LECTURE NOTE
DYNAMICS OF SOILS AND FOUNDATIONS
SECOND SEMESTER
M.TECH (GTE)

Dr. Debabrata Giri, Associate Professor
Department of Civil Engineering
Veer Surendra Sai University of Technology, Burla
Email: dgiri_ce@vssut.ac.in

DISCLAIMER

These lecture notes are being prepared and printed for the use in training the students. No commercial use of these notes is permitted and copies of these will not be offered for sale in any manner. This lecture notes will provide the basic knowledge about the course. The students are advised to refer Text and Reference books to gather more knowledge. The readers are encouraged to provide written feedbacks for improvement of course materials.

Dr. Debabrata Giri

Subject Name: DYNAMICS OF SOILS AND FOUNDATIONS	MCEGT201
Course Content	
Module-I	
Fundamentals of vibrations: single, two and multiple degree of freedom systems, vibration isolation, vibration absorbers, vibration measuring instruments.	
Module-II	
Wave propagation: elastic continuum medium, semi-infinite elastic continuum medium, soil behaviour under dynamic loading.	
Module-III	
Liquefaction of soils: liquefaction mechanism, factors affecting liquefaction, studies by dynamic tri-axial testing, shake table and blast tests, assessment of liquefaction potential.	
Module-IV	
Dynamic elastic constants of soil: determination of dynamic elastic constants, various methods including block resonance tests, cyclic plate load tests, wave propagation tests, oscillatory shear box test.	
Module-V	
Theory of Vibration of Foundation: Vertical, sliding, torsional and rocking oscillation of footing resting on Elastic half space. Oscillation of rigid circular footing supported by an elastic layer. Introduction of bearing capacity of dynamically loaded shallow foundation.	
Reference Books:	
• Das, B.M., “Fundamentals of Soil Dynamics”, Elsevier, 1983. • Steven Kramer, “Geotechnical Earthquake Engineering”, Pearson, 2008. • Prakash, S., Soil Dynamics, McGraw Hill, 1981. • Kameswara Rao, N.S.V., Vibration analysis and foundation dynamics, Wheeler Publication Ltd., 1998. • Richart, F.E. Hall J.R and Woods R.D., Vibrations of Soils and Foundations, Prentice Hall Inc., 1970. • Prakash, S. and Puri, V.K., Foundation for machines: Analysis and Design, John Wiley & Sons, 1998	
COURSE OUTCOME	
Students can interpret theory of vibration and resonance phenomenon, dynamic amplification.	
Students can investigate propagation of body waves and surface waves through soil.	
Students can predict dynamic bearing capacity and assess liquefaction potential of any site.	
Student exposed to different methods for estimation of dynamic soil properties required for design purpose.	
Students apply theory of vibrations to design machine foundation based on dynamic soil properties and bearing capacity	

1.0 FUNDAMETALS OF VIBRATION

In order to understand the behaviour of a structure subjected to dynamic load lucidly, one must study the mechanics of vibrations 'caused by the dynamic load. The pattern of variation of a dynamic load with respect to time may be either periodic or transient. The periodical motions can be resolved into sinusoidally varying components e.g. vibrations in the case of reciprocating machine foundations. Transient vibrations may have very complicated non-periodic time history e.g. vibrations due to earthquakes and quarry blasts.

A structure subjected to a dynamic load (periodic or transient) may vibrate in one of the following four ways of deformation or a combination there-of:

(i) Extensional

(ii) Bending

(iii) Shearing

(iv) Torsional

The forms of vibration mainly depend on the mass, stiffness distribution and end conditions of the system.

To study the response of a vibratory system, in many cases it is satisfactory to reduce it to an idealized system of lumped parameters. In this regard, the simplest model consists of mass, spring and dashpot. This chapter is framed to provide the basic concepts and dynamic analysis of such systems. Actual field problems which can be idealized to mass-spring-dashpot systems, have also been included.

1.1 Important Definition

Vibrations: If the motion of the body is oscillatory in character, it is called vibration.

Degrees of Freedom: The number *of* independent co-ordinates which are required to define the position of a system during vibration, is called degrees *of* freedom (Fig. 1)

Periodic Motion: If motion repeats itself at regular intervals of time, it is called periodic motion.

Free Vibration: If a system vibrates without an external force, then it is said to undergo free vibrations. Such vibrations can be caused by setting the system in motion initially and allowing it to move.

Natural Frequency: This is the property of the system and corresponds to the number of free oscillations made by the system in unit time.

Forced Vibrations: Vibrations that are developed by externally applied exciting forces are called forced vibrations. These vibrations occur at the frequency of the externally applied exciting force.

Forcing Frequency: This refers to the periodicity of the external forces which acts on the system during forced vibrations. This is also termed as operating frequency.

Frequency Ratio: The ratio of the forcing frequency and natural frequency of the system is referred as frequency ratio.

Amplitude of Motion: The maximum displacement of a vibrating body from the mean position is amplitude of motion.

Time Period: Time taken to complete one cycle of vibration is known as time period.

Resonance: A system having *n* degrees of freedom has *n* natural frequencies. If the frequency of excitation coincides with anyone of the natural frequencies of the system, the condition of resonance occurs. The amplitudes of motion are very excessive at resonance.

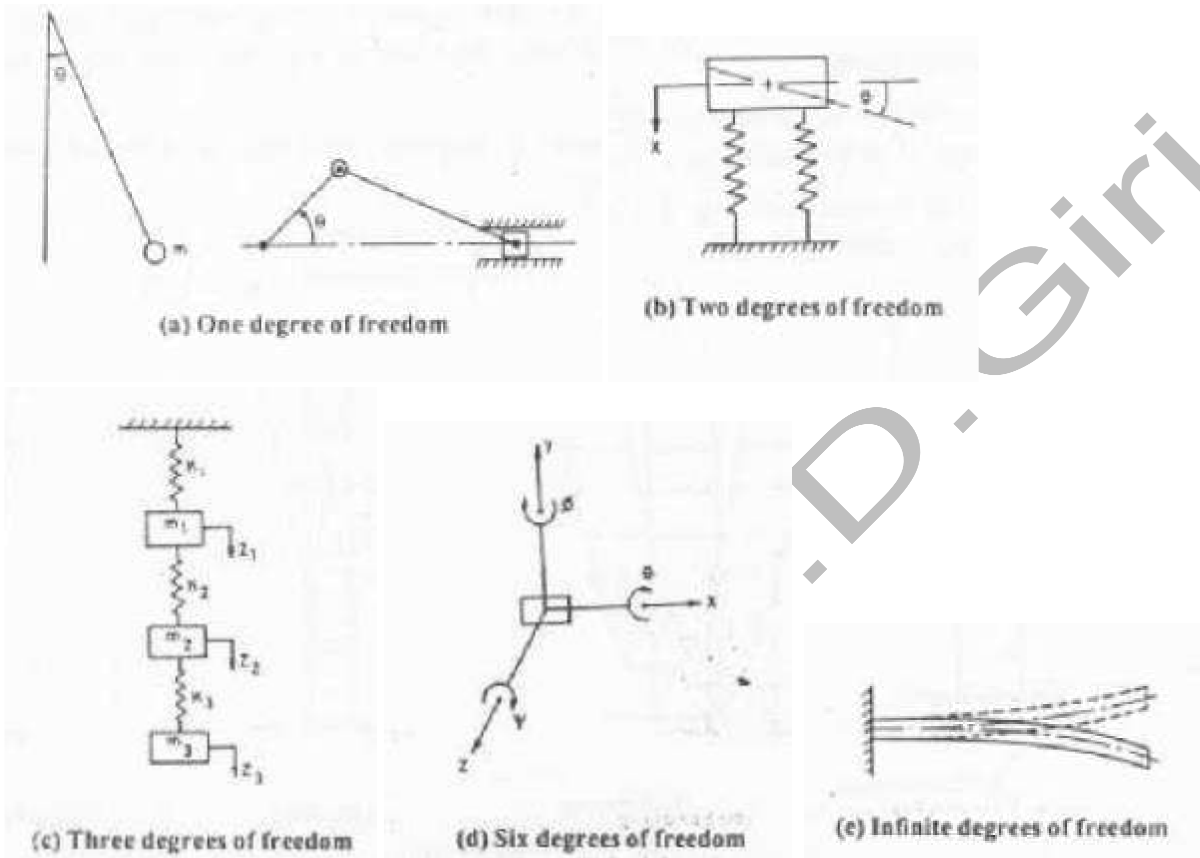


Fig.1.1: System with different degrees of freedom

Damping: All vibration systems offer resistance to motion due to their own inherent properties. This resistance is called damping force and it depends on the condition of vibration, material and type of the system. If the force of damping is constant, it is termed as Coulomb damping. If the damping force is proportional to the velocity, it is termed viscous damping. If the damping in a system is free from its material property and is contributed by the geometry of the system, it is called geometrical or radiation damping.

A typical concrete block is regarded as rigid as compared to the soil over which it rests. Therefore, it may be assumed that it undergoes only rigid-body displacements and rotations. Under the action of unbalanced forces, the rigid block may thus undergo displacements and oscillations as follows (Fig. 2)

1. Translation along *Z* axis
2. Translation along *X* axis
3. Translation along *Y* axis
4. Rotation about *Z* axis
5. Rotation about *X* axis
6. Rotation about *Y* axis

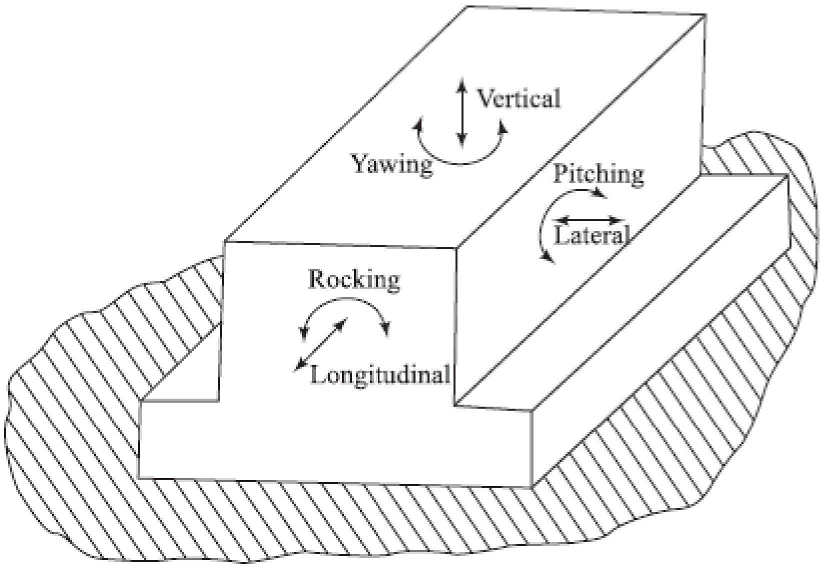


Fig.1.2: Modes of vibration of a rigid block foundation

Any rigid-body displacement of the block can be resolved into these six independent displacements. Hence, the rigid block has six degrees of freedom and six natural frequencies. Of six types of motion, translation along the Z axis and rotation about the Z axis can occur independently of any other motion. However, translation about the X axis (or Y axis) and rotation about the Y axis (or X axis) are coupled motions. Therefore, in the analysis of a block, we have to concern ourselves with four types of motions. Two motions are independent and two are coupled. For determination of the natural frequencies, in coupled modes, the natural frequencies of the system in pure translation and pure rocking need to be determined. Also, the states of stress below the block in all four modes of vibrations are quite different. Therefore, the corresponding soil-spring constants need to be defined before any analysis of the foundations can be undertaken

1.2 HARMONIC MOTION

Harmonic motion is the simplest form of vibratory motion. It may be described mathematically by the following equation:

$Z = A\sin(\omega t - \theta)$----------------- Eq.1.1

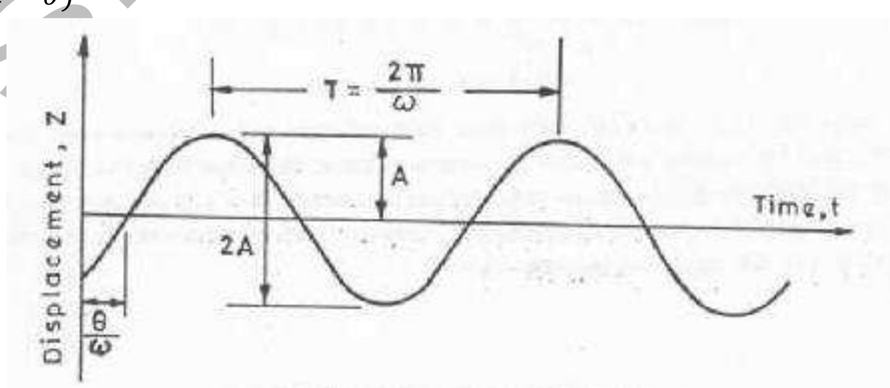


Fig.1.3: Quantities describing harmonic motion

The Eq. (1.1) is plotted as function of time in Fig.3. The various terms of this equation are as follows:

Z = Displacement of the rotating mass at any time *t*

A = Displacement amplitude from the mean position, sometimes referred as single amplitude. The distance **2A** represents the peak-to-peak displacement amplitude, sometimes referred to as double amplitude, and is the quantity most often measured from vibration records.

ω= Circular frequency in radians per unit time. Because the motion repeats itself after 2π radians, the frequency of oscillation in terms of cycles per unit time will be ${}^{\omega}/_{2\pi}$. It is denoted by f

θ= Phase angle. It is required to specify the time relationship between two quantities having the same frequency when their peak values having like sign do not occur simultaneously. In Eq. (1) the phase angle is a reference to the time origin.

The time period, T is given by

$T = \frac{1}{f} = \frac{2\pi}{\omega}$------------------- Eq.1.2

The velocity and acceleration of motion are obtained from the derivatives of Eq. (1.1)

Velocity = $\frac{dZ}{dt} = A\omega cos(\omega t - \theta)$----------------- Eq.1.3

$= A\omega \sin(\omega t - \theta + {}^{\pi}/_{2})$

Acceleration = $\frac{d^2Z}{dt^2} = \omega^2 Asin(\omega t - \theta)$--------------- Eq.1.4

$= \omega^2 Asin(\omega t - \theta + \pi)$

Equations (1.3) and (1.4) show that both velocity and acceleration are also harmonic and can be represented by vectors ωA and $\omega^2 A$, which rotate at the same speed as A, *i.e.* ω rad/unit time. These, however, lead the displacement and acceleration vectors by ${}^{\pi}/_{2}$ and π respectively. In Fig.4 vector representation of harmonic displacement, velocity and acceleration is presented considering the displacement as the reference quantity (θ = *0)*

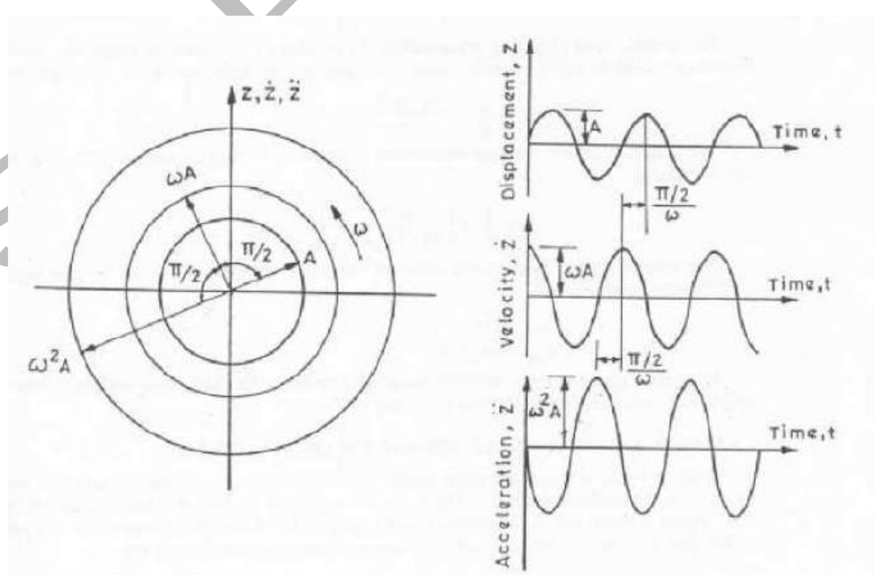


Fig.1.4: Vector representation of harmonic displacement, velocity, acceleration

1.3 VIBRATIONS OF A SINGLE DEGREE FREEDOM SYSTEM

The simplest model to represent a single degree of freedom system consisting of a rigid mass *m* supported by a spring and dashpot is shown in Fig. 1.5 *a.* The motion of the mass *m* is specified by one co-ordinate, Z. Damping in this system is represented by the dashpot, and the resulting damping force is proportional to the velocity. The system is subjected to an external time dependent force F *(t).*

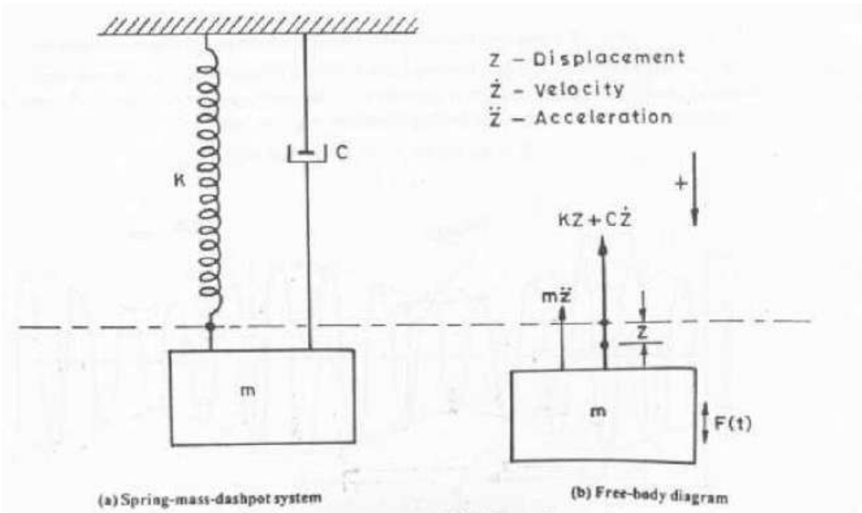


Fig.1.5: Single degree freedom system

Figure 1.5 *(b)* shows the free body diagram of mass ‘’*m* at any instant during the course of vibrations. The forces acting on the mass *m* are:

(i) Exciting force, F *(t):* It is the externally applied force that causes the motion of the system.

(ii) Restoring force, F_r.: It is the force exerted by the spring on the mass and tends to restore the mass, to its original position. For a linear system, restoring force is equal to K Z, where K is the spring constant and indicates the stiffness. This force always acts towards the equilibrium position of the system.

(iii) Damping force, F_d The damping force is considered directly proportional to the velocity and given by $C\dot{Z}$, where C is called the coefficient of viscous damping; this force always opposes the motion.

In some problems in which the damping is not viscous, the concept of viscous damping is still used by defining an equivalent viscous damping which is obtained so that the total the energy dissipated per cycle is same as for the actual damping during a steady state of motion.

(iv) Inertia force, F.: It is due to the acceleration of the mass and is given by $m\ddot{Z}$. According to De-Alembert’s principle, a body which is not in static equilibrium by virtue of some acceleration which it possess, can be brought to static equilibrium by introducing on it an inertia force. This force acts through the centre of gravity of the body in the direction opposite to that of acceleration.

The equilibrium of mass m gives

$m\ddot{Z} + C\dot{Z} + KZ = F(t)$------------- Eq.1.5

which is the equation of motion of the system.

1.3.1 Undamped Free Vibrations

For undamped free vibrations, the damping force and the exciting force are equal to zero. Therefore the equation of motion of the system becomes

$m\ddot{Z} + KZ = 0$------------ Eq.1.6

Or $\ddot{Z}+\frac{K}{m}Z = 0$

The solution of this equation can be obtained by substituting

$Z = A_1 cos\omega_n t + A_2 sin\omega_n t$--------------- Eq.1.7

where A_1 and A_2 are both constants and ω_n undamped natural frequency.

Now Substituting Eq. (7) in Eq. (6), we get;

$-\omega_n^2(A_1 cos\omega_n t + A_2 sin\omega_n t) + \frac{K}{m}(A_1 cos\omega_n t + A_2 sin\omega_n t)$------------- Eq.1.8

Or $\omega_n = \pm\sqrt{\frac{K}{m}}$

The values of constants A_1 and A_2 are obtained by substituting proper boundary conditions. We may nave the following two boundary conditions:

(i) At time *t* = 0, displacement Z = Z_0 and

(ii) At time t= 0, velocity $\dot{Z} = V_0$

Substituting the first boundary condition in Eq. (1.7), we get

$A_1 = Z_0$ and

$\dot{Z} = -A_1\omega_n sin\omega_n t + A_2\omega_n cos\omega_n t$--------------- Eq.1.9

Substituting the second boundary conditions in Eq. (1.9), we have

$A_2 = \frac{V_0}{\omega_n}$--------------- Eq.1.10

Hence

$Z = Z_0 cos\omega_n t + \frac{V_0}{\omega_n}\text{sin}\omega_n t$----------------- Eq.1.11

Now let $Z_0 = A_Z cos\theta$ -------------------- Eq.1.12

and $\frac{V_0}{\omega_n}$=$A_Z sin\theta$--------------------------- Eq.1.13

Substitution of Eqs. (1.12) and (1.13) into Eq. (1.11) yields

$Z = A_Z cos(\omega_n t - \theta)$-------------------- Eq.1.14

Where $\theta = tan^{-}\left(\frac{V_0}{\omega_n Z_0}\right)$------------------- Eq.1.15

And $A_Z = \sqrt{Z_0^2 + (\frac{V_0}{\omega_n})^2}$---------------------- Eq.1.16

The displacement, velocity and acceleration of mass as expressed in above eqs can be graphically shown as

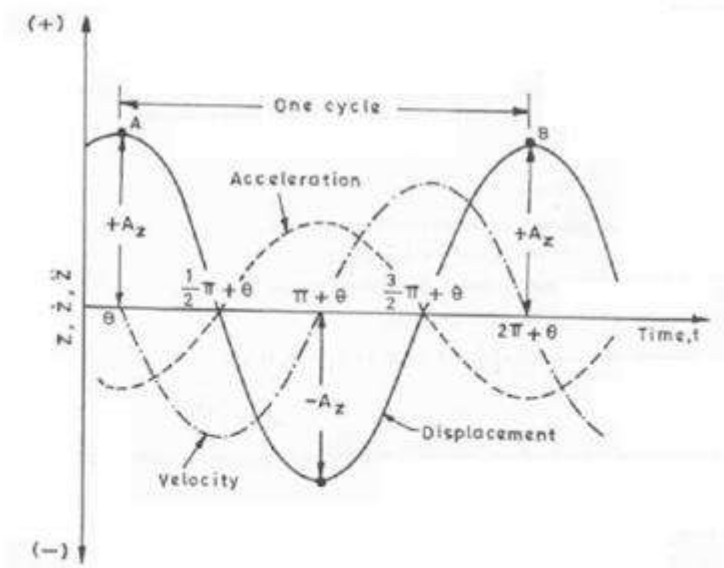


Fig.1.6: Plot of displacement, velocity and acceleration of vibrating mass-spring system

It is evident from Fig. 1.6 that nature of foundation displacement is sinusoidal. The magnitude of Maximum displacement is Az. The time required for the motion to repeat itself is the period of vibration,

T and is therefore given by. $T = \frac{1}{f} = \frac{2\pi}{\omega}$

The natural frequency of oscillation, $f_n$is given by

$f_n = \frac{1}{T} = \frac{\omega_n}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{K}{m}}$----------------- Eq.1.17

It can be shown that $f_n = \frac{1}{2\pi}\sqrt{\frac{g}{\delta_{st}}}$----------- Eq.1.18

Where, $\delta_{st}$is the static deformation of spring.

1.3.2 Free Vibrations with Viscous Damping

For damped free vibration system *(i.e.*, the excitation force F_0 sin$\omega_n t$ on the system is zero), the differential equation of motion can be written as

$m\ddot{Z} + C\dot{Z} + KZ = 0$------------------------ Eq.1.19

where C is the damping constant or force per unit velocity. The solution of Eq. (1.19) may be written as

$Z = Ae^{\lambda t}$---------------------- Eq.1.20

where A and λ are arbitrary constants. By substituting the value of Z given by Eq. (1.20) in Eq. (1.19), we get

$mA\lambda^2 e^{\lambda t} + CA\lambda e^{\lambda t} + KAe^{\lambda t} = 0$------------ Eq.1.21

Or $\lambda^2 + \left(\frac{C}{m}\right)\lambda + \frac{K}{m} = 0$-------------------------- Eq.1.22

By solving Eq. (22)

$\lambda_{1,2} = -\frac{C}{2m} \pm \sqrt{(\frac{C}{2m})^2 - \frac{K}{m}}$---------------- Eq.1.23

The complete solution of Eq.(1.19) is given by

$Z = A_1 e^{\lambda_1 t} + A_2 e^{\lambda_2 t}$------------------- Eq.1.24

The physical significance of this solution depends upon the relative magnitudes of $(\frac{C}{2m})^2$ and *(K/m)*, which determines whether the exponents are real or complex quantities.

Case I: $(\frac{C}{2m})^2 > (K/m$

The roots λ_1 and λ_2 are real and negative. The motion of the system is *not* oscillatory but is an exponential as shown in Fig.1.7).

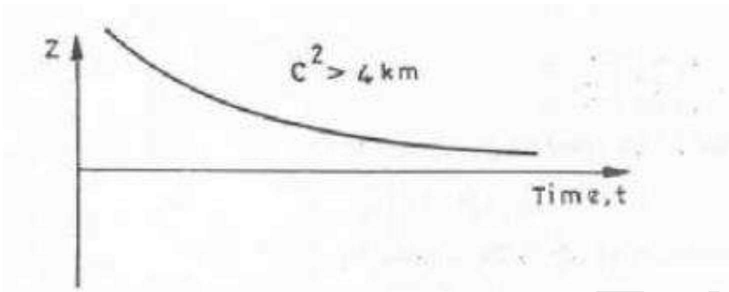


Fig.1.7: Free Vibration of over Damped Viscous system

Because of the relatively large damping, so much energy is dissipated by the damping force that there is sufficient kinetic energy left to carry the mass and pass the equilibrium position. Physically this means a relatively large damping and the system is said to be **over damped.**

Case II: $(\frac{C}{2m})^2 = (\frac{K}{m})$

The roots λ_1 and λ_2 are equal and negative. Since the equality must be fulfilled, the solution is given by

$Z = (A_1 + A_2) e^{\lambda t}$--------------------- Eq.1.25

In this case also, there is no vibratory motion. It is similar to over damped case except that it is possible for the sign to change once as shown in Fig.1. 8.

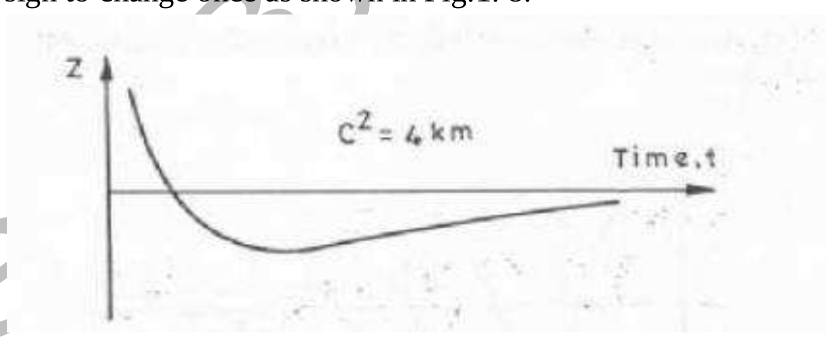


Fig.18: Free Vibration Critically damped viscous system

This case is of little importance in itself; it assumes greater significance as a measure of the damping capacity of the system.

When $(\frac{C}{2m})^2 = (\frac{K}{m})$, C=C_c

And $C_c = 2\sqrt{Km}$-------------------- Eq.1.26

The system in this condition is known as critically damped system and C_c is known as critical damping constant.' The ratio of the actual damping constant to the critical damping constant is defined as damping ratio:

Damping ratio, $\xi = \frac{C}{C_c}$

By substituting this value of' $\xi = \frac{C}{C_c}$ in Eq. (1.23), we get

$\lambda_{1,2} = (-\xi \pm \sqrt{(\xi)^2 - 1})\omega_n$------------------ Eq.1.27

Case III: $(\frac{C}{2m})^2 < (\frac{K}{m})$

The roots λ_1 and λ_2 are complex and are given as

$\lambda_{1,2} = (-\xi \pm i\sqrt{1 - (\xi)^2})\omega_n$------------------ Eq.1.28

The complete solution to the Eq.27, gives

$Z = A_1 e^{(-\xi + i\sqrt{1-\xi^2})\omega_n t} + A_2 e^{(-\xi - i\sqrt{1-\xi^2})\omega_n t}$------------------ Eq.1.29

Or $Z = e^{-\xi\omega_n t}[A_1 e^{(i\sqrt{1-\xi^2})\omega_n t} + A_2 e^{(-i\sqrt{1-\xi^2})\omega_n t}]$--------------- Eq.1.30

The above equation can be written as

$Z = e^{-\xi\omega_n t}[C_1 \sin(\omega_n\sqrt{1-\xi^2}t) + C_2 \cos(\omega_n\sqrt{1-\xi^2}t)]$-------------- Eq.1.31

Or $Z = e^{-\xi\omega_n t}[C_1 \sin(\omega_{nd} t + C_2 \cos(\omega_{nd} t]$-------------- Eq.1.32

Where $\omega_{nd} = \omega_n(1 - \xi^2)$is known as damped natural frequency

The motion of the system is oscillatory (Fig.1.9) and the amplitude of vibration goes on decreasing in an exponential fashion.

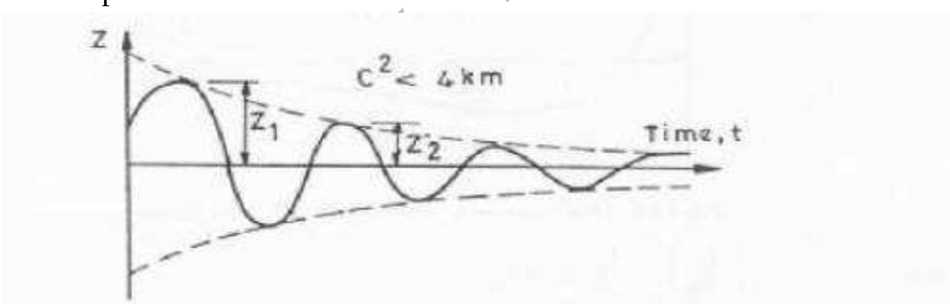


Fig. 1.9: Free Vibration under damped viscous system

As a convenient measure of damping, we may compute the ratio of amplitudes of the successive cycles of vibration.

$\frac{Z_1}{Z_2} = \frac{e^{-\omega_n \xi t}}{e^{-\omega_n \xi(1+2\pi/\omega_n t)}}$------------ Eq.1.33

Or $\frac{Z_1}{Z_2} = \frac{2\pi\xi}{e^{\sqrt{1-\xi^2}}}$----------------- Eq.1.34

Now taking logarithm, we get

$ln\frac{Z_1}{Z_2} = \frac{2\pi\xi}{\sqrt{1-\xi^2}}$------------------ Eq.1.35

The natural logarithm of ratio of two consecutive peak amplitudes is known **as Logarithmic decrement.**

Thus, damping of a system can be obtained from a free vibration record by knowing the successive amplitudes which are one cycle apart.

If the damping is very small, it may be convenient to measure the differences in peak amplitudes for a number of cycles, say *n*, as

$\xi = \frac{1}{2\pi n} ln \frac{Z_0}{Z_n}$------------------ Eq.1.36

Therefore, a system is

Over damped if ξ> 1;

Critically damped if ξ = 1 and

Under damped if ξ< 1

1.3.2 Forced Vibrations of Single Degree Freedom System

In many cases of vibrations caused by rotating parts of machines, the systems are subjected to periodic exciting forces. Let us consider the case of a single degree freedom system: which is acted upon by a steady state sinusoidal exciting force having magnitude F and frequency ω i.e. *F(t)* =F_0 sin *ωt*. For this case the equation of motion (Eq.1. 5) can be written as

$m\ddot{Z} + C\dot{Z} + KZ = F_0 sin\omega t$------------- Eq.1.37

Eq.(37) is a linear, non-homogeneous, second order differential equation. The solution of this equation consists of two parts namely *(i*) complementary function, and *(ii)* particular integral. The complementary function is obtained by considering no forcing function. Therefore the equation of motion in this case will be:

$m\ddot{Z}_1 + C\dot{Z}_1 + KZ_1 = 0$----------------- Eq.1.38

The solution of Eq. (1.38) has already been obtained in the previous section and is given by,

$Z_1 = e^{-\xi\omega_n t}[C_1 \sin(\omega_{nd}t + C_2 \cos(\omega_{nd}t]$-------------- Eq.1.39

Here Z_1 represents the displacement of mass m at any instant *t* when vibrating without any forcing function. .

The particular integral is obtained by rewriting Eq. (1.37) as

$m\ddot{Z}_2 + C\dot{Z}_2 + KZ_2 = F_0 sin\omega t$--------------------- Eq.1.40

Where, Z_2= displacement of mass m at any instant of time t when vibrating with forcing function.

The, solution of Eq. (40) is given as

$Z_2 = A_1 cos\omega_n t + A_2 sin\omega_n t$------------------- Eq.1.41

where A_1 and A_2 are two, arbitrary constants. Substituting Eq. (1.41) in Eq.1.40

$m(-A_1\omega^2 sin\omega t - A_2\omega^2 cos\omega t) + C(A_1\omega cos\omega t - A_2\omega sin\omega t) + K(A_1 sin\omega t + A_2 cos\omega t) = F_0 sin\omega t$----------------------------- Eq.1.42

Considering sine and cosine functions in Eq. (1.42) separately,

$(-mA_1\omega^2 + KA_1 - CA_2\omega)sin\omega t = F_0 sin\omega t$-------------- Eq.1.43

$(-mA_2\omega^2 + KA_2 + CA_1\omega)cos\omega t = 0$-------------- Eq.1.44

From Eq.1.43

$A_1\left(\frac{K}{m} - \omega^2\right) - A_2\left(\frac{C}{m}\right)\omega = \frac{F_0}{m}$------------ Eq.1.45

From Eq.1.44

$A_1\left(\frac{C}{m}\omega\right)+A_2(\frac{K}{m}-\omega^2)=0$-------------- Eq.1.46

By solving these equations, we have

$A_1=\frac{(K-m\omega^2)F_0}{(K-m\omega^2)^2+C^2\omega^2}$---------------- Eq.1.47

$A_2=\frac{-C\omega F_0}{(K-m\omega^2)^2+C^2\omega^2}$-------------- Eq.1.48

Let us assume

$x=Xcos(\omega t+\alpha)$-------------- Eq.1.49

Where $\alpha=tan^{-1}\frac{A_1}{A_2}=tan^{-1}\left(\frac{K-m\omega^2}{C\omega}\right)=tan^{-1}\left(\frac{1-(\frac{\omega}{\omega_n})^2}{2\xi\frac{\omega}{\omega_n}}\right)$---------------- Eq.1.50

Amplitude $X=\sqrt{{A_1}^2+{A_2}^2}=\frac{F_0/_K}{\sqrt{(1-\frac{\omega^2}{\omega_n^2})^2+4\xi^2(\frac{\omega}{\omega_n})^2}}$---------------------------- Eq.1.51

Now complete solution is given as

$x(t)=e^{-\xi\omega_n t}(C_1cos\omega_d t+C_2sin\omega_d t)+Xcos(\omega t+\alpha)$---------------- Eq.1.52

Finally after some time 1^{ST} part vanishes and vibration is due to steady state which is due to 2^{nd} term only.

