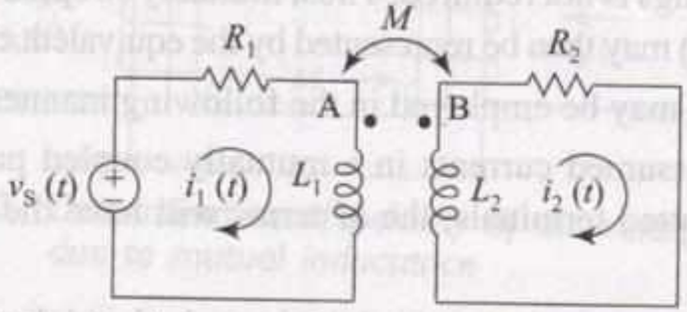


Fig. 9.5 *Application of the dot convention to write voltage equations*

Eq. Circuit of Mutually Coupled Coils

If the voltage source in Fig.9.5 is a sinusoidal voltage source whose operating frequency is f Hz, Eqs (9.18) and (9.19) take the following form:

$$\begin{aligned} R_1 \boldsymbol{I}_1 + j\omega L_1 \boldsymbol{I}_1 - j\omega M \boldsymbol{I}_2 &= \boldsymbol{V}_S \\ R_2 \boldsymbol{I}_2 + j\omega L_2 \boldsymbol{I}_2 - j\omega M \boldsymbol{I}_1 &= 0 \end{aligned} \quad (9.20)$$

where $\omega = 2\pi f$ rad/s and $\boldsymbol{V}_S$, $\boldsymbol{I}_1$, and $\boldsymbol{I}_2$ are the effective values of the voltage source and currents in the coils, respectively. In the matrix form Eq. (9.20) can be expressed as

$$\begin{bmatrix} (R_1 + j\omega L_1) & -j\omega M \\ -j\omega M & (R_2 + j\omega L_2) \end{bmatrix} \begin{bmatrix} \boldsymbol{I}_1 \\ \boldsymbol{I}_2 \end{bmatrix} = \begin{bmatrix} \boldsymbol{V}_S \\ 0 \end{bmatrix} \quad (9.21)$$

or

$$\begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} \boldsymbol{I}_1 \\ \boldsymbol{I}_2 \end{bmatrix} = \begin{bmatrix} \boldsymbol{V}_S \\ 0 \end{bmatrix} \quad (9.22)$$

where $Z_{11} = (R_1 + j\omega L_1)$, $Z_{12} = Z_{21} = -j\omega M$, and $Z_{22} = (R_2 + j\omega L_2)$. The mesh currents $\boldsymbol{I}_1$ and $\boldsymbol{I}_2$ can be computed by solving Eq. (9.22).

The equivalent network representation of Eq. (9.20) is shown in Fig. 9.6. The veracity of the representation can be verified by developing mesh equations for the network.

Mutual Coupling and Dot Conversion

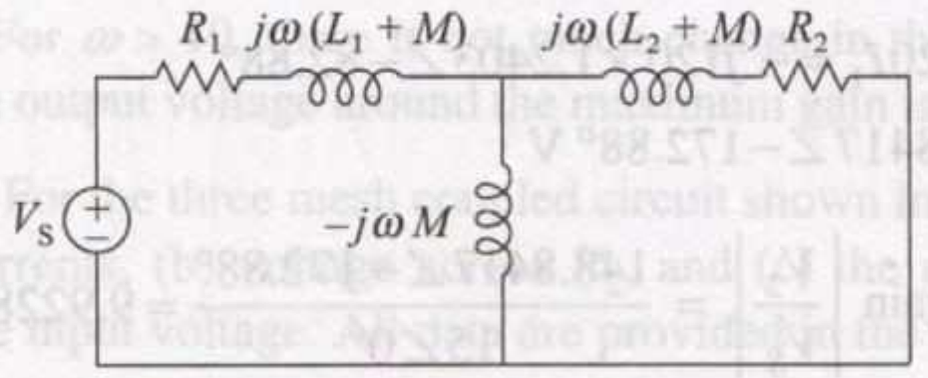
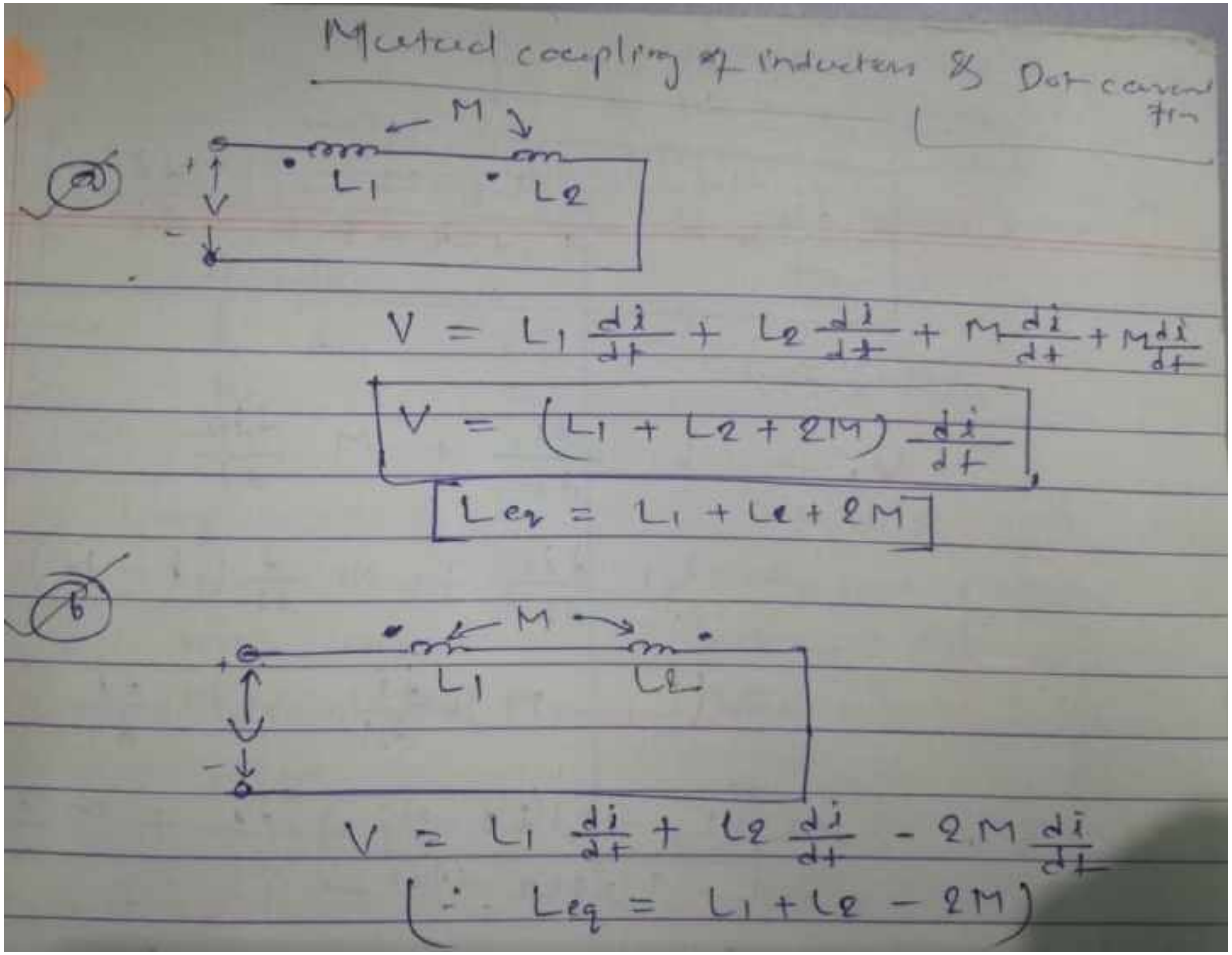


Fig. 9.6 *T-equivalent circuit of a mutually coupled circuit*

Mutual Coupling and Dot Conversion



Mutual coupling of inductors & Dot convention

(a)

$$V = L_1 \frac{di}{dt} + L_2 \frac{di}{dt} + M \frac{di}{dt} + M \frac{di}{dt}$$

$$V = (L_1 + L_2 + 2M) \frac{di}{dt}$$

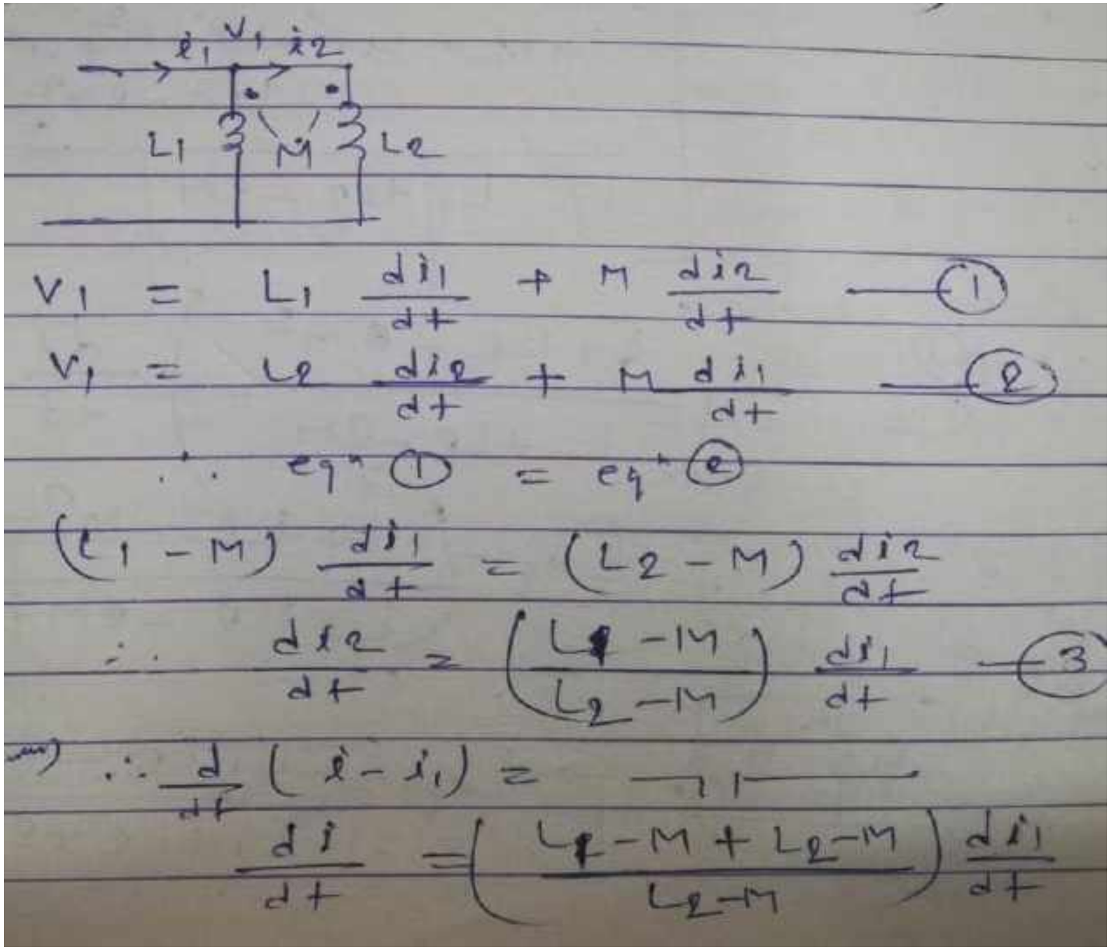
$$L_{eq} = L_1 + L_2 + 2M$$

(b)

$$V = L_1 \frac{di}{dt} + L_2 \frac{di}{dt} - 2M \frac{di}{dt}$$

$$(\therefore L_{eq} = L_1 + L_2 - 2M)$$

Mutual Coupling and Dot Conversion



$$V_1 = L_1 \frac{di_1}{dt} + M \frac{di_2}{dt} \quad \text{—— (1)}$$

$$V_1 = L_2 \frac{di_2}{dt} + M \frac{di_1}{dt} \quad \text{—— (2)}$$

$\therefore$ eqn (1) = eqn (2)

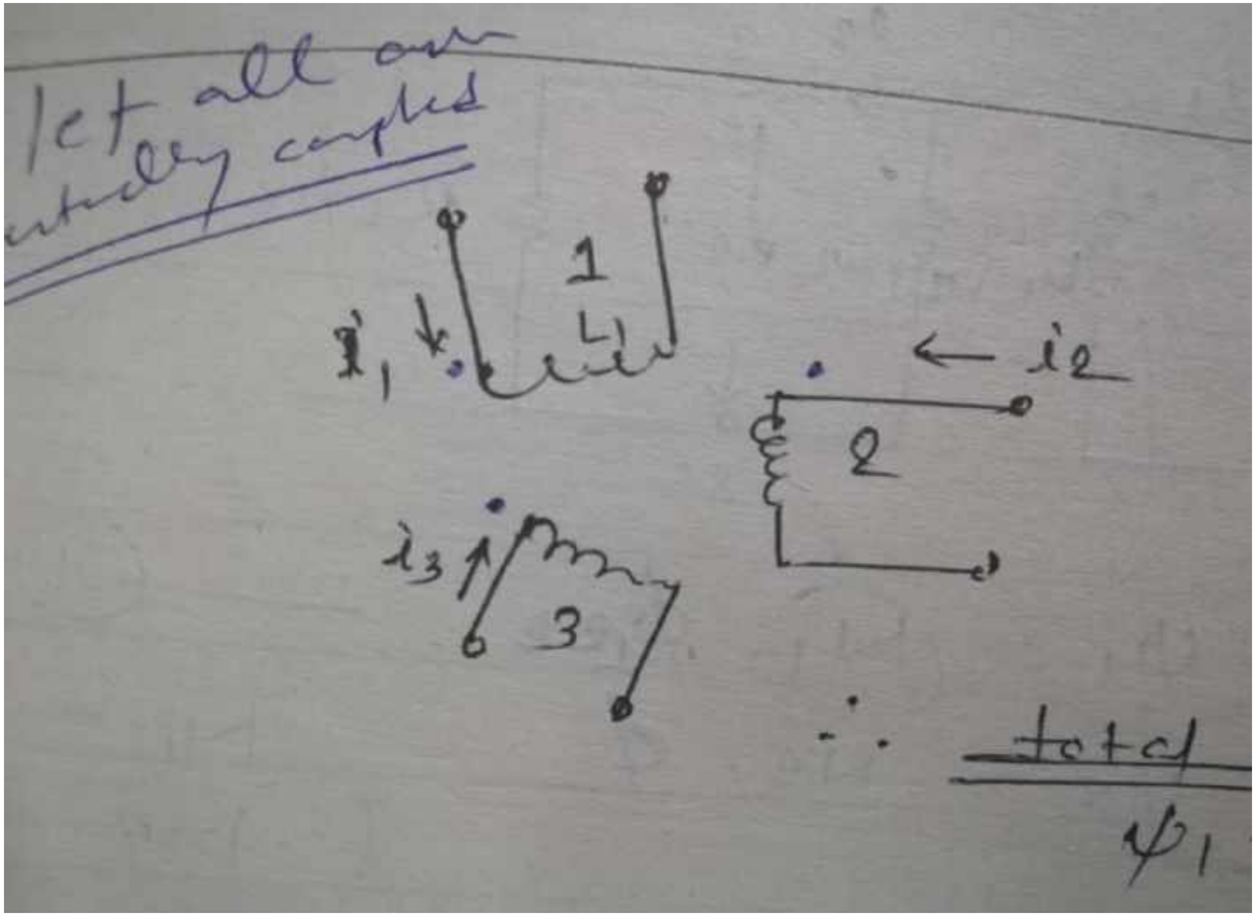
$$(L_1 - M) \frac{di_1}{dt} = (L_2 - M) \frac{di_2}{dt}$$

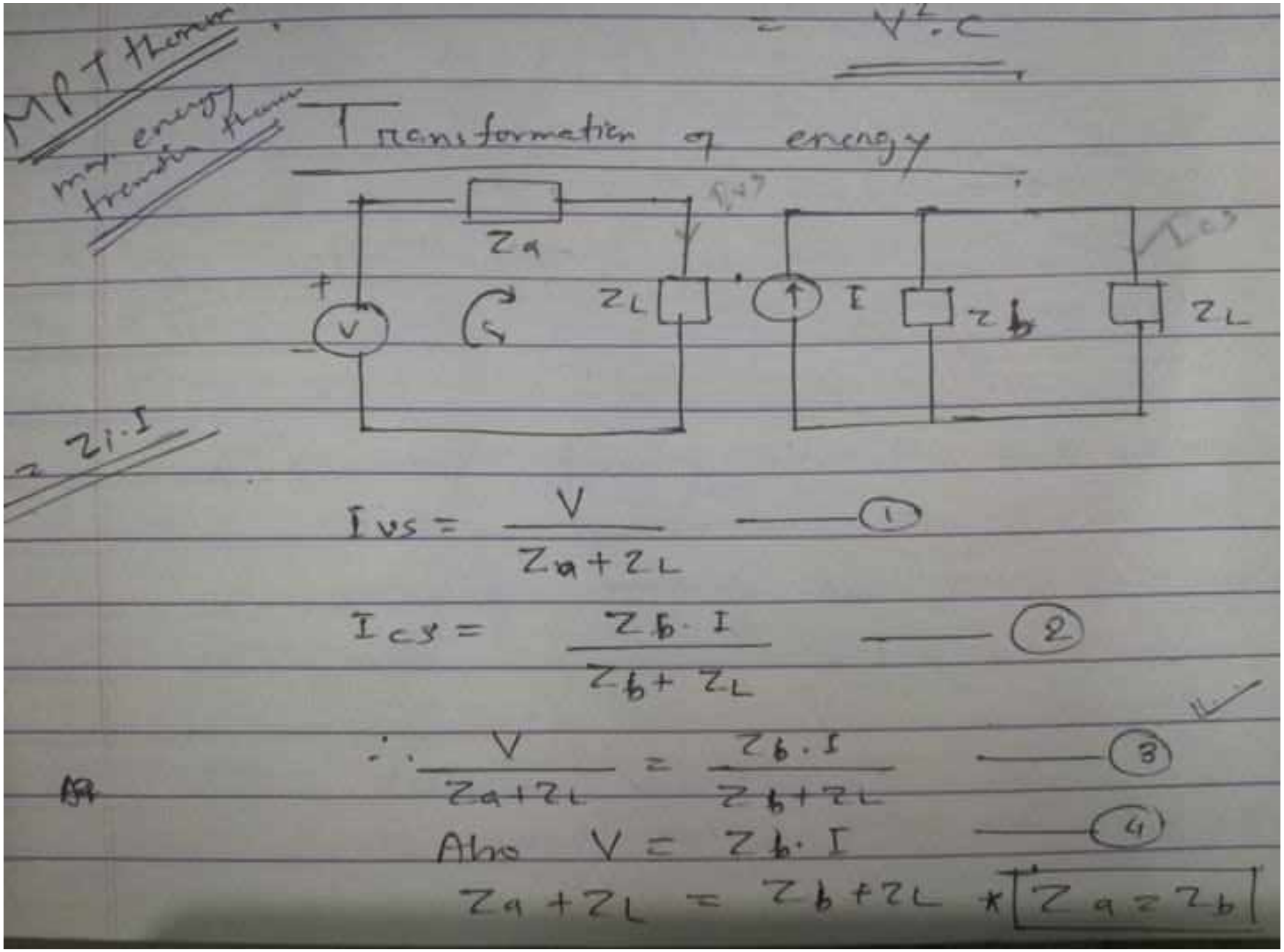
$$\therefore \frac{di_2}{dt} = \left(\frac{L_1 - M}{L_2 - M} \right) \frac{di_1}{dt} \quad \text{—— (3)}$$

$$\therefore \frac{d}{dt}(i - i_1) = \text{——''——}$$

$$\frac{di}{dt} = \left(\frac{L_1 - M + L_2 - M}{L_2 - M} \right) \frac{di_1}{dt}$$

Mutual Coupling and Dot Conversion





MPT theorem

max. energy transfer theorem

Transformation of energy

$$I_{vs} = \frac{V}{Z_a + Z_L} \quad \text{———} (1)$$

$$I_{cs} = \frac{Z_b \cdot I}{Z_b + Z_L} \quad \text{———} (2)$$

$$\therefore \frac{V}{Z_a + Z_L} = \frac{Z_b \cdot I}{Z_b + Z_L} \quad \text{———} (3)$$

Also $V = Z_b \cdot I$ ——— (4)

$$Z_a + Z_L = Z_b + Z_L \Rightarrow \boxed{Z_a = Z_b}$$

Coupling and Energy Transformation

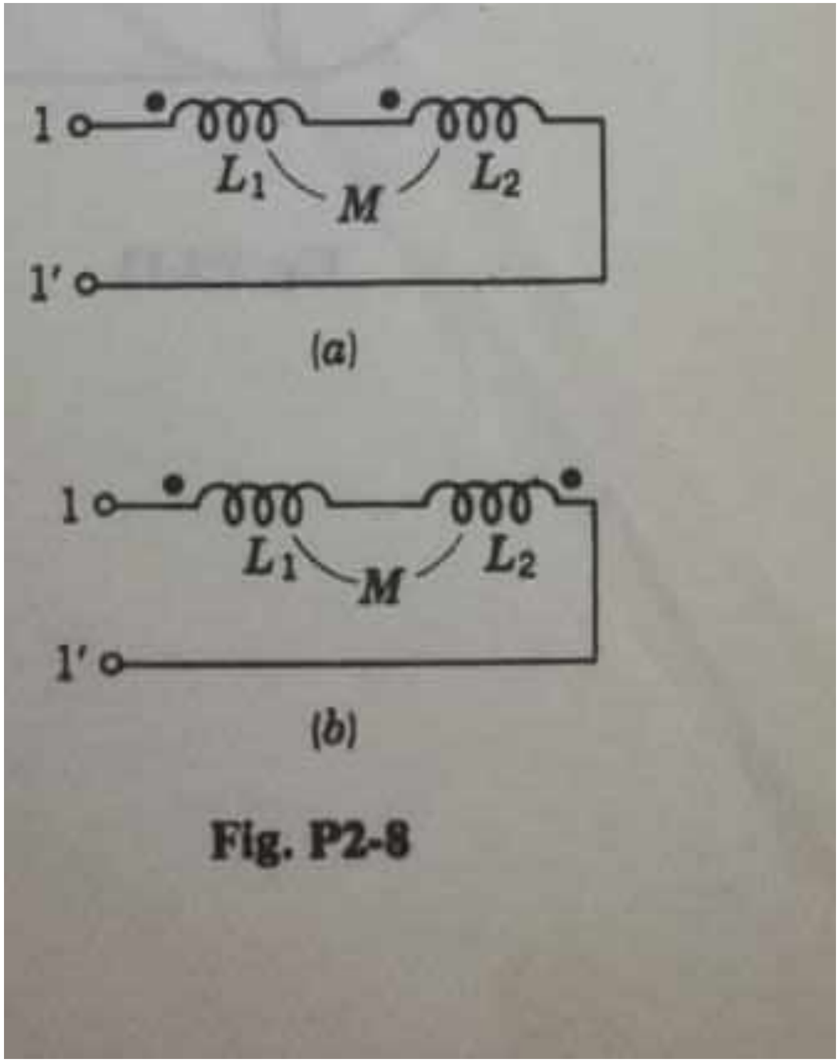


Fig. P2-8

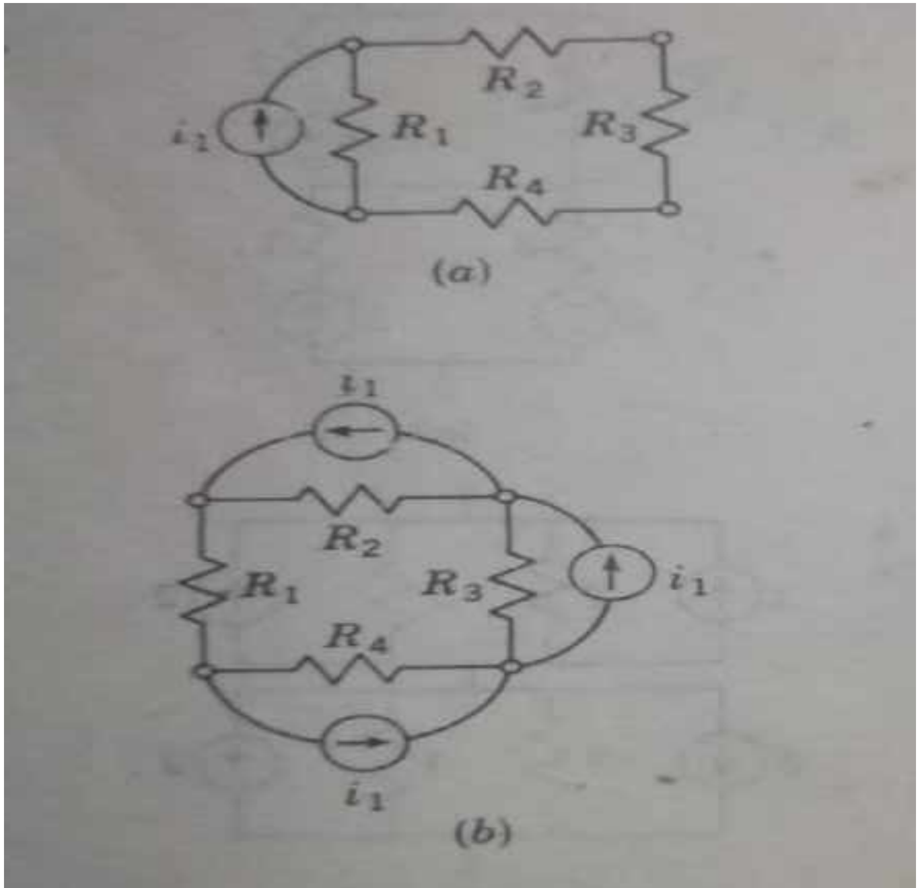


Fig. 3-22. An example illustrating the procedure by which a current source is shifted in a network.

Energy in Two Linearly Coupled Coils

Consider a pair of mutually coupled coils as shown in Fig. 9.10. Let it be assumed that initially currents $i_1(t)$ and $i_2(t)$ in the coils are zero at $t = 0$, and the initial energy stored in the circuit is zero. The voltage equations for the circuit are

$$v_1(t) = L_1 \frac{di_1(t)}{dt} + M \frac{di_2(t)}{dt} \quad (9.23)$$

$$v_2(t) = L_2 \frac{di_2(t)}{dt} + M \frac{di_1(t)}{dt} \quad (9.24)$$

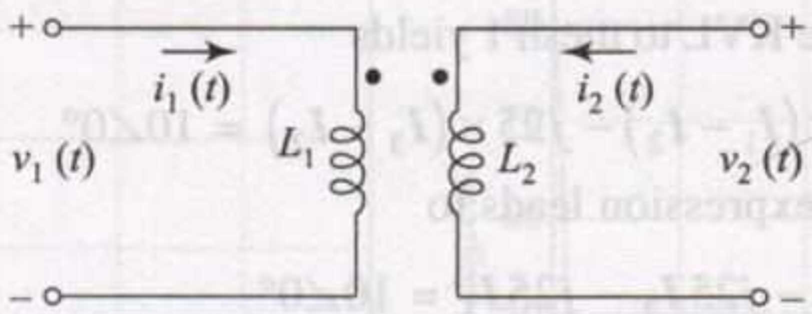


Fig. 9.10 *Stored energy relationship with two mutually coupled coils with* $M_{12} = M_{21} = M$ *and* $k = \sqrt{L_1 L_2}$

Mutual Coupling and Dot Conversion

The net energy input to the coupled circuit at any instant of time t is given by

$$w(t) = \int_0^t \left[v_1(t) i_1(t) + v_2(t) i_2(t) \right] dt \tag{9.25}$$

Substituting for $v_1(t)$ and $v_2(t)$ from Eqs (9.23) and (9.24), respectively, yields

$$w(t) = \int_0^t \left[\left\{ L_1 \frac{di_1(t)}{dt} + M \frac{di_2(t)}{dt} \right\} i_1(t) + \left\{ L_2 \frac{di_2(t)}{dt} + M \frac{di_1(t)}{dt} \right\} i_2(t) \right] dt \tag{9.26}$$

from which

$$w_T(t) = \left(\frac{1}{2} L_1 \left[i_1(t) \right]^2 + \frac{1}{2} L_2 \left[i_2(t) \right]^2 + M \left[i_1(t) \right] \left[i_2(t) \right] \right) \text{J} \tag{9.27}$$

In case current enters a coil at the dot-marked terminal and leaves at the other dot-marked terminal, then the energy input is given by

$$w_T(t) = \left(\frac{1}{2} L_1 \left[i_1(t) \right]^2 + \frac{1}{2} L_2 \left[i_2(t) \right]^2 - M \left[i_1(t) \right] \left[i_2(t) \right] \right) \text{J} \tag{9.28}$$

Maximum Value of 'M'

Since the mutually coupled coils are passive elements, the energy stored $w_T(t)$ can never be negative for any values of $i_1(t)$, $i_2(t)$, L_1, L_2, or M. At best, the stored energy in the circuit can be greater than or equal to zero. If both currents $i_1(t)$ and $i_2(t)$ are both positive or both negative, their product will always be positive. It may be inferred from Eqs (9.27) and (9.28) that $w_T(t)$ could possibly be negative only in the case of Eq. (9.28).

Adding and subtracting $\sqrt{L_1 L_2}\, i_1(t) i_2(t)$ to Eq. (9.28) and rearranging give

$$w_T(t) = \frac{1}{2}\left[\sqrt{L_1}\, i_1(t) - \sqrt{L_2}\, i_2(t)\right]^2 + \left[\sqrt{L_1 L_2} - M\right] i_1(t) i_2(t) \qquad (9.29)$$

Equation (9.29) indicates that in order that the stored energy $w_T(t) \geq 0$, the second term should never be negative since the squared term can only have a minimum value of zero. Hence the necessary condition for $w_T(t) \geq 0$ is obtained when

$$\sqrt{L_1 L_2} \geq M \qquad (9.30)$$

Equation (9.30) implies that

$$M = k\sqrt{L_1 L_2}, \quad \text{for } 0 \leq k \leq 1$$

Maximum Value of 'M'

Equation (9.30) implies that

$$M = k\sqrt{L_1 L_2}, \quad \text{for } 0 \le k \le 1$$

The inequality in Eq. (9.30) provides a basis for a physical interpretation of the magnetic coupling. If coil 2 is on open circuit $i_2(t) = 0$, the leakage flux which links both the coils is established by current $i_1(t)$ alone. Therefore, it is evident that the leakage flux within coil 2 cannot be greater than the flux contained in coil 1 which signifies the total flux. As such, there has to be an upper limit to the mutual coupling and hence the mutual inductance between the two coils.