The system will vibrate harmonically, with the same frequency as the forcing and the peak amplitude is given by

$A_Z=\frac{F_0/_K}{\sqrt{(1-\frac{\omega^2}{\omega_n^2})^2+4\xi^2(\frac{\omega}{\omega_n})^2}}$------------------------- Eq.1.53

The quantity $F_0/_K$ equals to the static deflection of the mass under force F_0. Dynamic magnification factor M is the ratio of the dynamic amplitude A_z to the static deflection and is given by

$M=\frac{1}{\sqrt{(1-\frac{\omega^2}{\omega_n^2})^2+4\xi^2(\frac{\omega}{\omega_n})^2}}$----------------------- Eq.1.54

It would be seen that the frequency ratio near ($\frac{\omega}{\omega_n}$=η=) 1, the value of frequency is maximum. This is called resonance and the forcing frequency at which this occurs is called as the **resonant frequency.**

Differentiating Eq. (1.53) with respect to η and equating to zero, it can be shown that resonance will occur at a frequency ratio given by

$\eta=\sqrt{1-2\xi^2}$------------------- Eq.1.55

which is approximately equal to unity for small values of ξ

Now $\omega_{nd}=\omega_n\sqrt{1-2\xi^2}$------------------------ Eq.1.56

This is known as damped resonance frequency.

Maximum value of magnification factor can be obtained as

$M_{max}=\frac{1}{2\xi\sqrt{1-\xi^2}}$------------------------ Eq.1.57

Example:1

An unknown weight W is attached to the end of an unknown spring k and natural frequency of the system was found to be 90 cpm. If 1 kg weight is added to W, the natural frequency reduced to 75 cpm. Determine the unknown weight W and spring constant k.

Sol:

$$\omega_n = 90\ cpm$$

When 1 kg added to the weight W, the natural frequency reduced to 75 cpm

ω_n = 90 cpm

Or f= 90/60=1.5 cps

$$\omega = 2\pi f = 2\pi \times 1.5\ r/s$$

$\omega^2 = {K}/{m} = {Kg}/{W} = 88.92$--------------1

Again, *f= 75/60=1.25 cps*

And

$$\omega = 2\pi f = 2\pi \times 1.25 = 61.88$$

${Kg}/{(W+1)} = 61.88$----------------2

Solving for 1 and 2, we get W=2.27kg

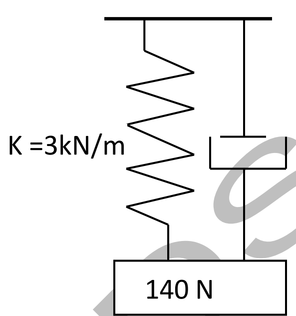
And Spring constant K=201 kg/cm

Example 2:

A spring and dashpot are attached to a body weighing 140 N. The spring constant is 3.0 kN/m. The dashpot has a resistance of 0.75 N at a velocity of 0.06 m/s. Determine the following for free vibration:

(i) whether the system is over damped, under damped or critically damped

Sol:



Given:

K=3 kN/m, Damping force = 0.75 N at a velocity of 0.06 m/s

Hence damping coefficient C= 0.75/0.06=12.5 N.s/m

We know:

for over damped vibration

$$\left(\frac{C}{2m}\right)^2 > \left(\frac{K}{m}\right)$$

For critical damping

$$\left(\frac{C}{2m}\right)^2 = \left(\frac{K}{m}\right)$$

For under damped

$\left(\frac{C}{2m}\right)^2 < (K/m)$

Now checking for damping condition, we have $\frac{C}{2m} = \frac{12.5\times 9.81}{2\times 140} = 0.437$

Again, $\sqrt{\frac{K}{m}} = \sqrt{\frac{3000\times 9.81}{140}} = 14.5$

So the system is under damped.

Example 3:

An SDF system is excited by a sinusoidal force. At resonance the amplitude of displacement was measured to be 2 mm. At an exciting frequency of one-tenth of the natural frequency of the system, the displacement amplitude was measured to be 0.2 mm. Estimate the damping ratio of the system.

Sol:

Given:

U_{max}= 2 mm

u=0.2mm at the exciting frequency of one-tenth of the natural frequency (At small frequency)

We know that

$$u = \frac{F_0/K}{\sqrt{(1-\frac{\omega^2}{\omega_n^2})^2 + 4\xi^2(\frac{\omega}{\omega_n})^2}}$$

At low frequency ratio $\frac{u}{F_0/k}$=1

And $\frac{u_{max}}{F_0/k}$=$\frac{1}{2\xi\sqrt{1-\xi^2}} \sim \frac{1}{2\xi}$

Hence $\frac{0.2}{F_0/K} = 1$

So $\frac{F_0}{K}$=0.2

Now $\frac{2}{F_0/K} = \frac{1}{2\xi}$, which gives $\frac{2}{0.2} = \frac{1}{2\xi}$

Hence ξ=$\frac{0.2}{4} = 0.05$ or 5%

Example 4:

A body weighing 600 N is suspended from a spring which deflects 12 mm under the load. It is subjected to a damping effect adjusted to a value 0.2 times that required for critical damping. Find the natural frequency of the un-damped and damped vibrations, and in the latter case, determine the ratio of successive amplitudes.

Sol:

$$K = \frac{W}{\delta} = \frac{600}{12 \times 10^{-3}} = 5 \times 10^4 \, N/m$$

m=60 kg

Damping ratio $\xi = \frac{C}{C_c} = 0.2$

Natural Frequency=$\sqrt{\frac{K}{m}}$=$\sqrt{\frac{5\times10^4}{60}}$=28.86 rpm

Damping frequency $\omega_d = \omega_n\sqrt{1-\xi^2} = 28.86 \times \sqrt{1-0.2^2} = 28.27$

Now $\delta = 2\pi\xi = ln\frac{Z_1}{Z_2}$

So $2\pi \times 0.2 = \ ln\frac{Z_1}{Z_2}$

Or $\frac{Z_1}{Z_2} = e^{2\pi\times 0.2} = 3.51$

Problem No.1

For a machine foundation, given weight = 60 kN, spring constant = 11,000 kN/m, and *c* = 200 kN-s/m, determine

(a) whether the system is overdamped, underdamped, or critically damped,

(b) the logarithmic decrement, and

(c) the ratio of two successive amplitudes.

Problem No.2

For Problem No.1, determine the damped natural frequency.

Problem No. 3

A machine and its foundation weight 140 kN. The spring constant and the damping ratio of the soil supporting the soil may be taken as 12 × 104 kN/m and 0.2, respectively. Forced vibration of the foundation is caused by a force that can be expressed as Q (kN) = Q_0 sin ωt

Q_0 = 46 kN,ω = 157 rad/s

Determine

(a) the undamped natural frequency of the foundation,

(b) amplitude of motion, and

(c) maximum dynamic force transmitted to the sub-grade.

1.4 TWO DEGREES OF FREEDOM SYSTEMS

1.4.1 Undamped free vibration

Figure 1.10 shows a mass-spring system with two degrees of freedom.

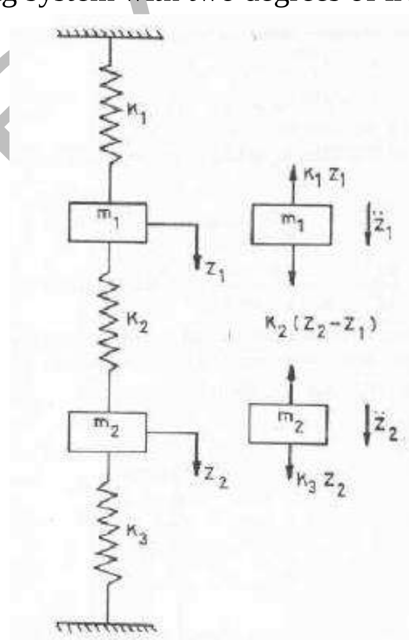


Fig.1.10: Free vibration of two degree freedom system

Let Z_1 and Z_2 be the displacements of mass m_1 and mass m_2 respectively. The equations of motion of the system can be written:

$m_1\ddot{Z}_1 + K_1Z_1 + K_2(Z_1 - Z_2 = 0)$---------------- Eq.1.58

AND

$m_2\ddot{Z}_2 + K_3Z_2 + K_2(Z_2 - Z_1 = 0)$---------------- Eq.1.59

The solutions of Eq. (1.58) and (1.59) will be of the following form

$Z_1 = A_1\sin(\omega_n t)$--------------- -------------------- Eq.1.60

$Z_2 = A_2\sin(\omega_n t)$---------------------------------- Eq.1.61

Substitution of Eqs. (1.20) and (1.61), into Eqs. (1.58) and (1.59) yields:

$(K_1 + K_2 - m_1\omega_n^2)A_1 - K_2A_2 = 0$-------------- Eq.1.62

$(K_2 + K_3 - m_2\omega_n^2)A_2 + K_2A_1 = 0$-------------- Eq.1.63

For nontrivial solutions of ω_n in Eqs. (1.62) and (1.63),

$$\begin{vmatrix} K_1 + K_2 - m_1\omega_n^2 & -K_2 \\ -K_2 & K_2 + K_3 - m_2\omega_n^2 \end{vmatrix} = 0\text{--------} \quad \text{Eq.1.64}$$

Or

$$\omega_n^4 - \left(\frac{K_1+K_2}{m_1} + \frac{K_2+K_3}{m_2}\right)\omega_n^2 + \frac{K_1K_2+K_2K_3+K_3K_1}{m_1m_2} = 0\text{-----------} \quad \text{Eq.1.65}$$

Equation (1.65) is quadratic in ω_n^2, and the roots of this equation are:

$$\omega_n^2 - \frac{1}{2}\left[\frac{K_1+K_2}{m_1} + \frac{K_2+K_3}{m_2}\right] \pm \sqrt{\left(\frac{K_1+K_2}{m_1} - \frac{K_2+K_3}{m_2}\right)^2 + \frac{4K_2^2}{m_1m_2}}\text{---------------} \quad \text{Eq.1.66}$$

From Eq.(9),two values of natural frequencies (ω_{n1})and (ω_{n2}) can be obtained.

ω_{n1}, is corresponding to the first mode and ω_{n2} is of the second mode of vibration

The general equation of motion of the two masses can now be written as

$Z_1 = A_1^1 sin\omega_{n1}t + A_1^2 sin\omega_{n2}t$------------------ Eq.1.67

$Z_2 = A_2^1 sin\omega_{n1}t + A_2^2 sin\omega_{n2}t$------------------ Eq.1.68

The superscripts in A represent the mode.

The relative values of amplitudes A_1 and A_2 for the two modes can be obtained using Eqs.1.62 and 1.63. Thus

$$\frac{A_1^1}{A_2^1} = \frac{K_2}{K_1+K_2-m_1\omega_{n1}^2} = \frac{K_2+K_3-m_2\omega_{n1}^2}{K_2}\text{--------------} \quad \text{Eq.1.69}$$

$$\frac{A_1^2}{A_2^2} = \frac{K_2}{K_1+K_2-m_1\omega_{n2}^2} = \frac{K_2+K_3-m_2\omega_{n2}^2}{K_2}\text{--------------} \quad \text{Eq.1.70}$$

1.4.2 Undamped forced vibrations

Consider the system shown in Figure 1.11 with excitation force

F_0 sin (ω t) acting on mass m_1. In this case, equations of motion will be:

$m_1\ddot{Z}_1 + K_2Z_1 + K_2(Z_1 - Z_2) = F_0 sin\omega t$------------ Eq.1.71

AND

$m_2\ddot{Z}_2 + K_3Z_2 + K_2(Z_2 - Z_1)$ =0---------------- Eq.1.72

For steady state, the solutions will be as

$Z_1 = A_1 sin\omega t$------------ Eq.1.73

AND

$Z_2 = A_2 sin\omega t$-------- Eq.1.74

Substituting Eqs. (1.73) and (1.74) in Eqs. (1.71) and (1.72), we get

$(K_1 + K_2 - m_1\omega^2)A_1 - K_2A_2 = F_0$--------------- Eq.1.75

AND

$-K_2A_1 + (K_2 + K_3 - m_2\omega^2)A_2 = 0$---------- Eq.1.76

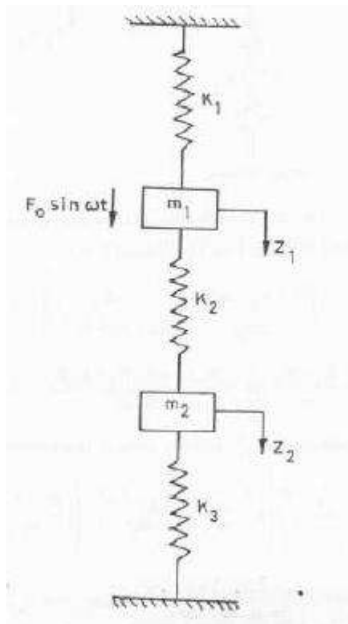


Fig. 1.11: Mass spring arrangement for Two degree of freedom

Solving for A_1 and A_2 from the above two equations, we get

$A_1 = \frac{(K_1+K_2-m_2\omega^2)F_0}{m_1m_2\left[\omega^4-\left(\frac{K_1+K_2}{m_1}+\frac{K_1+K_2}{m_2}\right)\omega^2+\frac{K_1K_2+K_2K_3+K_3K_1}{m_1m_2}\right]}$----------- Eq.1.77

$A_2 = \frac{(K_3F_0}{m_1m_2\left[\omega^4-\left(\frac{K_1+K_2}{m_1}+\frac{K_2+K_3}{m_2}\right)\omega^2+\frac{K_1K_2+K_2K_3+K_3K_1}{m_1m_2}\right]}$--------------- Eq.1.78

The above Two equations give steady state amplitude of vibration of the two masses respectively, as a function of ω. The denominator of the two equations is same. It may be noted that:

(i) The expression inside the bracket of the denominator of Eqs.1.77 *and* 1.78 is of the same type as the expression of natural frequency given by Eq. (1.66). Therefore at $\omega = \omega_{n1}$ and $\omega = \omega_{n2}$ values of A_1 and A_2 will be infinite as the denominator will become zero.

(ii) The numerator of the expression for Al becomes zero when

$\omega = \sqrt{\frac{K_1+K_3}{m_2}}$-------------------- Eq.1.79

Thus it makes the mass m_1 motionless at this frequency. No such stationary condition exists for mass m_1. The fact that the mass which is being excited can have zero amplitude of vibration under certain conditions by coupling it to another spring-mass system forms the principle of dynamic vibration absorbers which will be discussed latter on.

1.5 SYSTEM WITH n DEGREES OF FREEDOM

1.5.1 Undamped free vibrations

Consider a system shown in Figure 1.12 having n-degree of freedom.

If Z_1, Z_2, Z3 ... Z_n *are* the displacements of the respective masses at any instant, then equations of motion are:

$m_1\ddot{Z}_1 + K_1 Z_1 + K_2(Z_1 - Z_2) = 0$--------------------- Eq.1.80

$m_2\ddot{Z}_2 - K_2(Z_1 - Z_2) + K_3(Z_2 - Z_3) = 0$------------ Eq.1.81

$m_n\ddot{Z}_n - K_n(Z_{n-1} - Z_n) = 0$--------------------------- Eq.1.82

The solution of Eqs. (1.80) to (1.82) will be of as follows;

$Z_1 = A_1 sin\omega_n t$------------------ Eq.1.83

$Z_2 = A_2 sin\omega_n t$------------------- Eq.1.84

$Z_n = A_n sin\omega_n t$------------------ Eq.1.85

Substitution of Eqs. (1.83) to (1.85) into Eqs. (1.80) to (1.82), yields:

$[(K_1 + K_2) - m_1{\omega_n}^2]A_1 - K_2A_2 = 0$------------------------------- Eq.1.86

$-K_2A_1 + [(K_2 + K_3) - m_2{\omega_n}^2]A_2 - K_3A_3 = 0$-------- Eq.1.87

$-K_3A_2 + [(K_2 + K_4) - m_3{\omega_n}^2]A_3 - K_4A_4 = 0$-------------- Eq.1.88

$-K_nA_{n-1} + [K_n - m_n{\omega_n}^2]A_n = 0$------------------- Eq.1.89

The nontrivial solution of ω_n is in the form of

$$\begin{vmatrix} [(K_1 + K_2) - m_1{\omega_n}^2] & -K_2 & 0 & 0 \\ -K_2 & [(K_2 + K_3) - m_2{\omega_n}^2] & 0 & 0 \\ 0 & 0 & -K_n & [K_n - m_n{\omega_n}^2] \end{vmatrix} = 0 \text{--Eq.1.90}$$

Equation (1.90) is of n^{th} degree in ω_n^2 and therefore gives *n* values of ω_n corresponding to *n* natural frequencies. The mode shapes can be obtained from Eq. (1.86 to 1.89) by using, at one time, one of the various values of ω_n obtained from Eq. (1.90).

When the number of degrees of freedom exceeds three, the problem of forming the frequency equation and solving it for determination of frequencies and mode shapes becomes tedious. Numerical techniques are more useful in such cases. ,

Holzer's numerical technique is a convenient method of solving the problem for an idealized system

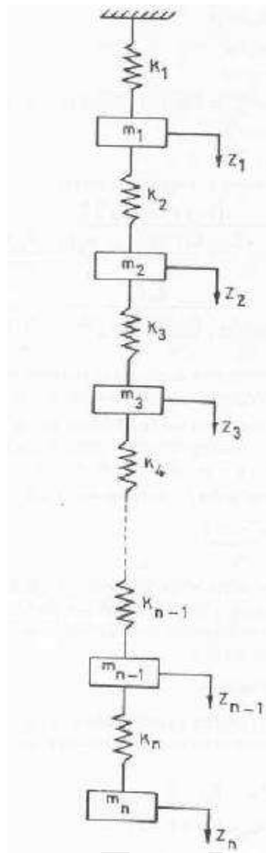
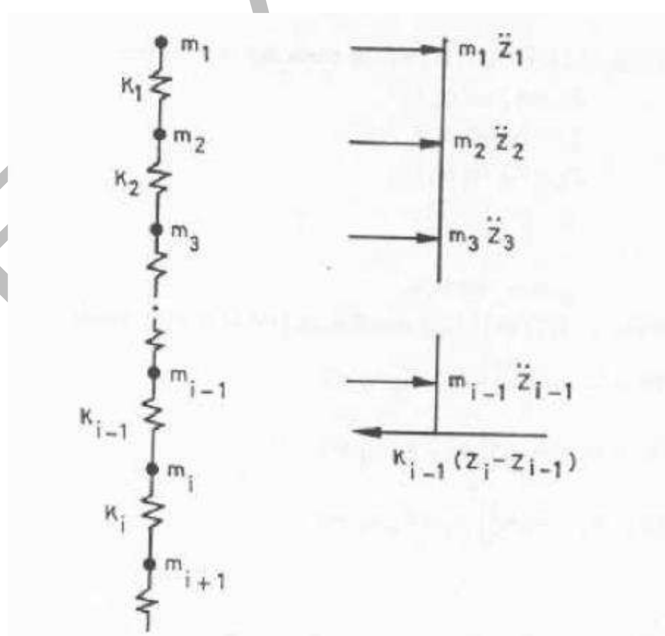


Fig. 1.12: Undamped free vibrations of a multi-degree freedom system



.

Fig. 1.13: An idealized multiple degree of freedom system

Inertia force at a level below mass $m_{i-1} = \sum_{j=1}^{i-1} m_j \ddot{Z}_j$-------------------- Eq.1.91

Spring force at that level corresponding to the difference of adjoining masses

$$K_{i-1}(Z_i - Z_{i-1})\text{-------------------------} \qquad \text{Eq.1.92}$$

Equating the above eqs, we obtain

$$\sum_{j=1}^{i-1} m_j \ddot{Z}_j = K_{i-1}(Z_i - Z_{i-1})\text{------------------} \qquad \text{Eq.1.93}$$

Putting $Z_i = A_i sin\omega_n t$ in Eq.1.93, we get

$$\sum_{j=1}^{i=1} m_j\,(-A_i\omega_n^2 sin\omega_n t) = K_{i-1}(A_i sin\omega_n t - A_{i-1} sin\omega_n t)\text{--------------} \qquad \text{Eq.1.94}$$

$$\text{Or } A_i = A_{i-1} - \frac{\omega_n^2}{K_{i-1}}\sum_{j=1}^{i-1} m_j \ddot{Z}_j\text{------------------} \qquad \text{Eq.1.95}$$

Equation (1.95) gives a relationship between any two successive amplitudes. Starting with any arbitrary value of A_i amplitude of all other masses can be determined. A plot of A_{n+1} versus ω_n^2 would have the shape as shown in Figure 1.14. Finally A_{n+1} should worked out to zero because of base fixity.

The intersection of the curve with (ω_n^2) axis would give various ω_n^2. The mode shape can be obtained by substituting the value of ω_n^2 in Eq. (1.95).

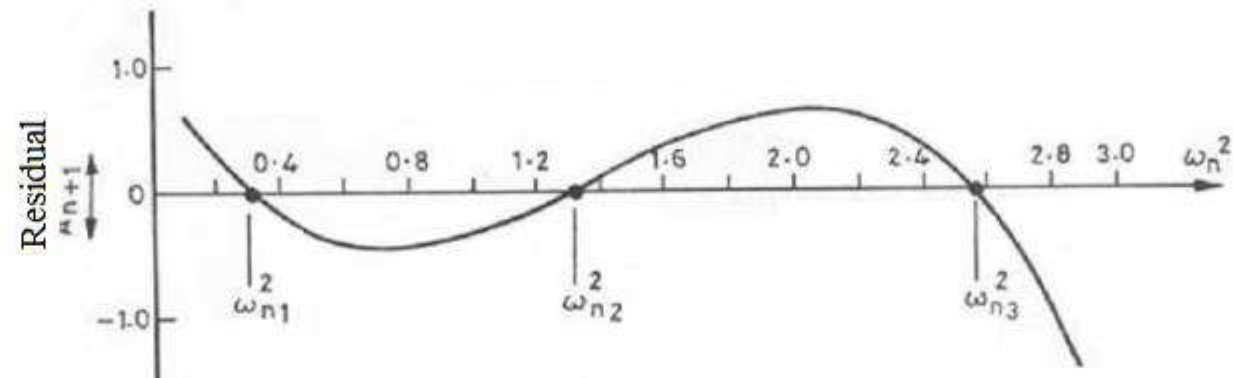


Fig.1.14: Residual as a function of frequency in Holzer method

1.5.2 Forced vibration

Let an undamped *n* degree of freedom system be subjected to forced vibration, and $F_i\,(t)$ represents the force on mass m_i. The equation of motion for the mass m_i will be

$$m_i\ddot{Z}_i + \sum_{i=1}^{n} K_{ij}Z_J = F_i(t)\text{-------------} \qquad \text{Eq.1.96}$$

Where i=1,2,3-------n

The amplitude of vibration of a mass is the algebraic sum of the amplitudes of vibration in various modes. The individual modal response would be some fraction of the total response with the sum of fractions being equal to unity. If the factors by which the modes of vibration are multiplied are represented by the coordinates "*d*", then for mass m_i

$$Z_i = A_i^{(1)}d_1 + A_i^{(2)}d_2 + -- + A_i^{(r)}d_r + --- + A_i^{(n)}d_n\text{--} \qquad \text{Eq.1.97}$$

The above equation can be rewritten as

$$Z_i = \sum_{r=1}^{n} A_i^{(r)}d_r\text{--------------} \qquad \text{Eq.1.98}$$

Substituting Eq.1.98 in 1.96, yields

$$\sum_{r=1}^{n} m_i\,A_i^{(r)}\ddot{d}_r + \sum_{r=1}^{n}\sum_{j=1}^{n} K_{ij}\,A_i^{(r)}d_r = F_i(t)\text{-----} \qquad \text{Eq.1.99}$$

For free vibration, it can be shown

$$\sum_{j=1}^{n} K_{ij}\,A_i^{(r)}d_r = \omega_{nr}^2 m_i A_i^{(r)}\text{---------------------------} \qquad \text{Eq.1.100}$$

Substituting Eq. 1.100 in 1.99, we

$\sum_{r=1}^{n} m_i\, A_i^{(r)} \ddot{d}_r + \sum_{r=1}^{n} \omega_{nr}^2 m_i\, A_i^{(r)} d_r = F_i(t)$------- Eq.1.101

Or $\sum_{r=1}^{n} m_i\, A_i^{(r)} (\ddot{d}_r + \omega_{nr}^2 d_r) = F_i(t)$------------ Eq.1.102

Since the left hand side is a summation involving different modes of vibration, the right hand side should also be expressed as a summation of equivalent force contribution in corresponding modes.

Let $F_i(t)$ be expressed as

$F_i(t) = \sum_{r=1}^{n} m_i\, A_i^{(r)} f_r(t)$----------------- Eq.1.103

Where $f_r(t)$ is the modal force and is given by

$f_r(t) = \frac{\sum_{r=1}^{n} F_i(t) A_i^{(r)}}{\sum_{i=1}^{n} m[A_i^{(r)}]^2}$------------------ Eq.1.104

Substituting Eq.1.103 in Eq.1.102 we have

$\ddot{d}_r + \omega_{nr}^2 d_r = f_r(t)$------------------------ Eq.1.105

Now the equation 1.105 is a single degree freedom equation and solution can be expressed as

$d_r = \frac{1}{\omega_{nr}} \int_0^t f_r(t)\, sin\omega_{nr}(t-\tau) d\tau$--------------

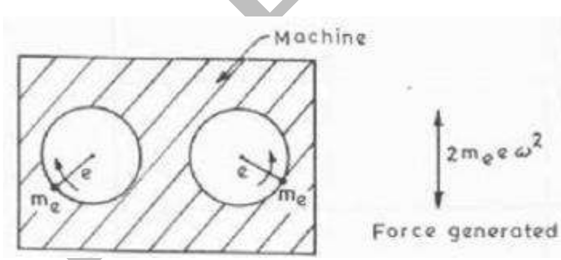
Where, $0 < \tau < 1$

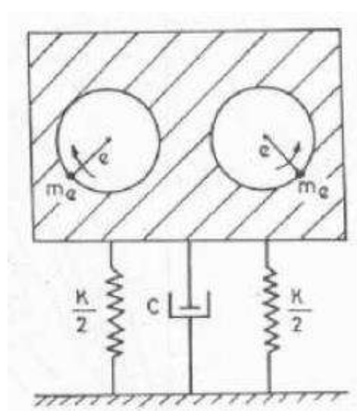
It is observed that the co-ordinate *d*, uncouples the *n* degree of freedom system into *n* systems of single degree of freedom. The *d's* are termed as normal co-ordinates and this approach is known as normal mode theory. Therefore the total solution is expressed as a sum of contribution of individual modes.

1.6 APPLICATION OF VIBRATION THEORY

1.6.1 Rotating mass type excitation

Machines with unbalanced rotating masses develop alternating force as shown in Fig. 1.15 *a*. Since horizontal forces on the foundation at any instant cancel, the net vibrating force on the foundation is vertical and equal to $2m_e e\omega^2 sin\omega t$,where m_e is the mass of each rotating element, placed at eccentricity *e* from the centre of rotating shaft and ω is the angular frequency of masses. Fig. 1.15 b shows such a system mounted on elastic supports with dashpot representing viscous damping.





(a) Rotating mass type excitation (b) Mass-spring-dash pot system

Fig.1.15: Single degree freedom system with rotating mass type excitation

The equation of motion can be written as

$m\ddot{Z} + C\dot{Z} + KZ = 2m_e e\omega^2 sin\omega t$-------------------- Eq.1.106

Where, m is the mass of foundation including $2m_e$. The solution of Eq. (1.106) may be written as,

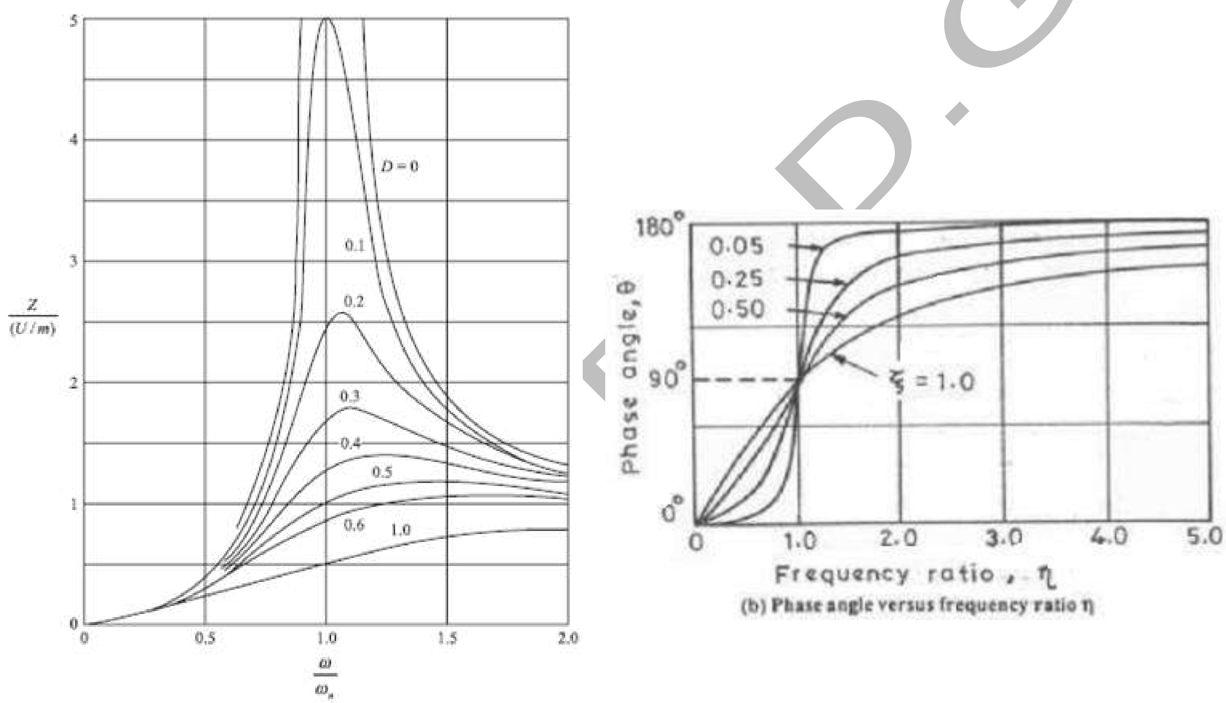
$Z = A_Z \sin(\omega t + \theta)$------------------------- ---------- Eq.1.107

Where

$A_Z = \frac{(2m_e e/m)\eta^2}{\sqrt{(1-\frac{\omega^2}{\omega_n^2})^2 + 4\xi^2(\frac{\omega}{\omega_n})^2}}$-- Eq.1.108

$\frac{F_0}{k} = 2m_e e\frac{\omega^2}{K} = 2m_e e\frac{\omega^2}{m\omega_n^2} = (2m_e\frac{e}{m})\xi^2$---------- Eq.1.109

$\theta = tan^{-1}\left(\frac{2\eta\xi}{1-\eta^2}\right)$-------------- --------------------- Eq.1.110



(b) Phase angle versus frequency ratio η

Fig.1.16: Response of a mass rotating system

The Eq. (1.108) can be expressed in non-dimensional form as given below:

$\frac{A_Z}{(2m_e e/m)} = \frac{\eta^2}{\sqrt{(1-\frac{\omega^2}{\omega_n^2})^2+4\xi^2(\frac{\omega}{\omega_n})^2}}$--------------------------------- Eq.1.111

Differentiating Eq. (1.111) with respect to η and equating to zero. It can be shown that resonance will occur at a frequency ratio given by:

$\eta = \frac{1}{\sqrt{1-2\xi^2}}$------------------ --------------- Eq.1.112

Or $\omega_d = \frac{\omega_n}{\sqrt{1-2\xi^2}}$------------------------- ------ Eq.1.113

By substituting Eq. (1.113) in Eq. (1.111), we get:

$$\frac{A_Z}{(2m_e e/_m)_{max}} = \frac{1}{2\xi\sqrt{1-\xi^2}}$$ ----------------- Eq.1.114

$=\frac{1}{2\xi}$ For small damping

1.7 VIBRATION ISOLATION

In case a machine is rigidly fastened to the foundation, the force will be transmitted directly to the foundation and may cause objectionable vibrations. It is desirable to isolate the machine from the foundation through a suitably designed mounting system in such a way that the transmitted force is reduced.

For example, the inertial force developed in a reciprocating engine or unbalanced forces produced in any other rotating machinery should be isolated from the foundation so that the adjoining structure is not set into heavy vibrations. Another example may be the isolation of delicate instruments from their supports which may be subjected to certain vibrations. In either case the effectiveness of isolation may be measured in terms of the force or motion transmitted to the foundation. The first type is known as ***force isolation*** and the second type as ***motion isolation.***

1.7.1 Force Isolation

Figure 1.17 shows a machine of mass m supported on the foundation by means of an isolator having an equivalent stiffness *K* and damping coefficient *C*. The machine is excited with unbalanced vertical force of magnitude $2m_e e\omega^2 sin\omega t$ .The equation of motion of the machine can be written as:

$$m\ddot{Z} + C\dot{Z} + KZ = 2m_e e\omega^2 sin\omega t$$ -------------------- Eq.1.115

The steady state motion of the mass of machine can be worked out as

$$Z = \frac{2m_e e\omega^2/_K}{\sqrt{\left(1-\frac{\omega^2}{\omega_n^2}\right)^2 + 4\xi^2\left(\frac{\omega}{\omega_n}\right)^2}}\sin(\omega t - \theta)$$ ----------------- Eq.1.116

$$= \frac{2m_e e\omega^2/_K}{\sqrt{(1-\eta^2)^2 + 4\xi^2(\eta)^2}}\sin(\omega t - \theta)$$

Where, $\theta = tan^{-1}\left[\frac{2\eta\xi}{1-\eta^2}\right]$ -------------- Eq.1.117

The only force which can be applied to the foundation is the spring force KZ and the damping force, $C\dot{Z}$; hence the total force transmitted to the foundation during steady state forced vibration is

$$F_t = KZ + C\dot{Z}$$ --------------------------- --Eq.1.118

Now substituting Eq. (1.116) in Eq. (1.118), we get

$$F_t = \frac{2m_e e\omega^2}{\sqrt{(1-\eta^2)^2 + 4\xi^2(\eta)^2}}\sin(\omega t - \theta) + C\omega\frac{2m_e e\omega^2/_K}{\sqrt{\left(1-\frac{\omega^2}{\omega_n^2}\right)^2 + 4\xi^2\left(\frac{\omega}{\omega_n}\right)^2}}\cos(\omega t - \theta)$$ ---------- Eq.1.119

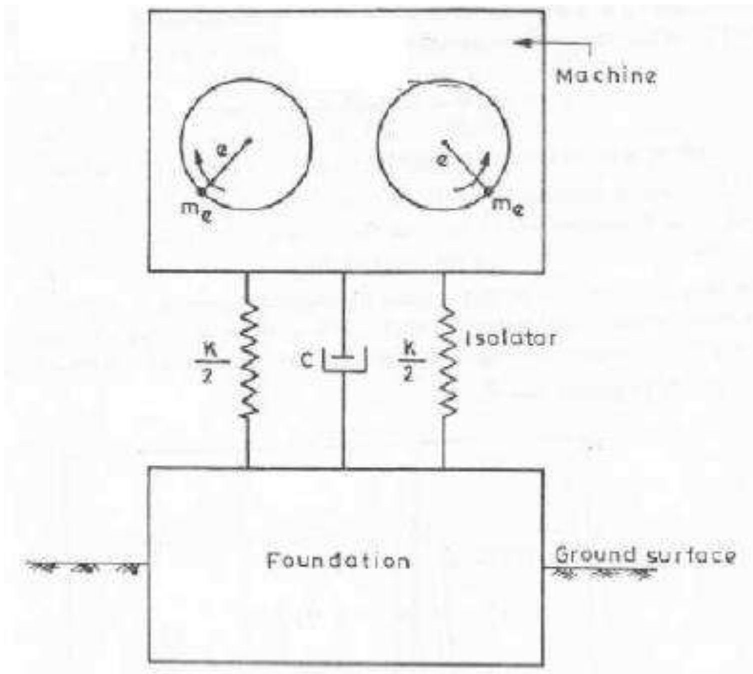


Fig.1.17: Machine isolation foundation system

Equation (1.119) can be written as:

$F_t = 2m_e e\omega^2 \frac{\sqrt{1+(2\eta\xi)^2}}{\sqrt{(1-\eta^2)^2+4\xi^2(\eta)^2}} \sin(\omega t - \beta)$---Eq.1.120

Where β is the phase difference between the exciting force and the force transmitted to the foundation and is given by,

$\beta = \theta - tan^{-1}\left[\frac{C\omega}{K}\right]$--Eq.1.121

Since the force $2m_e e\omega^2$ is the force which would be transmitted if springs are infinitely rigid, a measure of the ***effectiveness*** of the isolation mounting system is given by,

$\mu_T = \frac{F_t}{2m_e e\omega^2} = \frac{\sqrt{1+(2\eta\xi)^2}}{\sqrt{(1-\eta^2)^2+4\xi^2(\eta)^2}}$-------------- --Eq.1.122

μ_T is called the transmissibility of the system.