Maximum Value of ‘M’

Example 9.7 Figure 9.11 shows a mutually coupled linear circuit in which coils 1 and 2 have self-inductances of 0.5 H and 3.0 H, and the coefficient of coupling between the coils is 0.65. If coil 1 is excited by a current $i_1 = 2.5 \cos (8t)$ A, calculate the energy stored in the circuit at $t = 0$ s when terminals AB are (a) open-circuited, and (b) short-circuited.

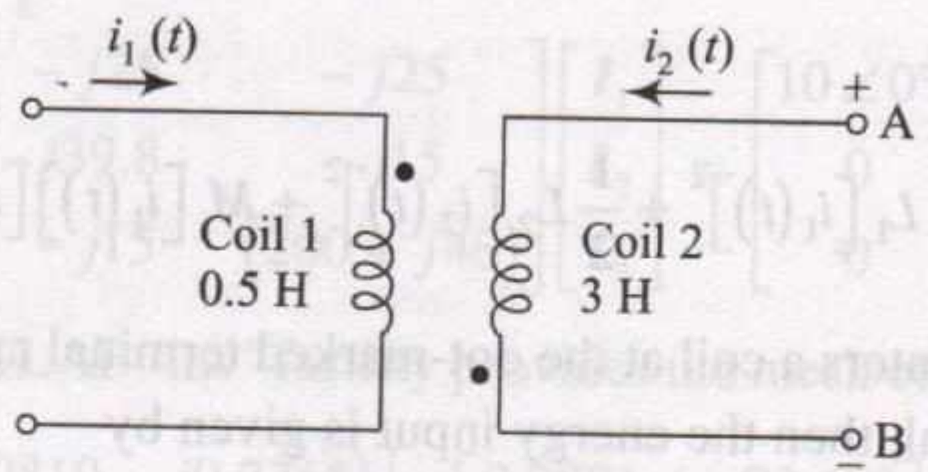


Fig. 9.11

Example 9.7 Figure 9.11 shows a mutually coupled linear circuit in which coils 1 and 2 have self-inductances of 0.5 H and 3.0 H, and the coefficient of coupling between the coils is 0.65. If coil 1 is excited by a current $i_1 = 2.5 \cos (8t)$ A, calculate the energy stored in the circuit at $t = 0$ s when terminals AB are (a) open-circuited, and (b) short-circuited.

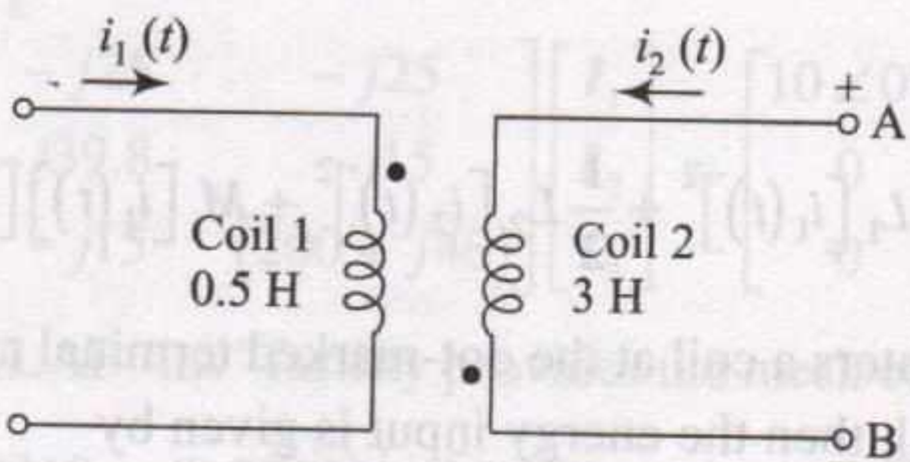


Fig. 9.11

Solution Using Eq. (9.17), the magnitude of M is computed as

$$M = 0.65\sqrt{0.5 \times 3.0} = 0.7961\ \text{H}$$

(a) Since terminals AB are open-circuited, no current flows through coil 2. All energy is stored in coil 1 is due to $i_1(0) = 2.5$ A. Hence,

The energy stored in the circuit $\frac{1}{2}L_1 i_1(0)^2 = \frac{1}{2} \times 0.5(2.5)^2 = 1.5625$ J

(b) The voltage across terminals AB of the mutually coupled circuit is given by

$$v_{AB}(t) = -M\frac{di_1(t)}{dt} + L_2\frac{di_2(t)}{dt} \tag{9.7.1}$$

Since current $i_1(t)$ is entering at the dotted terminal and current $i_2(t)$ is leaving at the dotted terminal, M has been assigned a negative sign in Eq. (9.7.1). Further, when the terminals AB are short-circuited, the voltage across the terminals is zero. Therefore, Eq. (9.7.1) is equated to zero and the values are substituted as given below

$$3\frac{di_2(t)}{dt} = 0.7961\frac{d[2.5\cos(8t)]}{dt} = -0.7961 \times 20\sin(8t)$$

or $$i_2(t) = -\frac{0.7961 \times 20}{3}\int_0^t \sin(8t)\,dt = \frac{0.7961 \times 20}{3 \times 8}\cos(8t)$$

$$= 0.6634\cos(8t)\ \text{A} \tag{9.7.2}$$

From Eq. (9.7.2), $i_2(0) = 0.6634$ A. The total energy stored in the circuit from Eq. (9.28) is

$$w_T(0) = \left(\frac{1}{2} \times 0.5 \times (2.5)^2 + \frac{1}{2} \times 3 \times (0.6634)^2 - 0.7961 \times (2.5) \times (0.6634)\right)$$

$$= 0.9023\ \text{J}$$

Transformer Applications

Ideal Transformer

Assumptions that leads to model above, are as follows

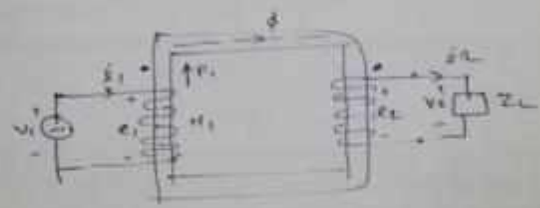
① Transformer windings are resistance less.
that is ohmic primary losses & resistance voltage drop are neglected

② " core materials are having an infinite permeability.
i.e. It requires 0 mmf to create flux in the core.

$F = NI = R\phi$

③ Leakage flux is neglected (i.e. no leakage ...)

④



Transformation Ratio

$$\frac{V_1}{V_2} = \frac{E_1}{E_2} = \frac{N_1}{N_2} = a \text{ (turns ratio)} = \frac{I_2}{I_1}$$

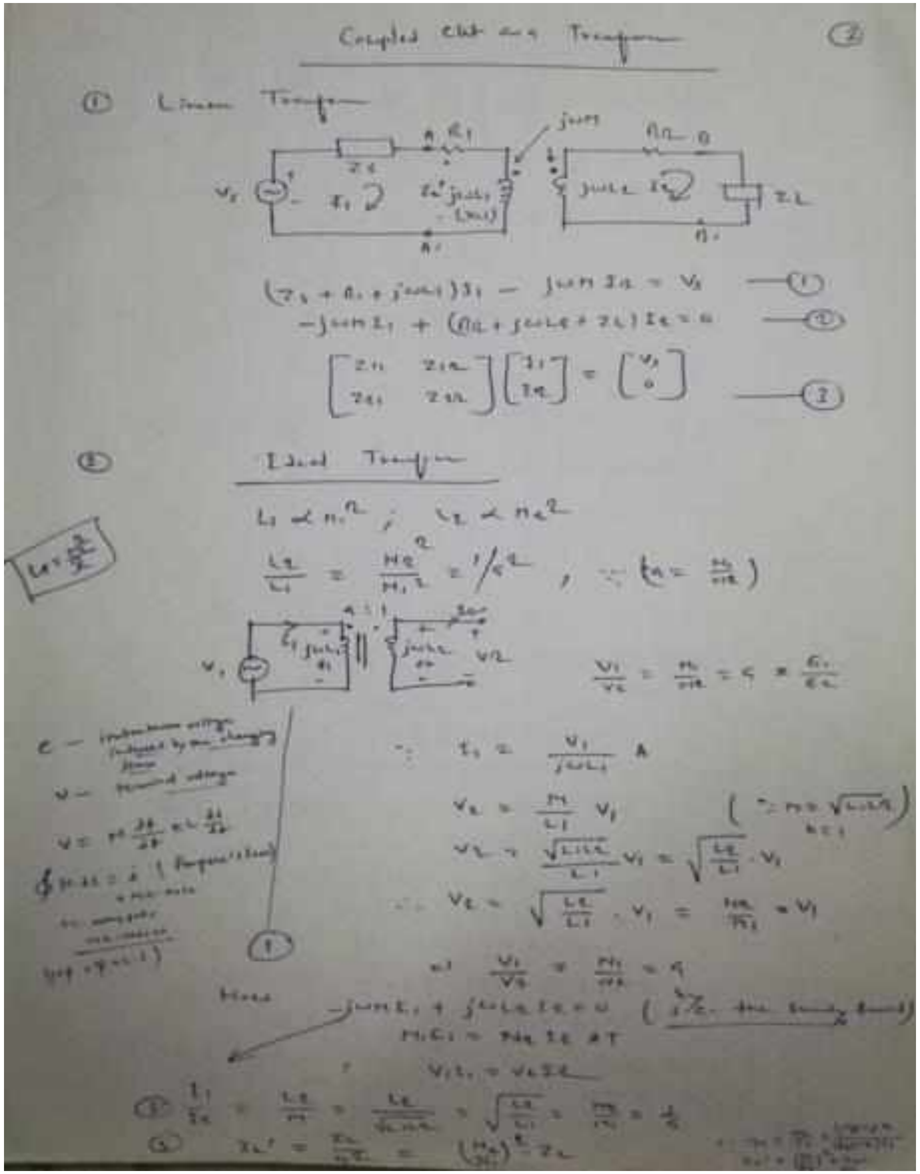
$\because V_1 = E_1 = \sqrt{2}\pi f N_1 \phi_{max}$

$V_2 = E_2 = \sqrt{2}\pi f N_2 \phi_{max}$

at $\phi = \phi_{max}$ —— (a)

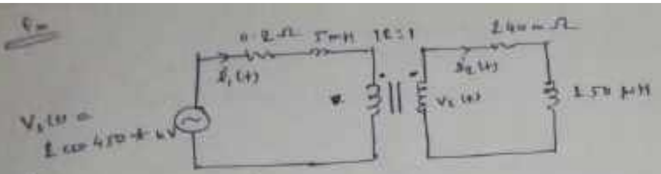
$\times$ (By $\frac{\text{Lenz's law}}{F = I N}$) $I_1 N_1 = I_2 N_2$ —— (b)

Transformer Applications



③

Q.



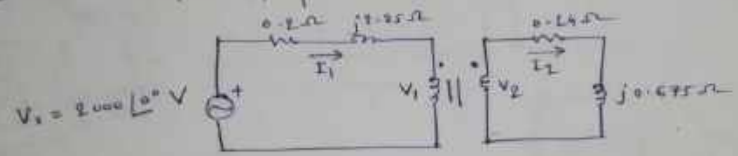
Find

(a) primary voltage & current
(b) secondary " "

in time domain

Soln

Equi frequency domain ckt is



$V_1 = [2000\angle 0^\circ - I_1(0.2 + j2.25)]\ V$ —— (1)

$V_2 = I_2(0.24 + j0.0675)\ V$ —— (2)

∴ (a) $\frac{V_1}{12} = 12 \cdot I_1(0.24 + j0.0675)\ V$

$\Rightarrow V_1 = 12^2 I_1(0.24 + j0.0675)$

$= 144\, I_1(0.24 + j0.0675)$

$= I_1(34.56 + j9.72)\ V$

∴ $I_1[(34.56 + j9.72) + (0.2 + j2.25)]$; from (1)

$\Rightarrow I_1(34.76 + 11.97) = 2000\angle 0^\circ\ V$

∴ $I_1 = 54.4021\angle -19^\circ\ A$

∴ $V_1 = 54.4021\angle -19^\circ \times (34.56 + j9.72)$

$= 54.4021\angle -19^\circ \times 35.90\angle 15.71^\circ$

$V_1 = 1953.16\angle -3.29^\circ\ V$

∴ $V_2 = \frac{V_1}{12} = 162.7633\angle -3.29^\circ\ V$

∴ $I_2 = I_1 \cdot 12 = 652.8252\angle -19^\circ\ V$

(b)

$v_1(t) = 1.95316 \cos(450t - 3.29^\circ)\ kV$

$i_1(t) = 54.4021 \cos(450t - 19.0^\circ)\ A$

$v_2(t) = 162.7633 \cos(450t - 3.29^\circ)\ V$

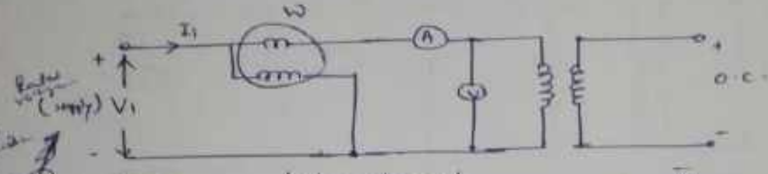
$i_2(t) = 652.8252 \cos(450t - 19^\circ)\ A$

} In time domain

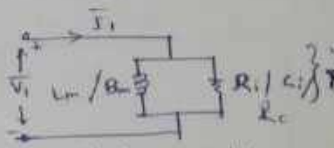
① Open-Circuit (OC) Test / No-load Test

i.e. transformer is excited at rated V & f on primary side (one side) & other is kept open at (other side) secondary side.

(used to det. shunt parameters of the model)



'ckt diagram'



'ckt model'

$$Y_0 = \frac{I_1}{V_1}$$

$$G_i = \frac{P_i}{V_1^2}$$

$$B_m = \sqrt{Y_0^2 - G_i^2}$$

$$\left(G_i = \frac{1}{R_i}\right)$$

The O.C. test yields (i) core loss and
(ii) parameters of shunt branch.
(iii) voltage ratio (i.e. turn ratio)

Ex

A 50 kVA, 2200/110 V transformer is connected to 110V supply with 2200 V side open circuited, the meter readings are 110V, 10A, 400W. Calculate the parameters of the shunt branch in equi ckt from both side (LV & HV)

Ans

LV side:

$$Y_0 = \frac{I_1}{V_1} = \frac{10}{110} = 0.091\ \Omega^{-1}$$

$$G_i = \frac{P_i}{V_1^2} = \frac{400}{110^2}$$

$$G_i = 0.033\ \Omega^{-1}$$

$$\therefore B_m = \sqrt{Y_0^2 - G_i}$$

$$B_m = 0.085\ \Omega^{-1}$$

HV side:

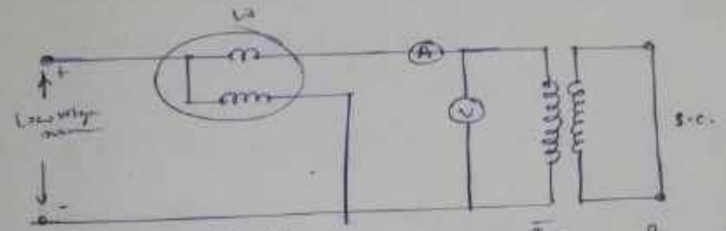
$$G_{i_{HV}} = G_{i_{LV}} \left(\frac{V_1}{V_2}\right)^2 = 0.033 \times \left(\frac{110}{2200}\right)^2 = 8.25\ m\Omega^{-1}$$

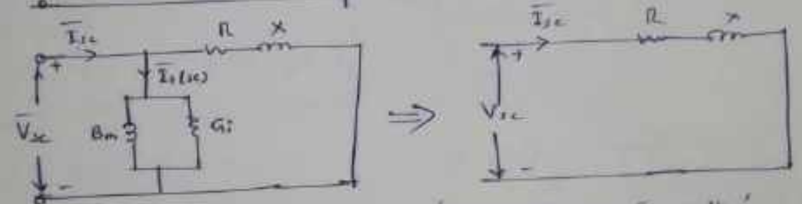
$$B_{m,HV} = B_{m,LV} \times \left(\frac{V_1}{V_2}\right)^2 = 0.085 \times \left(\frac{110}{2200}\right)^2 = 21.25\ m\Omega^{-1}$$

(8) Short Circuit (SC) Test

→ is used to determine the series parameters of the transformer ckt model.