A plot of μ_T versus η for different values of ξ is shown in Fig.1.18

It will be noted from the figure that for any frequency ratio greater than$\sqrt{2}$, the force transmitted to the foundation will be less than the exciting force. However in this case, the presence of damping reduces the effectiveness of the isolation system as the curves for damped case are above the undamped ones for η>$\sqrt{2}$. A certain amount of damping, however, is essential to maintain stability under transient conditions and to prevent excessive amplitudes when the vibrations pass through resonance during the starting or stopping of the machine. ***Therefore, for the vibration isolation system to be effective η should be greater than$\sqrt{2}$.***

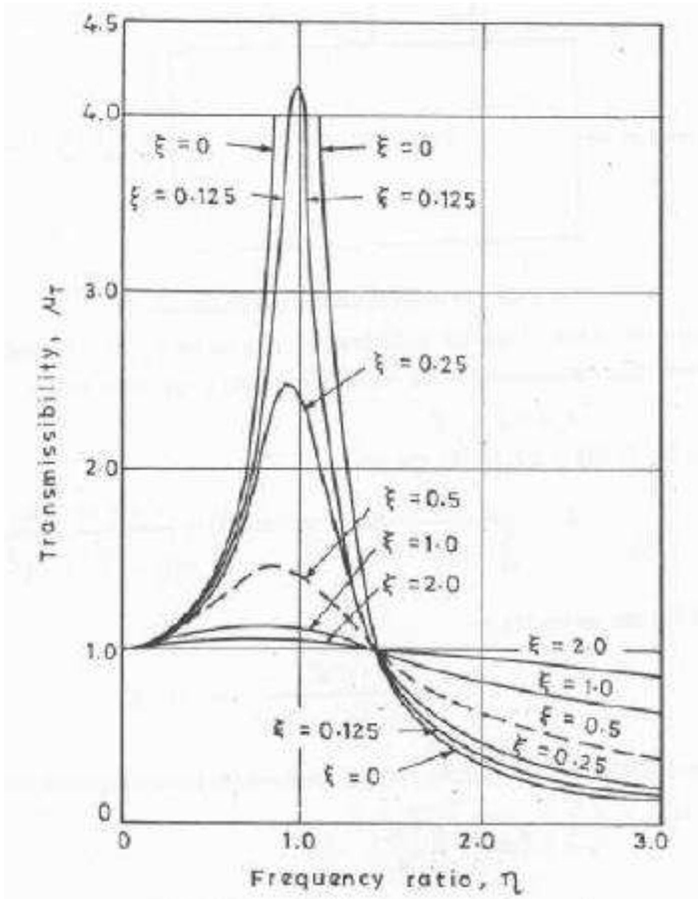


Fig.1.18: Transmisibilty versus frequency ratio plot

1.7.2 Motion Isolation

In many situations, *it would be necessary to isolate structure or mechanical systems from vibrations transmitted from the neighboring machines*. Again we require a suitable mounting system so that least vibrations are transmitted to the system due to the vibrating base. We consider a system mounted through a spring and dashpot and attached to the surface which vibrates harmonically with frequency (ω) and amplitude Y_0 as shown in Figure 1.19.

Let Z be the absolute displacement of mass; the equation of motion of the system can be written as:

$m\ddot{Z} + C(\dot{Z} - \dot{Y}) + K(Z - Y) = 0$-------------- --------------------------- Eq.1.123

OR $m\ddot{Z} + C\dot{Z} + KZ = C\dot{Y} + KY = C\omega Y_0 cos\omega t + KY_0 sin\omega t$------------ Eq.1.124

Or $m\ddot{Z} + C\dot{Z} + KZ = Y_0\sqrt{K^2 + (C\omega)^2}\sin(\omega t + \alpha)$---------------- ------- Eq.1.125

Where, $\alpha = tan^{-1}\left(\frac{C\omega}{K}\right)$--- Eq.1.126

The solution of Eq. (1.125) will give the maximum amplitude as:

$$Z_{max} = Y_0 \frac{\sqrt{1+(2\eta\xi)^2}}{\sqrt{(1-\eta^2)^2+(2\eta\xi)^2}} \quad \text{Eq.1.127}$$

The effectiveness of the mounting system (transmissibility) is given by

$$\mu_T = \frac{Z_{max}}{Y_0} = \frac{\sqrt{1+(2\eta\xi)^2}}{\sqrt{(1-\eta^2)^2+(2\eta\xi)^2}} \quad \text{Eq.1.128}$$

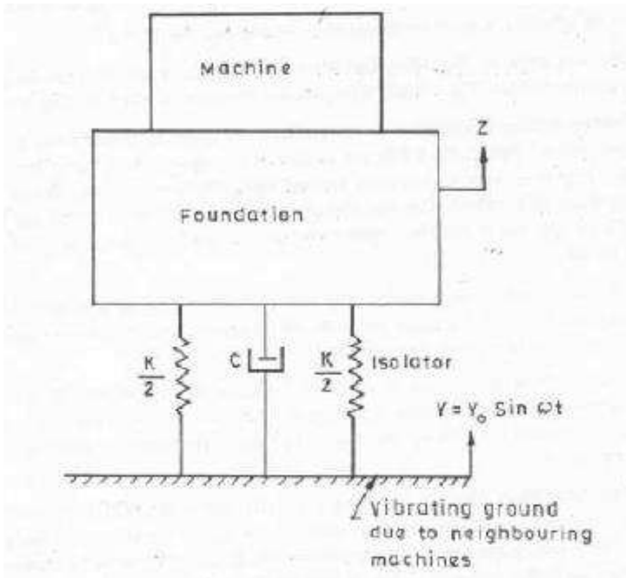


Fig.1.19: Motion isolation system

Equation (1.128) is the same expression as Eq. (1.122) obtained earlier. Transmissibility of such system can also be studied from the response curves shown in Fig.1.18. It is again noted that for the vibration isolation to be effective, it must be designed in such a way that $\eta>\sqrt{2}$.

1.7.3 Materials Used In Vibration Isolation

Materials used for vibration isolation are rubber, felt, cork and metallic springs. The effectiveness of each depends on the operating conditions.

i) ***Rubber:*** Rubber is loaded in compression or in shear; the latter mode gives higher flexibility. With loading greater than about 0.6 N per sq mm, it undergoes much faster deterioration. Its damping and stiffness properties vary widely with applied load, temperature, shape factor, excitation frequency and the amplitude of vibration. The maximum temperature up to which rubber can be used satisfactorily is about 65°c. It must not be used in presence of oil which attacks rubber. It is found very suitable for high frequency vibrations.

ii) ***Felt:*** Felt is used in compression only and is capable of taking extremely high loads. It has very high damping and so is suitable in the range of low frequency ratio. It is mainly used in conjunction with metallic springs to reduce noise transmission.

iii) ***Cork:*** Cork is very useful for acoustic isolation and is also used in small pads placed underneath a large concrete block. For satisfactory working it must be loaded from 10 to 25 N/sq mm. It is not affected by oil products or moderate temperature changes. However, its properties change with the frequency of excitation.

iv) ***Metallic springs****:* Metallic springs are not affected by the operating conditions or the environments. They are quite consistent in their behaviour and can be accurately designed for any desired conditions. They have high sound transmissibility which can be reduced by loading felt in conjunction with it. It has negligible damping and so is suitable for working in the range of high frequency ratio.

1.8 THEORY OF VIBRATION MEASURING INSTRUMENTS

The purpose of a vibration measuring instrument is to give an output signal which represents, as closely as possible, the vibration phenomenon. This phenomenon may be displacement, velocity or acceleration of the vibrating system and accordingly the instrument which reproduces signals proportional to these are called vibrometers, velometers or accelerometers.

There are essentially two basic systems of vibration measurement. One method is known as the directly connected system in which motions can be measured from a reference surface which is fixed. More often such a reference surface is not available. The second system, known as “Seismic System" does not require a fixed reference surface and therefore is commonly used for vibration measurement.

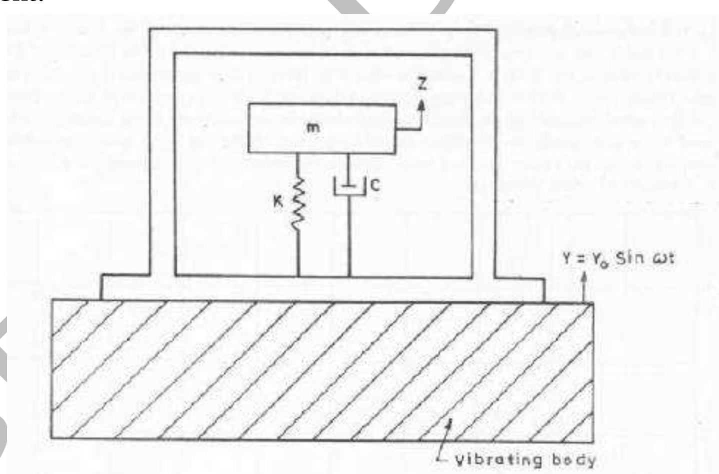


Fig.1.20: Schematic diagram of vibration measuring instrument

Figure 1.20 shows a Vibration measuring instrument which is used to measure any of the vibration phenomena. It consists of a frame in which the mass “m” is supported by means of a spring K and dashpot C. The frame is mounted on a vibrating body and vibrates along with it. The system reduces to a spring mass dashpot system having base on support excitation as discussed earlier in illustrating motion isolation.

Let the surface S of the structure be vibrating harmonically with unknown amplitude Y_0 and an unknown frequency ω. The output of the instrument will depend upon the relative motion between the mass and the structure, since it is this relative motion which is detected and amplified. Let Z be the absolute displacement of the mass, then the output of the instrument will be proportional to X =Z - Y.

The equation of motion of the system can be written as

$m\ddot{Z} + C(\dot{Z} - \dot{Y}) + K(Z - Y) = 0$------------------------- Eq.1.129

Subtracting $m\ddot{Y}$ from both sides,

$m\ddot{X} + C\dot{X} + KX = -m\ddot{Y} = mY_0\omega^2 sin\omega t$---------------- Eq.1.130

The solution can be written as

$X = \frac{\eta^2}{\sqrt{(1-\eta^2)^2+(2\eta\xi)^2}} Y_0 \sin(\omega t - \theta)$------------------ ------------------------ Eq.1.131

Where $\eta = \frac{\omega}{\omega_n}$= Frequency ratio

ξ = damping ratio

$\theta = tan^{-1}(\frac{2\eta\xi}{1-\eta^2})$------------------ -------------------------- Eq.1.132

Equation (1.131)can be rewritten as

$\boldsymbol{X = \eta^2 \mu Y_0 \sin(\omega t - \theta)}$----------- ------------------------- Eq.1.133

Where

$\mu = \frac{1}{\sqrt{(1-\eta^2)^2+(2\eta\xi)^2}}$-- Eq.1.134

1.8.1 Displacement Pickup

The instrument will read the displacement of the structure directly if $\eta^2\mu$ = I and θ =0.The variation of$\eta^2\mu$ with η and ξ is shown in Figure 1.21

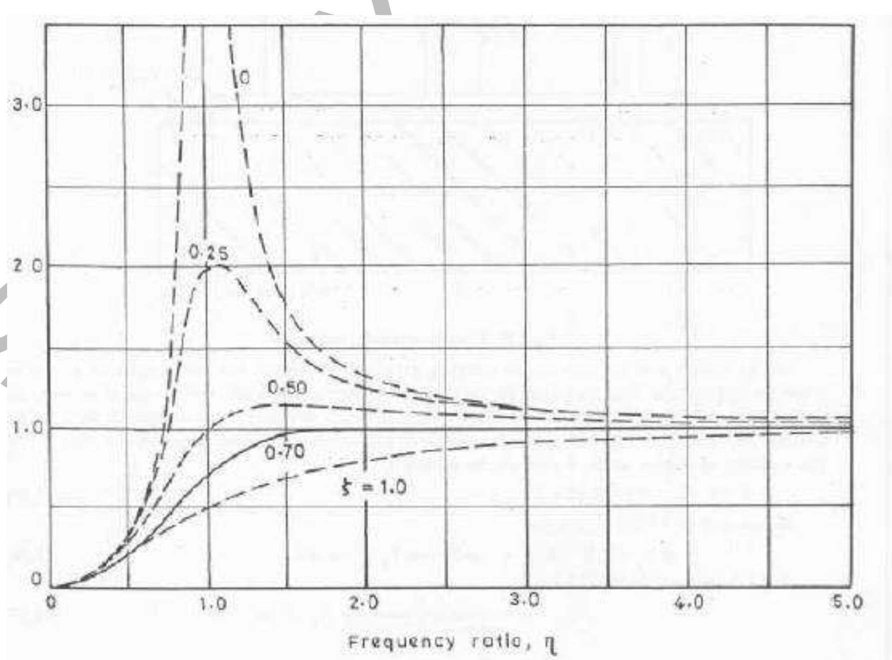


Fig.1.21: Response of a vibration measuring instrument to a vibrating base

It is seen when η is large, η^2μ is approximately equal to 1 and θ is approximately equal to 180°. Therefore to design a displacement pickup, η should be large which means that the natural frequency of the instrument itself 'should be low compared to the frequency to be measured. Or in other words, **the instrument should have a soft spring and heavy mass**. The instrument is sensitive, flimsy and can be used in a weak vibration environment. The instrument cannot be used for measurement of strong vibrations.

1.8.2 Acceleration Pickup (Accelerometer)

Equation (1.133) can be rewritten as

$\ddot{X} = \frac{1}{\omega_n^2}\mu\omega^2 Y_0 \sin(\omega t - \theta)$-- Eq.1.135

The output of the instrument will be proportional to the acceleration of the structure if μ is constant. It is seen that μ is approximately equal to unity for small values of η. Therefore to design an acceleration pick up, it should be small which means that the natural frequency of the instrument itself should be high compared to the frequency to be measured. In other words, the ***instrument should have a stiff spring and small mass***. The instrument is less sensitive and suitable for the measurement of strong motion. The instrument size is small.

1.8.3 Velocity Pickup

Equation (1.133) can be rewritten as

$\dot{X} = \frac{1}{\omega_n}\eta\mu Y_0 \omega \sin(\omega t - \theta)$-- Eq.1.136

The output of the instrument will be proportional to velocity of the structure if $\frac{1}{\omega_n}\eta\mu$ is a constant.

At η= 1, Eq. (1.136) can be written as

$\acute{X} = \frac{1}{\omega_n}\frac{1}{2\xi} Y_0 \omega \sin(\omega t - \theta)$--Eq.1.137 as at η=1,$\mu = \frac{1}{2\xi}$

Since ω_n and ξ are constant, the instrument will measure the velocity at η= 1.

It may be noted that the same instrument can be used to measure displacement, acceleration and velocity in different frequency ranges.

$X\alpha Y, if\ \eta \gg 1$, Displacement pickup (Vibrometer)

$X\alpha Y, if\ \eta \ll 1$, Acceleration pickup (Accelerometers)

$X\alpha Y, if\ \ \eta = 1$, Velocity pickup (Velometers)

Displacement and velocity pickups have the disadvantage of having rather a large size if motions having small frequency of vibration are to be measured. Calibration of these pickups is not simple. Further corrections have to be made in the observations as the response is not flat in the starting regions. From the point of view of small size, flat frequency response, sturdiness and ease of calibration, acceleration pickups are to be favored. They are relatively less sensitive and this disadvantage can easily be overcome by ***high gain electronic instrumentation.***

1.8.4 Transducer

A transducer is a device for converting the mechanical motion of vibration into an electrical signal, commonly called pickup.
There are three kinds of transducers: Displacement, Velocity, and Acceleration

1.8.5 Displacement Transducer

It is the most common type of transducer which is operated on the eddy current principle. It sets up a high-frequency electric field in the gap between the end of the Proximity Probe and the metal surface that is moving. It senses the change in the gap and measures relative displacement not absolute displacement.

Proximity Probe

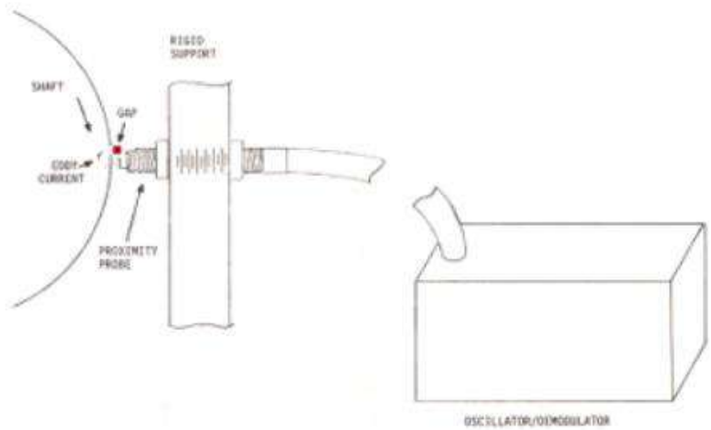


Fig.1.22: Schematic diagram of proximity probe

It is sensitive to shaft surface defects such as scratches, dents and vibrations in conductivity and permeability.
Senses shaft run out, and it is very difficult to distinguish vibration from run out.
The practical maximum frequency of proximity probes is about 1500Hz.The minimum frequency is zero. It can also measure static displacement. A useful application of proximity probes is to measure very slow relative movement like thermal expansion. It is useful in situations where the vibrating part cannot tolerate the mass of the pickup.

1.8.6 Velocity Transducer

Velocity transducer is also called seismic pickup.
The relative motion between the permanent magnet and the coil generates a voltage that is proportional to the velocity of the motion. The velocity transducer has an internal natural frequency of about 8 Hz. The velocity transducer is rather large. On small devices this added mass can significantly affect the vibration output. The coil in the velocity pickup is sensitive to external electromagnetic fields.

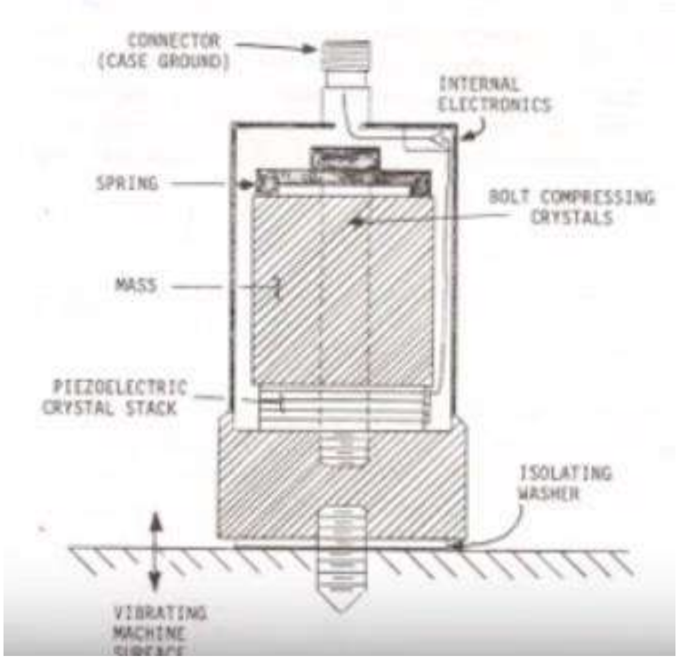


Fig.1.23: Velocity transducer

1.8.7 Acceleration Transducers

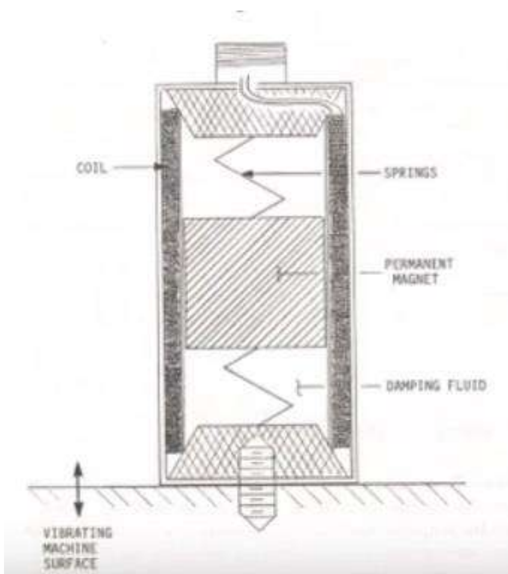


Fig.1.24: Accelerator transducer

The most common acceleration transducer is the piezoelectric accelerometer It consist of quartz crystal with a mass bolted on top and a spring compressing the quartz. A property of piezoelectric material is that it generates an electrical charge output when it is compressed. The charge output is proportional to force F= ma, force is also proportional to acceleration.

Typically accelerometer has very high natural frequency, typically 25000 Hz Its response is linear for about 1/3 of this range. It has a useful frequency range of from about 5 to approximately 100000 Hz depending on its size. The primary considerations in selecting an accelerometer are sensitivity and frequency response.

If high-amplitude motions are to be measured, i.e. greater than 10g, such as in shock measurement, then a low-sensitivity accelerometer is appropriate 10 mV/g or less.

If the level motion is to be measured, such as building or structural motions at low frequencies then a high sensitivity accelerometer should be chosen 1000 mv/g.

For most machinery monitoring,100 mV/g sensitivity accelerometer provide the right balance of sensitivity and frequency response. Other considerations in accelerometer selection or transducer are Temperature exposure

Linearity - It is expressed as the percent deviation from a constant value of the sensitivity. Transverse Sensitivity is the ability of the transducer to detect motion in directions perpendicular to its sensitive axis.

Damping is very low in piezoelectric accelerometer but can be significant in other types, such as piezo-resistive accelerometer. Strain sensitivity is the ability of the transducer to generate a signal when the base is distorted, such as when it is clamped against a non flat surface.

2.0 WAVE PROPAGATION; *BASIC ELASTIC PROPERTIES AND RELATIONSHIP*

2.1 Elastic Constants

An elastic material is one which obeys Hook's law of proportionally between stress and strain. For an isotropic elastic material subjected to normal stress $\sigma_x$in the x-direction, the strains in *x, y,* z directions are given as

$\varepsilon_x = \frac{\sigma_x}{E}$--------------- Eq.2.1

$\varepsilon_y = \varepsilon_z = -\mu\frac{\sigma_x}{E}$---------- Eq.2.2

If the element of the material is subjected to normal stress $\sigma_x, \sigma_y, \sigma_z,$ then by superposition we obtain

$\varepsilon_x = \frac{1}{E}\left[\sigma_x - \mu(\sigma_y + \sigma_z)\right]$-------------- Eq.2.3

$\varepsilon_y = \frac{1}{E}\left[\sigma_y - \mu(\sigma_x + \sigma_z)\right]$------------------ Eq.2.4

$\varepsilon_z = \frac{1}{E}\left[\sigma_z - \mu(\sigma_x + \sigma_y)\right]$---------------- Eq.2.5

In the above expressions, E is the modulus of elasticity and μ is Poisson's ratio. It may be noted that here E is dynamic modulus of elasticity.

Equations (3to 5) can be rearranged so, that the stresses are expressed in terms of the strains as follows: (Timoshenko and Goodier, 1951; Kolsly, 1963).

$\sigma_x = \frac{\mu E}{(1+\mu)(1-2\mu)}\left[\varepsilon_x + \varepsilon_y + \varepsilon_z\right] + \frac{E}{1+\mu}\varepsilon_x$----------- Eq.2.6

$\sigma_y = \frac{\mu E}{(1+\mu)(1-2\mu)}\left[\varepsilon_x + \varepsilon_y + \varepsilon_z\right] + \frac{E}{1+\mu}\varepsilon_y$----------- Eq.2.7

$\sigma_z = \frac{\mu E}{(1+\mu)(1-2\mu)}\left[\varepsilon_x + \varepsilon_y + \varepsilon_z\right] + \frac{E}{1+\mu}\varepsilon_z$----------- Eq.2.8

For simplicity the equations may be written

$\sigma_x = \lambda\bar{\varepsilon} + 2G\varepsilon_x$-- Eq.2.9

$\sigma_y = \lambda\bar{\varepsilon} + 2G\varepsilon_y$-- Eq.2.10

$\sigma_z = \lambda\bar{\varepsilon} + 2G\varepsilon_z$-- Eq.2.11

In which

$\bar{\varepsilon} = \varepsilon_x + \varepsilon_y + \varepsilon_z$------------------ -------------------- Eq.2.12

$\lambda = \frac{\mu E}{(1+\mu)(1-2\mu)}$-- Eq.2.13

$G = \frac{E}{2(1+\mu)}$-- Eq.2.14

Similarly in an isotropic elastic material, there exists linear relation between shear stress and shear strain. Thus

$\gamma_{xy} = \frac{\tau_{xy}}{G}$------------------ ------------------------------ Eq.2.15

$\gamma_{yz} = \frac{\tau_{yz}}{G}$-- Eq.2.16

$\gamma_{xz} = \frac{\tau_{xz}}{G}$-- Eq.2.17

G is the shear modulus or rigidity modulus and is the same as given by Eqs. (2.9 to 2.11).

Equations (2.9 to 2.11) and (2.15 to 2.17) comprise six equations that define the stress-strain relationship

2.2 WAVE PROPAGATION IN AN INFINITE, HOMOGENEOUS, ISOTROPIC, ELASTIC MEDIUM

In this section, the propagation of stress waves in an infinite, homogeneous, isotropic medium presented in Figure.2.1 shows the stresses acting on a soil element with sides d_x, d_y, d_z. For obtaining the differential equations of motion, the sum of the forces acting parallel to each axis is considered.

In the x-direction the equilibrium equation is given as

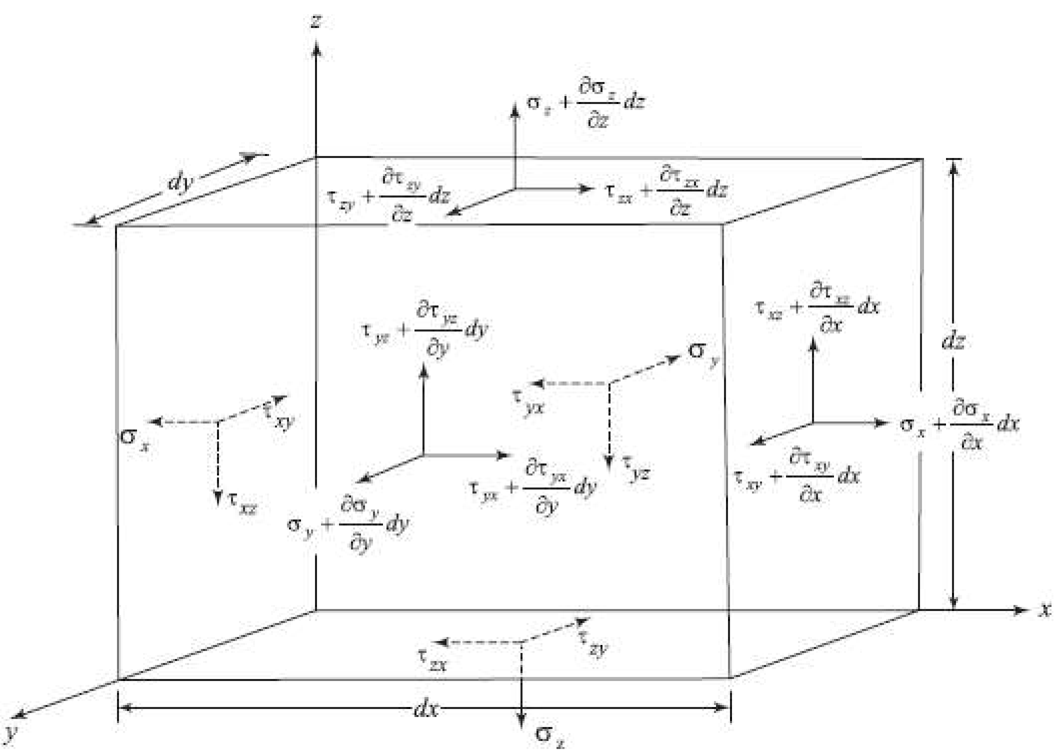


Fig. 2.1: Stress on an element of an infinite elastic medium

$$\left[\sigma_x - \left(\sigma_x + \frac{\partial \sigma_x}{\partial x} dx\right)\right](dy.dz) + \left[\tau_{xz} - \left(\tau_{xz} + \frac{\partial \tau_{xz}}{\partial z} dz\right)\right](dx.dy) + \left[\tau_{yx} - \left(\tau_{yx} + \frac{\partial \tau_{yx}}{\partial y} dy\right)\right](dx.dz) + \rho(dx.dy.dz)\frac{\partial^2 u}{\partial t^2} = 0 \text{---------------------------} \qquad \text{Eq.2.18}$$

Or,

$$\rho \frac{\partial^2 u}{\partial t^2} = \frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} + \frac{\partial \tau_{xz}}{\partial z} \text{---} \qquad \text{Eq.2.19 (a)}$$

Equations similar to Eq. (1), it can be written for the y -and z -directions. These will give

$$\rho \frac{\partial^2 v}{\partial t^2} = \frac{\partial \tau_{yz}}{\partial x} + \frac{\partial \sigma_y}{\partial y} + \frac{\partial \tau_{xz}}{\partial z} \text{---} \qquad \text{Eq.2.19 (b)}$$

$$\rho \frac{\partial^2 \omega}{\partial t^2} = \frac{\partial \tau_{xz}}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} + \frac{\partial \sigma_z}{\partial z} \text{---} \qquad \text{Eq.2.19 (c)}$$

In the above expressions, ρ is the mass density of the soil; *u*, v and ω are displacements in the *x*, *y*, and z directions respectively. To express the right hand sides of these Eqs., the relationship for an elastic medium given is used. The equations for strains and rotations *of* elastic and isotropic materials in terms of displacements are as follows:

2.2.1 Axial Strains

$\varepsilon_x = \frac{\partial u}{\partial x}$----------------- Eq.2. 20(a)

$\varepsilon_y = \frac{\partial v}{\partial y}$---------------- Eq.2. 20(b)

$\varepsilon_z = \frac{\partial \omega}{\partial z}$--------------- Eq.2. 20(c)

Shearing Strains:

$\gamma_{xy} = \frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}$------------ Eq.2. 21(a)

$\gamma_{yz} = \frac{\partial \omega}{\partial y} + \frac{\partial v}{\partial z}$---------------- Eq.2 .21(b)

$\gamma_{xz} = \frac{\partial \omega}{\partial x} + \frac{\partial u}{\partial z}$-------------------- Eq.2. 21(c)

Rotations:

$2\overline{\omega_x} = \frac{\partial \omega}{\partial y} - \frac{\partial v}{\partial z}$------------ Eq.2. 22(a)

$2\overline{\omega_y} = \frac{\partial u}{\partial z} - \frac{\partial \omega}{\partial x}$------------- Eq.2 .22(b)

$2\overline{\omega_z} = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}$---------------- Eq.2. 22(c)

2.2.2 Compression Waves

Substitution of Eq.2. 9, 2 .15 and 2.17 in Eq.2. 19 (a) gives

$\rho \frac{\partial^2 u}{\partial t^2} = \frac{\partial}{\partial x}(\lambda \bar{\varepsilon} + 2G\varepsilon_x) + \frac{\partial}{\partial y}(G\gamma_{xy}) + \frac{\partial}{\partial z}(G\gamma_{xz})$-------------- Eq.2. 23

Or

$\rho \frac{\partial^2 u}{\partial t^2} = \frac{\partial}{\partial x}(\lambda \bar{\varepsilon} + 2G\varepsilon_x) + G\frac{\partial}{\partial y}\left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}\right) + G\frac{\partial}{\partial z}\left(\frac{\partial u}{\partial z} + \frac{\partial \omega}{\partial x}\right)$----------- Eq.2. 24

$\rho \frac{\partial^2 u}{\partial t^2} = \frac{\partial}{\partial x}(\lambda \bar{\varepsilon} + 2G\varepsilon_x) + G\frac{\partial}{\partial y}\left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}\right) + G\frac{\partial}{\partial z}\left(\frac{\partial u}{\partial z} + \frac{\partial \omega}{\partial x}\right)$----------- Eq.2. 25

$\rho \frac{\partial^2 u}{\partial t^2} = \lambda\frac{\partial \bar{\varepsilon}}{\partial x} + G\left[\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 v}{\partial x \partial y} + \frac{\partial^2 \omega}{\partial x \partial z} + \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right]$--------------- Eq.2. 26

As $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 v}{\partial x \partial y} + \frac{\partial^2 \omega}{\partial x \partial z} = \frac{\partial \bar{\varepsilon}}{\partial x}$

The equation Eq.2. 26 can be rewritten as

$\rho \frac{\partial^2 u}{\partial t^2} = (\lambda + G)\frac{\partial \bar{\varepsilon}}{\partial x} + G\nabla^2 u$--- Eq. 2. 27(a)

Where $\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$

Similarly corresponding equations in other directions can be written as

$\rho \frac{\partial^2 v}{\partial t^2} = (\lambda + G)\frac{\partial \bar{\varepsilon}}{\partial y} + G\nabla^2 v$------------ -------------------------------- Eq.2. 27(b)

$\rho \frac{\partial^2 \omega}{\partial t^2} = (\lambda + G)\frac{\partial \bar{\varepsilon}}{\partial z} + G\nabla^2 \omega$-- Eq. 2. 27(c)

Equations (2. 27) are the **equations of motion of an infinite homogeneous, isotropic, and elastic medium.** On differentiating these equations with respect to *x*, *y* and *z*, respectively, and adding, we get

$\rho \frac{\partial^2}{\partial t^2}\left[\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial \omega}{\partial z}\right] = (\lambda + G)\left[\frac{\partial^2 \bar{\varepsilon}}{\partial x^2}+\frac{\partial^2 \bar{\varepsilon}}{\partial y^2}+\frac{\partial^2 \bar{\varepsilon}}{\partial z^2}\right] + G\nabla^2\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial \omega}{\partial z}\right)$------------- Eq.2. 28

$\rho \frac{\partial^2 \bar{\varepsilon}}{\partial t^2} = (\lambda + G)(\nabla^2 \bar{\varepsilon}) + (G\nabla^2 \bar{\varepsilon})$---------------------- Eq.2. 29

Hence, $\rho \frac{\partial^2 \bar{\varepsilon}}{\partial t^2} = (\lambda + 2G)(\nabla^2 \bar{\varepsilon})$-------------- Eq.2. 30

0r

$\frac{\partial^2 \bar{\varepsilon}}{\partial t^2} = \frac{(\lambda+2G)}{\rho}(\nabla^2 \bar{\varepsilon}) = V_p^2 \nabla^2 \bar{\varepsilon}$----------------- Eq.2. 31

Where $V_p^2 = \frac{(\lambda+2G)}{\rho}$---------------------------- Eq.2.32

V_p is the ve1ocity of compression waves which is also referred as primary wave or, P-wave. It is important to note the difference in the wave velocities for an infinite elastic medium with those obtained for an elastic rod is, $V_c = \sqrt{E/\rho}$: but in the infinite medium, $V_p = \sqrt{\frac{(\lambda+2G)}{\rho}}$. ***This means that $V_p > V_c$, that is compression wave travels faster in infinite medium. It is due to the fact that in infinite medium, there are no lateral displacements, while in the elastic rod lateral displacements are possible.***

2.2.3 Shear-Waves

Differentiating Eq. (2.27,b) with respect to *z* and Eq. (2.27,c) with respect to *y*, we get

$\rho \frac{\partial^2}{\partial t^2}\left(\frac{\partial v}{\partial z}\right) = (\lambda + G)\frac{\partial \bar{\varepsilon}}{(\partial y)(\partial z)} + G\nabla^2 \frac{\partial v}{\partial z}$----------------- Eq. 2.33

$\rho \frac{\partial^2}{\partial t^2}\left(\frac{\partial \omega}{\partial y}\right) = (\lambda + G)\frac{\partial \bar{\varepsilon}}{(\partial y)(\partial z)} + G\nabla^2 \frac{\partial \omega}{\partial y}$--------------- Eq.2.34

Subtracting Eq.2.34 from Eq.2.33, we get

$\rho \frac{\partial^2}{\partial t^2}\left(\frac{\partial \omega}{\partial y} - \frac{\partial v}{\partial z}\right) = G\nabla^2\left(\frac{\partial \omega}{\partial y} - \frac{\partial v}{\partial z}\right)$---------------------- Eq.2.35

FromEq.(2.22,a)

$$2\overline{\omega_x} = \frac{\partial \omega}{\partial y} - \frac{\partial v}{\partial z}$$

Therefore,

$\rho \frac{\partial^2 \overline{\omega_x}}{\partial t^2} = G\nabla^2 \overline{\omega_x}$--------------------- Eq.2.35

Or,

$\frac{\partial^2 \overline{\omega_x}}{\partial t^2} = \frac{G}{\rho}\nabla^2 \overline{\omega_x} = V_s^2 \nabla^2 \overline{\omega_x}$--------------------- Eq.2.36 (a)

Similar expression can be obtained for $\bar{\omega}_y$ *and* $\overline{\omega_z}$ as

$\frac{\partial^2 \overline{\omega_y}}{\partial t^2} = \frac{G}{\rho}\nabla^2 \overline{\omega_y} = V_s^2 \nabla^2 \overline{\omega_y}$---------------------- Eq.2. 36 (b)

$\frac{\partial^2 \overline{\omega_z}}{\partial t^2} = \frac{G}{\rho}\nabla^2 \overline{\omega_z} = V_s^2 \nabla^2 \overline{\omega_z}$--------------------- Eq.2. 36 (c)

The above expressions indicate that the Rotation is propagated with velocity V_s which is equal to

$\sqrt{G/\rho}$. Shear wave is also referred as distortion wave or S-wave. ***It may be noted that shear wave propagates at the same velocity in both the rigid elastic medium like rod or bar and the infinite-medium.***

2.3 WAVEPROP AGATION IN ELASTIC HALF-SPACE

In an elastically homogeneous ground, stressed suddenly at a point 'S' near its surface as shown in (Figure 2.2), three elastic waves travel outwards at different speeds. Two are body waves; which are propagated as spherical, fronts affected only a minor extent by the free surface of the ground, and the third is a surface wave which is confined to the region, near the free surface.