The transformer is short circuited on one side & is excited from a rated voltage source from other side.





'ckt model under SC condition'

$V \rightarrow V_{sc}$

$I \rightarrow I_{sc}$

W (power) $= P_{sc} = I^2R$ loss / copper loss P_c

$$Z = \frac{V_{sc}}{I_{sc}} = \sqrt{R^2 + x^2}$$

$$R_{eq} = R = \frac{P_{sc}}{I_{sc}^2}$$

$$X_{eq} = X = \sqrt{Z^2 - R^2}$$

SC test yields (i) full-load copper loss

(ii) equivalent 'R' & 'x' of the transformer

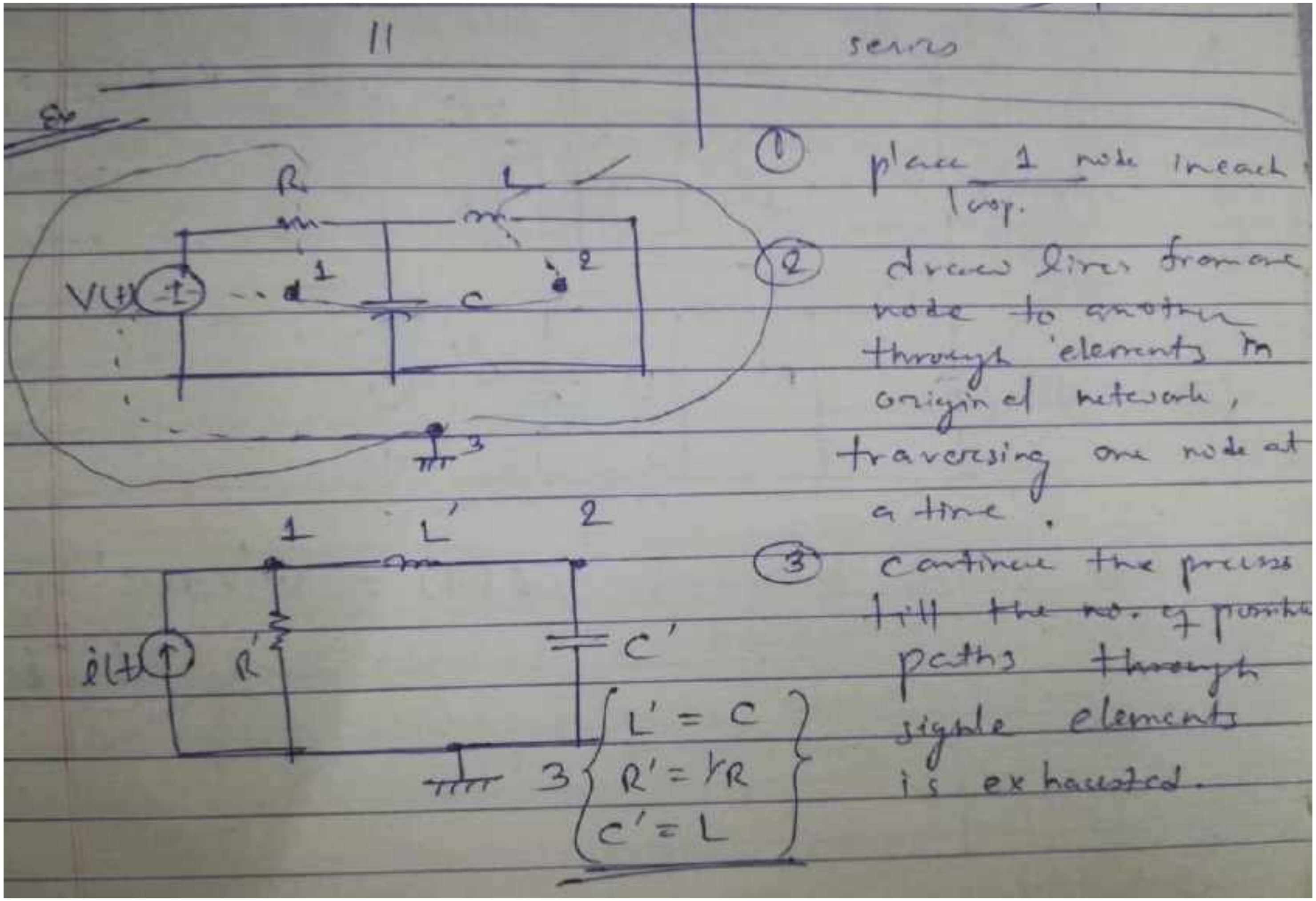
Duality Principle

Duality

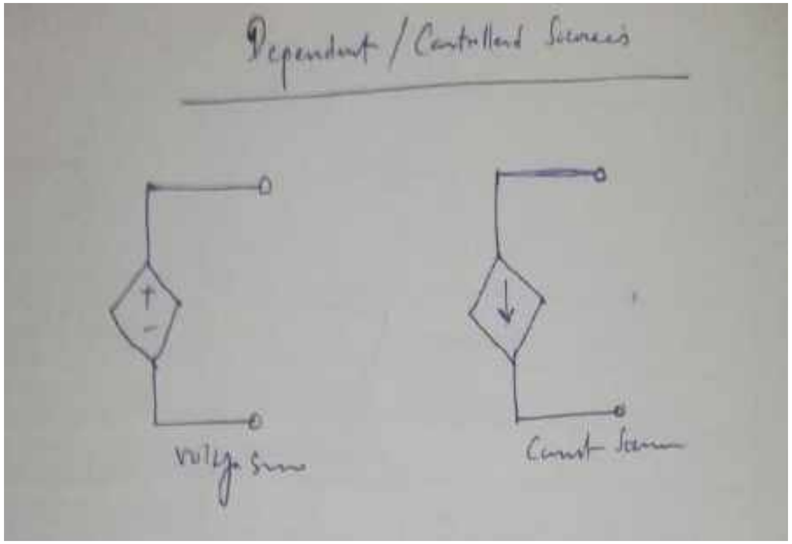
If an elet. network is govern by the same type of eqn of the network under consideration then that network is known as the dual of the network.

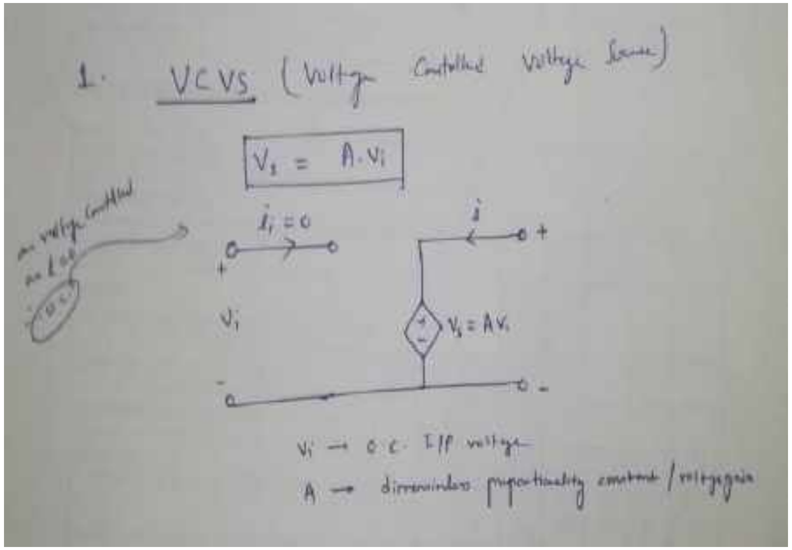
Quantity	Dual
q	φ
i	V
R	G (1/R)
L	C
Loop current (I)	Node pair voltage (V
No. of loops	No. of node pair
mesh current	node potential
Link / cord	Tree branch
S.C.	O.C.
series path	Shunt (∥el) path
∥	series