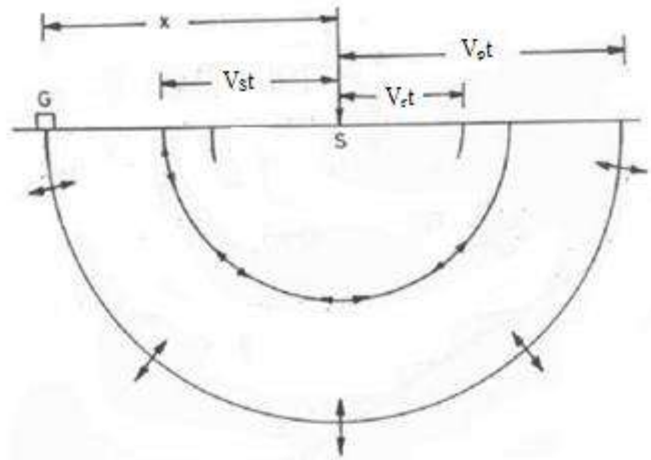


Fig.2.2: Pulse fronts of the P, S and R waves

The stresses in the P wave, which is a longitudinal wave like a sound wave in air, are thus due to uniaxial compression, while during the passage of an S wave the medium is subjected to shear stress. The surface wave travels more slowly than either body wave, and is generally complex. This wave was first studied by Rayleigh (1885) and later was, described in detail by Lamb (1904). It is referred as Rayleigh wave or R-wave. The influence of Raleigh wave decreases rapidly with depth.

The half space is defined as the *x-y* plane with z assumed to be positive toward the interior of the half-space as shown in Figure 2.3. Let *u* and *w* represent the displacements in the directions *x* and z, respectively and are independent of *y*, then

$u = \frac{\partial \emptyset}{\partial x} + \frac{\partial \varphi}{\partial z}$---Eq.2.37

$\omega = \frac{\partial \emptyset}{\partial z} - \frac{\partial \varphi}{\partial x}$--- Eq.2.38

Where ∅ and φ are two potential function. As $\frac{\partial v}{\partial y}$=0, the dilation $\bar{\varepsilon}$ of the wave can be written as

$\bar{\varepsilon} = \frac{\partial u}{\partial x} + \frac{\partial \omega}{\partial z} = \left[\frac{\partial^2 \emptyset}{\partial x^2} + \frac{\partial^2 \varphi}{\partial x \partial z}\right] + \left[\frac{\partial^2 \emptyset}{\partial z^2} - \frac{\partial^2 \varphi}{\partial x \partial z}\right]$------------------------------- Eq.2.38

Or, $\bar{\varepsilon} = \frac{\partial^2 \emptyset}{\partial x^2} + \frac{\partial^2 \emptyset}{\partial z^2} = \nabla^2 \emptyset$-- Eq.2.39

Similarly the rotation in x-z plane is given by

$2\overline{\omega_y} = \frac{\partial u}{\partial z} - \frac{\partial \omega}{\partial x} = \frac{\partial^2 \varphi}{\partial x^2} + \frac{\partial^2 \varphi}{\partial z^2} = \nabla^2 \varphi$---Eq.2.40

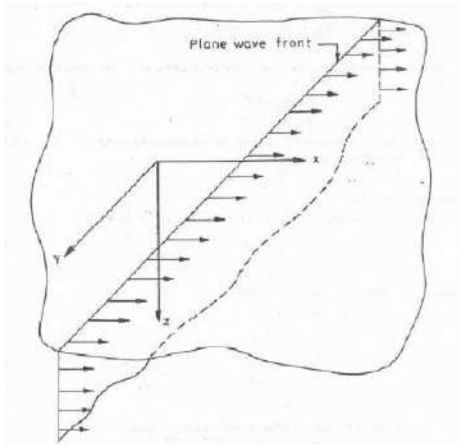


Fig.2.3: Wave propagation in Elastic half space

$u = \frac{\partial \emptyset}{\partial x} + \frac{\partial \varphi}{\partial z}$--- Eq.2.41

$\omega = \frac{\partial \emptyset}{\partial z} - \frac{\partial \varphi}{\partial x}$--- Eq.2.42

Where $\emptyset$ and φ are two potential function. As $\frac{\partial v}{\partial y}$=0, the dilation $\bar{\varepsilon}$ of the wave can be written as

$\bar{\varepsilon} = \frac{\partial u}{\partial x} + \frac{\partial \omega}{\partial z} = \left[\frac{\partial^2 \emptyset}{\partial x^2} + \frac{\partial^2 \varphi}{\partial x \partial z}\right] + \left[\frac{\partial^2 \emptyset}{\partial z^2} - \frac{\partial^2 \varphi}{\partial x \partial z}\right]$------------------------------- Eq.2.43

Or, $\bar{\varepsilon} = \frac{\partial^2 \emptyset}{\partial x^2} + \frac{\partial^2 \emptyset}{\partial z^2} = \nabla^2 \emptyset$--- Eq.2.44

Similarly the rotation in x-z plane is given by

$2\overline{\omega_y} = \frac{\partial u}{\partial z} - \frac{\partial \omega}{\partial x} = \frac{\partial^2 \varphi}{\partial x^2} + \frac{\partial^2 \varphi}{\partial z^2} = \nabla^2 \varphi$-- Eq.2.45

Substituting *u* and *w* from Eq.1 and 2 in, we get

$\rho \frac{\partial}{\partial x}\left(\frac{\partial^2 \emptyset}{\partial t^2}\right) + \rho \frac{\partial}{\partial z}\left(\frac{\partial^2 \varphi}{\partial t^2}\right) = (\lambda + 2G)\frac{\partial}{\partial x}(\nabla^2 \emptyset) + G\frac{\partial}{\partial z}(\nabla^2 \varphi)$------------- Eq.2.46

And $\rho \frac{\partial}{\partial z}\left(\frac{\partial^2 \emptyset}{\partial t^2}\right) - \rho \frac{\partial}{\partial x}\left(\frac{\partial^2 \varphi}{\partial t^2}\right) = (\lambda + 2G)\frac{\partial}{\partial z}(\nabla^2 \emptyset) - G\frac{\partial}{\partial x}(\nabla^2 \varphi)$-------- Eq.2.47

The above Eqs (2.46 and2.47) are satisfied if

$\frac{\partial^2 \emptyset}{\partial t^2} = \frac{(\lambda+2G)}{\rho}\nabla^2 \emptyset = V_p^2 \nabla^2 \emptyset$--- Eq.2.48

And $\frac{\partial^2 \varphi}{\partial t^2} = \frac{G}{\rho}\nabla^2 \varphi = V_s^2 \nabla^2 \varphi$--- Eq.2.49

Now, consider a sinusoidal wave traveling in the positive *x* direction. Let the solution of $\emptyset$ and φ be expressed as

$\emptyset = F(z)\exp[i(\omega t - fx)]$--------------------- Eq.2.50

And

$\varphi = G(z)\exp[i(\omega t - fx)]$---------------------- Eq.2.51

Where F(z) and G(z) are function of depth

And $f = \frac{2\pi}{wave\ length}$

Now substituting Eq. 2.50 into Eq. 2.48 , we get

$\left(\frac{\partial^2}{\partial t^2}\right)\{F(z)\exp[i(\omega t - fx)]\}=V_p^2\nabla^2\{F(z)\exp[i(\omega t - fx)]\}$---------- Eq.2.52

Or $-\omega^2 F(z) = V_p^2[F^{"}(z) - f^2 F(z)]$-------------------------------------- Eq.2.53

Where F(z) and G(z) are functions of depth

Similarly, substituting Eq. (2.51) into Eq. (2.49) results in

$-\omega^2 G(z) = V_s^2[G^{"}(z) - f^2 G(z)]$--------------- Eq.2.54

Where

$F^{"}(z) = \frac{\partial^2 F(z)}{\partial z^2}$-------------------------------------- Eq.2.55

and

$G^{"}(z) = \frac{\partial^2 G(z)}{\partial z^2}$-- Eq.2.56

Now Equations (2.53) and (2.54) can be rearranged to the form

$F^{"}(z) - q^2 F(z) = 0$------------------------------ Eq.2.57

$G^{"}(z) - s^2 F(z) = 0$------------------------------ Eq.2.58

Where

$q^2 = f^2 - \frac{\omega^2}{V_p^2}$--- Eq.2.59

$s^2 = f^2 - \frac{\omega^2}{V_s^2}$------------------ Eq.2.60

Solutions to Eqs. (2.57) and (2.48) can be given as

$F(z) = A_1 e^{-qz} + A_2 e^{qz}$--------------- Eq.2.61

$G(z) = B_1 e^{-sz} + B_2 e^{sz}$---------------- Eq.2.62

where A_1, A_2, B_1, and B_2 are constants.

It can be seen from Eqs. (2.61) and (2.62) that A_2 and B_2 must equal zero; otherwise $F(z)$ and $G(z)$ will approach infinity with depth, which is not the type of wave that is considered here. With A_2 and B_2 equal zero, we have

$F(z) = A_1 e^{-qz}$--------------- Eq.2.63

$G(z) = B_1 e^{-sz}$---------------- Eq.2.64

Now combining Eqs. (2.50) and (2.63) and Eqs. (2.51) and (2.64), we get

$\emptyset = (A_1 e^{-qz})[\exp i(\omega t - fx)]$--------------------- Eq.2.65

$\varphi = (B_1 e^{-sz})[expi(\omega t - fx)]$----------------------- Eq.2.66

The boundary conditions for the two preceding equations are at $z = 0$, $\sigma_z = 0$, $\tau_{zx} = 0$, and $\tau_{zy} = 0$. We have

$\sigma_{z(z=0)} = \lambda\bar{\varepsilon} + 2G\varepsilon_z = \lambda\bar{\varepsilon} + 2G\left(\frac{\partial\omega}{\partial z}\right) = 0$------------ Eq.2.67

Combining Eqs. (2.42), (2.444), and (2.65)–(2.67), one obtains

$A_1[(\lambda + 2G)q^2 - \lambda f^2] - 2iB_1 Gfs = 0$---------- Eq.2.68

And $\frac{A_1}{B_1} = \frac{2iGfs}{(\lambda+2G)q^2-\lambda f^2}$----------------- Eq.2.69

Similarly, $\tau_{zx(z=0)} = G\gamma_{zx} = G\left(\frac{\partial\omega}{\partial x} + \frac{\partial u}{\partial z}\right) = 0$----- Eq.2.70

Again, combining Eqs. (2.21), (2.23), (2.65), (2.66), and (2.70),

$2iA_1fq + (s^2+f^2)B_1 = 0$------------- Eq.2.71

OR $\frac{A_1}{B_1} = \frac{(s^2+f^2)}{2ifq}$--------------- Eq.2.72

Equating the right-hand sides of Eqs.(2.69) and (2.72),

$$\frac{2iGfs}{(\lambda+2G)q^2-\lambda f^2} = \frac{(s^2+f^2)}{2ifq}$$ -------------- Eq.2.73

$$4Gf^2sq = \left(s^2+f^2\right)\left[(\lambda+2G)q^2-\lambda f^2\right]$$ ------- Eq.2.74

Or, $16G^2f^4s^2q^2 = (s^2+f^2)^2[(\lambda+2G)q^2-\lambda f^2]$----------Eq.2.75

Substituting for q and s and then dividing both sides of Eq. (2.75) by G^2f^8, we get

$16\left(1-\frac{\omega^2}{V_p^2f^2}\right)\left(1-\frac{\omega^2}{V_s^2f^2}\right) = \left[2-\left(\frac{\lambda+2G}{G}\right)\frac{\omega^2}{V_p^2f^2}\right]^2\left[2-\frac{\omega^2}{V_s^2f^2}\right]^2$---------------------- Eq.2.76

However, $wave\ length = \frac{velocity\ of\ wave}{\omega/2\pi}$

$f = \frac{\omega}{V_r}$----------------- Eq.2.77

So, $\frac{\omega^2}{V_p^2f^2} = \frac{\omega^2}{V_p^2(\omega^2/V_r^2)} = \frac{V_r^2}{V_p^2} = \alpha^2V^2$---------------- Eq.2.78

Similarly, $\frac{\omega^2}{V_s^2f^2} = \frac{\omega^2}{V_s^2(\omega^2/V_r^2)} = \frac{V_r^2}{V_s^2} = V^2$----------------- Eq.2.79

Where $\alpha^2 = \frac{V_s^2}{V_p^2}$

However, $V_p^2 = \lambda + 2G/\rho$ and $V_s^2 = G/\rho$

So, $\alpha^2 = \frac{V_s^2}{V_p^2} = \frac{G}{\lambda+2G}$------------------ Eq.2.80

The term α^2 can also be expressed in terms of Poisson's ratio. From the relations given in Eq. (2.81),

$\frac{2\mu G}{1-2\mu}$---------------------- Eq.2.81

Substitution of this relation in Eq. (2.80) yields,

$\alpha^2 = \frac{G}{\lambda+2G} = \frac{(1-2\mu)}{2(1-\mu)}$--------------- Eq.2.82

Again, substituting Eqs. (2.78), (2.79), and (2.80) into Eq. (2.76),

$$16(1-\alpha^2V^2)(1-V^2) = (2-V^2)(2-V^2)^2$$

Or, $V^6 - 8V^4 - (16\alpha^2-24)V^2 - 16(1-\alpha^2) = 0$------------ Eq.2.83

Equation (2.83) is a cubic equation in V^2. For a given value of Poisson's ratio, the proper value of V^2 can be found and, hence, so can the value of V_r in terms of V_p or V_s.

Example 1:

Given μ = 0.25, determined the value of the Rayleigh wave velocity in terms of V_s

Solution:

$$V^6 - 8V^4 - (16\alpha^2 - 24)V^2 - 16(1 - \alpha^2) = 0$$

For μ=0.25

$$\alpha^2 = \frac{1-2\mu}{2-2\mu} = 1/3$$

$$V^6 - 8V^4 - \left(16 \times \frac{1}{3} - 24\right)V^2 - 16\left(1 - \frac{1}{3}\right) = 0$$

$$3V^6 - 24V^4 + 56V^2 - 32 = 0$$

$$(V^2 - 4)(3V^4 - 12V^2 + 8) = 0$$

Therefore, $V^2 = 4, 2 + \frac{2}{\sqrt{3}}, 2 - \frac{2}{\sqrt{3}}$

If $V^2=4$, $\frac{S^2}{f^2} = 1 - V^2 = 1 - 4 = -3$

So S/f is imaginary. This is also the case for $V^2=2 + \frac{2}{\sqrt{3}}$

It can be seen that when q/f and s/f are imaginary, it does not yield the primary and secondary waves as discussed.

For $V^2=2 - \frac{2}{\sqrt{3}}$, $v = \frac{v_r}{v_s}$=0.9194

Or v_r=0.9194v_s

3.0 LIQUEFACTION OF SOIL

Previous earthquake devastation was an illustration of catastrophic damages to structures and resulting in loss of life which was due to liquefaction phenomenon. Liquefaction is defined as a condition where a soil will undergo continuation of deformation at a constant low residual stress or with no residual resistance, due to the build-up and maintenance of high pore water pressure which reduces the effective confining pressure to a very low value. The pore pressure so build-up leading to true liquefaction of this type may be due either to static or cyclic stress applications.

3.1 Initial Liquefaction

It denotes a condition where, during the course of cyclic stress applications, the residual pore water pressure on completion of any full stress -cycle becomes equal to the applied confining stress.

MECHANISM OF LIQUEFACTION:

The strength of sand is primarily due to internal friction. In saturated state it may be expressed as

$s = \overline{\sigma_n}$tanφ------ Eq.3.1

Where S= Shear strength of sand

$\overline{\sigma_n}$= Effective normal stress on any plane at a depth of z

Φ= Angle of internal friction

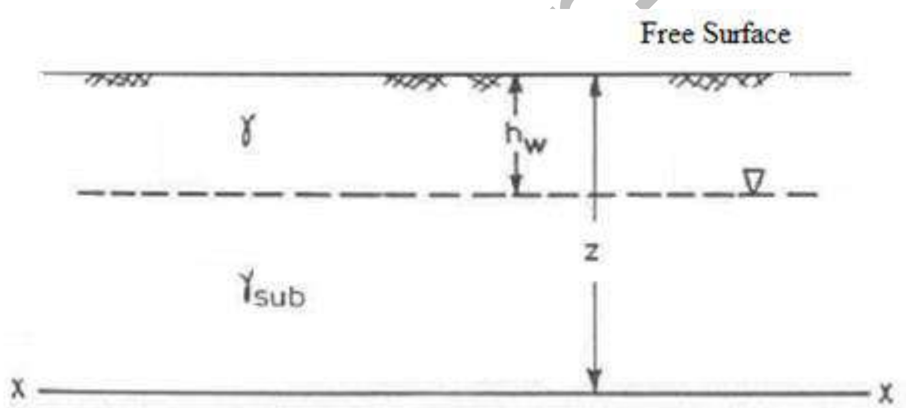


Fig.3.1. Section of ground showing the position of water table

When a saturated sand is subjected to ground vibrations, it tends to compact and decrease volume, if drainage is restrained the tendency to decrease in volume results in an increase in pore pressure.

The strength may now be expressed as,

$S_{dyn} = (\overline{\sigma_n} - u_{dyn}) tan\,\emptyset_{dyn}$------------- Eq.3.2

S_{dyn}=Shear strength of soil under vibration

u_{dyn}=Excess pore water due to ground vibration

$\emptyset_{dyn}$=angle of internal of friction under vibration

It is observed that with development of additional positive pore pressure, the strength of sand is reduced.

For complete loss of strength, shear strength becomes zero, S_{dyn}=0

Thus, $\overline{\sigma_n} - u_{dyn}$=0

Or $\overline{\sigma_n}=u_{dyn}$, hence $\frac{u_{dyn}}{\overline{\sigma_n}}=1$--------- Eq.3.3

Now u_{dyn} can be expressed in terms of rise in water head and be written as

$\frac{\gamma_w h_w}{\frac{G-1}{1+e}\gamma_w Z}=1$---- Eq.3.4

Or $\frac{h_w}{Z}=\frac{G-1}{1+e}$--------- Eq.3.5

This is the critical hydraulic gradient.

It is seen that, because of increase in pore water pressure the effective stress reduces, resulting in loss of strength. Transfer of inter granular stress takes place from soil grains to pore water. Thus if this transfer is completed, there is complete loss of strength, resulting in what is known as complete liquefaction. However, if only partial transfer of stress from the grains to the pore water occurs, there is partial loss of strength resulting in partial liquefaction. In case of complete liquefaction, the effective stress is lost and the sand-water mixture behaves as a viscous material and process of consolidation starts. Due to surface settlement, resulting in closer packing of sand grains occurs. Thus the structures resting on such a material start sinking. The rate of sinking of structures depends upon the time for which the sand remains in liquefied state. Liquefaction of sand may develop at any zone of a deposit, where the necessary combination of in-situ density, surcharge conditions and vibration characteristics occur. Such a zone may be at the surface or at some depth below the ground surface, depending only on the state of sand and the induced motion.

3.3 FACTORS AFFECTING LIQUEFACTION

The factors affecting liquefaction are summarised below

a) **Soil Type:** Liquefaction occurs in cohesion-less soils as they lose their strength completely under vibration due to the development of pore pressures which in turn reduce the effective stress to zero. Liquefaction does not occur in case of cohesive soils. Only highly sensitive clays may lose their strength substantially under vibration.

b) **Grain Size and Its Distribution:** Fine and uniform sands are more prone to liquefaction than coarser ones. Since the permeability of coarse sand is greater than fine sand, the pore pressure developed during vibrations can dissipate faster.

c) **Initial Relative Density:** It is one of the most important factors controlling liquefaction. Both pore pressures and settlement are considerably reduced during vibrations with increase in initial relative density and hence chances of liquefaction and excessive settlement reduce with increased relative density.

d) **Vibration Characteristics:** Out of the four parameters of dynamic load namely (i) frequency; (ii) amplitude; (iii) acceleration; and (iv) velocity; frequency and acceleration are more important. Frequency .of the dynamic load plays vital role, only if it is close to the natural frequency of the system. Further the liquefaction depends on the type of the dynamic load i.e. whether it is a transient load or the load that causes steady vibration. For a given acceleration, liquefaction occurs only after a certain number of cycles imparted to the deposit. Further, horizontal vibrations have more severe effect than vertical vibrations. Multi directional shaking is more severe than one directional loading (Seed at al.1977), as the pore water pressure build up is much faster and the stress ratio required is about 10 percent less than that required for unidirectional shaking.

e) **Location of Drainage and Dimension of Deposit**: Sands are more pervious than fine grained soil. However, if a pervious deposit has large dimensions, the drainage path increases and the deposit may behave as un-drained, thereby, increasing the chances of liquefaction of such a deposit. The drainage path is reduced by the introduction of drains made out of highly pervious material.

f) **Surcharge Load**: If the surcharge load, i.e. the initial effective stress is large, then transfer of stress from soil grains to pore water will require higher intensity vibrations or vibration for a longer duration. If the initial stress condition is not isotropic as in field, then stress condition causing liquefaction depends upon K_0 (coefficient of earth pressure at rest) and for $K_0 > 5$, the stress condition required to cause liquefaction increases by at least 50%.

g) **Method of Soil Formation:** Sands unlike clays do not exhibit a characteristics structure. But recent investigations show that liquefaction characteristics of saturated sands under cyclic loading are significantly influenced by method of sample preparation and by soil structure.

h) **Period under Sustained Load:** Age of sand deposit may influence liquefaction characteristics. A 75% increase in liquefaction resistance has been reported on liquefaction of undisturbed sand compared to its freshly prepared sample which may be due to some form of cementation or welding at contact points of sand particles and associated with secondary compression of soil.

i) **Previous Strain History**: Studies on liquefaction characteristics of freshly deposited sand and of similar deposit previously subjected to some strain history reveal, that although the prior strain history caused no significant change in the density of the sand, it increased the stress that causes liquefaction by a factor of 1.5.

j) **Trapped Air**: If air is trapped in saturated soil and pore pressure develop, a part of it is dissipated due to the compression of air, hence trapped air helps to reduce the possibility of liquefaction.

k) **Groundwater Table**: The most conducive condition to liquefaction is near the surface of ground water table. Unsaturated soil located above the groundwater table will not liquefy. At the location where groundwater table significantly fluctuates, the liquefaction will also fluctuate.

3.4 EVALUATION OF ZONE OF LIQUEFACTION IN FIELD

At a depth below the ground surface, liquefaction will occur if shear stress induced by earthquake is more than the shear stress predicted. By comparing the induced and predicted shear stresses at various depths, liquefaction zone can be obtained.

In a sand deposit consider a column of soil of height h and unit area of cross section subjected to maximum ground acceleration Q_{max}(Fig.3.2).

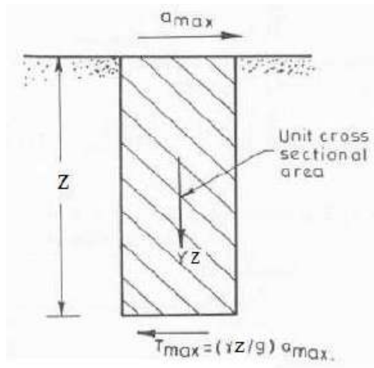


Fig.3.2: Maximum shear stress at a depth for a rigid soil column

Assuming the soil column to behave as a rigid body, the inertia force F can be obtained as

$F = \frac{W}{g} a_{max}$--------- Eq.3.6

Or $= \frac{\gamma z}{g} a_{max} = \sigma_0 \frac{a_{max}}{g}$-------------- Eq.3.7

The maximum shear stress τ_{max} at a depth h is given by

$\tau_{max} = \frac{F}{A} = \sigma_0 \frac{a_{max}}{g}$----------------- Eq.3.8

As base area of soil column is taken as unity

Where g = Acceleration due to gravity and γ = Unit weight of soil

Since the soil column behaves as a deformable body, the actual shear stress at depth h, (τ_{max}) is taken as

$\tau_{act} = r_d \tau_{max} = r_d (\frac{\gamma h}{g}) a_{max}$------ Eq.3.9

Where r_d= Depth reduction factor

If linear variation is assumed between reduction factor and depth, than r_d can be taken as

$r_d = 1 - 0.012z$------- Eq.3.10

The above relation is valid for depth up to 15 m.

According to Seed and Idriss (1971), the average equivalent uniform shear stress τ_{avg} is about 65 percent of the maximum shear stress τ_{max} .Therefore

$\tau_{avg} = 0.65 \frac{\gamma z}{g} a_{max} r_d$-------- Eq.3.11

The corresponding number of significant cycles N_s for τ_{avg} is given in table Table.3.1

Table 3.1: Significant cycles N_s corresponding to τ_{avg}

Earthquake magnitude, M on Richter's scale	N_s
7	10
7.5	20
8.0	30

In order to felicitate liquefaction analysis one non dimensional parameter known as Cyclic Stress Ratio (CSR) or Seismic Stress Ratio (SSR), can be defined as

$$CSR or\ SSR = \frac{\tau_{avg}}{\sigma_0'} \text{----------} \qquad \text{Eq.3.12}$$

Thus, $CSR or\ SSR = 0.65 r_d (\frac{\sigma_0}{\sigma_0'}) \frac{a_{max}}{g}$------ Eq.3.13

Seed and Idriss (1971) suggested the value of cyclic stress ratio values C_r as given in Table 3.2

Table 3.2: Values of C_r corresponding to Relative density

Relative density D_R (%)	C_r
0-50	0.57
60	0.60
80	0.68

It was observed that up to a relative density of 80%, the peak pulsating shear stress causing liquefaction increases almost linearly with the increase in relative density. Keeping this fact in view, the following general relation is suggested:

$$(\frac{\tau_h}{\sigma_v})_{field D_R} = (\frac{\sigma_d}{2\sigma_3})_{triax,50} C_r \frac{D_R}{50} \text{----------} \qquad \text{Eq.3.14}$$

Where, $(\frac{\tau_h}{\sigma_v})_{field D_R}$= Cyclic shear stress ratio in field at relative density of D_R percentage

$(\frac{\sigma_d}{2\sigma_3})_{triax,50}$=Stress ratio obtained from triaxial test at relative density of 50%. It can be determined from Fig.3.3

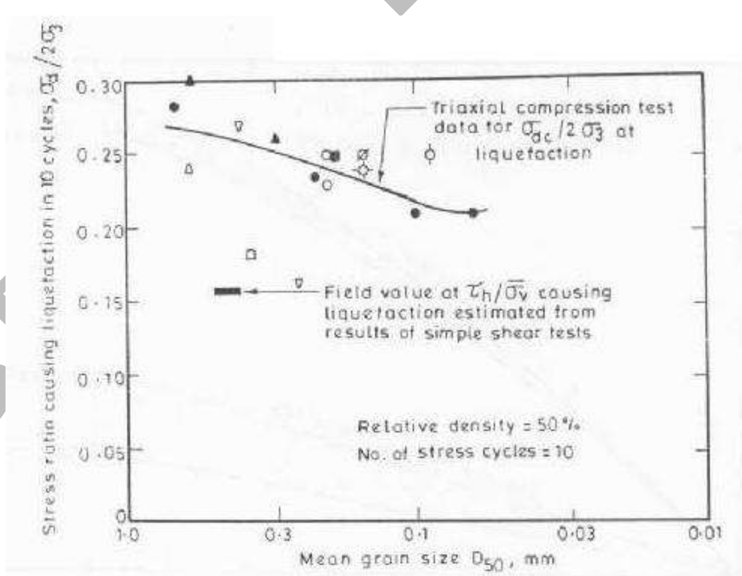


Fig. 3.3: Stress condition causing liquefaction of sands (Seed and Idriss, 1971)

3.5 THE PROCEDURE OF LOCATING LIQUEFACTION ZONE CAN BE SUMMARISED IN FOLLOWING STEPS

i) Establish the design earthquake, and obtain peak ground acceleration a_{max}. Also obtain number of significant cycles *Ns* corresponding to earthquake magnitude using Table.2

ii) Using Eq. 11 determine τ_{avg} at depth *h* below ground surface.

iii) Using Fig.2, determine the value of $\frac{\sigma_d}{2\sigma_3}$ for given value of D_{50} of soil and number of equivalent cycles N_S for the relative density of 50%.

iv) Using Eq. 12, determine the value of $(\frac{\tau_h}{\sigma_v})_{field D_R}$ for the relative density of D_R of the soil at site. Multiplying $(\frac{\tau_h}{\sigma_v})_{field D_R}$ with effectives stress at depth *h*, we can obtain the value of shear stress τ_h required for causing liquefaction.

v) At depth *h*, liquefaction will occur if $\tau_{avg} > \tau_h$

vi) Repeat steps *(ii)* to *(iv)* for other values of *h* to locate the zone of liquefaction.

τ_{avg} and τ_h can be plotted in a graph to identify the zone of liquefaction.

Problem No.1

At a given site, a boring supplemented with standard penetration tests was done up to 15.0m depth. The results of the boring are as given below:

Depth (m)	Classification of soil	D_{50} (mm)	N-Value	D_R (%)	Remarks
1.5	SP	0.18	3	19	Position of ground water lies 1.5 m below the ground surface
3.0	SP	0.2	5	30	
4.5	SM	0.12	6	35	
6.0	SM	0.14	9	40	
7.5	SM	0.13	12	45	γ_{moist}=19 kN/m^3 γ_{sub}=10 kN/m^3
9.0	SP	0.16	17	52	
10.5	SW	0.2	20	52	
12.0	SW	0.22	18	46	
13.0	SW	0.22	24	60	
15.0	SW	0.24	30	65	

The site is located in seismically, active region, and is likely to be subjected by an earthquake of Magnitude 7.5. Determine the zone of liquefaction using Seed and ldriss (1971) method.

3.6 EVALUATION OF LIQUEFACTION POTENTIAL USING STANDARD PENETRATION RESIST ANCE

The standard penetration test is most commonly used in-situ test in a bore hole to have fairly good estimation of relative density of cohesion-less soil. Since liquefaction primarily depends on the initial relative density of saturated sand, many researchers have made attempt to develop correlations in liquefaction potential and standard penetration resistance. IS: 2131-1981 gives the standard procedure to carry out standard penetration test. SPT values (N) obtained in the field for sand have to be corrected for accounting the effect of over burden pressure as below:

$N_{corrected} = C_N N_{field}$------------ Eq.3.15

C_N = Correction factor obtained from Figure 3.4 or it can be also be found from the expression

$C_N = \sqrt{\frac{100}{\bar{\sigma}}}$------ Eq.3.16

$\bar{\sigma}$ is the effective stress in kN/m^2

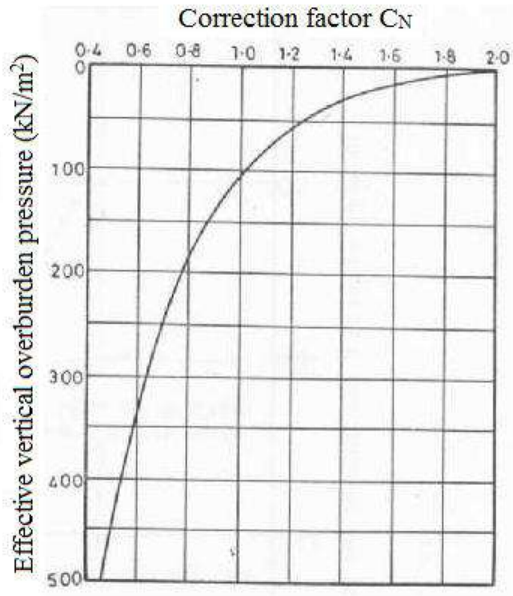


Fig. 3.4: Chart for correction of N value for over burden pressure

3.7 FOLLOWING PROCEDURE FOR LIQUEFACTION ANALYSIS IS USED

i) Establish the design earthquake, and obtain the peak ground acceleration a_{max}. Also obtain number of significant cycles corresponding to the magnitude of earthquake using Table 3.1.

ii) Using Eq. 3.11 determine τ_{avg} at depth *h* below ground surface.

iii) Determine the value of standard penetration resistance value (N) at depth *h* below ground surface. Obtain corrected $N_{corrected}$ value after applying overburden correction to N using Fig.3.4

iv) Using Fig.3.3, determine $\frac{\tau_h}{\sigma_v}$ for the given magnitude of earthquake and $N_{corrected}$ value obtained in step (iii). Multiplying $\frac{\tau_h}{\sigma_v}$ with effective stress at depth h below ground surface, obtain the value of shear stress τ_h required for causing liquefaction.

v) At depth *h*, liquefaction will occur if

$$\tau_{avg} > \tau_h$$

vi) Repeat steps *(ii)* to *(v)* for other values of *h* to locate the zone of liquefaction.

Example No.1

At a given site boring supplement with SPT was done up to 20 m depth. The results of the boring are given below. Water table lies 2 m below the ground surface. Take γ_{sub}=10 kN/m^3. The site is located in seismically active zone and the likely to be subjected by an earthquake of magnitude 7.5 and maximum ground acceleration is 0.15g. Find the zone of liquefaction if any.

Depth(m)	2.0	4.0	6.0	8.0	10.0	12.0	14.0	16.0	18.0	20.0
N_{field}	4	4	5	7	9	10	12	14	16	18

Solution:

The effective stress, $\sigma' = \gamma_{sub} Z$, Reduction factor $r_d = 1 - 0.012z$

The calculation are tabulated as below

Depth (m)	N	σ'	$C_N = \sqrt{\frac{100}{\sigma'}}$	$N_{correct}$ =NC_N	$\frac{\tau_h}{\sigma'}$	σ_0	r_d	$\frac{a_{max}}{g} 0.65 \times r_d$	τ_h	τ_{avg}
2	4	20	2.24	8.96	0.09	20	0.976	0.0952	1.8	1.9
4	4	40	1.58	6.32	0.07	60	0.952	0.0928	2.8	5.56
6	5	60	1.29	6.45	0.075	100	0.928	0.0905	4.5	9.05
8	7	80	1.12	7.84	0.08	140	0.904	0.0881	6.4	12.33
10	9	100	1.0	9.0	0.1	180	0.88	0.0858	10	15.44
12	10	120	0.91	9.1	0.1	220	0.856	0.0835	12	18.37
14	12	140	0.84	10.8	0.12	260	0.832	0.0811	16.8	21.08
16	14	160	0.79	11.06	0.13	300	0.808	0.0788	20.8	23.64
18	16	180	0.74	11.84	0.14	340	0.784	0.0764	25.2	25.97
20	18	200	0.707	12.73	0.16	380	0.76	0.0741	32	28.15

From the above calculation, it is found that up to 18 m depth, $\tau_{avg} > \tau_h$. Hence liquefaction can occur up to depth of 18 m from the ground surface.