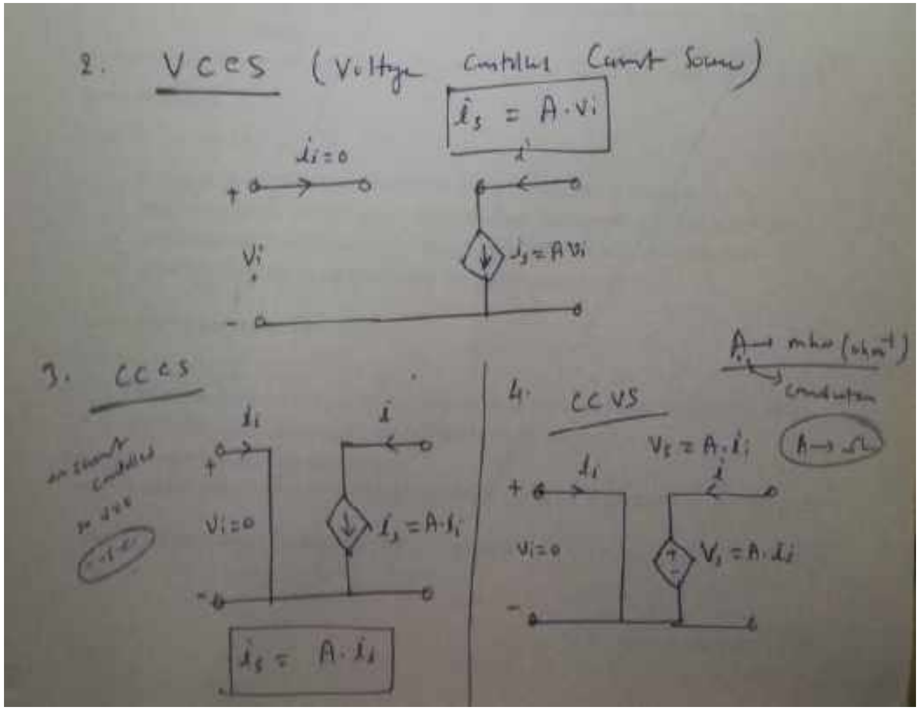
Dual Circuit



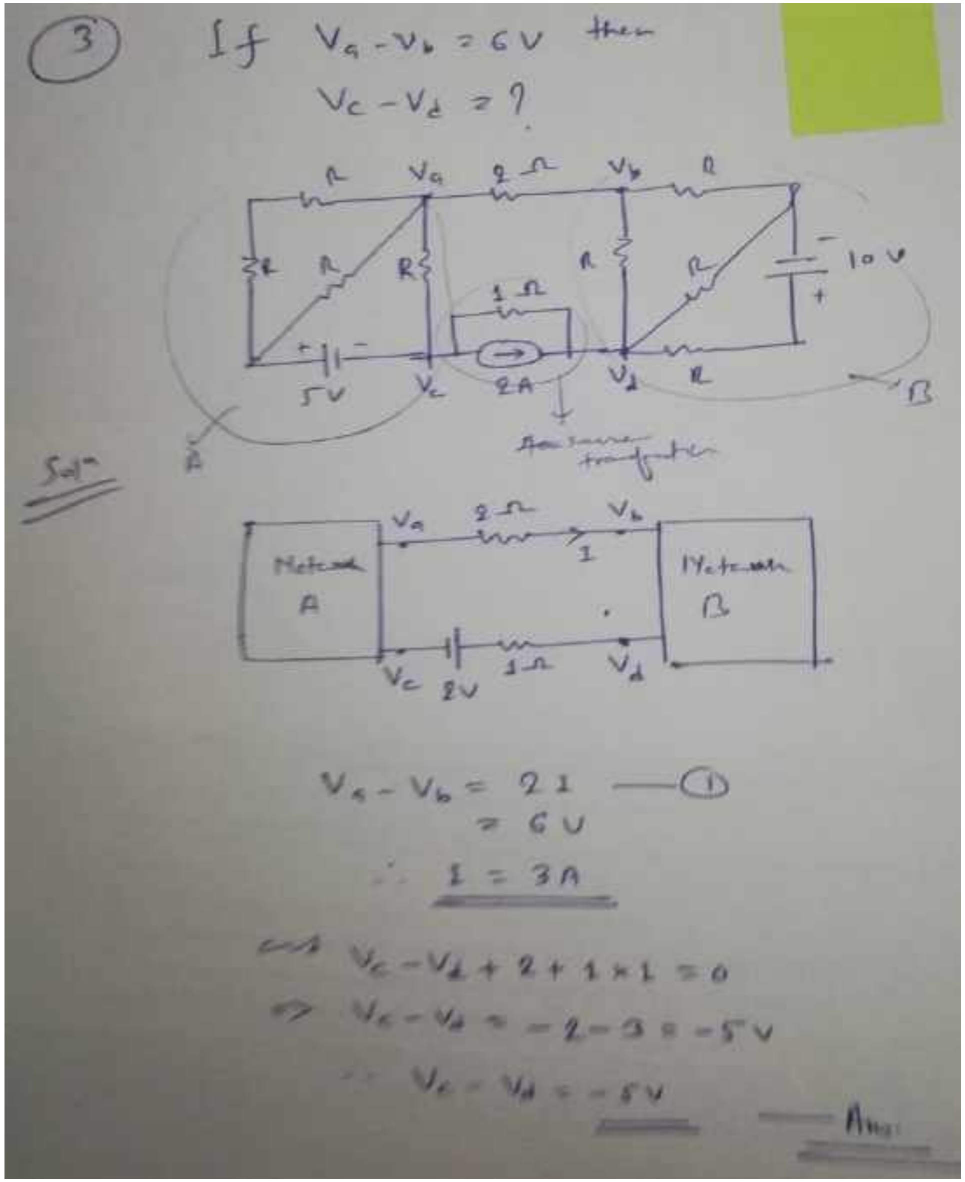
Dependent Sources

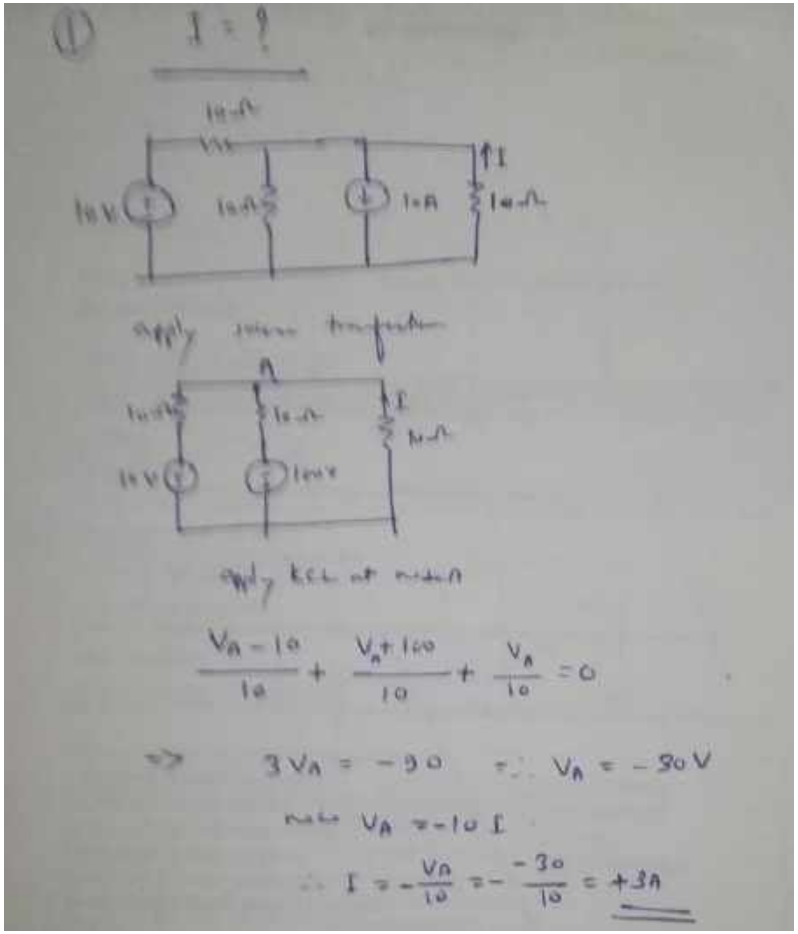


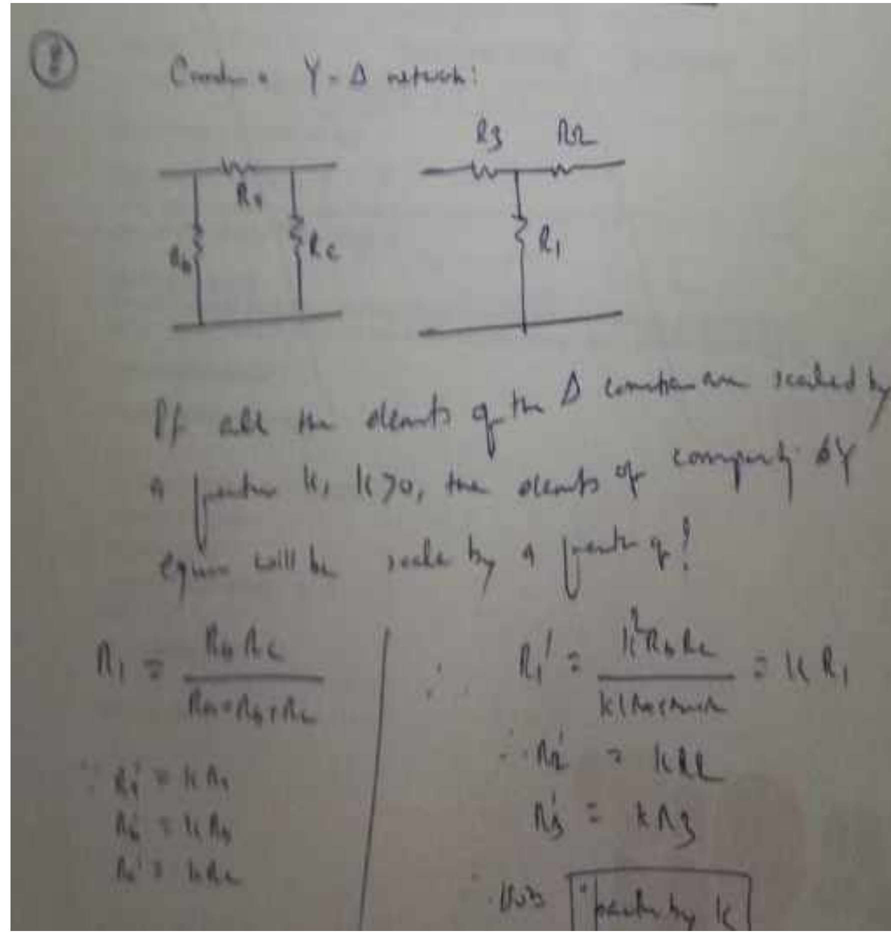


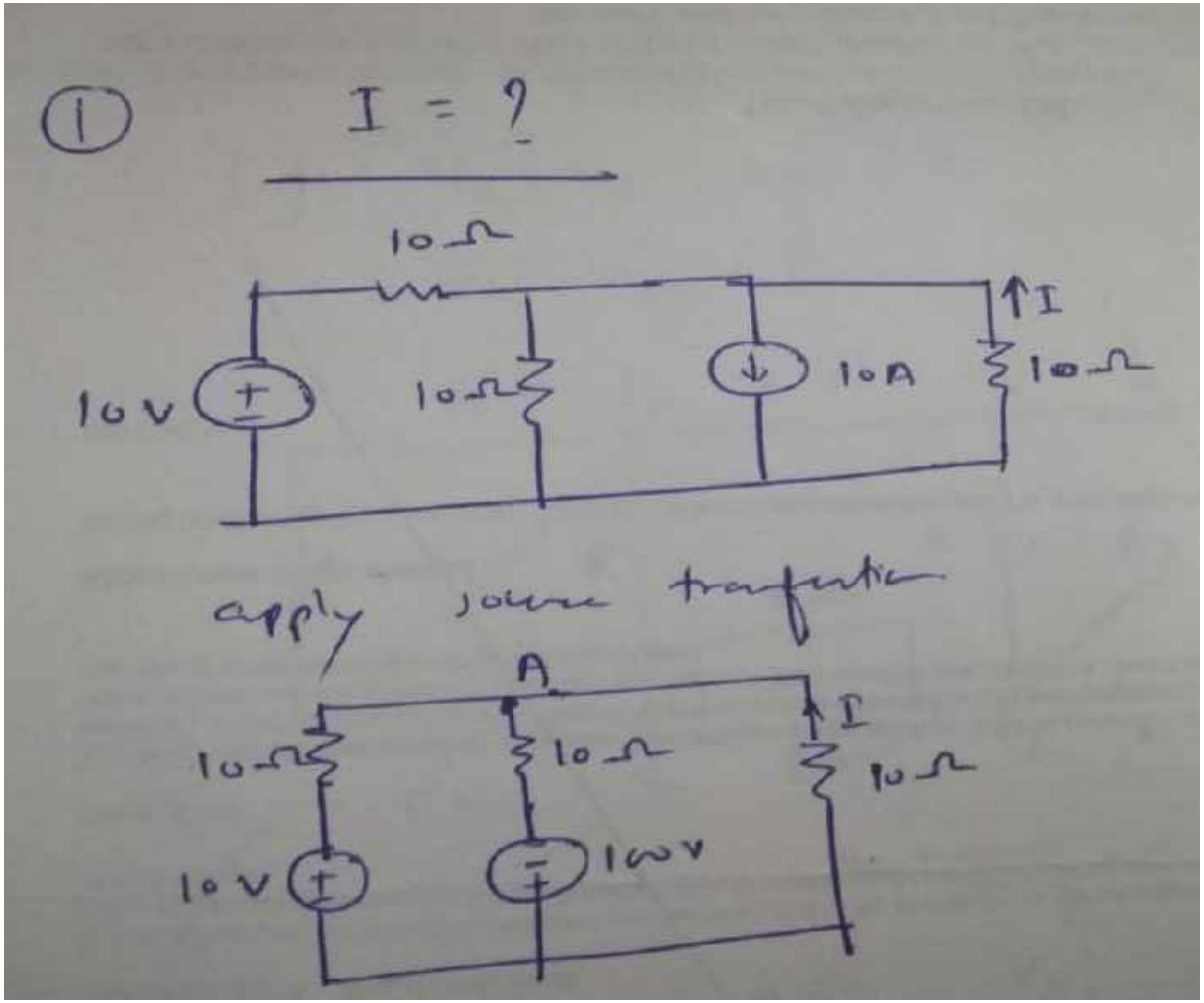


Dependent Sources



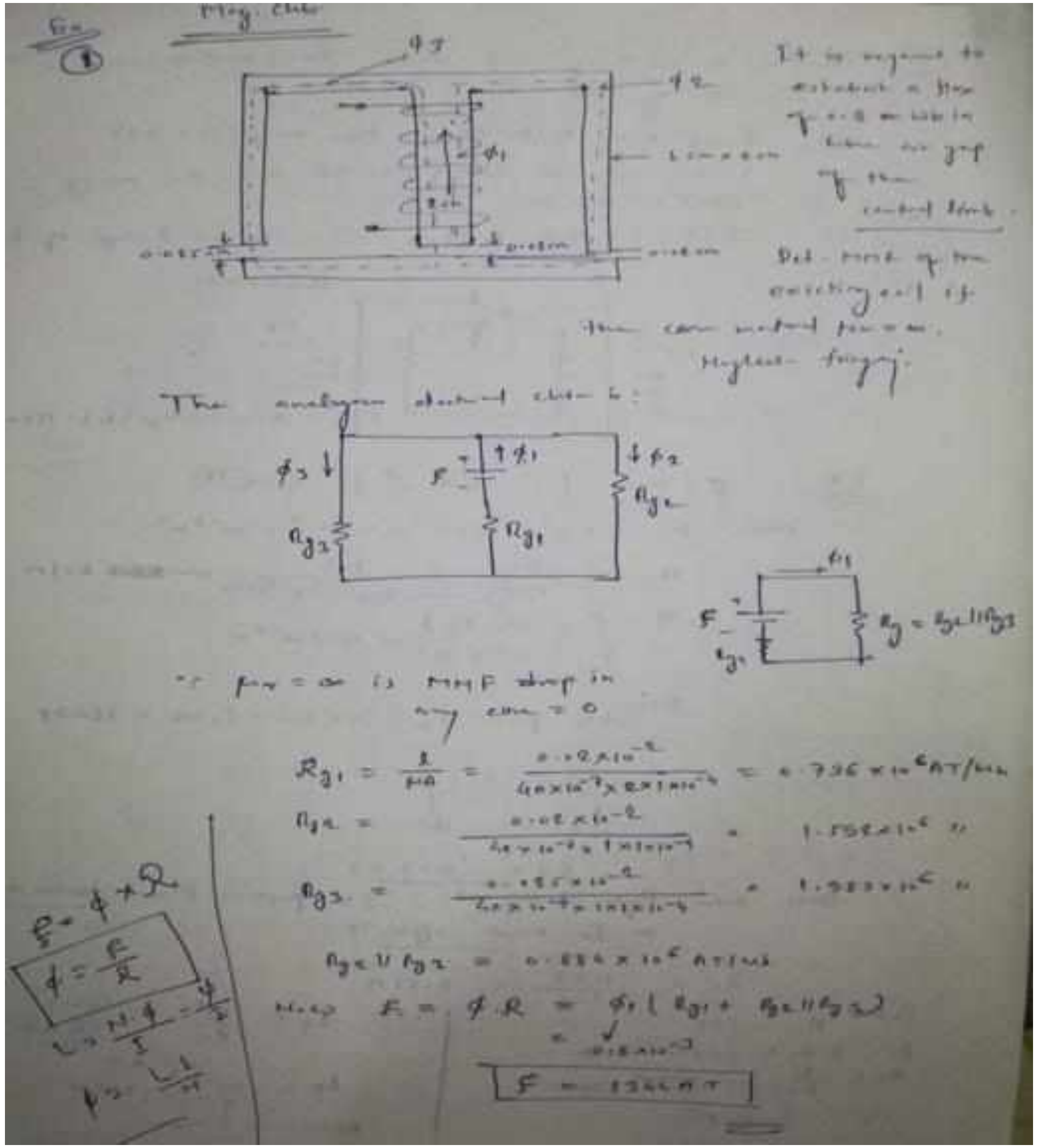

I = ?
10 Ω
10 V
10 Ω
10 A
10 Ω
I
apply KCL at node A
(V_A − 10)/10 + (V_A + 100)/10 + V_A/10 = 0
⇒ 3V_A = −90 ∴ V_A = −30 V
∴ I = −V_A/10 = −(−30)/10 = +3A


Consider a Y-Δ network:
R3
R2
R1
Rc
∴ R3' = kR3

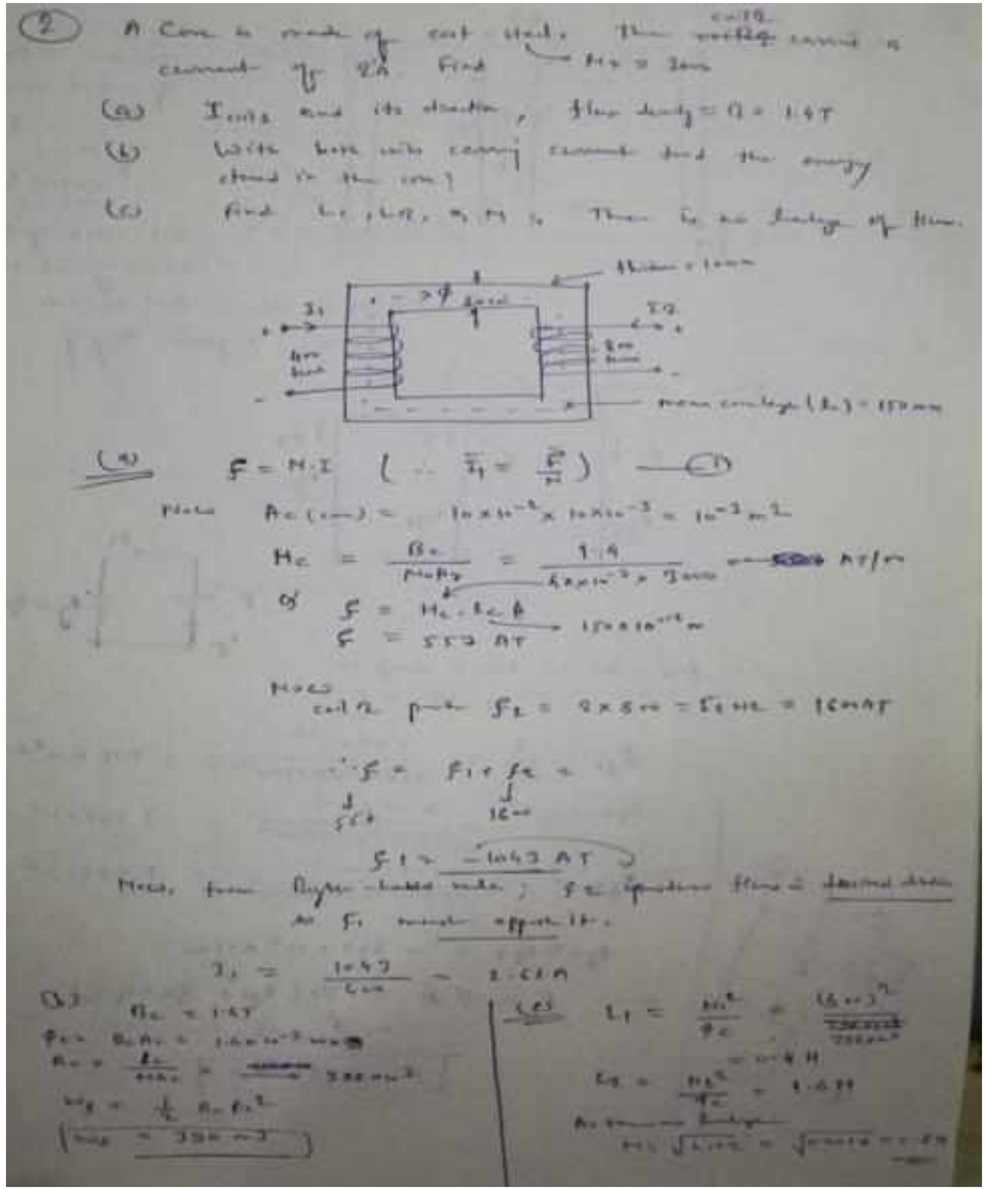

① I = ?
10Ω
↑I
10V
10Ω
10A
10Ω
apply source transformation
A
I
10Ω
10Ω
10Ω
10V
100V