3.8 FACTOR OF SAFETY AGAINST LIQUEFACTION

The liquefaction analysis can also provide the determination of factor of safety against liquefaction. If the cyclic stress ratio (CSR) caused by the anticipated earthquake is greater than the cyclic resistance ratio (CRR) of the in-situ soil, the liquefaction could occur during the earthquake. The cyclic resistance ratio represents the liquefaction resistance of the soil, which can be obtained from the standard penetration test. It was observed that the resistance to liquefaction increased with increase in the corrected N value. Figure 3.5 presents a chart that can be used to obtain the cyclic resistance of the in-situ soil.

So the factor of safety (FS) against liquefaction may be defined as

$FS = \frac{CRR}{CSR}$--------------- Eq.3.17

Higher the factor of safety more is the resistance of the soil to liquefaction. However, soil having FS slightly more than 1.0 may still liquefy during the earthquake.

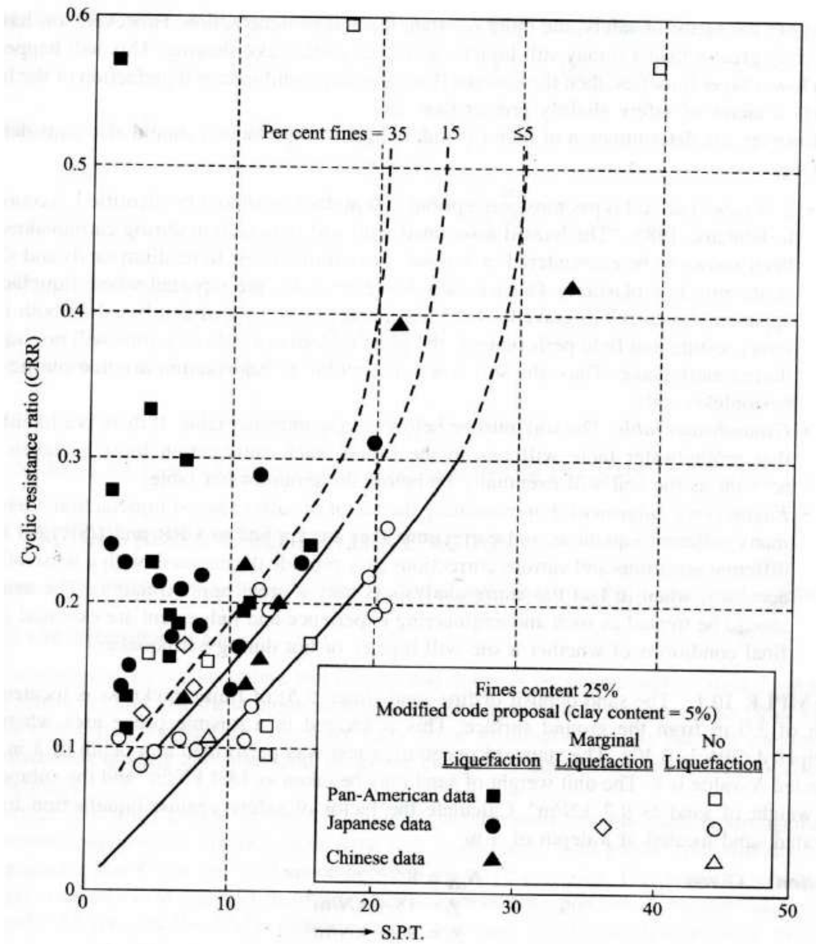


Fig.3.5: Chart to determine the cyclic resistance ratio for clean and silty sand for M=7.5 earthquake (After Seed et al.1975)

Example No.2

The sand deposit of fine sand (finer ≤ 5%) of finite thickness is located at a depth of 3.0 m from the ground surface and ground water table is located at 1.5m below the ground surface. This is located in seismic prone area where the anticipated GPA is 0.40g.The standard penetration test was performed at depth of 3.0m. The corrected N value is 8. The unit weight of sand is taken as18.4 kN/m^3. Calculate the factor of safety against liquefaction for the saturated sand.

Sol:

$N_{corrected}$ = 8, Unit weight of sand = 18.4 kN/m^3

Submerged unit weight = 8.59 kN/m^3

PGA=0.4g

Effective stress $\sigma_0' = \sigma_0 - u$=18.4x1.5+1.5x8.59=40.485 kN/m^3

Total stress $\sigma_0 = \sigma_0' + u$=40.485+1.5x9.81=55.2 kN/m^3

Now using the linear relationship for the stress, depth reduction factor can be computed as

$r_d = 1 - 0.012z$=1-0.012x3=0.964

So CSR=0.65x$r_d\left(\frac{\sigma_0}{\sigma_0'}\right)(\frac{a_{max}}{g})$=$0.65 \times 0,.964 \times \frac{55.2}{40.485} \times 0.4$=0.342

Using Fig.4 with $N_{corrected}$=8, CRR can be obtained as 0.09

Hence $FS = \frac{0.09}{0.342}$=0.263

So based on the calculation of factor of safety against liquefaction, the sand deposit liquefy.

Problem No 1:

A 10 m thick loose sand deposit (D_r=42%, finer≤5%) is saturated below a depth of 4 m. The sand layer region is highly prone to liquefaction. Estimate the ground acceleration that would be required to produce sand soils in a M=7.5 earthquake.

4.0 DYNAMIC SOIL PROPERTIES

4.1 Laboratory Method

The soil properties which are needed in analysis and design of a structure subjected to dynamic loading are:

(a) Dynamic moduli, such as Young's modulus E, shear modulus G, and bulk modulus K

(b) Poisson's ratio μ

(c) Dynamic elastic constants, such as coefficient of elastic uniform compression C_u, coefficient of elastic uniform shear, C_τ, coefficient of elastic non-uniform Compression $C_\emptyset$ and coefficient of elastic non-uniform shear C_φ

(d) Damping ratio, ξ

(e) Liquefaction parameters, such as cyclic stress ratio, cyclic deformation and pore pressure response.

f) Strength-deformation characteristics in terms of strain rate effects.

Since the dynamic properties of soils are strain dependent various laboratory and field techniques have been developed to measure these properties over a wide range of strain amplitudes.

4.2 LABORATORY TECHNIQUES

The laboratory methods used for determining the dynamic properties of soils are:

i) Resonant column test,

ii) Ultrasonic pulse test,

iii) Cyclic simple shear test,

iv) Cyclic torsional simple shear test, and

v) Cyclic triaxial compression test

4.2.1 Resonant Column Test:

The resonant column test is used to obtain the elastic modulus E, shear modulus G and damping characteristics of soils at low strain amplitudes. This test is based on the theory of wave propagation in prismatic rods (Richart, Hall and Woods, 1970). Either a cyclically varying axial load or torsional load is applied to one end of the prismatic or cylindrical specimen of soil. This in turn will propagate either a compression wave or a shear wave in the specimen. In this technique the excitation frequency generating the wave is adjusted until the specimen experiences resonance. The value of the resonant frequency is used to find the value of E and G depending on the type of the excitation (axial or torsional).

i) Fixed-free end Condition:

Hall and Richart (1963) described the apparatus with fixed-free end condition. In this arrangement one end of the specimen is fixed against rotation and the other end is free to rotate under the applied torsion (Figure 4.1a).Anode occurs at the fixed end and the distribution of angular rotation θ along the specimen is a $\frac{1}{4}sine$ wave.

As shown in Figure 4.1*b*, by adding a mass at the free end, the variation of θ along the specimen becomes nearly linear. J and J_0 are respectively the polar moment of inertias of the specimen and the added mass respectively. Dmevich (1967) used the concept of added mass to obtain a uniform strain distribution 'throughout the length of the specimen.

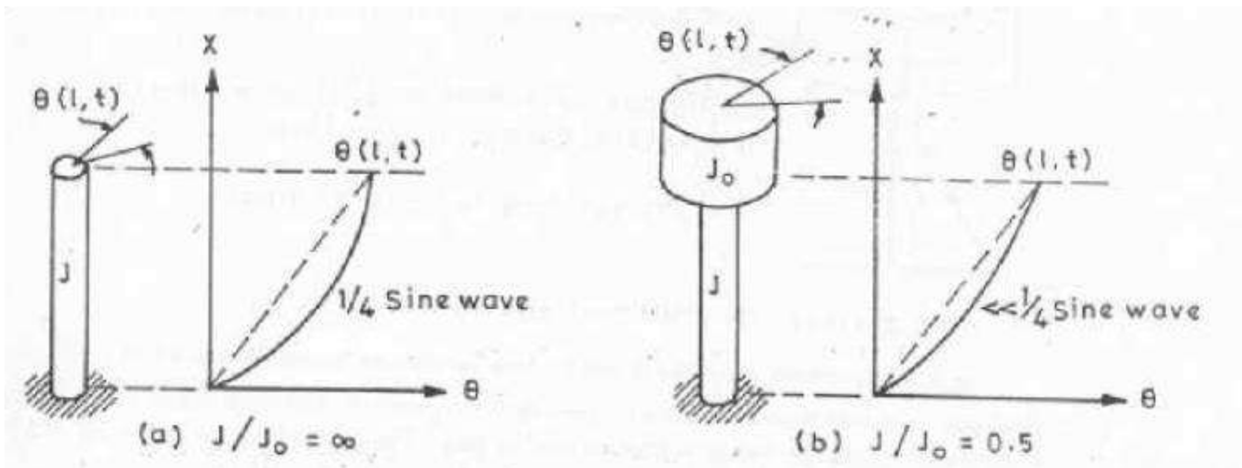


Fig.4.1: Resonance Column test

Calibration and determination of G and ξ:

Hardin (1970) suggested the following procedure of calibration of the apparatus described in Fig. 1b:

(i) For this model the vibration excitation device itself, without a specimen attached is a single degree of freedom system. Firstly remove the specimen cap and the additional rigid mass, connect the sine wave generator to the vibration excitation device and vary the excitation frequency to determine the resonant frequency f_{n1} of the device.

(ii) Attach the additional rigid mass of polar moment of inertia Jo' and determine the resonant frequency f_{nA} of the new system.

The rotational spring constant (torque per unit rotation), K_0, of the spring about the axis of specimen can be obtained using Eq.4. 1

$$K_0 = \frac{4\pi - J_A f_{nA}^2}{[1-(\frac{f_{nA}}{f_{n1}})^2}\text{---------------------} \qquad \text{Eq.4.1}$$

(iii) With the added mass removed and with the specimen cap, specimen and all apparatus, determine the resonant frequency, f_{n0}. The value of mass polar moment of inertia of the rigid mass, J_0 can be computed using Eq.4.2.

$$J_0 = \frac{K_0}{4\pi f_{n0}^2}\text{----------------------} \qquad \text{Eq.4.2}$$

Now at resonance cut off the power and record the decay curve for the vibration, From the decay curve compute the logarithmic decrement for the apparatus, as follows

$$\delta = \frac{1}{n} log_e \frac{A_1}{A_2}\text{----------------} \qquad \text{Eq.4.3}$$

Under steady state vibrations, the apparatus damping constant, D is given by

$$D = \frac{\delta}{\pi}\sqrt{K_0 J_0}\text{--------------------} \qquad \text{Eq.4.4}$$

The procedure of obtaining G and ξ has been explained in the following steps:

Calculate the mass density of the specimen, ρ, from Eq. (4.5),

$$\rho = \frac{4W}{\pi d^2 lg}\text{--------------------------} \qquad \text{Eq.4.5}$$

Where W =Total weight of specimen

l= Length of specimen
d = Diameter of specimen
g =Acceleration due to gravity,
(ii) Calculate the inertia of the specimen about its axis *J*, *as* follows:

$J = \rho \frac{\pi}{32} d^4 l$-------------------- Eq.4.6

(iii) Calculate the system factor, T as follows:

$T = \frac{J_0}{J} - \frac{K_0}{4\pi^2 f_{nR}^2 J}$-------------------- Eq.4.7

Where J_0 = Mass polar moment of inertia of the apparatus
K_0 = Rotational spring constant,
J = Inertia of the specimen
f_{nR} = Resonant frequency of the complete system.
iv) To measure the torque current constant, K_t excite the apparatus successively at frequencies $(\sqrt{2}, 2), f_{n0}\sqrt{2}$ and $2f_{n0}$, during the steady state vibration at each of these frequencies measure the current flowing through the coils, C in amperes, and the displacement amplitude of vibration, θ in radians. For each frequency compute the torque-current constant K_t as follows

$K_t = \frac{K_0 \theta}{C M_f}$---------------- Eq.4.8

Where M_f is given in Table 4.1

Frequency	M_f
$(\frac{\sqrt{2}}{2}) f_{n0}$	2
$(\sqrt{2})\, f_{n0}$	1
$2f_{n0}$	1/3

(iv) Using Figure 4.2, determine the dimensionless frequency F for the value of T computed in step (ii).

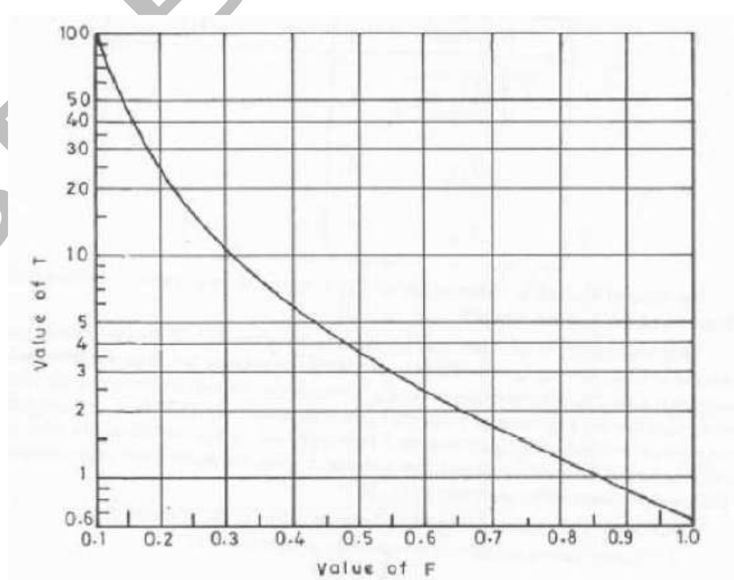


Fig.4.2: System factor T versus F

$G = 4\pi^2 \rho [\frac{f_{nR}}{F} l]^2$----------------- Eq.4.9

For steady state vibration, the damping factor of the system is given as

$D_s = \frac{1}{4\pi^2 J^2 f_{nR}^2} \left[\frac{K_t C_R}{\theta_R} - 2\pi D_A F_{nR}\right]$-------------- Eq.4.10

C_R is the mode shape

4.2.2 Cyclic Triaxial Compression Test

In general the stress-deformation and strength characteristics of a soil depend on the following factors:

1. Type of soil
2. Relative density in case of cohesionless soils; consistency limits, water content and state of disturbance in cohesive soils
3. Initial static stress level i.e. sustained stress
4. Magnitude of dynamic stress
5. Number of pulses of dynamic load
6. Frequency of loading
7. Shape of wave form of loading
8. One directional or two directional loading

In one directional loading only compression of the sample is done while in two directional loading both compression and extension is done. All the factors listed above can be studied lucidly on a triaxial set up.

Casagrande and Shannon (1948, 1949) developed the following three types of apparatus for studying the strength of soils under transient loading (Table.4.2)

Table 4.2: Type of Apparatus

Type of apparatus	Time of loading (seconds)	Remarks
(i) Pendulum loading	0.05 to 0.01	Suitable for performing fast transient tests
(ii) Falling beam	0.5 to 300	
(iii) Hydraulic loading	0.05 to any desired larger value	

Time of loading was defined as the time between the beginning of test and the point at which the maximum compressive stress is reached (Figure 4.3). The pendulum loading apparatus (Figure 4. 4) utilizes the energy of a pendulum which, when released from a selected height, strikes a spring connected to the piston rod of a hydraulic lower cylinder. This lower cylinder is connected hydraulically to an upper cylinder, which is mounted on a loading frame.

i) Pendulum Loading Test: A pendulum loading system was first developed by Casagrande and Shannon (1948-49). The loading mechanism is based on the utilization of energy of a pendulum when released from a selected height and striking a spring connected to the piston rod of a hydraulic cylinder as shown in the Fig.4. 4(a)

ii) The falling beam apparatus consists essentially of a beam with a weight and rider, a dashpot to control the velocity of the fall of the beam, and a yoke for transmitting the load from the beam to the specimen (Fig. 4.4 b). A small beam mounted above the yoke counter-balances the weight of the beam.

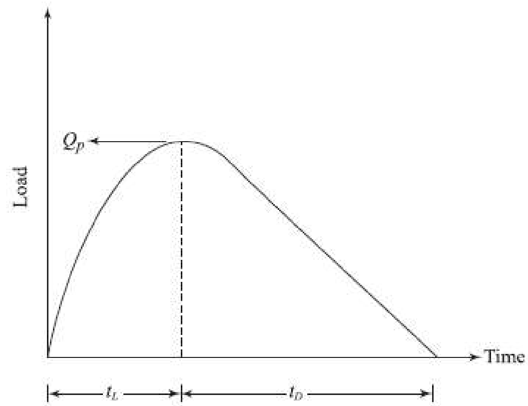


Fig.4.3 Time of loading in transient tests

iii) Pendulum Loading Test: A pendulum loading system was first developed by Casagrande and Shannon (1948-49). The loading mechanism is based on the utilization of energy of a pendulum when released from a selected height and striking a spring connected to the piston rod of a hydraulic cylinder as shown in the Fig. 4.4(a)

iv) The falling beam apparatus consists essentially of a beam with a weight and rider, a dashpot to control the velocity of the fall of the beam, and a yoke for transmitting the load from the beam to the specimen (Fig 4.4 b). A small beam mounted above the yoke counter-balances the weight of the beam.

v) The hydraulic loading apparatus (Fig. 4.4 c) consists of a constant volume vane-type hydraulic pump connected to a hydraulic cylinder through valves by which either the pressure in the cylinder or the volume of the liquid delivered to the cylinder can be controlled. The peak load that can be produced by this apparatus is much greater than can be obtained by either the pendulum type or falling beam apparatus.

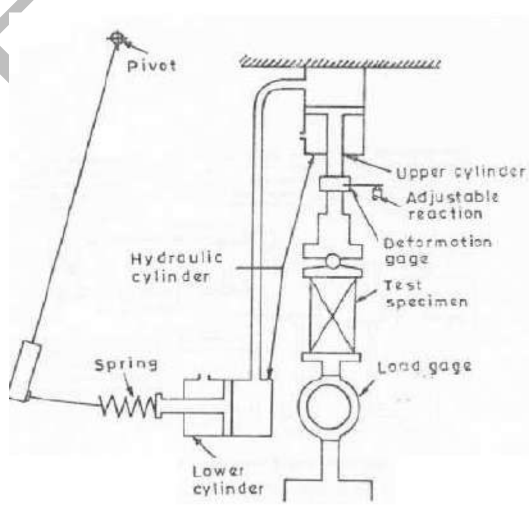


Fig. 4.4 (a) Pendulum loading apparatus (Casagrande and Shannon. 1948)

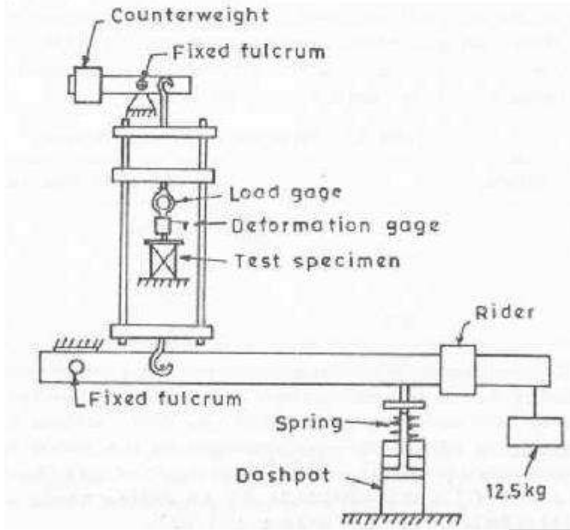


Fig.4.4 (b) Falling beam apparatus (Casagrande and Shannon. 1948)

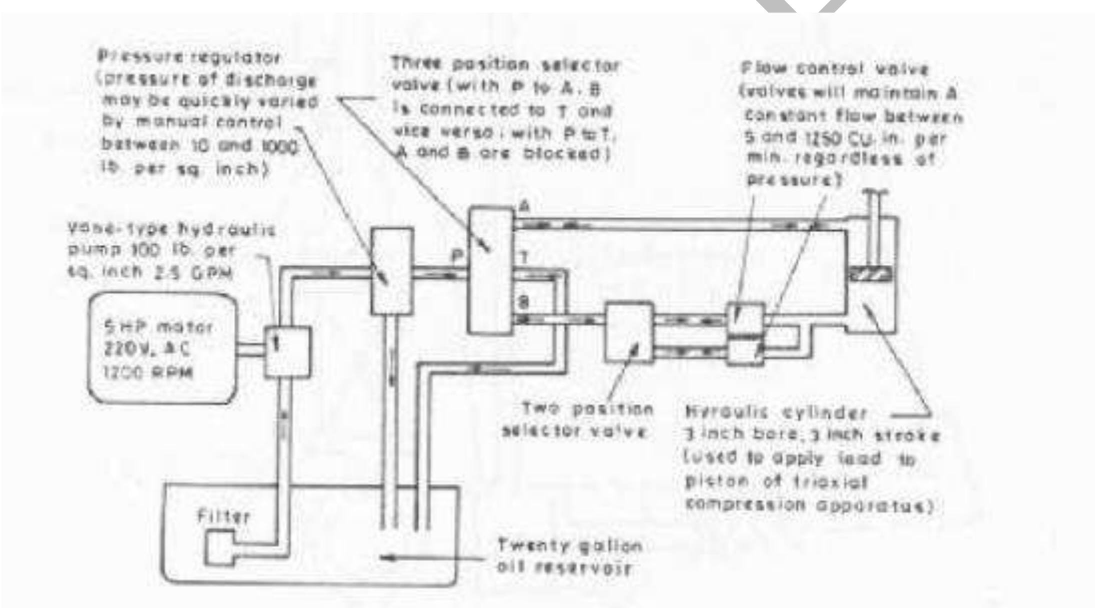


Fig. 4.4 c: Hydraulic loading apparatus (Casagrande & Shannon, 1948)

For measuring load, a load gage of rectangular or cylindrical shape is used, with four strain gages mounted on the inside face. For measuring deflection, a thin flexible steel spring cantilever is used with strain gages mounted on the cantilever, the base of which is clamped to the loading piston.

A simultaneous plot of stress and strain versus time from an unconfined compression test with a time of loading of 0.02 *s* on cambridge clay is shown in Figure 4.5. Similar plots were prepared for other times of loading on Manchester sand. Using this data, stress-strain plots were obtained as shown in Figs.6 *a* and *b*. In these figures, stress-strain curves for corresponding static tests are also shown. Typical plots of maximum compressive stress versus time of loading (or unconfined and confined transient tests on Cambridge clay are shown in Fig. 4.7 *a* and *b* respectively. A typical plot in terms of principal stress ratio a failure and time of loading for Manchester sand is shown in Fig. 4.8.

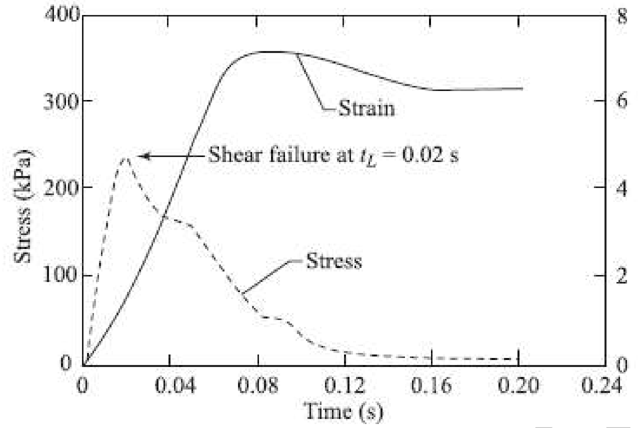


Fig.4.5: Time Vs stress and strain in an unconfined transient test on Cambridge clay (Casagrande & Shannon, 1948)

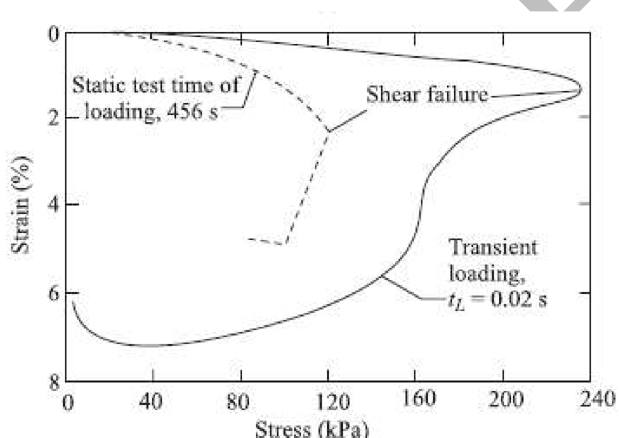


Fig 4.6 (a): Stress vrs Strain curves

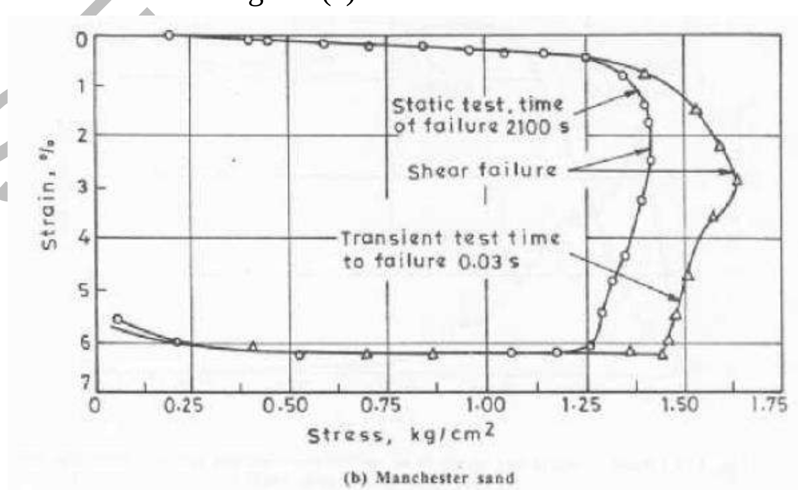


Fig. 4.6(b): Stress Vs Strain Curves (Manchester sand)

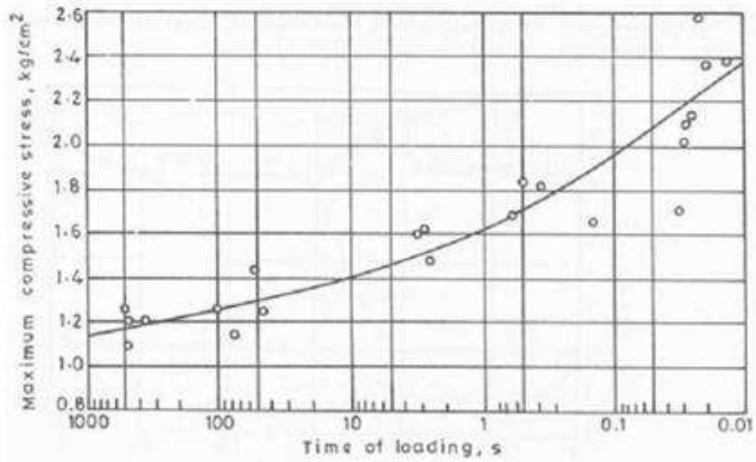


Fig. 4.7(a): Maximum Compressive Stress (unconfined) versus Time of Loading for Cambridge clay

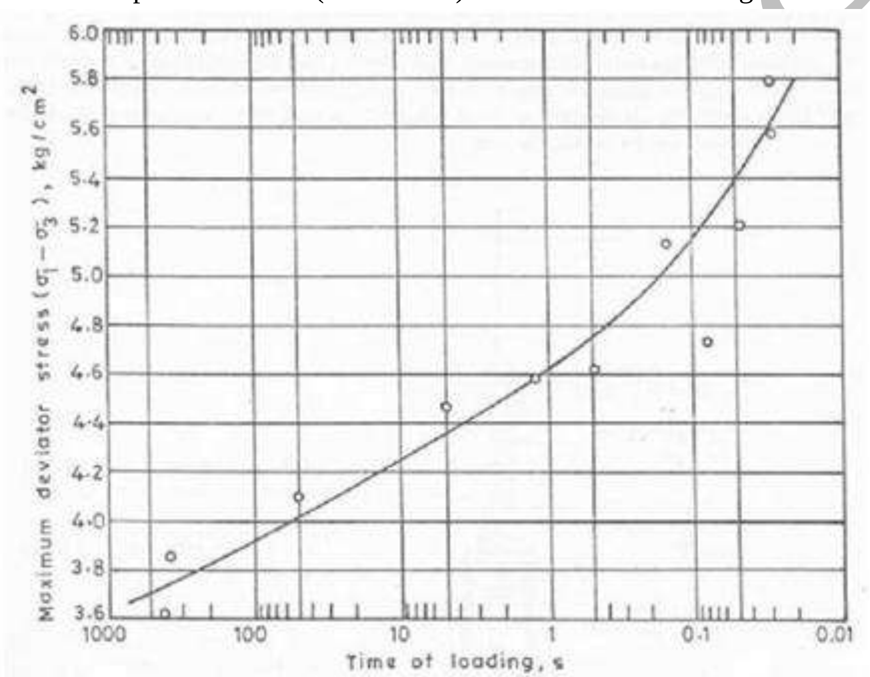


Fig.4.7 (b): Maximum Compressive Stress (confined) versus Time of Loading for Cambridge clay

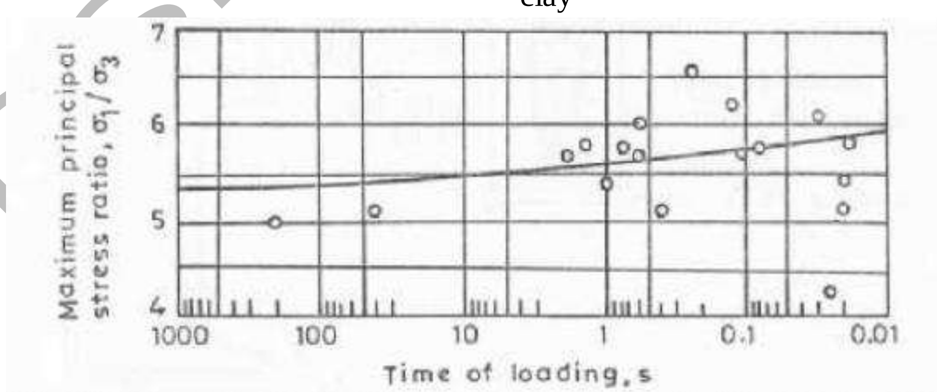


Fig. 4.8: Maximum principal stress ratio versus time of loading for Manchestor sand

Modulus of deformation is defined as the slope of a line drawn from the origin through the point on the stress-deformation curve and corresponding to stress of one-half the strength. It is found that in case of clays, modulus of deformation in fast transient tests was about two times that obtained in static tests. In case of sands, modulus of deformation was found independent of the time of loading.

4.2.3 Summary of Cyclic Tests

In the preceding sections, various types of laboratory test methods were presented, from which the fundamental soil properties such as the shear modulus, modulus of elasticity, and damping ratio are determined. These parameters are used in the design and evaluation of the behavior of earthen, earth-supported, and earth-retaining structures. As was discussed in the preceding sections, the magnitudes of G and ξ are functions of the shear strain amplitude γ'. Hence, while selecting the values of G and ξ for a certain design work, it is essential to know the following:

a. Type of test from which the parameters can be obtained

b. Magnitude of the shear strain amplitude at which these parameters needs to be measured For example, strong ground motion and nuclear explosion can develop large strain amplitudes whereas some sensitive equipment such as electron microscopes may be very sensitive to small strain amplitudes.

Figure 9 provides is a useful reference table for geotechnical engineers; as it gives the amplitude of shear strain levels, type of applicable dynamic tests, and the area of applicability of these test results. Despite the fact that laboratory testing is not ideal, it will continue to be important because soil conditions can be better controlled in the laboratory. Parametric studies necessary for understanding the soil behaviour of soils under dynamic loading conditions must be performed in the laboratory conditions. Table 4.3 provides a comparison of the relative qualities (what property can be measured and what is the degree of quality of the measured property) of various laboratory techniques for measuring dynamic soil properties. Similarly, Table 4.4 gives a summary of the different engineering parameters that can be measured in different dynamic or cyclic laboratory tests

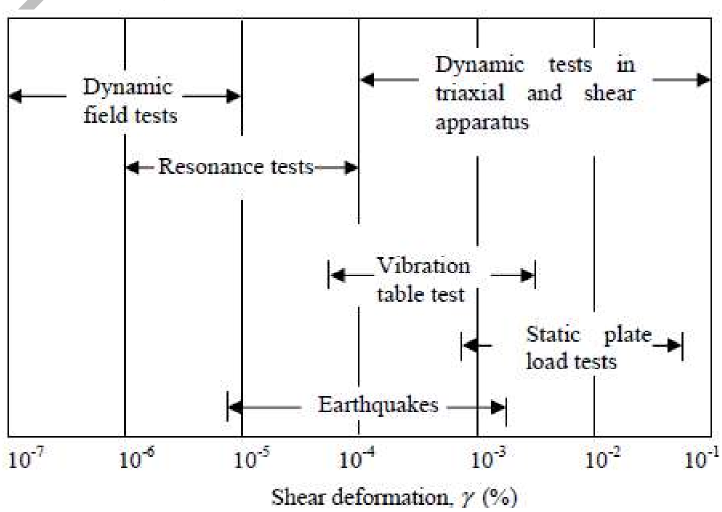


Fig. 4.9: Range and applicability of dynamic laboratory tests

Table 4.3: Relative Quality of Laboratory Techniques for Measuring Dynamic Soil Properties[a]

	Relative Quality of Test Results				
	Shear modulus	Young's modulus	Material damping	Effect of number of cycles	Attenuation
Resonant column	Good	Good	Good	Good	-
with application	-	-	-	-	Fair
Ultrasonic pulse	Fair	Fair	-	-	Poor
Cyclic triaxial	-	Good	Good	Good	-
Cyclic simple shear	Good	-	Good	Good	-
Cyclic torsional shear	Good	-	Good	Good	-

a After Silver (1981)

Table 4.4: Parameters Measured in Dynamic or Cyclic Laboratory Tests[a]

	Resonant column	Cyclic triaxial	Cyclic simple shear	Torsional shear
Load	Resonant frequency	Axial force	Horizontal force	Torque
Deformation				
Axial	Vertical displacement	Vertical displacement	Vertical displacement	Vertical displacement
Shear	Acceleration	Not measured	Horizontal displacement	Rotation
Lateral	Not usually measured	Not usually measured	Often controlled	Not usually measured
Volumetric	None for undrained tests Volume of fluid moving into or out of the sample for drained tests			
Pore water pressure	Not usually measured	Measured at boundary	Measured at boundary	Measured at boundary

[a]After Silver (1981)

4.3 FIELD TEST METHOD

Field methods generally depend on the measurement of velocity of waves propagating through the soil or on the response of soil structure systems to dynamic excitation. The following methods are in use for determining dynamic properties of soil:

1. Seismic cross-bore hole survey
2. Seismic up-hole survey
3. Seismic down-hole survey
4. Seismic refraction survey
5. Vertical block resonance test

6. Horizontal block resonance test
7. Cyclic plate load test
8. Standard penetration test

4.3.1 Seismic Cross-borehole Survey

This method is based on the measurement of velocity of wave propagation from one borehole to another. Figure 4.10 shows the essentials of seismic cross-hole method outlined by Stoke and Woods (1972).A source of seismic energy is generated at the bottom of one borehole and the time of travel of the shear wave from this borehole to another at known distance is measured. Shear wave velocity is then computed by dividing the distance between the boreholes by the travel time.

As discussed above, seismic cross-borehole survey can be done using two boreholes one has the source for causing wave generation and another having geophone for recording travel time. However, for extensive investigations and better accuracy, three or more boreholes arranged in a straight line should be used.

In this case the wave velocities can be calculated from the time intervals between succeeding pairs of holes, eliminating most of the concern over triggering the timing instruments and the effects of borehole casing and backfilling (Stokoe and Hour, 1978). Also this arrangement of bore holes in a straight line overcomes problems of site anisotropy by examining one direction only at a time.

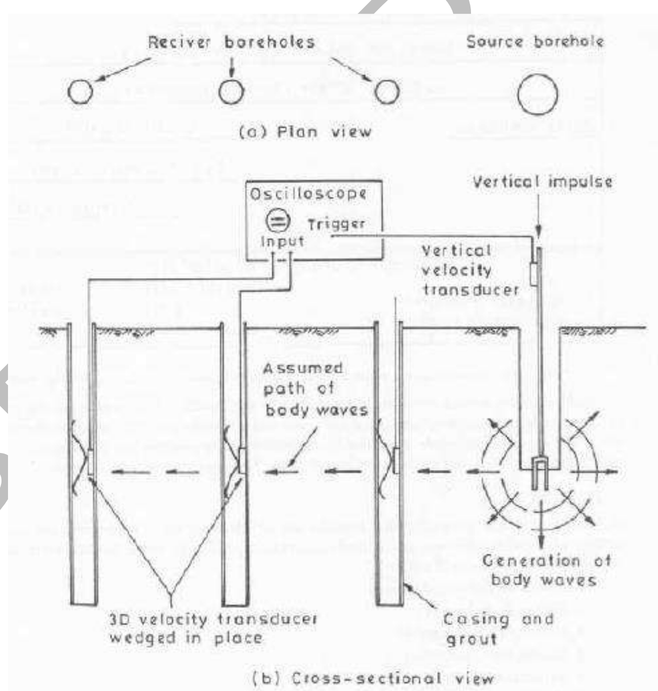


Fig. 4.10: Multiple hole seismic cross hole survey

4.3.2 Seismic Up-Hole Survey

Seismic up-hole survey is done by using only one borehole. In this method the receiver is placed at the surface, and shear waves are generated at different depths within the borehole. Figure 4.11shows the schematic presentation of the arrangement used in seismic up-hole survey (Gote et al., 1977).This method gives the average value of wave velocity for the soil between the excitation and the receivers if one receiver is used, or between the receivers.

The major disadvantage in seismic up-hole survey is that it is more difficult to generate waves of the desired type.

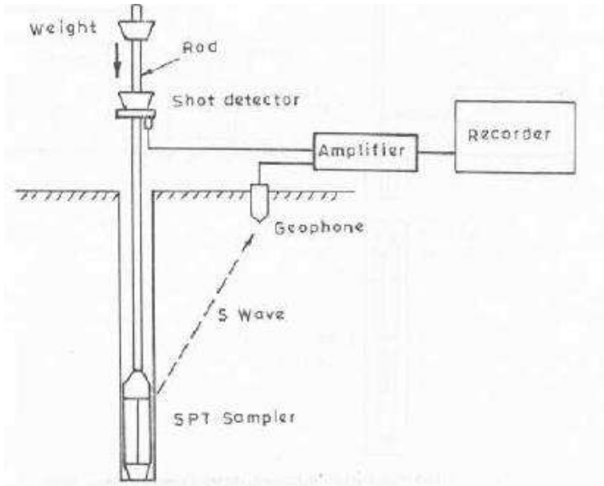


Fig.4.11: Seismic Up-hole survey

4.3.3 Seismic Down-hole Survey

In this method, seismic waves are generated at the surface of the ground near the top of the borehole, and travel times of the body waves between the source and the receivers which have been clamped to the borehole wall at predetermined depths are obtained. The arrangement used in seismic down-hole survey is shown schematic ally in Figure 4.12. This also requires only one borehole.

The main advantage of this method is that low velocity layers can be detected even if trapped between layers of greater velocity provided the geophone spacings are close enough.

4.3.4 Vertical Block Resonance Test

The vertical block resonance test is used for determining the values of coefficient of elastic uniform compression (C_u), Young's modulus (E) and damping ratio (ξ) of the soil.

According to IS 5249: 1984, a test block of size 1.5 *m* x 0.75 *m* x 0.70 *m* high is casted in M15concrete in a pit of plan dimensions 4.5 *m* x 2.75 m and depth equal to the proposed depth of foundation. Foundation bolts should be embedded into the concrete block at the time of casting for fixing the oscillator assembly. The oscillator assembly is mounted on the block so that it generates purely vertical sinusoidal vibrations. The line of action of vibrating force should pass through the centre of gravity of the block. Two acceleration or displacement pickups are mounted on the top of the block as shown in Figure 4.13 such that they sense the vertical motion of the block. A schematic diagram of the set up is shown in Figure 4.13.

The mechanical oscillator works on the principle of eccentric masses mounted on two shafts rotating in opposite directions. The force generated by the oscillator is given by

$F_d = 2m_e e^2\omega$--------------- Eq.4.11

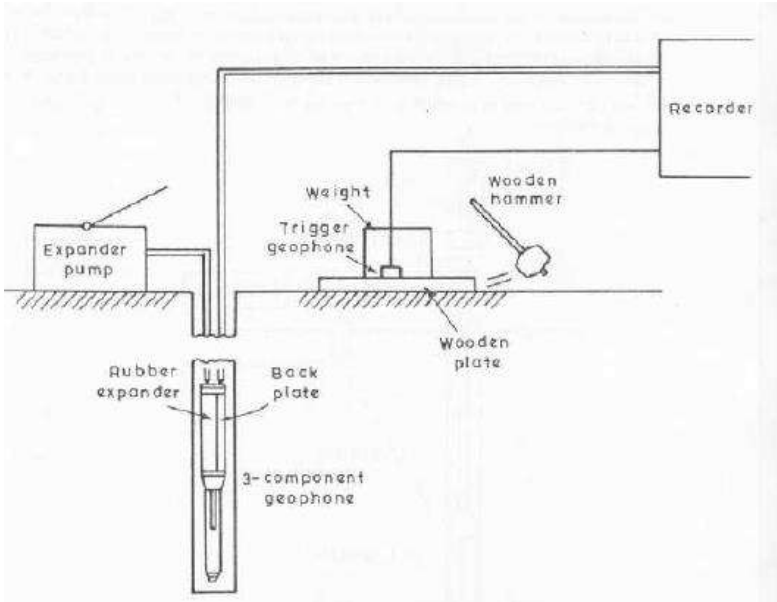


Fig. 4.12: Seismic Down-hole survey

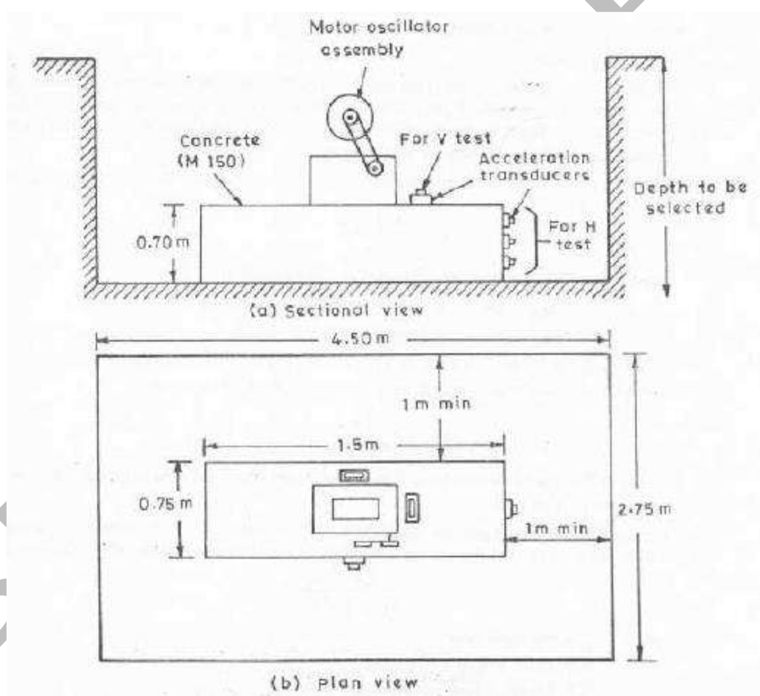


Fig.4.13: Set-up for block resonance test

The oscillator is first set at a particular eccentricity (e).As evident from Eq. (4.11) higher the eccentricity more will be the force level. It is then operated at constant frequency and the acceleration of the oscillatory motion of the block is monitored. The oscillator frequency is increased in steps, and the signals *of* monitoring pickups are recorded. At any eccentricity and frequency the dynamic force should not exceed 20 percent of the total mass of the block and oscillator assembly. The amplitude *of* vibration (A_Z) at a given frequency is given by

$$A_Z = \frac{a_z}{4\pi^2 f^2} \quad \text{Eq.4.12}$$

a_z = Vertical acceleration of the block, mm/s^2

f = frequency, Hz.

Amplitude versus frequency curves are plotted for each eccentricity to determine the natural frequency of the foundation-soil system (Fig.4.14).

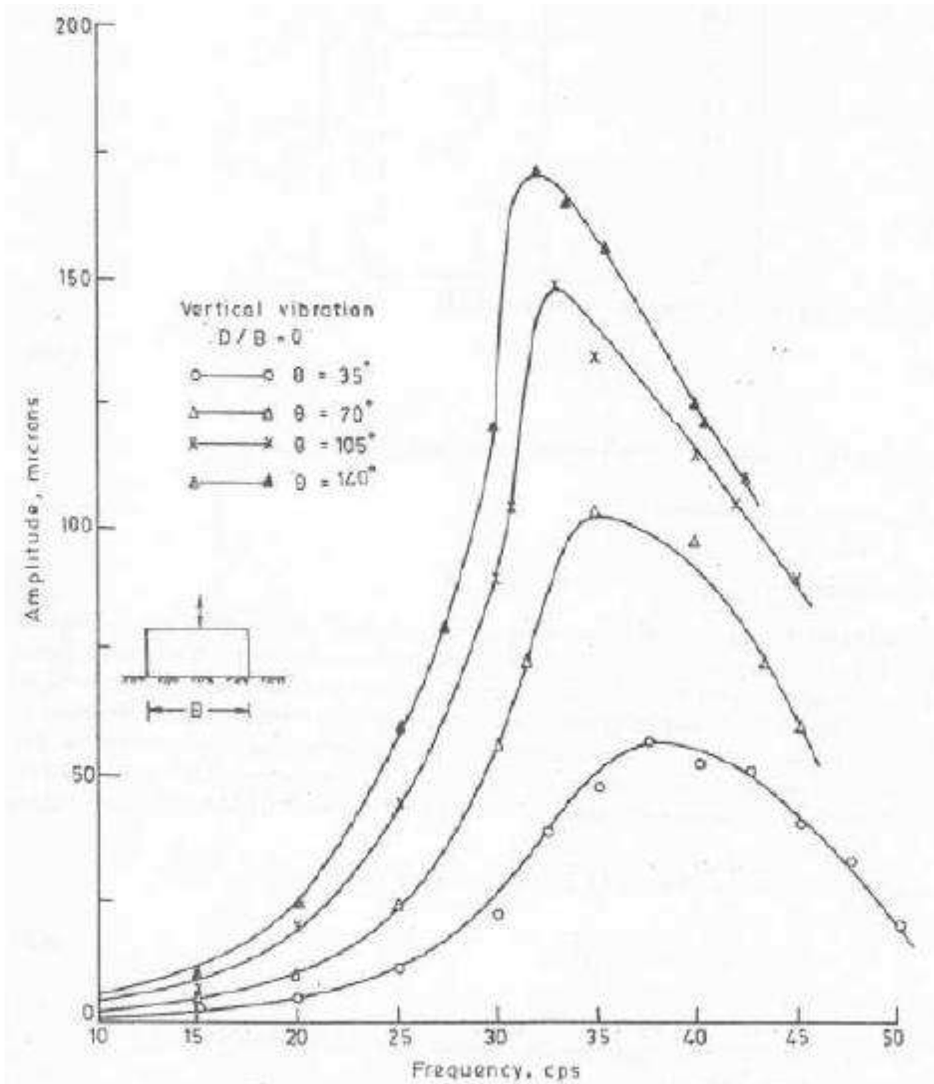


Fig.4.14: Amplitude versus frequency plot for vertical vibration test

The natural frequency, f_{nz}, at different eccentricity (i.e. force level) is different because different forces cause different strain levels *of* the block which may be accounted for when appropriate design parameters are being chosen.

The coefficient *of* elastic uniform compression *(C_u)* of the soil is then determined using Eq. (4.13)

$C_u = \frac{4\pi^2 {f_{nz}}^2 m}{A}$---------- Eq.4.13

Where, f_{nz} = Natural frequency of foundation-soil system, Hz

m = Mass of the block oscillator and motor, Kg –sec^2/m

A = Base contact area of the block, m^2

From the value of C_u obtained from Eq. (3) for the test block of contact area A the value of C_{u1} for the actual foundation having contact area A_1 may be obtained from Eq. (4.14)

$$C_{u1} = C_u\sqrt{\frac{A}{A_1}} \text{----------------} \qquad \text{Eq.4.14}$$

The Eq. (4.14) is valid for base areas of foundations up to 10 m². For areas larger than 10 m², the value Cu obtained for 10 m² is used.

The value of damping ratio ξ is determined using Eq. (4.15) as

$$\xi = \frac{f_2 - f_1}{2f_{nz}} \text{----------------} \qquad \text{Eq.4.15}$$

Where, f_1, f_2= Two frequencies at which amplitudes is equal to $\frac{A_{max}}{\sqrt{2}}$

A_{max}= Maximum amplitude

f_{nz} = Resonant frequency

The coefficient of elastic uniform compression *(C_u)* is related to the elastic Young's modulus (E) by Eq (6) which is in the form of Boussinesq relationship for the elastic settlement *of* a surface footing.

$$C_u = \frac{E}{1-\mu^2}\frac{C_s}{\sqrt{BL}} \text{-------------------} \qquad \text{Eq.4.16}$$

where μ = Poisson's ratio

B = Width of base of the block

L = Length of base of the block

C_s = Coefficient depending on ${}^{L}/_{B}$ ratio

Barkan (1962) recommended the values of C_s for various ${}^{L}/_{B}$ ratios as listed in Table 4.5

Table 4.5: values of C_s for various ${}^{L}/_{B}$ ratios

${}^{L}/_{B}$	*Cs*
1.0	1.06
1.5	1.07
2.0	1.09
3.0	1.13
5.0	1.22
10.0	1.41

The value of damping ratio ξ is determined using Eq. (4.17) as

$$\xi = \frac{f_2 - f_1}{2f_{rz}} \text{--------------------------} \qquad \text{Eq.4.17}$$

Where, f_1,f_2 = Two frequencies at which amplitudes is equal to $\frac{A_{max}}{\sqrt{2}}$

A_{max} = Maximum amplitude

f_{rz}= Resonant frequency

Example No.1

A vertical vibration test was conducted on a 1.5m x 0.75 m x 0.70 m high concrete block in an open pit having depth 2.0 which is equal to the anticipated depth of actual foundation. The test was repeated at different settings (e) of eccentric masses.
The data obtained from the tests are given below:

Sl.No	θ(degree)	f_{nz}	Amplitude at Resonance (Micron)
1	36	41.0	13.0
2	72	40.0	24.0
3	108	34.0	32.0
4	144	31.0	40.0

The soil is sandy in nature having angle of internal friction is 35° and saturated density is 20 kN/ m^3). The water table lies at a depth of 3.0 m below the ground surface. Probable size of the actual foundation is 4.0 x 3.0 x 3.5 m high. Determine the values of C_u, E and G to be adopted for the design of actual foundation.
Limiting vertical amplitude of the machine is 150 microns.

Sol:

1. Area of Block=1.5x0.75=1.125 m^2
   Mass of Block=1.125x0.75x2400=1890 kg
   Mass of oscillator and motor=100 kg (assumed)
   Mass of block, oscillator and motoa=1890+100=1990 kg
2. $C_u = \frac{4\pi^2 {f_{nz}}^2 m}{A} = \frac{4\pi^2 \times {f_{nz}}^2 \times 1990}{1.125\times 100} = 69.84{f_{nz}}^2\ kN/m^2$
   The calculated values of C_u for different observed resonance frequencies are tabulated as shown in Table 1

C_u can be evaluated from Eq.4.6 as $C_u = \frac{E}{1-\mu^2}\frac{C_S}{\sqrt{BL}}$ for L/B=2 C_S=1.09

Assume μ=0.35, $E = \frac{\sqrt{1.125}(1-0.35^2)}{1.09} \times C_u$=$0.854C_u\ kN/m^2$

$G = \frac{E}{2(1+\mu)} = \frac{0.854C_u}{2(1+0.35))}$=$0.316C_u\ kN/m^2$

For different values of C_u , E and G values are calculated and tabulated as shown in table 1

3. Correction for confining pressure and area
   The mean effective confining pressure $\overline{\sigma_{01}}$ at depth of' one -half the width below the centre of block is given by

$$\overline{\sigma_{01}} = \overline{\sigma_v}\frac{(1+2K_0)}{3}$$

Where $\overline{\sigma_v} = \overline{\sigma_{v1}} + \overline{\sigma_{v2}}$
$\overline{\sigma_{v1}}$ =Effective overburden pressure at the depth under consideration
$\overline{\sigma_{v2}}$=Increase in vertical pressure due to the weight of block
Assuming that the top 2.0 m soil has a moist unit weight of 18 kN/m^3, and the next 1.0 m soil i.e. up to water table is saturated then

$$\overline{\sigma_{v1}} = 18 \times 2.0 + 20 \times \frac{0.70}{2} = 43\ kN/m^2$$

$$\overline{\sigma_{v2}} = \frac{4q}{4\pi}\left[\frac{2mn\sqrt{m^2+n^2+1}}{m^2+n^2+1+m^2n^2} \times \frac{m^2+n^2+2}{m^2+n^2+1} + sin^{-1}\frac{2mn\sqrt{m^2+n^2+1}}{m^2+n^2+1+m^2n^2}\right]$$

$m = \frac{L/2}{Z} = \frac{1.5/2}{0.7/2}$=2.14

$n = \frac{B/2}{Z} = \frac{0.75/2}{0.7/2}$=1.07

Where q=24x0.7=16.8 kN/m^2

Substituting for the values of m and n, we get

$$\overline{\sigma_{v2}} = 13.44\ kN/m^2$$

Now $\overline{\sigma_v} = \overline{\sigma_{v1}} + \overline{\sigma_{v2}} = 43 + 13.44 = 56.44\ kN/m^2$

$$K_0 = 1 - sin\emptyset = 0.426$$

$$\overline{\sigma_{01}} = \overline{\sigma_v}\frac{(1+2K_0)}{3} = 56.44 \times \frac{1+2\times 0.426}{3} = 34.84\ kN/m^2$$

For the actual foundation, $\overline{\sigma_{v1}}$=18x2.0+20x1.0+ (20-10)x0.5= 61 $\frac{kN}{m^2}$

$$m = \frac{4.0/2}{3.0/2} = 1.33$$

$$n = \frac{3.0/2}{3.0/2} = 1.0$$

$$q = 24 \times 3.5 = 84\frac{kN}{m^2}$$

Substituting the values of *m*, *n* and *q* we get

$\overline{\sigma_{02}}$=124.76[$\frac{1+2\times0.426}{3}$] = 77.01 $\frac{kN}{m^2}$

Area of actual foundation =4.0x0.3=12.0 m^2 ($> 10m^2$)

Hence, $\frac{C_{u2}}{C_{u1}} = \frac{E_2}{E_1} = \frac{G_2}{G_1} = \left(\frac{\overline{\sigma_{02}}}{\overline{\sigma_{01}}}\right)^{0.5} \times \left(\frac{A_1}{A_2}\right)^{0.5} = \left(\frac{77.0}{34.84}\right)^{0.5} \times \left(\frac{1.125}{10}\right)^{0.5} = 0.4986$

For actual foundation C_u =0.4986x C_u for block

Table 1:

Sl. No	θ in deg.	f_{nz}	Amplitude at Resonance (micron)	For Test Block			For Actual Foundation		
				C_u $10^4 kN/m^2$	E $10^4 kN/m^2$	G $10^4 kN/m^2$	C_u $10^4 kN/m^2$	E $10^4 kN/m^2$	G $10^4 kN/m^2$
1	36	41	13	11.74	10.03	3.71	5.85	5.00	1.85
2	72	40	24	11.17	9.54	3.53	5.57	4.76	1.77
3	108	34	32	7.15	6.11	2.26	3.56	3.05	1.13
4	144	31	40	6.71	5.73	2.12	3.35	2.86	1.06

Strain Level Correction

The values of strain levels corresponding to values of C_u = amplitude at resonance per width of test block are given as

Sl.No	C_u (Test Block)	Strain Level(10^{-4})
1	11.74	0.173 ($13x10^{-6}/0.75$)
2	11.17	0.320 ($24x10^{-6}/0.75$)

3	7.15	0.427 (32x10^{-6}/0.75)
4	6.71	0.533 (40x10^{-6}/0.75)

Strain in Actual foundation=$\frac{150\times10^{-6}}{3.0} = 0.5\times10^{-4}$

The value of C_u, E and G corresponding to actual strain level of foundation can be obtained by interpolation as

$C_u = \left[3.56-(3.56-3.35)\frac{0.5-0.427}{0.533-0.427}\right]\times10^4$= 2.31x10^4 *kN/m^2*

$E = \left[3.05-(3.05-2.86)\frac{0.5-0.427}{0.533-0.427}\right]\times10^4$ =1.97x10^4 *kN/m^2*

$G = \left[1.13-(1.13-1.06)\frac{0.5-0.427}{0.533-0.427}\right]\times10^4$= 0.73x10^4 *kN/m^2*

4.3.5 Horizontal Block Resonance Test

Horizontal block resonance test is also performed on the block set up as shown in Figure 4.14. In this test, the mechanical oscillator is mounted on the block so that horizontal sinusoidal vibrations are generated in the direction of the longitudinal axis of the block. Three acceleration or displacement pickups are mounted along the vertical centre line of the transverse face of the block to sense horizontal vibrations (Fig.4.14 a). The oscillator is excited in steps starting from rest condition. The signal of each acceleration pick up is amplified and recorded. Rest of the procedure is same as described for vertical block resonance test. Similar tests can be performed by exciting the block in the direction of transverse axis.

The amplitude of Horizontal vibrations (Ax) is obtained using Eq. (4.18).

$A_x = \frac{a_x}{4\pi^2 f^2}$------------------------------ Eq.4.18

Where, a_x(mm) = Horizontal acceleration in the direction under consideration in mm/s^2

f = Frequency in Hz

Amplitude versus frequency curves are plotted for each force level to obtain the natural frequency, f_{nz} of the block soil system as done in vertical resonance test. A typical frequency versus amplitude plot is shown in Figure 4.15. It may be noted that the case of horizontal vibration is a problem of two degrees of freedom.

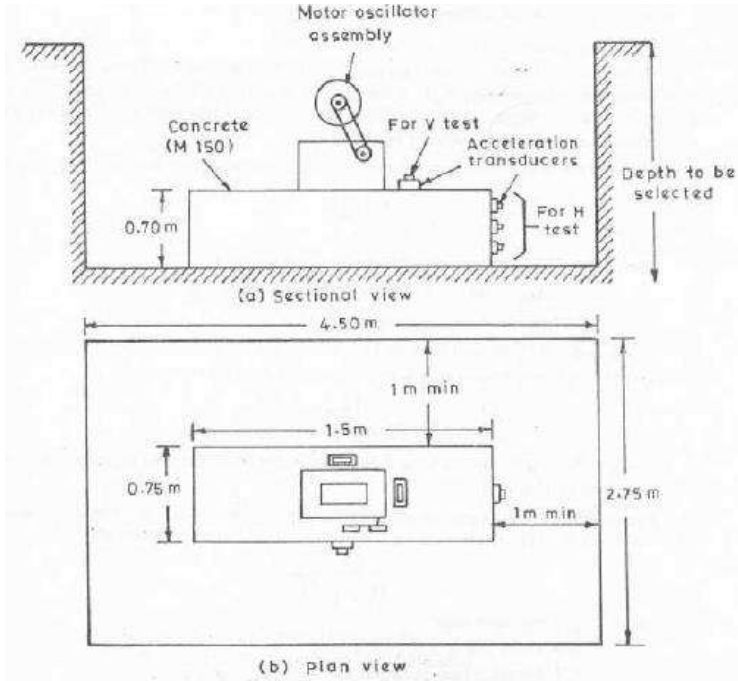


Fig.4.14 Experimental set up

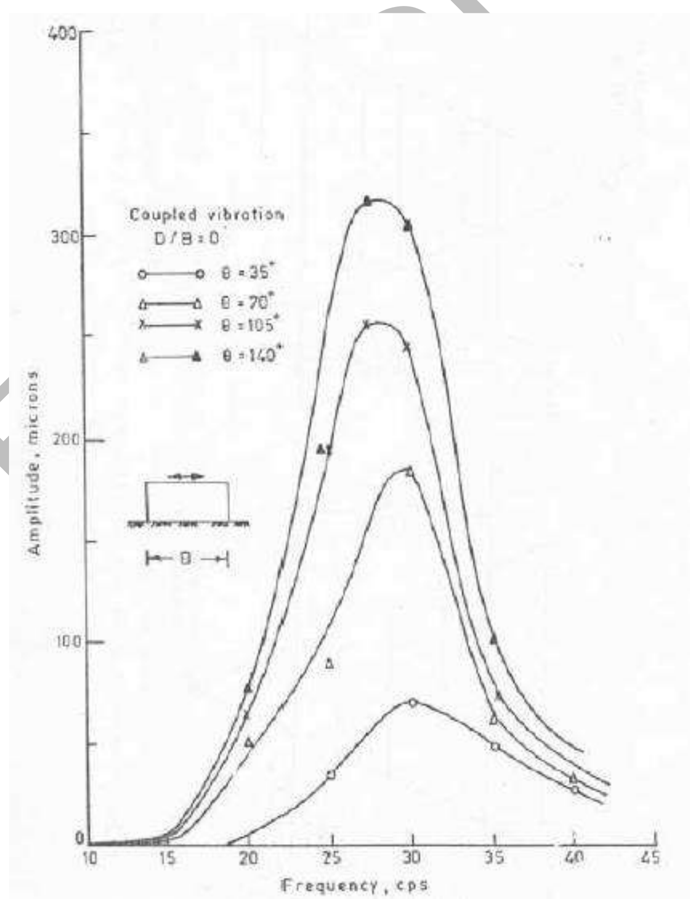


Fig.4.15: Amplitude versus frequency plots for horizontal resonance test

The coefficient of elastic uniform shear C_τ of soil is given as

$$C_\tau = \frac{8\pi^2 r f_{nx}^2}{(A_0+I_0)\pm\sqrt{(A_0+I_0)^2-4rA_0I_0}} \text{----------------------} \qquad \text{Eq.4.19}$$

Where $r = \frac{M_m}{M_{mo}}$

f_{nx} = Horizontal resonant frequency of block soil system

$A_0 = \frac{A}{M}$-

A = Contact area of block with soil

M = Mass of block, oscillator and soil

$I_0 = 3.46\frac{I}{M_{mo}}$------------------------------------- Eq.4.20

Mm = Mass moment of inertia of block, oscillator, motor, etc. about the horizontal axis passing through the centre of gravity of block and perpendicular to the direction of vibration

M_{mo} = Mass moment of inertia of the block, oscillator; motor etc. about the horizontal axis passing through centre of contact area of block and soil and perpendicular to the direction of vibration.

I = Moment of inertia of the foundation contact area about the horizontal axis passing through the centre of gravity of area and perpendicular to the direction of vibration.

In Eq. (4.19), negative sign is taken when the system vibrates in first mode and positive sign when the system vibrates in second mode. For the size of the block recommended in IS 5249-1977 and for first natural frequency, the Eq. (4.19) reduces to

$C_\tau = 92.3 f_{nx}^2$----------------- Eq.4.21 unit of C_τ in this equation is kN/m^2

The coefficient of elastic uniform shear ($C_{\tau 1}$) for actual area of foundation (A_1) is given by

$$C_{\tau 1} = C_\tau\sqrt{\frac{A}{A_1}} \text{--------------------} \qquad \text{Eq.4.22}$$

IS 5249: 1977 recommends the following relations between *Cu* and C_τ, C_φ and C_ψ

Cu = 1.5 to 2.0 C_τ -------------- Eq.4.23

$C_\emptyset$ = 3.46 C_τ ------------------ Eq.4.24

C_ψ = 0.75 Cu------------------- Eq.4.25

4.3.6 Cyclic Plate Load Test

The cyclic plate load test is performed in a test pit dug up to the proposed base level of foundation. *The equipment is same as used in static plate load test*. Circular or square bearing plates of mild steel not less than 25 *mm* thickness and varying in size from 300 to 750 *mm* with grooved bottom are used. The test pit should be at least ***five times the width*** of the plate. The equipment is assembled according to details given in **IS 1988-1982.** A typical set up is shown in Figure 4.16.

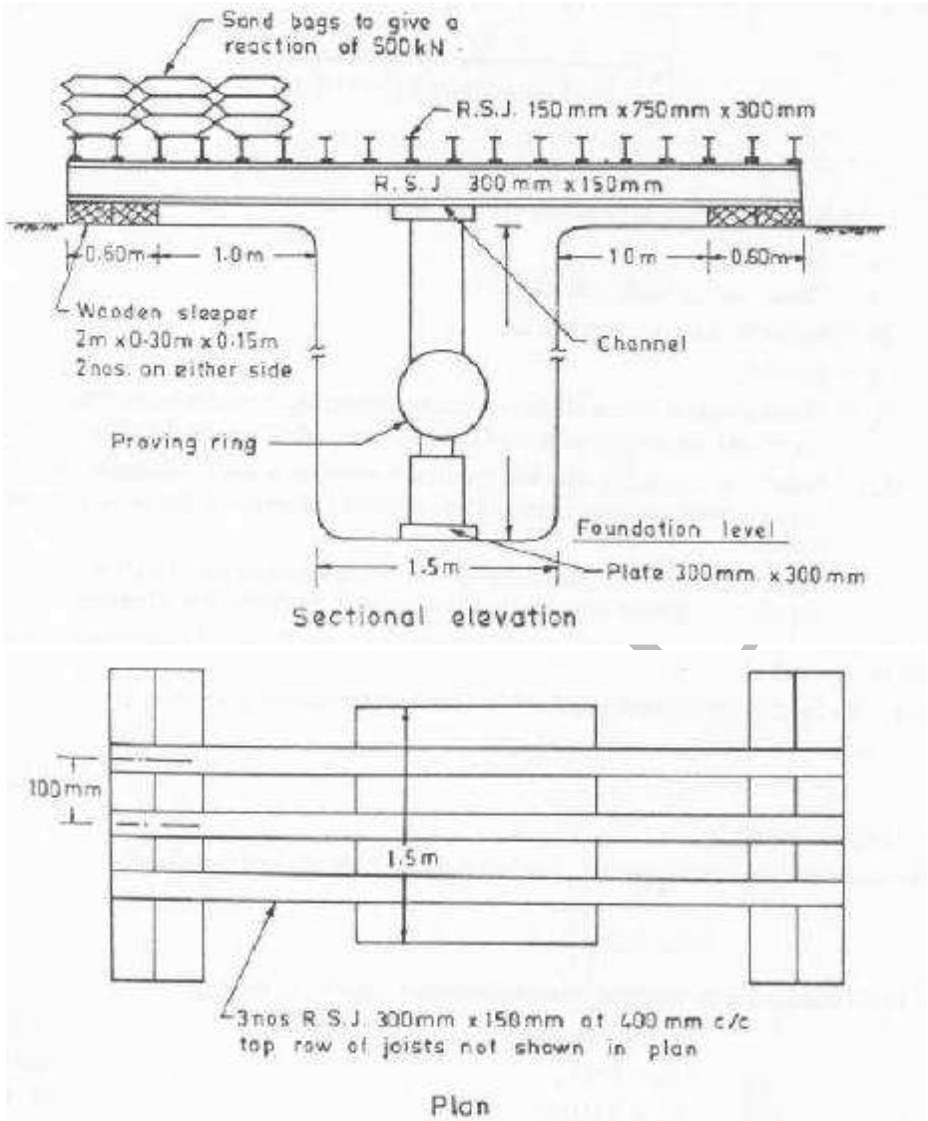


Fig. 4.16: Experimental set up for Cyclic Plate Load test

To commence the test, a seating pressure of about 7 *kPa* is first applied to the plate. It is then removed and dial gauges are set to read zero. Load is then applied in equal cumulative increments of ***not more than 100 kPa*** or of not more than ***one fifth of the estimated allowable bearing pressure***. In cyclic plate load test, each incremental load is maintained constant till the settlement of the plate is complete. The load is then released to zero and the plate is allowed to rebound. The reading of final settlement is taken. The load is then increased to next higher magnitude of loading and maintained constant till the settlement is complete, which is recorded. The load is then reduced to zero and the settlement reading is taken. The next increment of load is then applied. ***The cycles of unloading and reloading are continued till the required final load is reached.***

The data obtained from a cyclic plate load test is shown in Figure 4.17. From this data, the load intensity versus elastic rebound is plotted as shown in Figure 4.18, and the slope of the line is coefficient of elastic uniform compression.

$C_u = \frac{P}{S_e}(kN/_{m^3})$------------------ Eq.4.26

Where P=Load Intensity in kN/m^2

S_e= Elastic rebound corresponding to P in mm

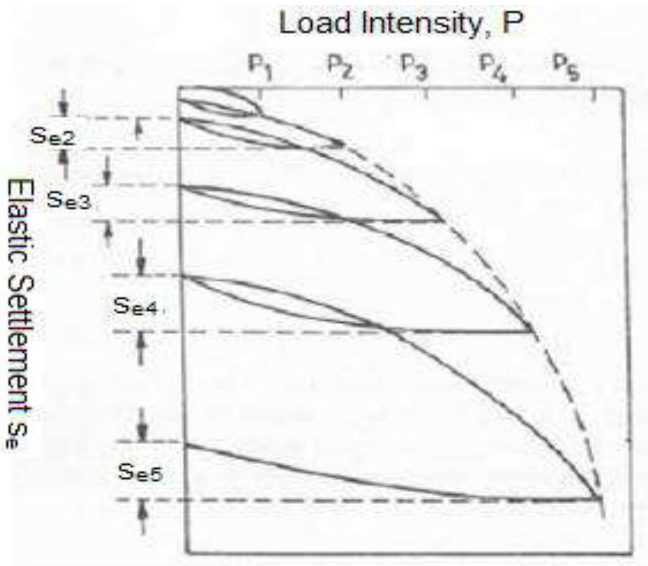


Fig. 4.17: Load Intensity versus Settlement

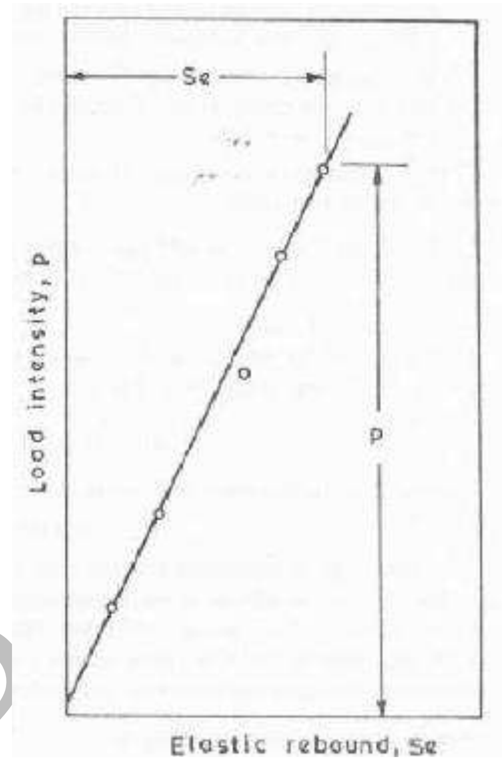


Fig.4.19: Load Intensity versus Elastic rebound

It can be shown theoretically (Barkan, 1962) that

$C_u = \frac{P}{S_e} = 1.13\frac{E}{1-\mu^2}\frac{1}{\sqrt{A}}$---------------- Eq.4.27

Where *Cu*= Sub-grade modulus, *E* = Modulus of elasticity, μ = Poisson's ratio and *A* = area of the plate.