Supplementary Sheets/ Problems

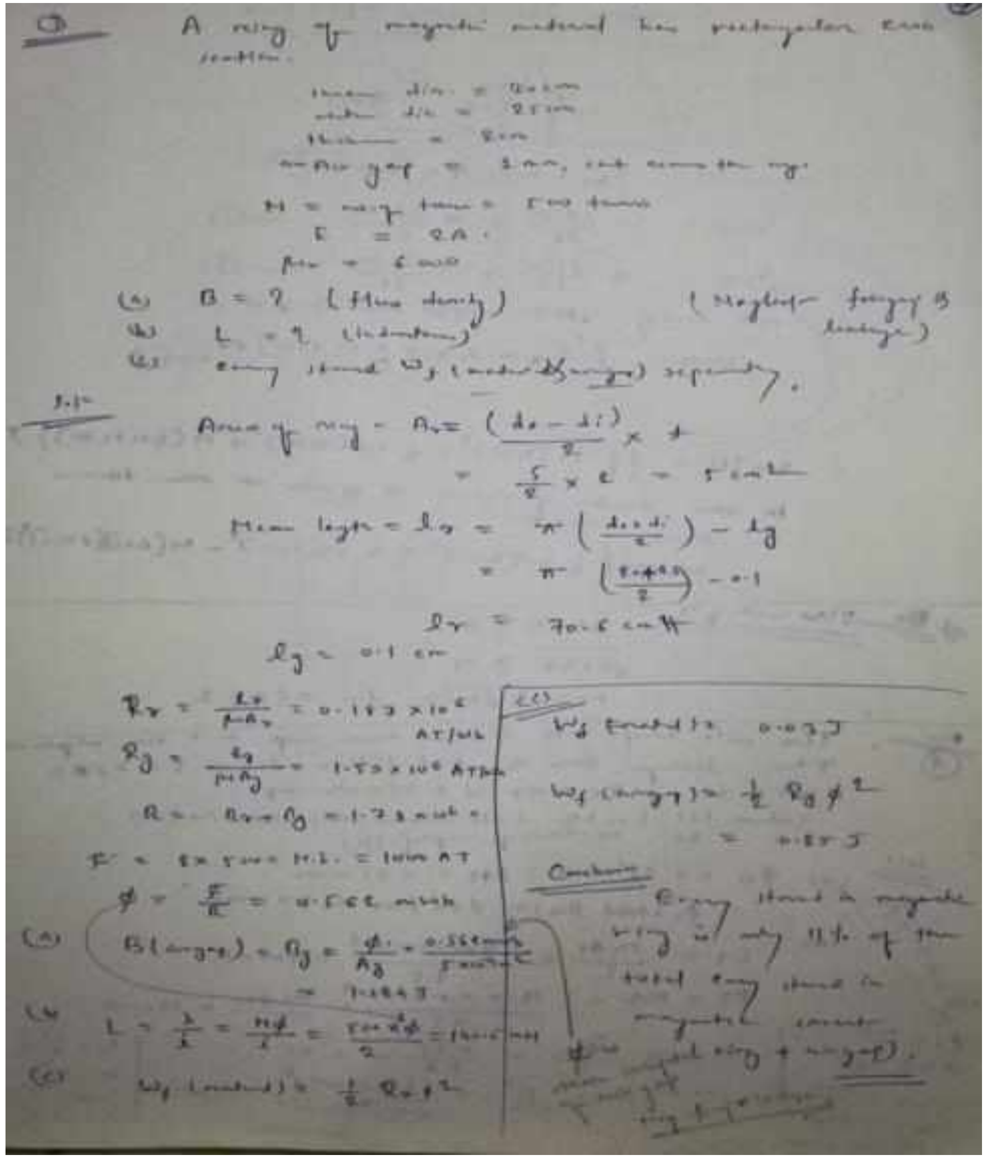
Mutual Coupling and Dot Conversion



Mutual Coupling and Dot Conversion



Mutual Coupling and Dot Conversion



Q. A ring of magnetic material has rectangular cross section.

Inner dia = 20 cm
outer dia = 25 cm
thickness = 2 cm
air gap = 1 mm, cut across the ring.
N = no. of turns = 500 turns
I = 2 A.
μ_r = 6000

(a) B = ? (flux density) (Neglect fringing & leakage)
(b) L = ? (inductance)
(c) energy stored W_f (core & air gap) separately.

Soln
Area of ring = $A_r = \frac{(d_o - d_i)}{2} \times t$
$= \frac{5}{2} \times 2 = 5\ cm^2$

Mean length $= l_r = \pi\left(\frac{d_o + d_i}{2}\right) - l_g$
$= \pi\left(\frac{25+20}{2}\right) - 0.1$
$l_r = 70.6$ cm
$l_g = 0.1$ cm

$R_r = \frac{l_r}{\mu A_r} = 0.187 \times 10^6$ AT/wb
$R_g = \frac{l_g}{\mu_0 A_g} = 1.59 \times 10^6$ AT/wb
$R = R_r + R_g = 1.78 \times 10^6$
$F = 2 \times 500 = N.I. = 1000$ AT
$\phi = \frac{F}{R} = 0.562$ mwb

(a) $B(\text{air gap}) = B_g = \frac{\phi}{A_g} = \frac{0.562\ mwb}{5\ cm^2}$
= 1.124 T

(b) $L = \frac{\lambda}{i} = \frac{N\phi}{i} = \frac{500 \times \phi}{2}$ = 140.5 mH

(c) $W_f(\text{core}) = \frac{1}{2} R_r \phi^2$

$W_f(\text{core})$ = 0.03 J

$W_f(\text{air gap}) = \frac{1}{2} R_g \phi^2$
= 0.25 J

Conclusion: Energy stored in magnetic ring is only 11% of the total energy stored in magnetic circuit (ring + air gap).

Example 9.3 In Fig. 9.5, the source voltage $V_S = 15\angle 0°$ V, at $\omega = 12$ rad/s, $R_1 = 1.5\ \Omega$, $L_1 = 1$ H, $R_2 = 450\ \Omega$, $L_2 = 100$ H, and $M = 10$ H. (a) Develop the voltage equations and compute the currents in the two coils. (b) Calculate the output voltage V_2 across the resistor R_2 and determine the voltage gain ratio $|V_2/V_S|$. (c) What is the minimum and maximum values of the voltage gain and the conditions at which these occur?

Solution

(a) Dot convention (ii) is applicable since the current in coil 1 is entering at the dotted terminal and current in coil 2 is leaving at the dotted terminal. Using Eq. (9.20), the voltage equations are

$$1.5\boldsymbol{I}_1 + j12\times 1\boldsymbol{I}_1 - j12\times 10\boldsymbol{I}_2 = 15\angle 0°$$

$$450\boldsymbol{I}_2 + j12\times 100\boldsymbol{I}_2 - j12\times 10\boldsymbol{I}_1 = 0$$

or $$(1.5 + j12)\boldsymbol{I}_1 - j120\boldsymbol{I}_2 = 15\angle 0°$$

$$-j120\boldsymbol{I}_1 + (450 + j1200)\boldsymbol{I}_2 = 0$$

Solving the above equations simultaneously yields

$$\boldsymbol{I}_1 = 2.6583\angle -15.20°\text{ A} \quad \text{and} \quad \boldsymbol{I}_2 = 0.2489\angle 5.36°\text{ A}$$

Mutual Coupling and Dot Conversion

(b) Output voltage across the resistor R_2,

$$V_2 = 450 \times 0.2489\angle 5.36° = 112.0089\angle 5.36° \text{ V}$$

and $$\left|\frac{V_2}{V_S}\right| = \frac{112.0089}{15} = 7.4673$$

It may be noted that the output voltage is greater than the input voltage. However, a phase shift also occurs in the output voltage.

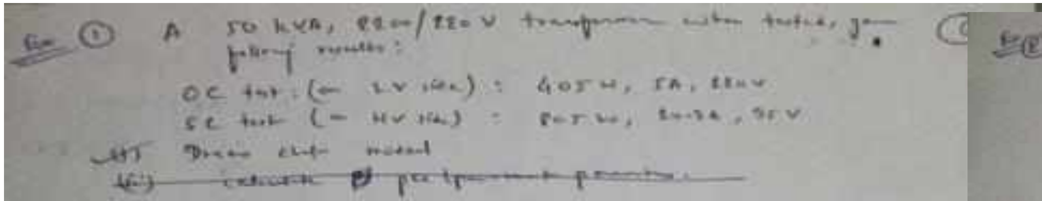
(c) If R_2 is made equal to zero, i.e., a short-circuit condition, $V_2 = 0$, hence the minimum ratio $\left|\frac{V_2}{V_S}\right| = 0$ results.

If R_2 is made equal to ∞, i.e., an open-circuit condition which results in $I_2 = 0$. Thus, the two voltage equations which are obtained from Eq. (9.20) are

$$(1.5 + j12)\, I_1 = 15\angle 0°, \text{ or } I_1 = \frac{15\angle 0°}{(1.5 + j12)} = 1.2403\angle -82.88° \text{ A}$$

$$V_2 = -j120 I_1 = -j120 \times 1.2403\angle -82.88°$$
$$= 148.8417\angle -172.88° \text{ V}$$

Maximum gain $$\left|\frac{V_2}{V_S}\right| = \frac{148.8417\angle -172.88°}{15\angle 0°} = 9.9228$$

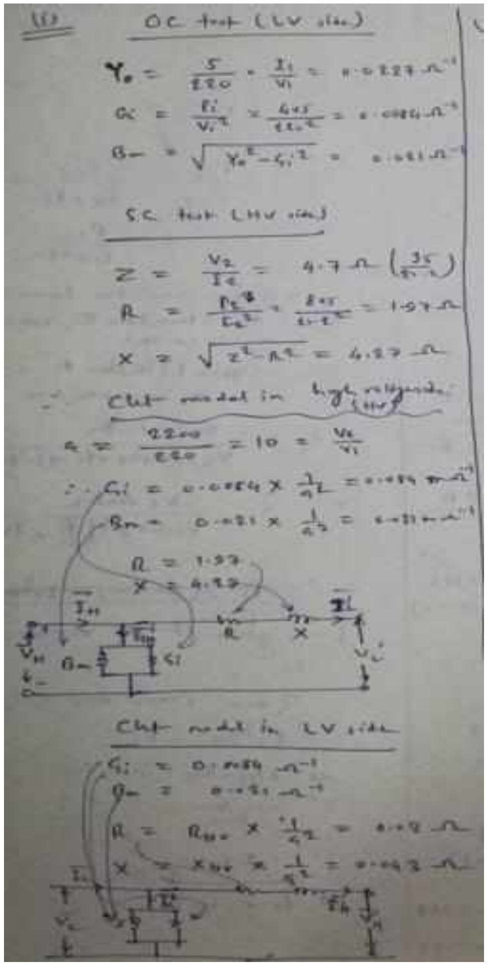


A 50 kVA, 2200/220 V transformer when tested, gave following results:

OC test (on LV side): 405 W, 5 A, 220 V

SC test (on HV side): 805 W, 20.3 A, 95 V

Draw ckt model



(i) OC test (LV side)

S.C test (HV side)

$$X = \sqrt{Z^2 - R^2} = 4.27\ \Omega$$

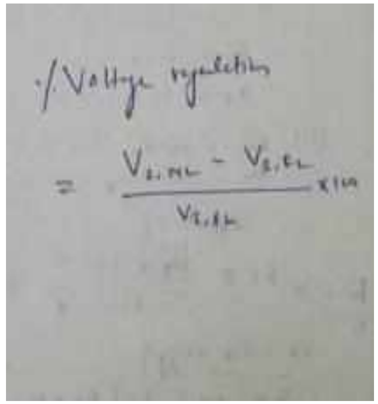
Ckt model in high voltage side

$$a = \frac{2200}{220} = 10 = \frac{V_2}{V_1}$$

R = 1.57

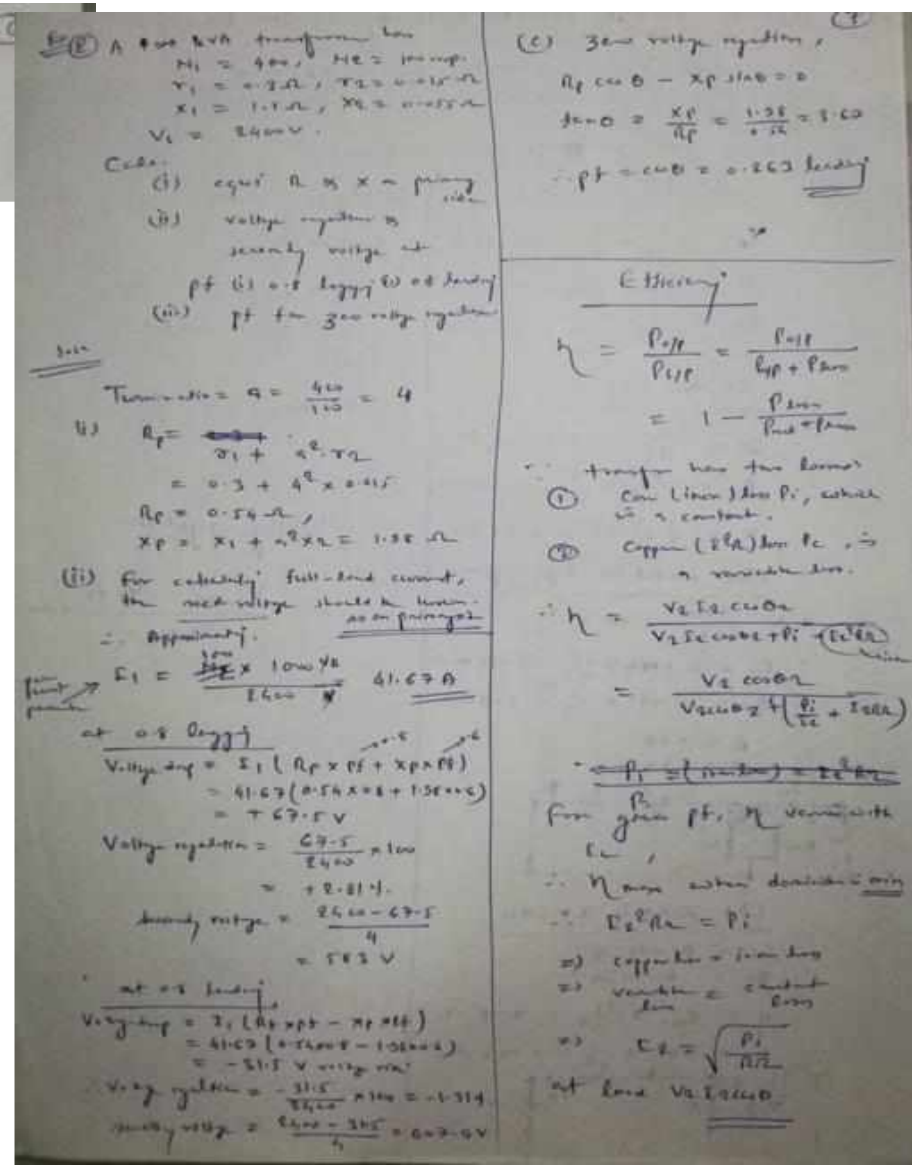
X = 4.27

Ckt model in LV side



% Voltage regulation

$$= \frac{V_{2,NL} - V_{2,FL}}{V_{2,FL}} \times 100$$



Transformation ratio = a = 4

$$R_p = 0.54\ \Omega$$

(ii) For calculating full-load current, the rated voltage should be known.