However $G = \frac{E}{2(1+\mu)}$

So, $C_u = 2.26\frac{G(1+\mu)}{1-\mu^2}\frac{1}{\sqrt{A}}$------------ Eq.4.28

OR, $G = \frac{(1-\mu)C_u\sqrt{A}}{2.26}$----------------- Eq.4.29

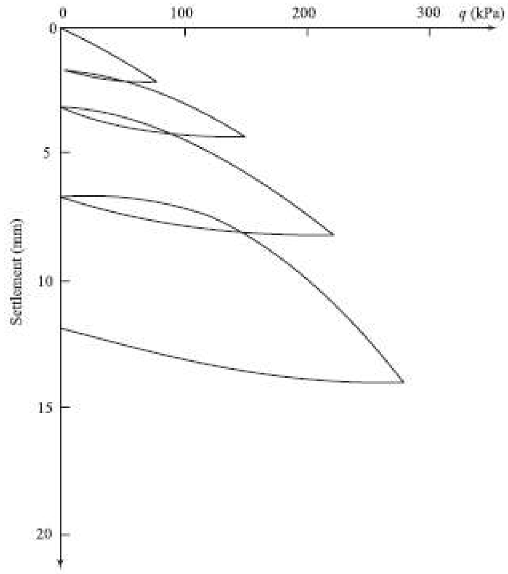
The magnitude of *Cu* can be obtained from the plot of *q* versus *se* (Figure 4). With the known value of *A* and *a* representative value of μ, the shear modulus can be calculated from Eq. (12). In non homogenous soils, it may be desirable to conduct the test at different depths or one may use different plate sizes to reflect the change in soil stiffness with depth. Again, it should be noted that this test suffers from the same limitations as reported in traditional geotechnical engineering practice for the design of foundations.

Example 1

The plot of q versus s (settlement) obtained from a cyclic plate load test is shown in Figure below. The area of the plate used for the test was 0.3 m^2. Calculate

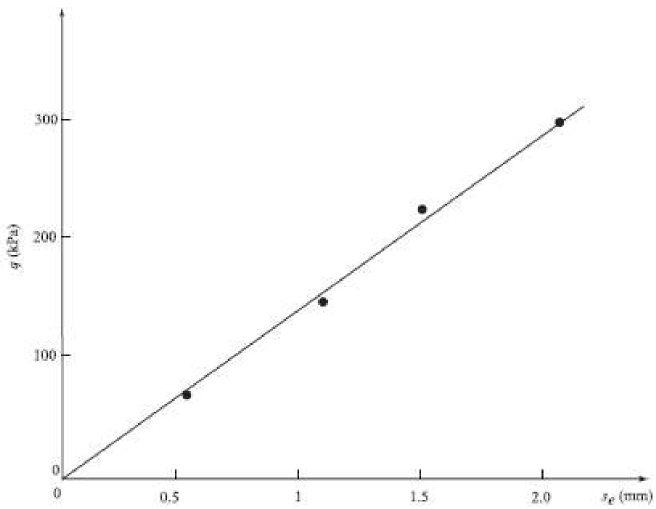
a. K_{plate}, and

b. Shear modulus G (assume $\mu = 0.35$).



Sol: From the settlement curve the following can be determined

Load per unit area, q (kPa)	Elastic settlement, s_e (mm).
75	0.53
150	1.10
225	1.50
300	2.10



From the graph

$$C_u = \frac{P}{S_e} = \frac{300}{0.0021} = 142.86\ {}^{MN}/_{m^3}$$

$$K_{plate} = \frac{qA}{S_e} = 142.86 \times 0.3$$

$$= 42.86 MN/m$$

$$G = \frac{(1-\mu)C_u\sqrt{A}}{2.26} = \frac{(1-0.35)142.86 \times \sqrt{0.3}}{2.26} = 22.5 MP_a$$

4.3.7 Standard Penetration Test

The standard penetration test (SPT) is the most extensively used situ test in India and many other countries. This test is carried in a bore hole using a split spoon sampler. As per IS: 2131-1981, steps involved in carrying out this test are as follows:

(i) The borehole is made to the depth at which the SPT has to be performed. The bottom of the borehole is cleaned.

(ii) The split-spoon sampler, attached to standard drill rods of required length is lowered into the borehole and rested at the bottom.

(iii) The split –spoon sampler is seated 150 mm by blows of a drop hammer of 65 kg falling vertically and

Freely from a height of 750 mm. Thereafter, the split spoon sampler shall be further driven 300 mm in two steps each of 150 mm. The number of blows required to effect each 150 mm of penetration shall be recorded. The first 150 mm of drive may be considered to be seating drive. The total blows required for the second and third 150 mm of penetration is termed the penetration resistance N.

If the split spoon sampler is driven less than 450 mm (total), then N-value shall be for the last 300 mm penetration. In case, the total penetration is less than 300 mm for 50 blows, it is entered as refusal in the bore log.

(iv) The split spoon sampler is then withdrawn and is detached from the drill rods. The split barrel is disconnected from the cutting shoe and the coupling. The soil sample collected inside the barrel is collected carefully and preserved for transporting the same to the laboratory for further tests.

(v) Standard penetration tests shall be conducted at every change in stratum or intervals of not more than 1.5 m whichever is less. Tests may be done at lesser intervals (usually 0.75 m) if specified or considered necessary.

The penetration test in gravelly soils requires careful interpretation since pushing a piece of gravel can greatly change the blow count.

4.3.7.1 Corrections to observed SPT values (N) in cohesionless soils

Following two types *of* corrections are normally applied to the observed SPT values (N) in cohesionless soils:

i) Corrections due to dilatancy:

In very fine, or silty, saturated sand, Terzaghi and Peck (1967) recommend that the observed N-values be

Corrected to N' if N was greater than 15 as

$N' = 15 + \frac{1}{2}(N - 15)$-----------------------Eq.4.30

Bazaraa (1967) recommended the correction as

N' = 0.6N (for N > 15) -------------------- Eq.4.31

This correction is introduced with the view that in saturated dense sand (N > 15); the fast rate of application of shear through the blows of drop hammer, is likely to induce negative pore pressures and thus temporary increase in shear strength will occur. This will lead to a N-value higher than the actual one. Since sufficient experimental evidence is not available to confirm this correction, many engineers are not applying this correction. However this correction has also been recommended in **IS: 2131-1981.**

ii) Correction due to overburden pressure:

On the basis of field tests, corrections to the N-value for overburden effects were proposed by many investigators (Gibbs and Holtz 1957; Teng 1965; Bazaraa 1967; Peck, Hanson and Thornburn 1974). The methods which are normally used are:

Bazaraa (1967)

For $\bar{\sigma}_0 < 75kPa$, $N' = \frac{4N}{1+0.04\overline{\sigma_0}}$------------------- Eq.4.32

For $\bar{\sigma}_0 > 75kPa$, $N' = \frac{4N}{3.25+0.01\overline{\sigma_0}}$------------------ Eq.4.34

where $\bar{\sigma}_0$ =effective over burden pressure, kPa

Peck, Hanson and Thornburm (1974) recommended

$N' = 0.77N\ log_{10}\frac{2000}{\bar{\sigma}_0}$---------------------- Eq.4.35

Figure 1 gives the correction factor based on Eq.4.35. Use of this figure has been recommended in IS: 2131-1981. In this figure,

$C_N = Correctio\ factor = 0.77N\ log_{10}\frac{2000}{\bar{\sigma}_0}$------------- Eq.4.36

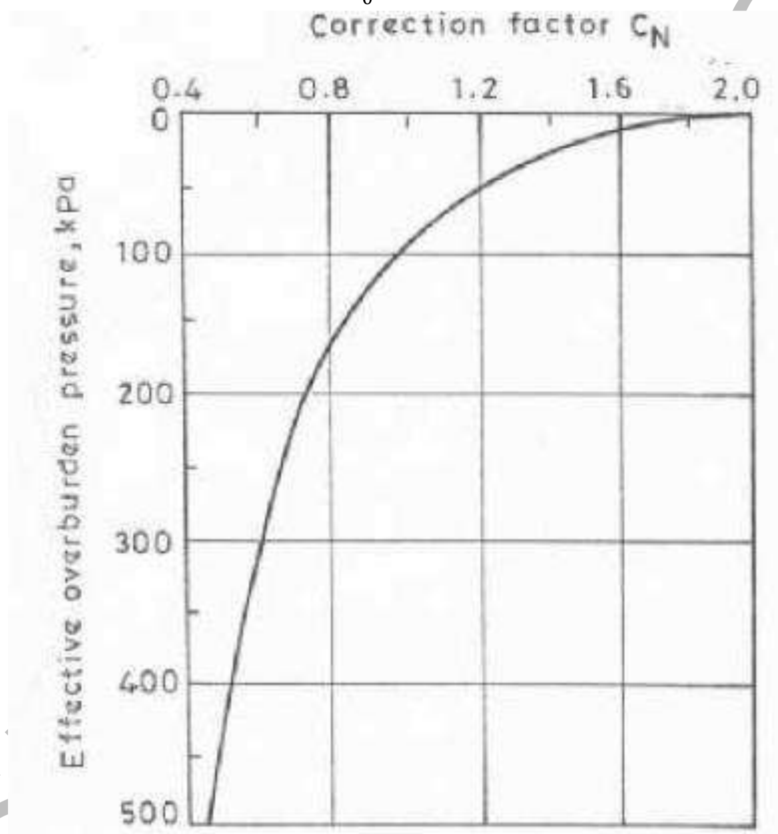


Fig. 4.20: Over burden correction

There is a controversy whether the correction due to dilatancy should be applied first and then the correction due to over burden pressure or vice-versa. However in ***IS: 2131-1981, it is recommended that the correction due to overburden should be applied first.***

4.4 FACTORS AFFECTING SHEAR MODULUS, ELASTIC MODULUS AND ELASTIC CONSTANTS

Hardin and Black (1968) have given the following factors which influence the shear modulus, elastic modulus and elastic constants:

(i) Type of soil including grain characteristics, grain shape, grain size, grading and mineralogy;
(ii) Void ratio
(iii) Initial average effective confining pressure;
(iv) Degree of saturation;
(v) Frequency of vibration and number of cycles of load
(vi) Ambient stress history and vibration history
(vii) Magnitude of dynamic stress; and
(viii) Time effects

Soil behavior over a wide range of strain amplitudes is nonlinear and on unloading follows a different stress-strain path forming a hysteresis loop as shown in Figure 4.21. The area inside this loop represents the energy absorbed by the soil during its deformation and is a measure of the internal damping within the soil.

At very low strain amplitudes (≪ 0.0001 %) the soil acts essentially as a linear elastic material with little or no loss of energy. The shear modulus under these conditions is maximum, but as the strain amplitude is increased, the shear modulus decreases and the damping within the soil increase.

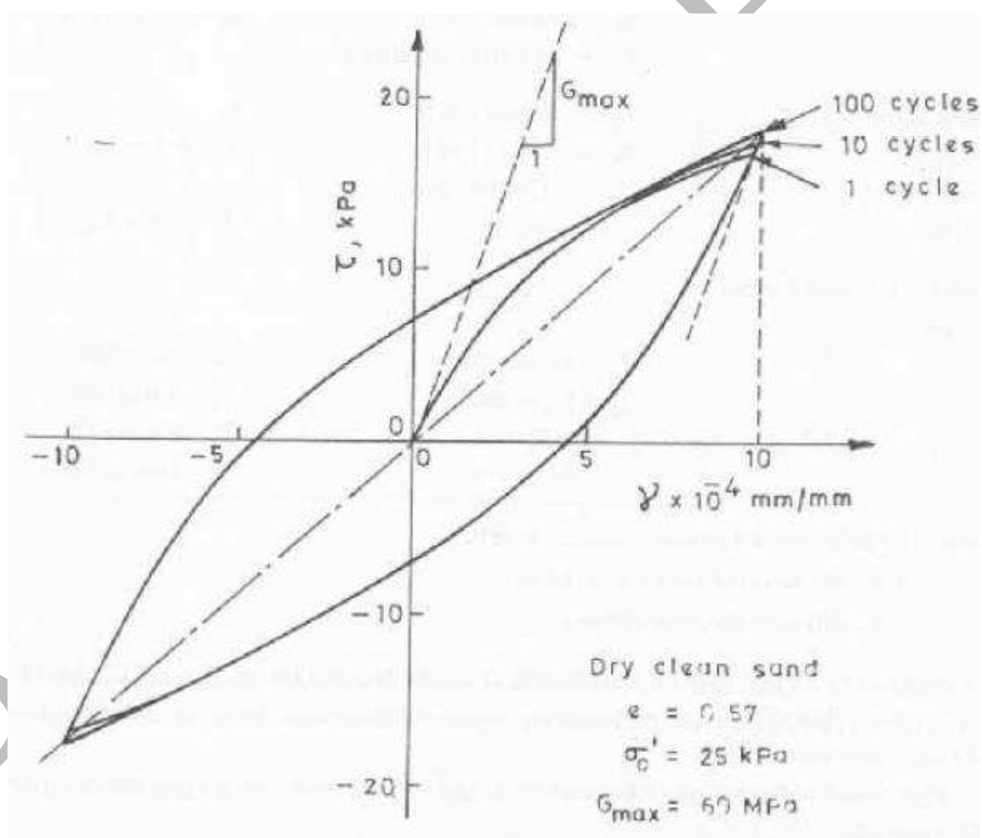


Fig.4.21: Stress-strain loop at different cycles of loading after Headley, 1985

5.0 DESIGN OF MACHINE FOUNDATIONS

5.1 Categories of machine foundations

Reciprocating machines:

It produces periodic unbalanced force and operating frequency is 600rpm. For designing unbalanced force is taken as varying sinusoidally.

Impact machines:

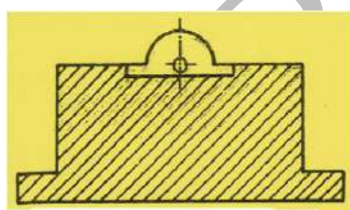
It produces impact loads at an operating frequency of 60-150 blows/min. Dynamic load attends the peak within short duration and then die out quickly. Designed as over tuned.

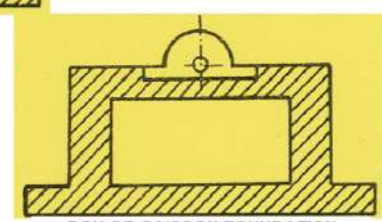
Rotary machines:

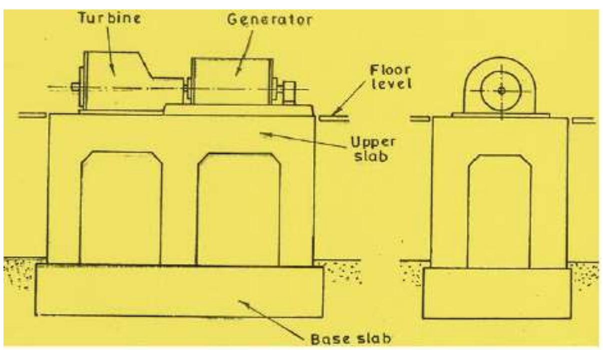
These are high speed machines with high operating frequency. Hence the foundations are designed as under tuned.

5.2 TYPES OF MACHINE FOUNDATIONS

Block type
Caisson type
Frame type


BLOCK FOUNDATION


BOX OR CAISSON FOUNDATION



Framed Foundation

Fig.5.1: Types of machine foundations

- Block type machine foundation is a solid block made of concrete block of huge mass, area and depth. The pedestal machine will be positioned on the top of the block.
- Box or caisson type is used to save some material of concrete in order to avoid an uneconomic design when the foundation requirement is huge size as well as height of the foundation is more.
- Framed foundation are used when the machine are subjected to very high operating frequency like turbo generator etc.

5.3 CRITERIA FOR THE DESIGN OF MACHINE FOUNDATIONS

- These foundations should be well design to take care of the static loads coming on the foundation.
- No shear or bearing capacity failure should occur. That means, the bearing capacity of the foundation against shear failure is to be checked.
- No excessive settlement, that is amount of settlement as calculated under static load and that has to be compared with different codal guidelines.
- Under the dynamic loading condition, the foundation should not resonate.
- The natural frequency of foundation soil must be far away from machine to avoid the resonance.
- Dynamic displacement amplitude must not exceed the permissible limit.
- Vibration of the machine and foundation system together must not be annoying to the person working in the environment and it should not damage the adjacent structures.
- Machine foundation should not design for a strong foundation to take care of the dynamic load but also safe against bearing capacity failure, as well as safe against the resonance criteria.

5.3.1 Suggested Foundation for Various Types of Machines

1. Machine producing Impulse: Block type foundation

Example: hammer, Presses etc

2. Rotating type machine with low to medium frequency: Block type foundation with large contact area.

Example: Large reciprocating engine, Compressor, large blower

3. Rotating type machine with medium to high frequency: Block type foundation resting on suitable elastic pad or spring.

Example: Medium sized reciprocating engine, diesel engine and gas engine

4. Rotating type with very high frequency: Framed foundation or massive block with minimum contact area.

Example: internal combustion engine, electric motors, turbo-generators

5.4 METHODS OF ANALYSIS

Linear elastic weightless spring MSD model

Linear elastic theory

Indian standard design code: IS 2974, Part 1 provisions

Machine foundation has to be designed by checking three criteria.

1 Dimensional criteria,

2 Vibration criteria

3 Displacement criteria

These are the three major criteria, which needs to be checked to design a machine foundation as per our Indian standard design code 2974.

5.4.1 Check the Dimension

For a block type of foundation, the criteria given that size of the foundation block must be larger than the base plate of the machine.

The second criteria says minimum all-around clearance of 150 mm. must be provided as per IS codal provision.

A third criterion is that, the foundation block should be placed deep enough on good bearing strata.

The combined centre of gravity of the machine plus foundation block should be as far below the top of the foundation as possible.

5.4.2 Vibration Check

Foundation which is having natural frequency either much higher or lower than the operating frequency of the machine, is called under tuned or over tuned respectively.

If the ratio of operating frequency to the natural frequency is less than or equal to 0.5 that can be designed as **Under tuned criterion**.

If the operating frequency is much higher than the natural frequency and frequency ratio

$\xi = \frac{\omega}{\omega_n} > 2$ for important machine and

$\xi = \frac{\omega}{\omega_n} > 1.5$ for less important machine

For both types of machine design criteria is **Over tuned.**

- To design machine for which operating frequency is very high, over tuned criteria is used because an under tuned type of foundation design result will provide negative value of mass or no mass because of the value of k which has to be excessively high.
- To design machine for which operating frequency is low, under tuned criteria is used.
- However for the range of say 1000 rpm or even in the range of 600 rpm, it is always better to check your design for both over tuned and under tuned.

5.4.3 *Displacement Criteria*

- The amplitude of permissible dynamic displacement should be less than or equals to 0.2 mm, If it exceeds, foundation is to be redesigned.
- The permissible displacement should be checked using Richart's chart. So, that it should not become annoying to the workers or adjacent structures.
- Y-axis of the chart shows dynamic displacement amplitude and X axis represents operating frequency of the machine.

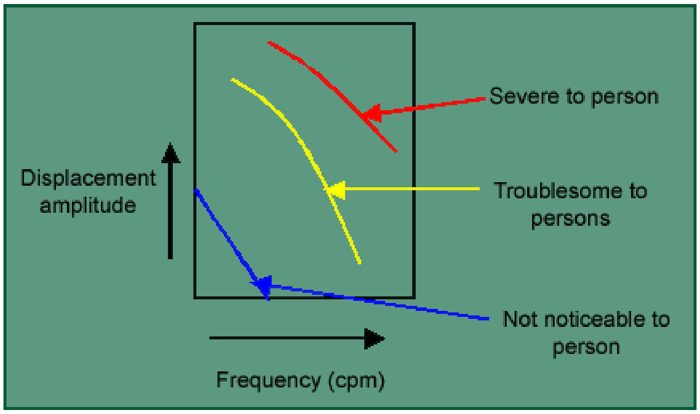
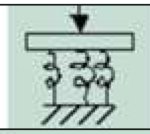
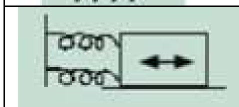
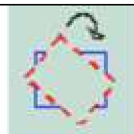
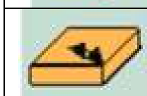


Fig.5.2: Displacement amplitude vs. frequency (Richard 1962

5.5 LINEAR ELASTIC WEIGHTLESS SPRING MSD MODEL

	$\omega_{nz} = \sqrt{\frac{K_z}{m}}$	Vertical Vibration
	$\omega_{nx} = \sqrt{\frac{K_x}{m}}$	Horizontal Vibration
	$\omega_{n\emptyset} = \sqrt{\frac{K_{n\emptyset}}{Mm_0}}$	Rocking Mode
	$\omega_{n\varphi} = \sqrt{\frac{K_{n\varphi}}{Mm_z}}$	Yawing Mode

Coefficient of uniform elastic compression, C_u

$C_u = \frac{P}{S_e}$---------------------- Eq.5.1

Where P and S_e are the load corresponding to elastic settlement

Coefficient of linear elastic shear, $C_\tau = \frac{\tau}{S_e}$------------------------ Eq.5.2

Barken (1962) proposed the following values:

$Cu=2C_z$, $C\varphi=2C_u$, $C_\tau = 1.5\ C_\varphi$------------------------ Eq.5.3

According to IS: 5249:

$C_u = 1.73C_\tau$ and $C_\tau = 1.5\ C_\varphi$------------------------------- Eq.5.4

Vertical Vibration of the Block

Load applied: $P_z = P_0 sin\omega t$------------------------------- Eq.5.5

Equation of motion: $m\ddot{Z} + K_z Z = P_0 sin\omega t$------------------- Eq.5.6

Natural Frequency = $\omega_{nz} = \sqrt{\frac{C_u A}{m}}$----------------------- Eq.5.7

Amplitude of Vertical Vibration, $A_z = \frac{P_O sin\omega t}{m(\omega_{nz}^2 - \omega^2)}$---------------- Eq.5.8

Maximum Amplitude of Vibration =$\frac{P_z}{m(\omega_{nz}^2 - \omega^2)}$-------------------- Eq.5.9

Sliding Vibration of the Block

Load applied: $P_x = P_0 sin\omega t$----------------------- Eq.5.10

Equation of motion: $m\ddot{x} + K_z x = P_0 sin\omega t$---------------- Eq.5.11

Natural Frequency = $\omega_{nx} = \sqrt{\frac{C_u A}{m}}$----------------------- Eq.5.12

Amplitude of Vertical Vibration, $A_x = \frac{P_O sin\omega t}{m(\omega_{nz}^2 - \omega^2)}$---------------- Eq.5.13

Maximum Amplitude of Vibration =$\frac{P_x}{m(\omega_{nz}^2 - \omega^2)}$----------------- Eq.5.14

5.4.3 LINEAR ELASTIC THEORY

(Based on Elastic Half Space Theory)

In 1904, Lamb studied the problem of vibration of single vibrating force acting at a point on the surface of an elastic half-space. This study included cases in which the oscillating force R acts in the vertical direction and in the horizontal direction, as shown in Figure 5.3 a and b. This is generally referred to as the ***dynamic Boussinesq problem***.

In 1936, Reissner analyzed the problem of vibration of a *uniformly loaded flexible circular area* resting on an elastic half-space. The solution was obtained by integration of Lamb's solution for a point load. Based on Reissner's work, the vertical displacement at the *center* of the flexible loaded area (Figure 5.4 a) can be given by

$Z = \frac{Q_O e^{i\omega t}}{G r_0}(f_1 + i f_2)$----------------------- Eq.5.15

Where Q_0=amplitude of the exciting force acting on the foundation

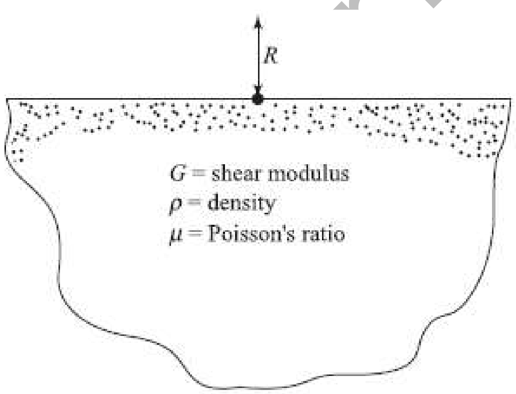
Z= periodic displacement at the centre of the loaded area

ω= circular frequency of the applied load

r_0 = radius of the loaded area

G= shear modulus of the soil

f_1, f_2=Reissner's displacement functions



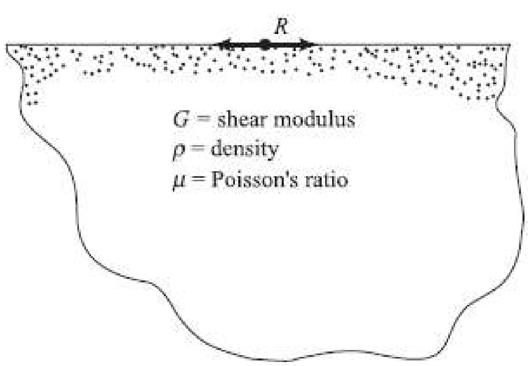


Fig. 5.3: Vibrating force on the surface of elastic half -space

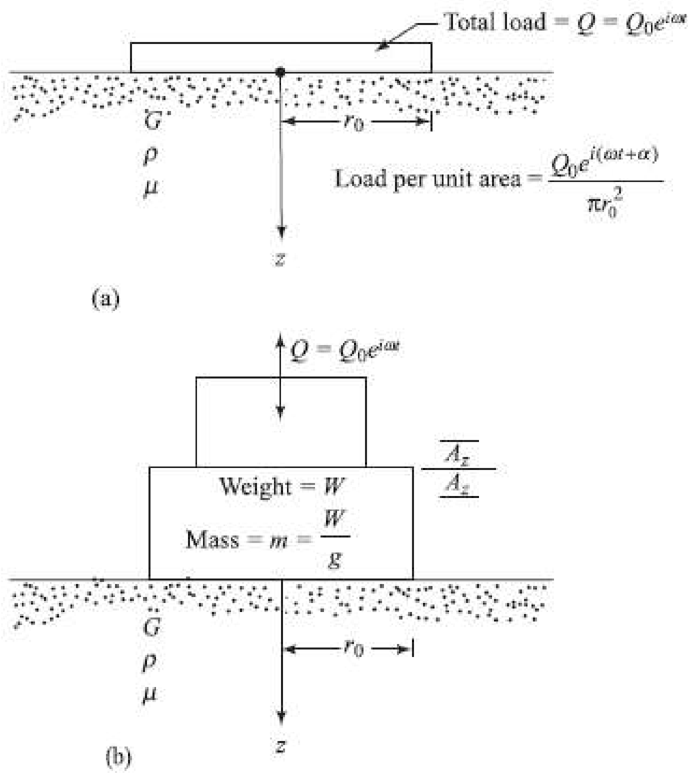


Fig.5.4: a) Vibration of uniformly loaded circular flexible area, b) Flexible circular area subjected to force vibration

The displacement functions f_1 and f_2 are related to the Poisson's ratio of the medium and the frequency of the exciting force. Now, consider a flexible circular foundation of weight W (mass = m = W/g) resting on an elastic half-space and subjected to an exciting force of magnitude of $(Q_0 e^{i(\omega t+\alpha)}$ as shown in Figure 5.4b. (*Note*: α is the phase difference between the exciting force and the displacement of the foundation.)

Using the displacement relation given in Eq. (5.15) and solving the equation of equilibrium of force, Reissner obtained the following relationships:

$A_z = \frac{Q_0}{Gr_0} Z$--------------------- Eq.5.16

Where A_z = the amplitude of vibration

Z= dimensionless amplitude

$= \sqrt{\frac{f_1^2+f_2^2}{(1-ba_0^2 f_1)^2+(ba_0^2 f_2))^2}}$--------------------- Eq.5.17

b= dimensionless mass ratio

$= \frac{m}{\rho r_0^3} = \frac{W}{g}\left[\frac{1}{(\gamma/g) r_0^3}\right] = \frac{W}{\gamma r_0^3}$--------------------- Eq.5.18

ρ= density of the elastic material

γ=unit of soil

a_0=dimensionless frequency=$\omega r_0\sqrt{\frac{\rho}{G}} = \frac{\omega r_0}{V_s}$------------------ Eq.5.19

V_s= velocity of shear wave in the elastic material on which the foundation is resting

The classical work of Reissner was further extended by Quinlan (1953) and Sung (1953). As mentioned before, Reissner's work related only to the case of flexible circular foundations where the soil reaction is uniform over the entire area (Figure 5.5a). Both Quinlan and Sung considered the cases of rigid circular foundations, the contact pressure of which is shown in Figure 5.3b, flexible foundations (Figure 5.5a), and the types of foundations for which the contact pressure distribution is parabolic, as shown in Figure 5.5c. The distribution of contact pressure q for all three cases may be expressed as follows.

For flexible circular foundations

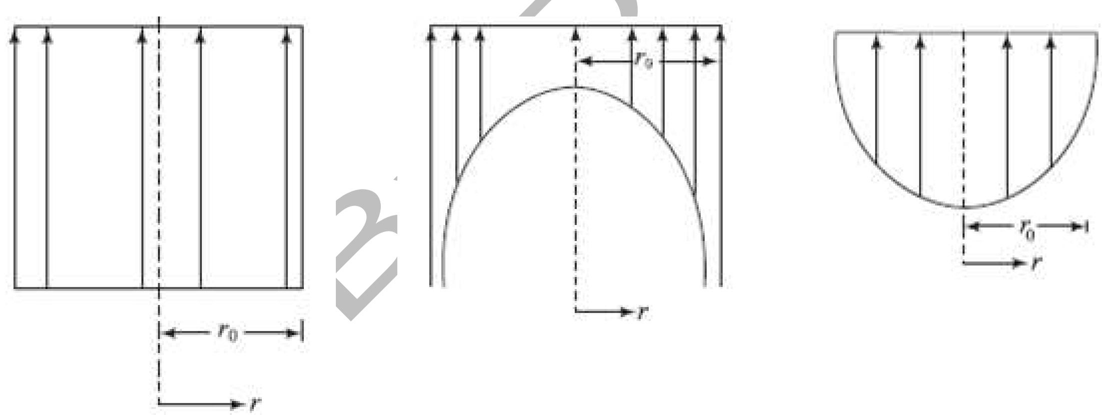
$q = \frac{Q_0 e^{i(\omega t+\alpha)}}{\pi r_0^2}$ for $r \leq r_0$--------------------- Eq.5.20

For rigid circular foundations

$q = \frac{Q_0 e^{i(\omega t+\alpha)}}{2\pi r_0\sqrt{r_0^2-r^2}}$ $(for\ r \leq r_0)$ --------------------- Eq.5.21

For foundations with parabolic contact pressure distribution

$q = \frac{2(r_0^2-r^2)Q_0 e^{i(\omega t+\alpha)}}{\pi r_0^4}$ $(for\ r \leq r_0)$ ----------------------- Eq.5.22



a) Uniform pressure distribution b) pressure distribution under rigid foundation c) parabolic pressure distribution

Fig. 5.5: Contact pressure distribution under circular footing of radius r_0

Quinlan derived the equations only for the *rigid circular* foundation; however, Sung presented the solutions for all the three class described. For all cases, the amplitude of motion can be expressed in a similar form to Eqs. (5.2 to 5.5). However, the displacement functions f_1 and f_2 will change, depending on the contact pressure distribution.

Foundations, on some occasions, may be subjected to a *frequency dependent excitation*, in contrast to the *constant-force* type of excitation just discussed. Figure 5.6 shows a foundation excited by two rotating masses. The amplitude of the exciting force can be given as

$Q = 2m_e e\omega^2 = m_1 e\omega^2$----------------------- Eq.5.23

Where $m_{1=}$ total of the rotating masses

ω= circular frequency of the rotating masses

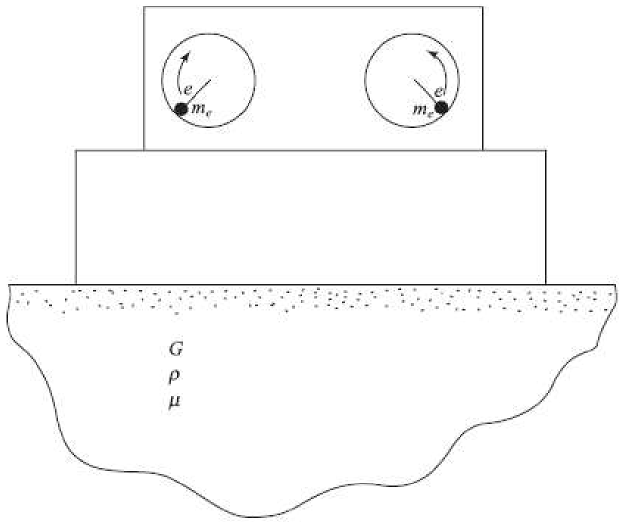


Fig. 5.6: Foundation vibration by a frequency dependent exciting force

For the above condition, the amplitude of vibration A_z can be expressed as

$$A_z = \frac{m_1 e\omega^2}{Gr_0}\sqrt{\frac{f_1^2+f_2^2}{(1-ba_0^2 f_1)^2+(ba_0^2 f_2))^2}} \text{-----------------} \quad \text{Eq.5.24}$$

Where $a_0 = \omega r_0\sqrt{\frac{\rho}{G}}$ ------------------------------- Eq.5.25

$$\omega^2 = \frac{Ga_0^2}{\rho r_0^2} \text{------------------------------------} \quad \text{Eq.5.26}$$

Substituting Eq. 5.26 into Eq. 5.24 we get

$$A_z = \frac{m_1 e a_0^2}{\rho a_0^3}\sqrt{\frac{f_1^2+f_2^2}{(1-ba_0^2 f_1)^2+(ba_0^2 f_2))^2}} = \frac{m_1 e}{\rho r_0^3}Z' \text{-------------} \quad \text{Eq.5.27}$$

Where $Z' = a_0^2\sqrt{\frac{f_1^2+f_2^2}{(1-ba_0^2 f_1)^2+(ba_0^2 f_2))^2}}$-------------------- Eq.5.28

Figures 5.7 and 5.8 show the plots of the variation of the dimensionless amplitude with a_0 (Richart, 1962) for *rigid circular* foundations (for μ = Poisson's ratio = 0.25 and b = 5, 10, 20, and 40).