at 0.8 lagging

Voltage regulation = + 2.81 %

at 0.8 leading

(c) Zero voltage regulation

$$\therefore pf = \cos\theta = 0.863 \text{ leading}$$

Efficiency

$$\eta = \frac{P_{o/p}}{P_{i/p}} = \frac{P_{o/p}}{P_{o/p} + P_{loss}}$$

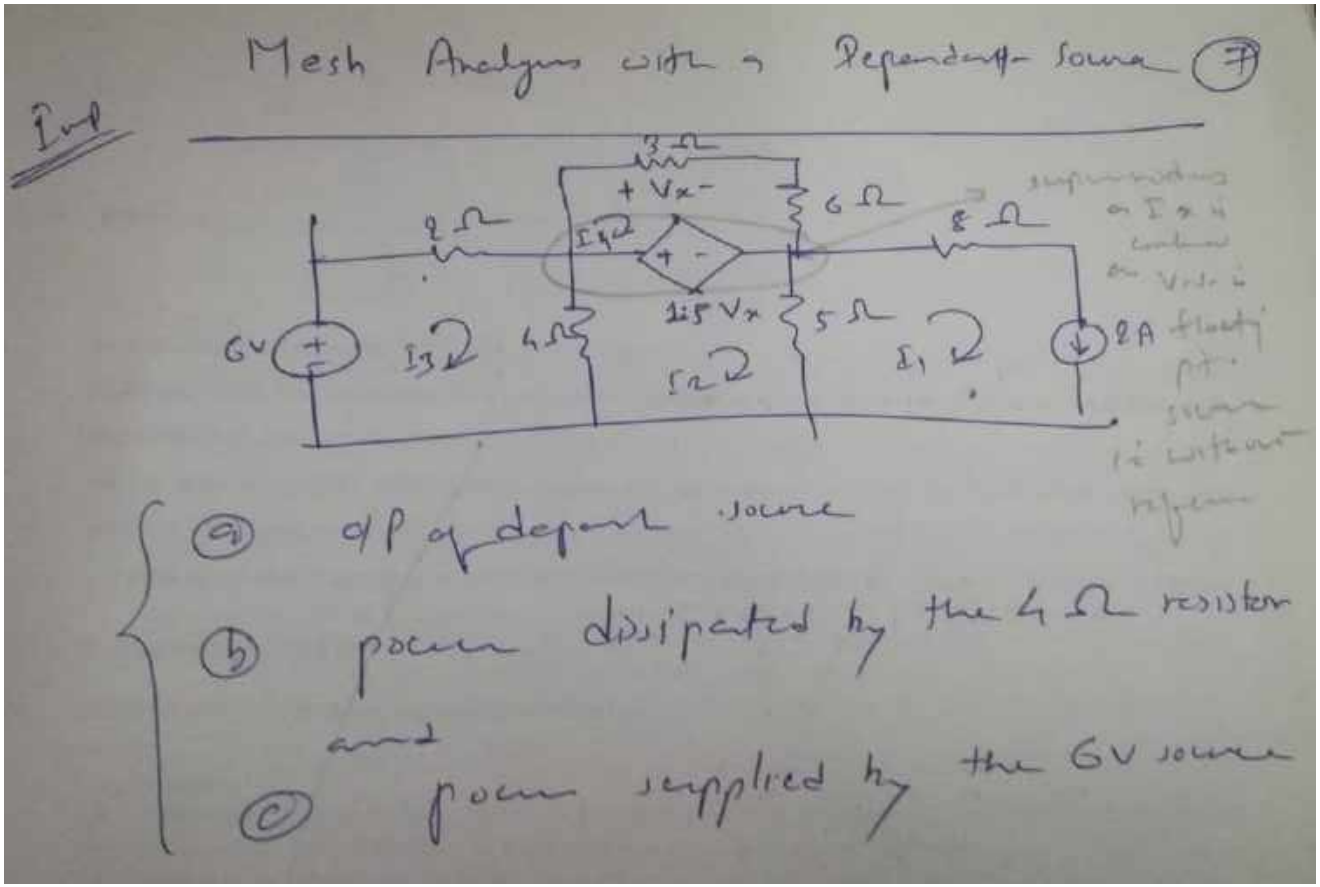
$$= 1 - \frac{P_{loss}}{P_{out} + P_{loss}}$$

Transformer has two losses:

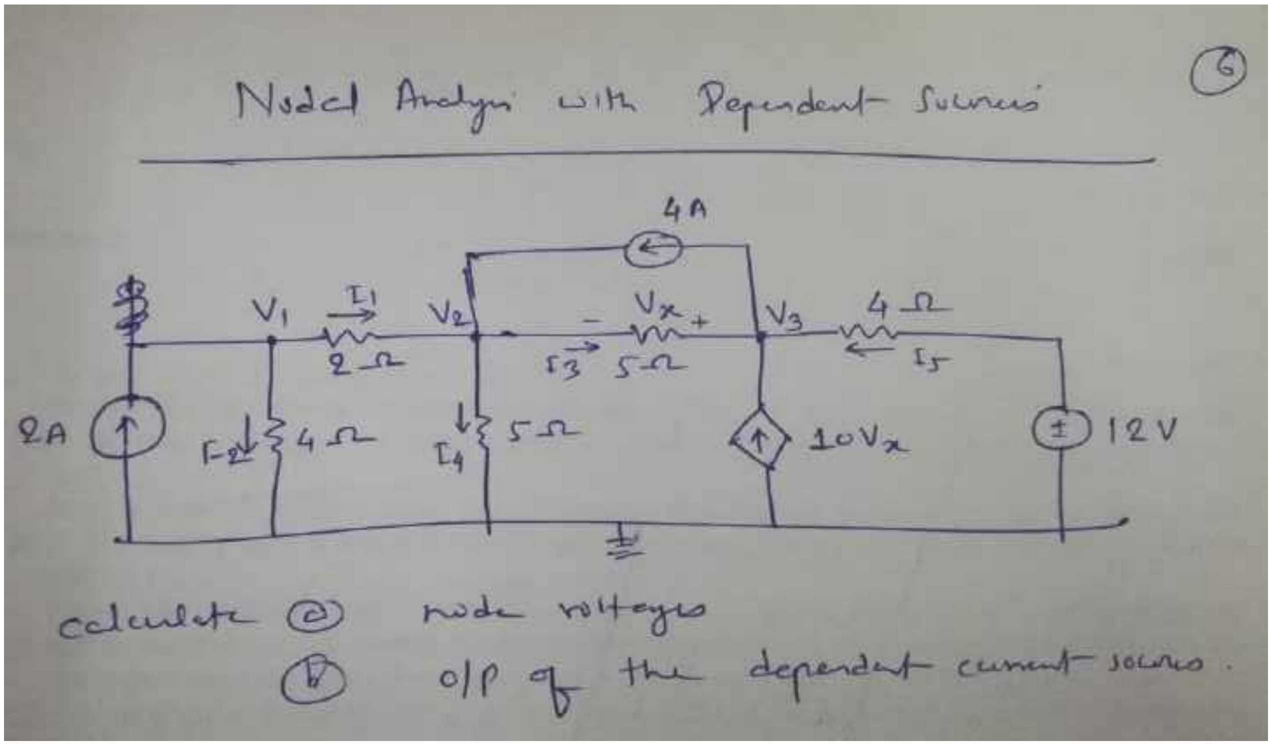
(1) Core (Iron) loss P_i, which is a constant.

(2) Copper (I^2R) loss P_c, a variable loss.

For given pf, η varies with I_2.


Mesh Analysis with a Dependent Source
3 Ω
+ Vx −
6 Ω
8 Ω
2 Ω
+ −
1.5 Vx
4 Ω
5 Ω
6V
I3
I2
I1
2A
(a) o/p of depent source
(b) power dissipated by the 4 Ω resistor
and
(c) power supplied by the 6V source

Dependent Sources



Dependent Sources

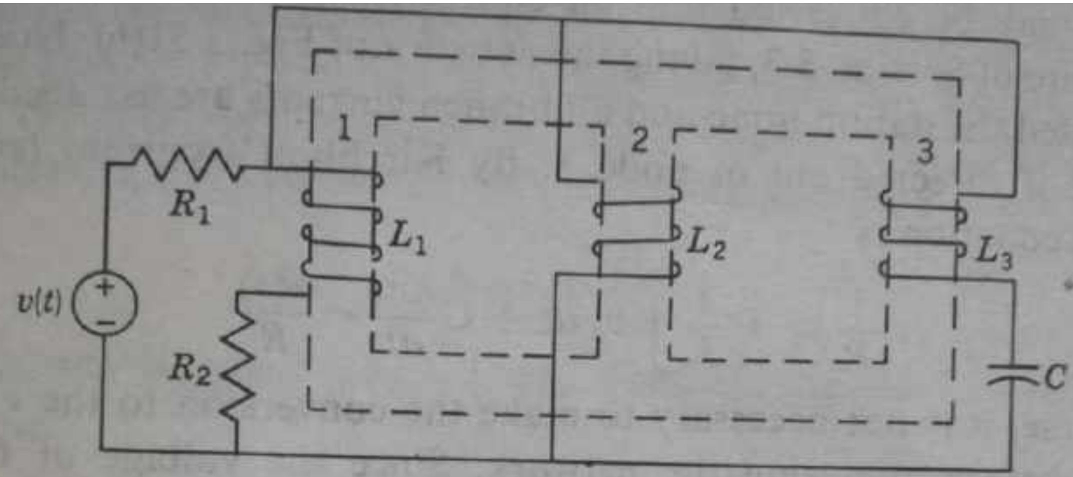


Fig. 3-29. Magnetically coupled network which is analyzed in Example 6.

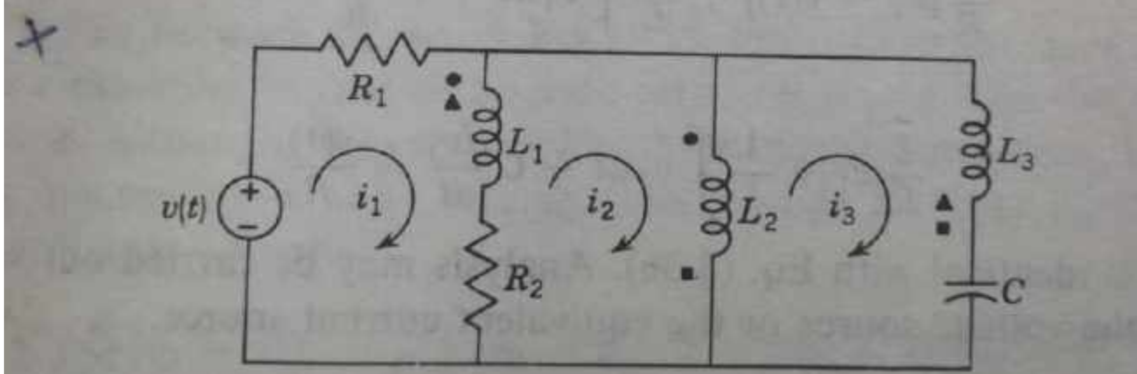


Fig. 3-30. A network representation which is equivalent to that shown in Fig. 3-29.

3-26. A network with magnetic coupling is shown in the figure. For the network, $M_{12} = 0$. Formulate the loop equations for this network using the Kirchhoff voltage law.

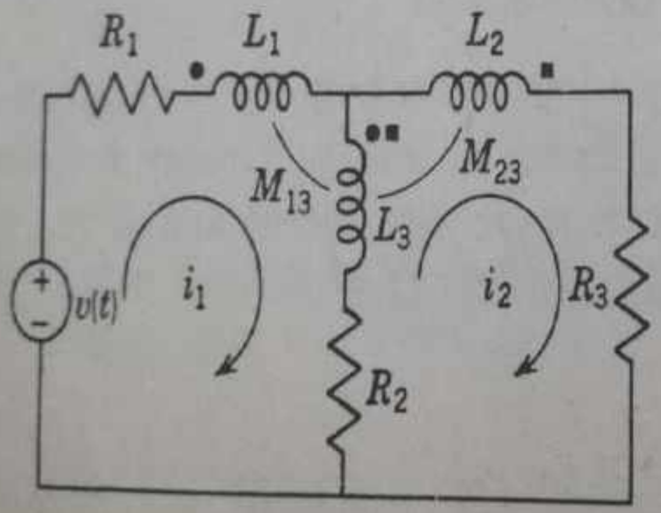
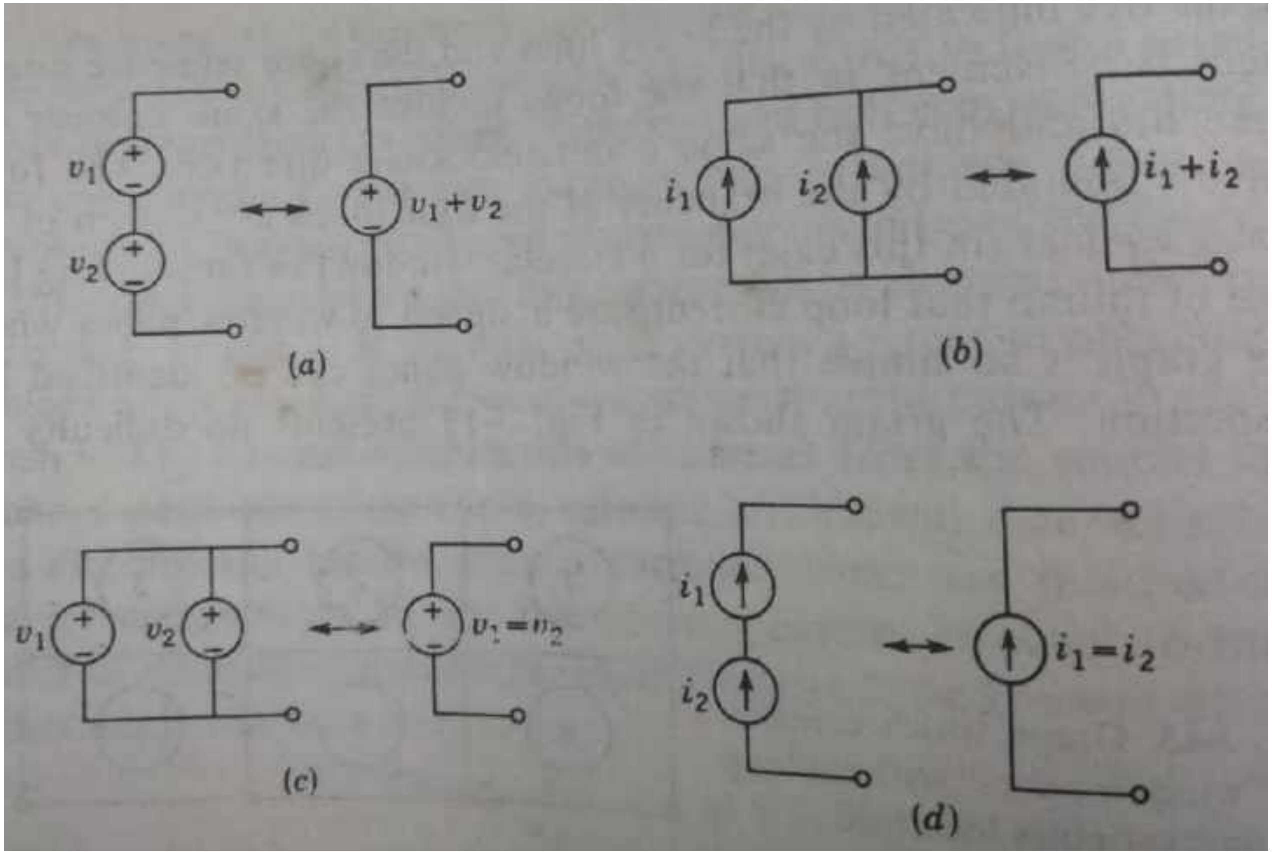


Fig. P3-26.


v_1
v_2
$v_1 + v_2$
(a)
i_1
i_2
$i_1 + i_2$
(b)
v_1
v_2
$v_1 = v_2$
(c)
i_1
i_2
$i_1 = i_2$
(d)

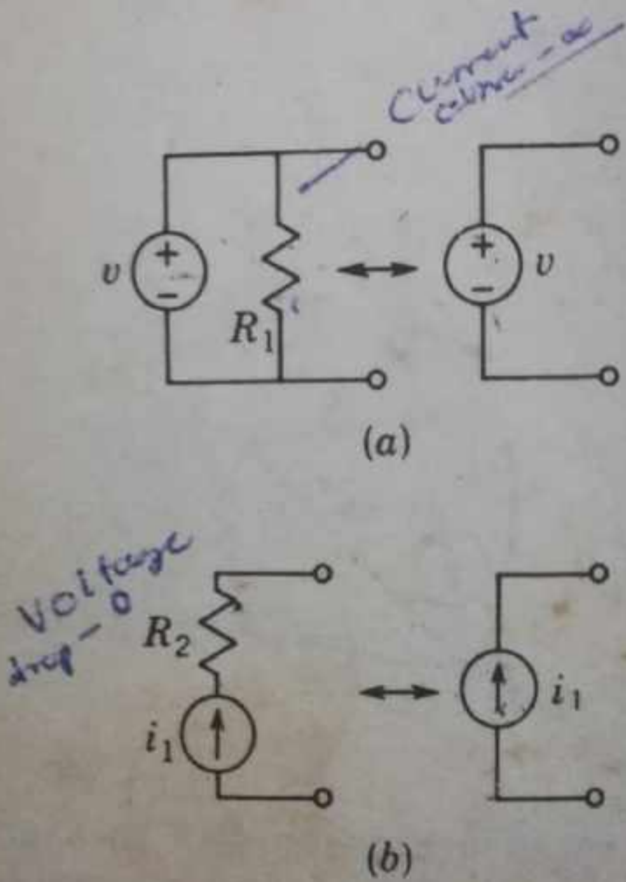


Fig. 3-15. Two examples of extraneous elements so far as terminal behavior is concerned.

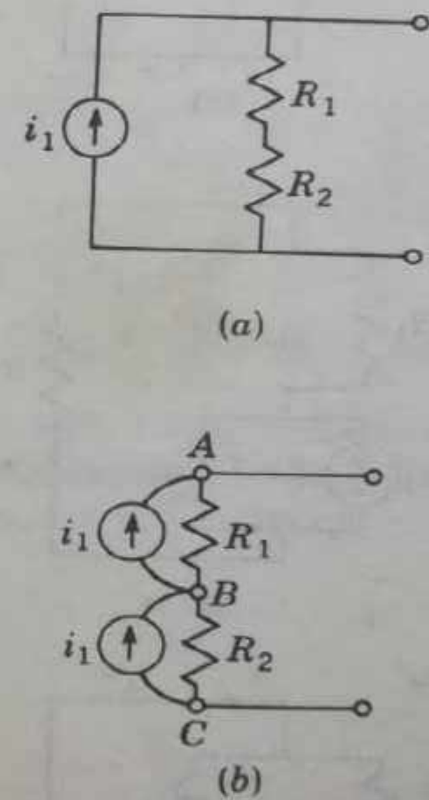


Fig. 3-21. Two equivalent networks illustrating the manner in which one source is replaced by two such that the Kirchhoff current law is still satisfied at each node.

this energy if the source ...

1-21. The current in a 1-H inductor follows the variation shown in the accompanying figure. The current increases from 0 at $t = 0$ at the rate of 1 amp/sec (for several seconds, at least). Find: (a) the flux linkages in the system after 1 sec, (b) the time rate of change of flux linkages in the system after 2sec, and (c) the quantity of charge having passed through the inductor after 1 sec.

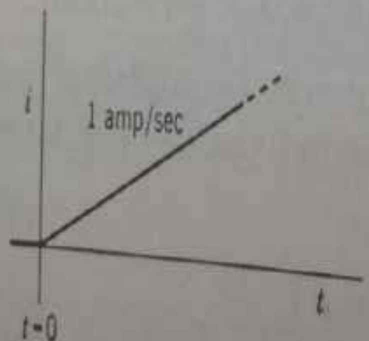


Fig. P1-21.

1-22. A time ...

1-24. In the circuit shown the switch K is closed at $t = 0$ (the reference time). The current flowing in the circuit is given by the equation $i(t) = (1 - e^{-t})$ amp, $t > 0$. At a certain time the current has a value of 0.63 amp. (a) At what rate is the current changing? (b) What is the value of the total flux linkages? (c) What is the rate of change of flux

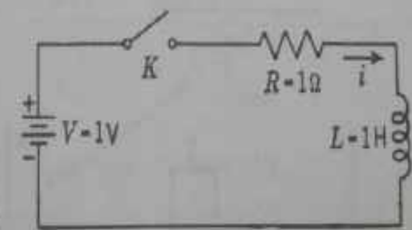


Fig. P1-24.

linkages? (d) What is the voltage across the inductor? (e) How much energy is stored in the magnetic field of the inductor? (f) What is the voltage across the resistor? (g) At what rate is energy being stored in the magnetic field of the inductor? (h) At what rate is energy being dissipated as heat? (i) At what rate is the battery supplying energy?

1-25. In the circuit shown the capacitor is charged to a voltage of 1 V, and at $t = 0$ the switch K is closed. The current in the circuit is known to be of the form $i(t) = e^{-t}$ amp, $t > 0$. At a certain time the current has a value of 0.37 amp. (a) At what rate is the voltage across the capacitor changing? (b) What is the value of the charge on the capacitor?

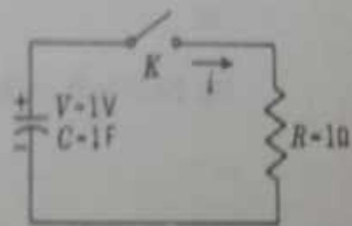


Fig. P1-25.

(c) What is the time rate of change of the product Cv? (d) What is the voltage across the capacitor? (e) How much energy is stored in the electric field of the capacitor? (f) What is the voltage across the resistor? (g) At what rate is energy being taken from the electric field of the capacitor? (h) At what rate is energy being dissipated as heat?

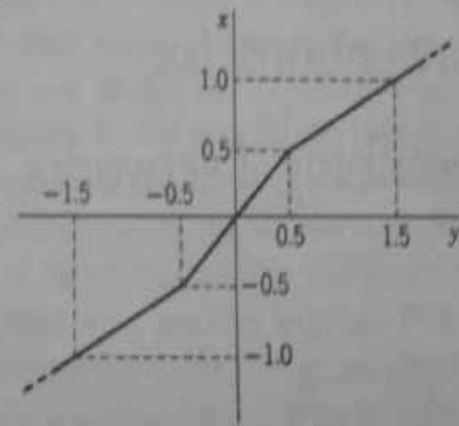


Fig. P1-39.

1-39. The figure shows a piecewise linear characteristic. Let $x = q_C$ and $y = v_C$ so that the characteristic represents a nonlinear capacitor. If the voltage applied to the capacitor is that shown in Fig. P1-32, plot the corresponding $i_C(t)$.

1-40. Repeat Prob. 1-39 if the voltage waveform is that shown in Fig. P1-30.

Source Transformations

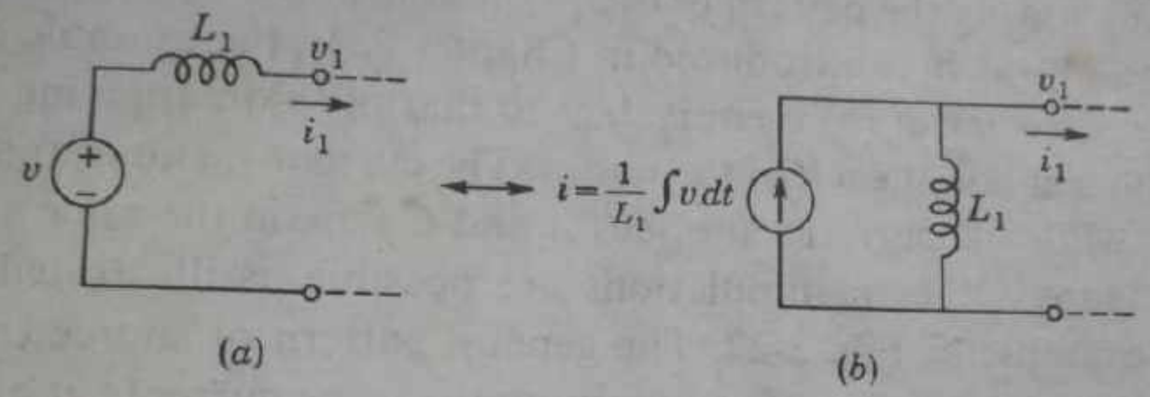


Fig. 3-17. Source transformation for a network with a single inductor.

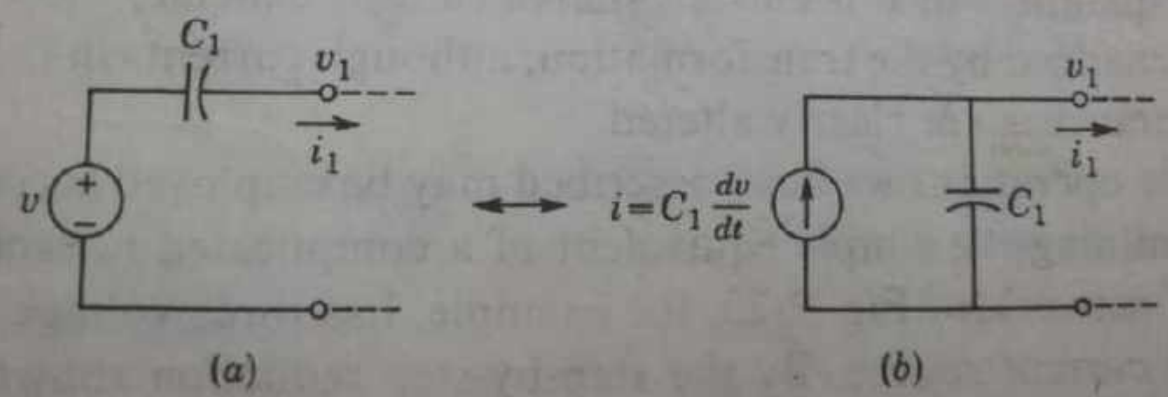


Fig. 3-18. Source transformation involving one capacitor.

3-36. For the network shown in the figure, determine the numerical value of the branch current i_1. All sources in the network are time invariant.

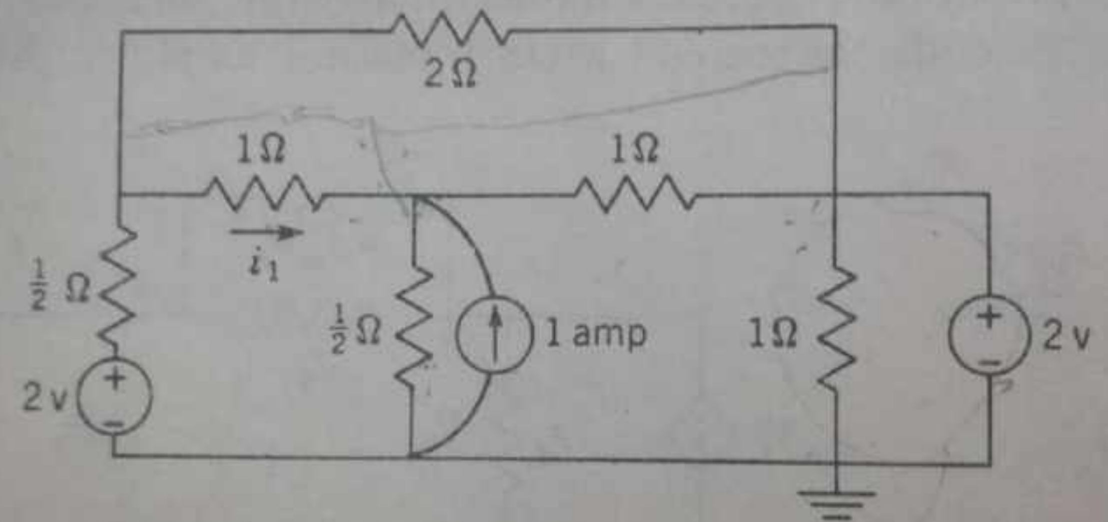


Fig. P3-36.

3-37. In the network of the figure, all sources are time invariant. Determine the numerical value of i_2.

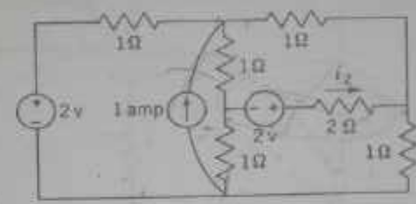


Fig. P3-37.

3-38. In the given network, all sources are time invariant. Determine the branch current in the 2-Ω resistor.

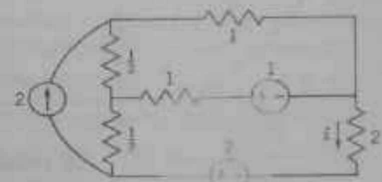
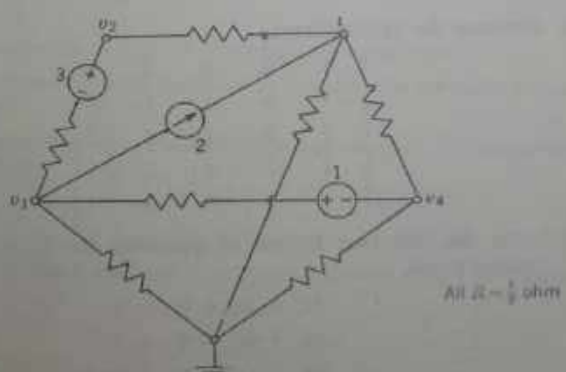


Fig. P3-38.

3-39. In the network of the figure, all voltage sources and current source are time invariant, and all resistors have the value $R = \frac{1}{2}\ \Omega$. Solve for the four node-to-datum voltages.



Thank you