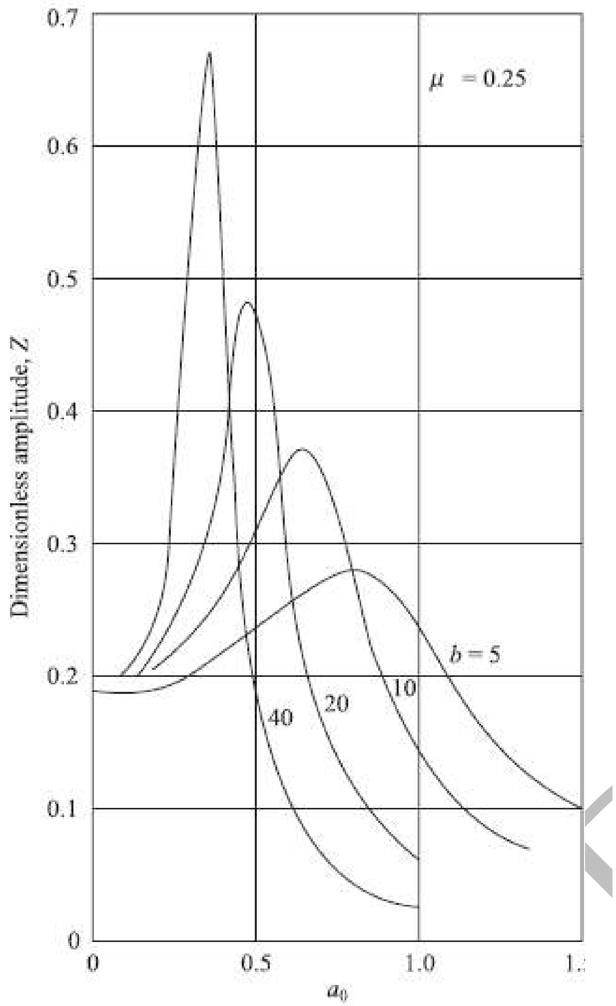


Fig. 5.7: Plot of Z versus a_0 for rigid circular foundation , Richart, 1962)

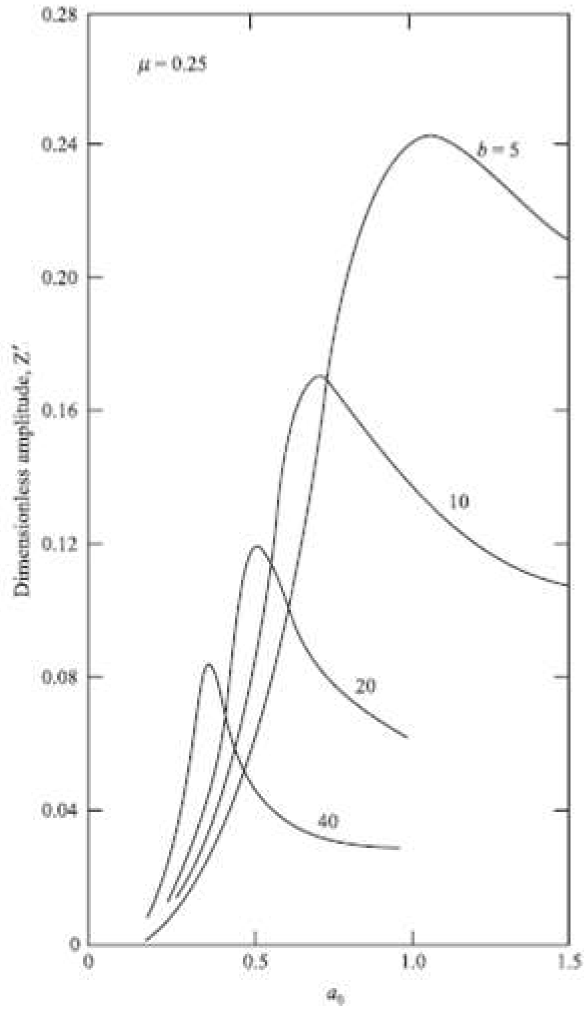


Fig. 5.8: Variation of Z' with a_0 for rigid circular foundation (redrawn after Richart, 1962)

5.4.4 *Effect of Contact Pressure Distribution and Poisson's Ratio*

The effect of the contact pressure distribution on the nature of variation of the non-dimensional amplitude Z' with a_0 is shown in Figure 5.9 (for $b = 5$ and $\mu = 0.25$). As can be seen, for a given value of a_0, the magnitude of the amplitude is highest for the case of parabolic pressure distribution and lowest for rigid bases.

For a given type of pressure distribution and mass ratio (b), the magnitude of Z' also greatly depends on the assumption of the Poisson's ratio μ. This is shown in Figure 5.10.

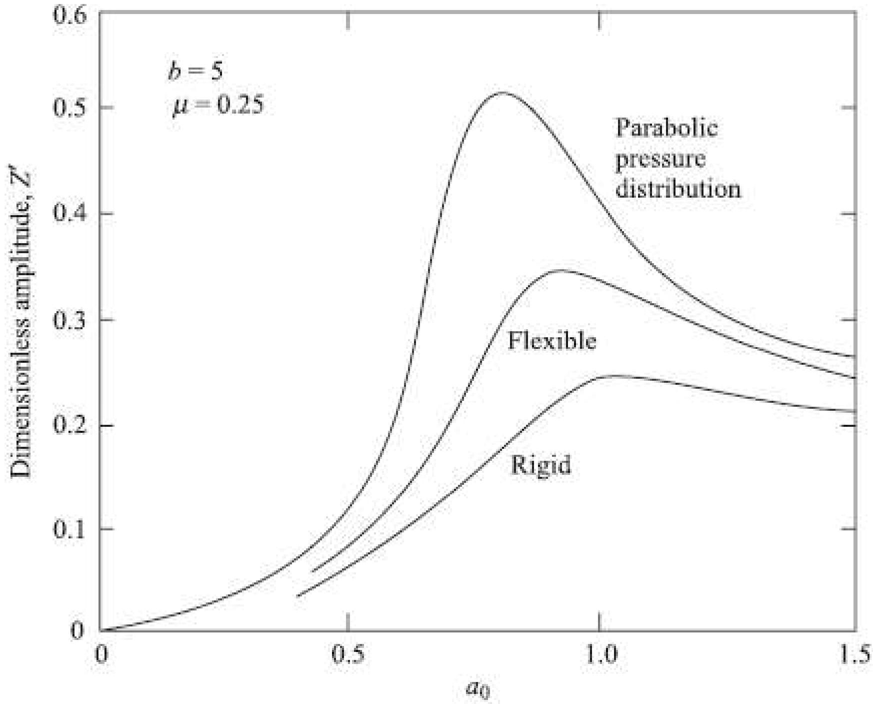


Fig. 5.9: Effect of contact pressure distribution variation of Z' with a_0 (redrawn after Richart and Whitman, 1967)

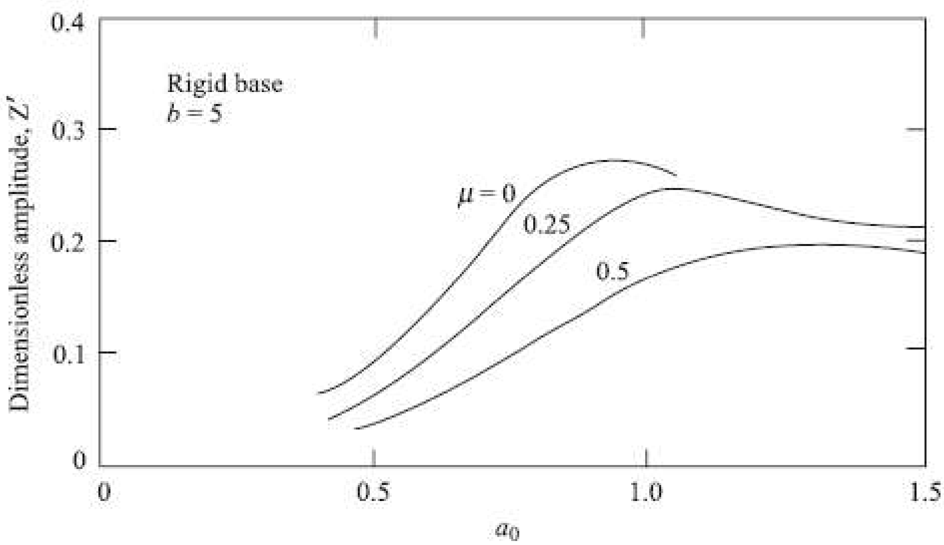


Fig.5.10: Effect of Poisson's ratio on the variation of Z' with a_0 (redrawn after Richart and Whitman, 1967)

5.4.5 Variation of Displacement Functions f_1 and f_2

As mentioned before, the displacement functions are related to the dimensionless frequency a_0 and Poisson's ratio μ. In Sung's original study, it was assumed that the contact pressure distribution remains the same throughout the range of frequency considered; however, for dynamic loading conditions, the rigid-base pressure distribution does not produce uniform displacement under the foundation. For that reason, Bycroft (1956) determined the weighted average of the displacements under a foundation. The variation of the displacement functions determined by the study is shown in Figure 5.11

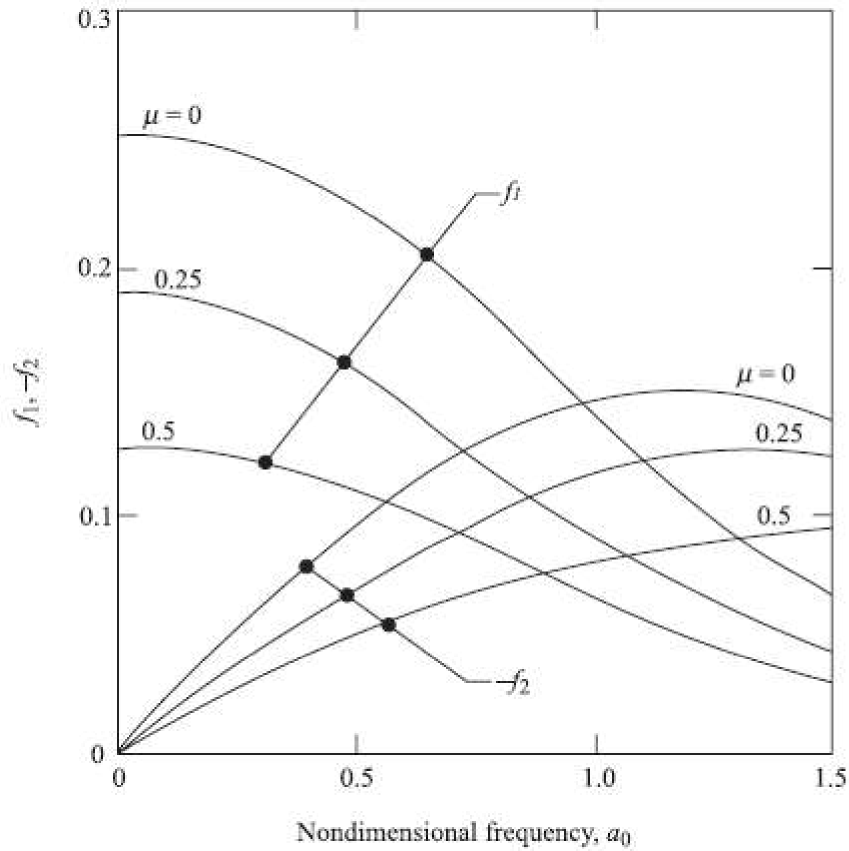


Fig.5.11: Variation of the displacement functions with a_0 and μ

5.5 Analog Solutions for Vertical Vibration of Foundations

Lysmer's Analog

A simplified model was also proposed by Lysmer and Richart (1966), in which the expressions for k_z and C_z were frequency independent. Lysmer and Richart (1966) redefined the displacement functions in the form

$$F = \frac{f}{\left[\frac{1-\mu}{4}\right]} = \frac{f_1 + i f_2}{\left[\frac{1-\mu}{4}\right]} = F_1 + iF_2 \text{------------------} \quad \text{Eq.5.29}$$

The functions F_1 and F_2 are practically independent of Poisson's ratio, as shown in Figure 5.11. The term *mass* ratio as expressed in Eq. (5.18) was also modified as

$$B_z = \left[\frac{1-\mu}{4}\right] b = \left[\frac{1-\mu}{4}\right] \frac{m}{\rho r_0^3} \text{-------------------------} \quad \text{Eq.5.30}$$

Where B_z = modified mass ratio

In this analysis, it was proposed that satisfactory results can be obtained within the range of practical interest by expressing the rigid circular foundation vibration in the form

$$m\ddot{Z} + C_z\dot{Z} + K_z Z = Q_0 e^{iwt} \text{--------------------} \quad \text{Eq.5.31}$$

Where

$K_z = static\ spring\ constant\ for\ rigid\ circular\ foundation = \frac{4Gr_0}{1-\mu}$--------- Eq.5.32

And

$C_z = \frac{3.4r_0^2}{1-\mu}\sqrt{G\rho}$---------------------------- Eq.5.33

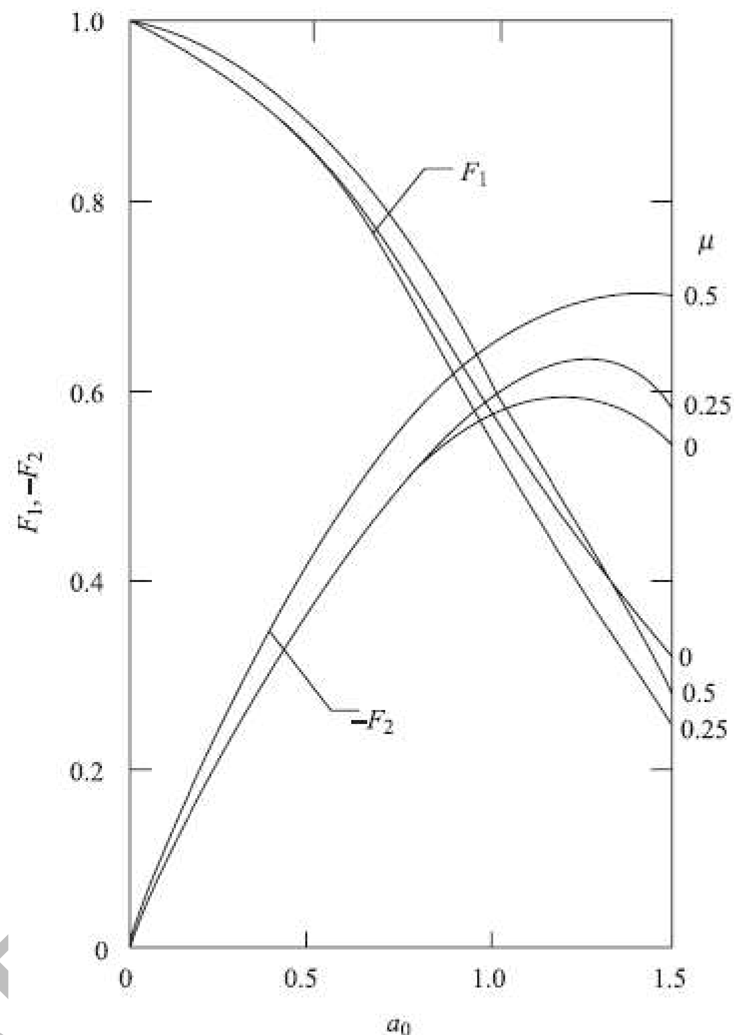


Fig.5.12: Plot of $F1$ and -$F2$ against $a0$ for rigid circular foundation subjected to vertical vibration (after Lysmer and Richart, 1966)

In Eqs. (5.32) and (5.33) the relationships for K_z and C_z are frequency independent. Equations (5.31 to 5.33) are referred to as ***Lysmer's analog***.

5.6 CALCULATION PROCEDURE FOR FOUNDATION RESPONSE, VERTICAL VIBRATION

Once the equation of motion of a rigid circular foundation is expressed in the form given in Equation (5.31), it is easy to obtain the resonant frequency and amplitude of vibration based on the mathematical expressions presented in earlier section. The general procedure is outlined next.

A. Resonant Frequency

1. *Calculation of natural frequency*. as

$f_n = \frac{1}{2\pi}\sqrt{\frac{K_z}{m}} = \frac{1}{2\pi}\sqrt{\frac{4Gr_0}{(1-\mu)m}}$-------------------- Eq.5.34

2. *Calculation of damping ratio ξ.* As given

Critical damping $C_c = 2\sqrt{K_z m} = 2\sqrt{\frac{4Gr_0}{(1-\mu)}}m$, substituting for m as $m = \frac{4B_z\rho r_0^3}{(1-\mu)}$

$= 4\sqrt{\left(\frac{4Gr_0}{(1-\mu)}\right)\left(\frac{B_z\rho r_0^3}{(1-\mu)}\right)} = \frac{8r_0^2}{(1-\mu)}\sqrt{G\rho B_z}$------------------------ Eq.5.35

Now , $\xi = \frac{C}{C_C} = \frac{\frac{3.4r_0^2}{1-\mu}\sqrt{G\rho}}{\frac{8r_0^2}{(1-\mu)}\sqrt{G\rho B_z}} = \frac{0.425}{\sqrt{B_z}}$-------------------------- Eq.5.36

3. Calculation of the resonance frequency (that is, frequency at maximum displacement). For constant force-type excitation,

$$f_m = f_n\sqrt{1-2\xi^2}$$

$= \frac{1}{2\pi}\sqrt{\frac{4Gr_0}{(1-\mu)m}} \times \sqrt{1-2(\frac{0.425}{\sqrt{B_z}})^2}$--------------------- Eq.5.37

It has also been shown by Lysmer that, for *Bz* ≥ 0.3, the following approximate relationship can be established:

$f_m = \frac{1}{2\pi}\sqrt{\left(\frac{G}{\rho}\right)\left(\frac{1}{r_0}\right)} \times \sqrt{\frac{B_z-0.36}{B_Z}}$---------------------- Eq.5.38

For rotating mass-type excitation, Lysmer's corresponding approximate relationship for *fm* is as follows:

$f_m = \frac{1}{2\pi}\sqrt{\left(\frac{G}{\rho}\right)\left(\frac{1}{r_0}\right)} \times \sqrt{\frac{0.9}{B_Z-0.45}}$------------------------ Eq.5.39

B. Amplitude of Vibration at Resonance

The amplitude of vibration *Az* at resonance for *constant force-type excitation* can be determined as

$A_{z_{resonance}} = \left(\frac{Q_z}{K_z}\right)\frac{1}{2\xi_z\sqrt{1-\xi_z^2}}$----------------- Eq.5.40

Now substituting for K_z and ξ_z we obtain

$A_{z_{resonance}} = \frac{Q_0(1-\mu)}{4Gr_0} \times \frac{B_z}{0.85\sqrt{B_Z-0.18}}$------------------------ Eq.5.41

The amplitude of vibration for *rotating mass-type vertical excitation* can be given as [see Eq. (2.99)]

$A_{z_{resonance}} = \frac{m_1 e}{m} \times \frac{B_z}{0.85\sqrt{B_Z-0.18}}$--------------------- Eq.5.42

C. Amplitude of Vibration at Frequencies Other Than Resonance

For *constant force-type excitation*, can be used for estimation of the amplitude of vibration, or

$$A_z = \frac{Q_0/K_z}{\sqrt{\left(1-\frac{\omega^2}{\omega_n^2}\right)^2 + 4(\xi_z)^2\left(\frac{\omega}{\omega_n}\right)^2}} \text{------------------ Eq.5.43}$$

Figure 5.13 shows the plot of $\frac{A_z}{Q_0/K_z}$ versus $(\frac{\omega}{\omega_n})$. So, with known values of ξ_z and $(\frac{\omega}{\omega_n})$, one can determine the value of $\frac{A_z}{Q_0/K_z}$ and, from that, *Az* can be obtained.

In a similar manner, for *rotating mass-type excitation*, Eq. (2.95) can be used to determine the amplitude of vibration, or

$$A_z = \frac{(m_e e/m)(\omega/\omega_n)^2}{\sqrt{\left(1-\frac{\omega^2}{\omega_n^2}\right)^2 + 4(\xi_z)^2\left(\frac{\omega}{\omega_n}\right)^2}} \text{--------------------------- Eq.5.44}$$

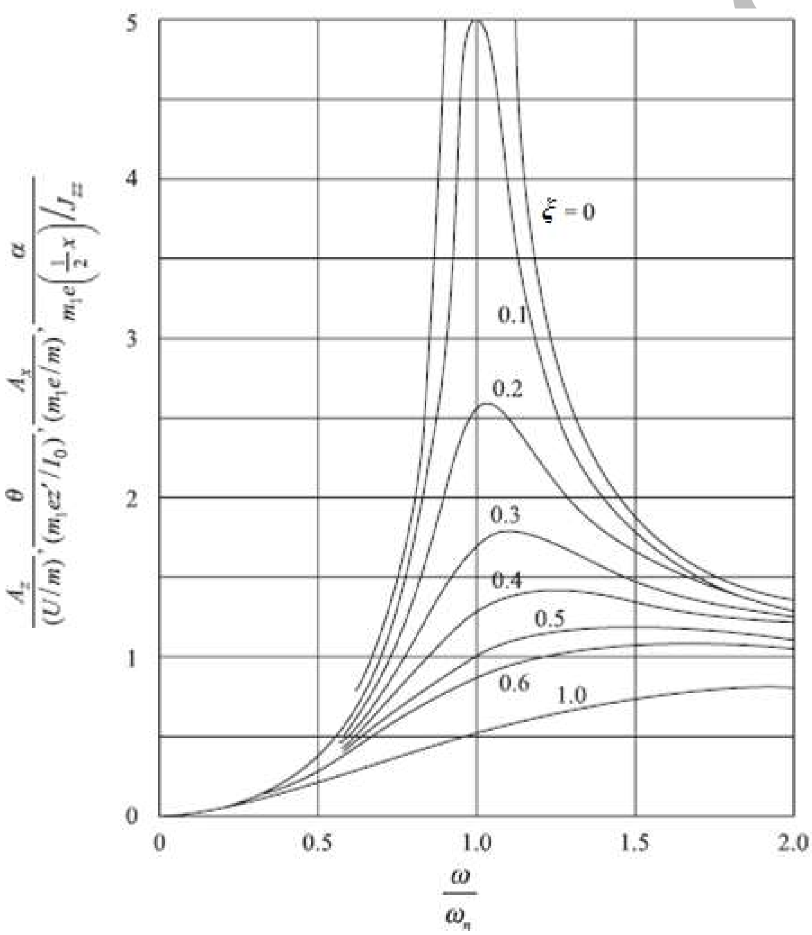


Fig.5.13: Plot of various non-dimensional parameters against (ω/ω_n) for constant force-type vibrator (*Note*: $\xi = \xi_z$ for vertical vibration, $\xi = \xi_\theta$ for rocking, $\xi = \xi_x$ for sliding; $\xi = \xi_\alpha$ for torsional vibration.)

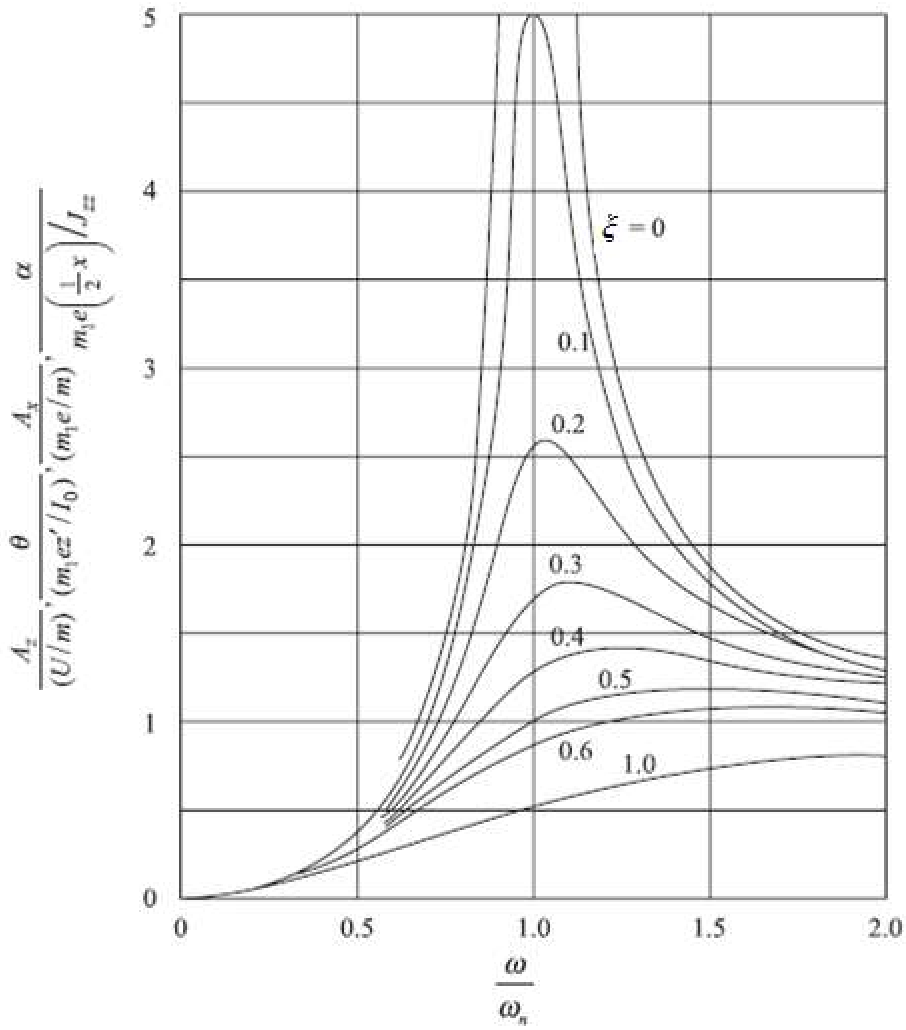


Fig.5.14: Plot of various non-dimensional parameters against ($^{\omega}/_{\omega_n}$) for rotating mass-type vibrator (*Note*: $\boldsymbol{\xi} = \xi_z$ for vertical vibration, $\xi = \xi_\theta$ for rocking, $\xi = \xi_x$ for sliding; $\xi = \xi_\alpha$ for torsional vibration.)

The procedure here described relates to a rigid circular foundation having a radius of r_0. If a foundation is rectangular in shape with length L and width B, it is required to obtain an equivalent radius, which can then be used in the preceding relationships as discussed in above. This can be done by equating the area of the given foundation to the area of an equivalent circle. Thus,

$$\pi r_0^2 = LB$$

where r_0 = radius of the equivalent circle.

It is obviously impossible to eliminate vibration near a foundation.

However, an attempt can be made to reduce the vibration problem as much as possible. Richart (1962) compiled guidelines for allowable vertical vibration amplitude for a particular frequency of vibration, and this is given in Figure 5.15.

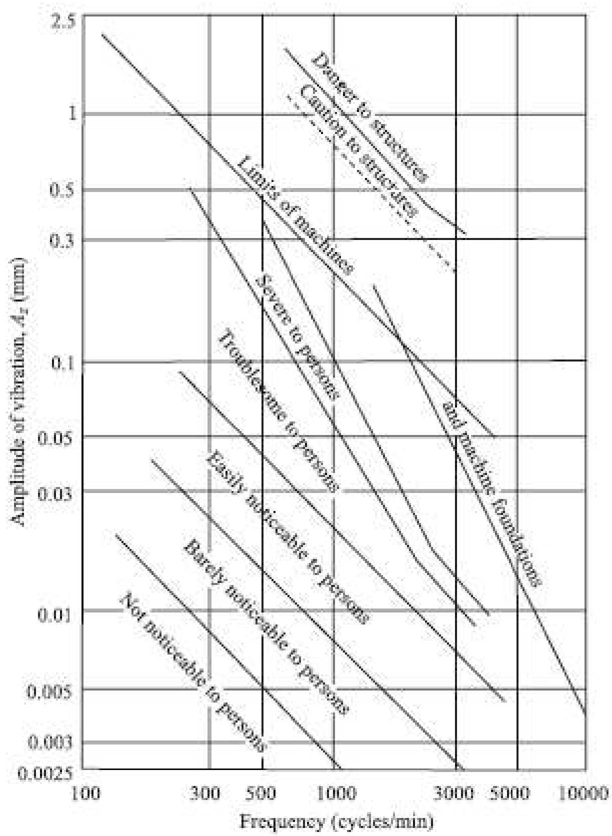


Fig.5.15:

The data presented in Figure 5.15 refer to the maximum allowable amplitudes of vibration. These can be converted to maximum allowable accelerations by

Maximum acceleration = (maximum displacement)ω^2

5.7 GENERAL RULES FOR DESIGNING MACHINE FOUNDATION

In the design of machine foundations, the following general rules may be kept in mind to avoid possible resonance conditions:

1. The resonant frequency of foundation-soil system should be less than half the operating frequency for high-speed machines (that is operating frequency ≥ 1000 cpm). For this case, during starting or stopping the machine will briefly vibrate at resonant frequency.
2. For low-speed machineries (speed less than about 350-400 cpm), the resonant frequency of the foundation-soil system should be at least two times the operating frequency.
3. In all types of foundations, the increase of weight will decrease the resonant frequency.
4. An increase of r_0 will increase the resonant frequency of the foundation.
5. An increase of shear modulus of soil (for example, by grouting) will increase the resonant frequency of the foundation.

5.7 SLIDING MODE OF VIBRATION FOR FOUNDATION

Arnold, Bycroft, and Wartburton (1955) have provided theoretical solutions for sliding vibration of *rigid circular* foundation (Figure 5.16) acted on by a force, $Q = Q_0 e^{i\omega t}$. Hall (1967) developed the mass-spring-dashpot analog for this type of vibration. According to this analog, the equation of motion of the foundation can be given in the form

$m\ddot{x} + C_x\dot{x} + K_x x = Q_0 e^{i\omega t}$------------------------ Eq.5.45

where m= mass of the foundation

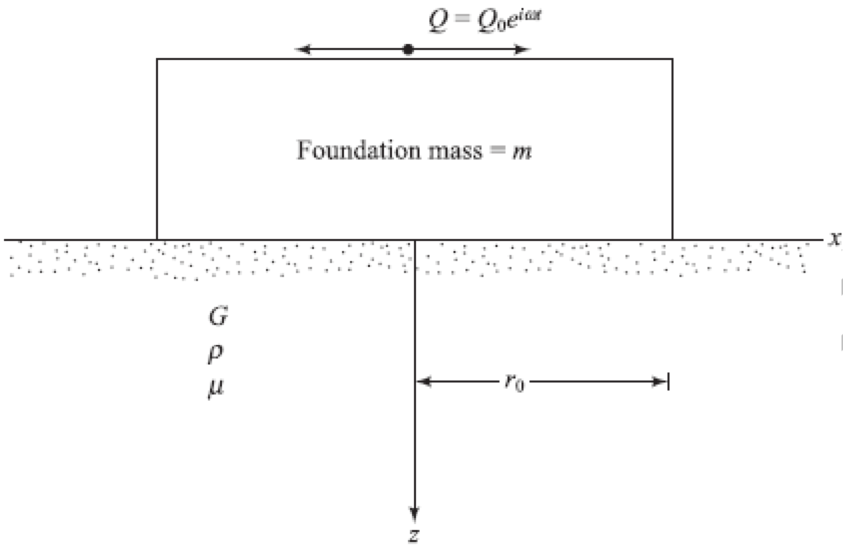


Fig.5.16: Sliding mode of vibration of rigid circular foundation

Spring constant for horizontal mode of vibration

$K_x = \frac{32.4(1-\mu)Gr_0}{7-8\mu}$------------------ Eq.5.46

Dash pot coefficient for horizontal mode of vibration

$C_x = \frac{18.4(1-\mu)r_0^2}{7-8\mu}\sqrt{\rho G}$---------------------- Eq.5.47

The natural frequency of the foundation for sliding can be calculated as

$f_n = \frac{1}{2\pi}\sqrt{\frac{K_x}{m}} = \frac{1}{2\pi}\sqrt{\frac{32(1-\mu)Gr_0}{(7-8\mu)m}}$-------------------------- Eq.5.48

The critical damping and damping ratio in sliding can be evaluated as

C_{cx} = critical damping in sliding

$C_{cx} = 2\sqrt{K_x m} = 2\sqrt{\frac{32(1-\mu)Gr_0}{(7-8\mu)}}m$-------------------- Eq.5.48

ξ_x = damping ratio in sliding

$\xi_x = \frac{C_x}{C_{cx}} = \frac{0.288}{\sqrt{B_x}}$----------------- Eq.5.49

Where , B_x is the dimensionless mass ratio expressed as

$B_x = \frac{7-8\mu}{32(1-\mu)}\frac{m}{\rho r_0^3}$-------------------------- Eq.5.50

Calculation Procedure for Foundation Response Using Eq. (5.51)

Resonant Frequency

1. Calculate the natural frequency f_n using Eq. (5.45)
2. Calculate the damping ratio ξ_x using Eq. (5.49). [Note**:** *Bx* can be obtained from Eq. (5.50)].

3. For constant force excitation (that is, Q_0 = constant), calculate

$f_m = f_n\sqrt{1-2\xi_x^2}$-------------------------Eq.5.51

4. For rotating mass type excitation, calculate

$f_m = \frac{f_n}{\sqrt{1-2\xi_x^2}}$------------------------------------Eq.5.52

Amplitude of Vibration at Resonance

1. For constant force excitation, amplitude of vibration at resonance is

$A_{x(resonance)} = \frac{Q_0}{K_x}\frac{1}{2\xi_x\sqrt{1-\xi_x^2}}$----------------------Eq.5.53

2. For rotating mass-type excitation

$A_{x(resonance)} = \frac{m_e e}{m}\frac{1}{2\xi_x\sqrt{1-\xi_x^2}}$--------------------Eq.5.54

where m_1 = total rotating mass causing excitation
e = eccentricity of each rotating mass

Amplitude of Vibration at Frequency Other than Resonance

1. For constant force-type excitation

$A_z = \frac{Q_0/K_x}{\sqrt{\left(1-\frac{\omega^2}{\omega_n^2}\right)^2+4(\xi_x)^2\left(\frac{\omega}{\omega_n}\right)^2}}$-------------------------- Eq.5.55

2. For rotating mass-type excitation,

$A_z = \frac{\left(m_e e/m\right)\left(\omega/\omega_n\right)^2}{\sqrt{\left(1-\frac{\omega^2}{\omega_n^2}\right)^2+4(\xi_x)^2\left(\frac{\omega}{\omega_n}\right)^2}}$---------------------------- Eq.5.56

Same figures 5.13 & 5.14 are used to calculate various non-dimensional parameters against (ω/ω_n) for constant force and rotating mass-type vibrator respectively.

5.8 TORSIONAL VIBRATION OF FOUNDATIONS

Figure 5.21a shows a circular foundation of radius *r*0 subjected to a torque $T = T_0\theta^{i\omega t}$ about an axis *z-z*. The vibration problem of this type was solved by Reissner (1937) solved considering a linear distribution of shear stress $\tau_{z\theta}$ (shear stress zero at center and maximum at the periphery of the foundation), as shown in Figure 5.17b which represents the case of a *flexible* foundation.

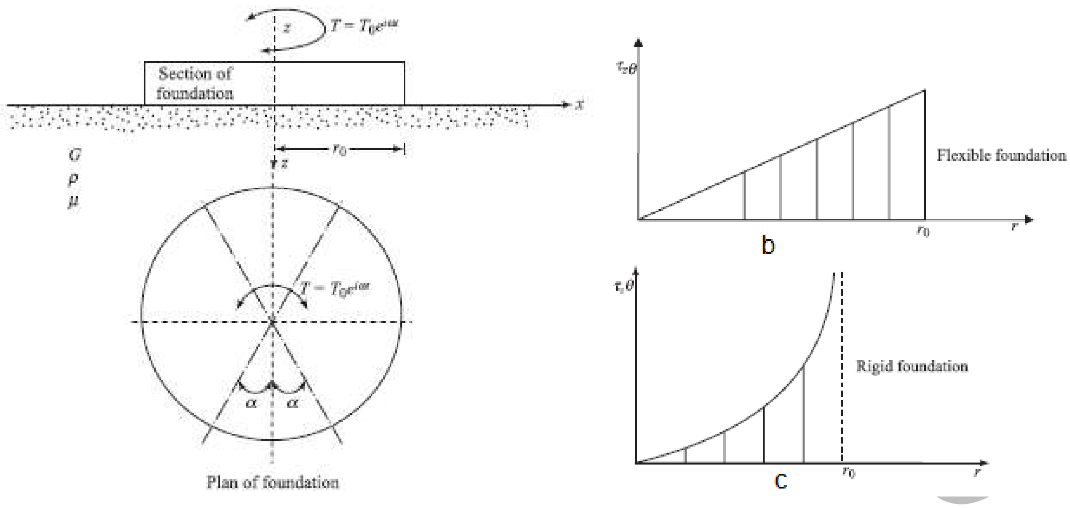


Fig.5.17 Torsional variation of rigid circular foundation

In 1944 Reissner and Sagoli solved the same problem for the case of a rigid foundation considering a *linear variation of displacement from the center to the periphery* of the foundation.

Similar to the cases of vertical, rocking, and sliding modes of vibration, the equation for the torsional vibration of a *rigid circular* foundation can be written as

$J_{zz}\ddot{\alpha} + C_\alpha \dot{\alpha} + K_\alpha \alpha = T_0 e^{i\omega t}$------------------------ Eq.5.57

Where, J_{zz}= mass moment of inertia of the foundation about the axis *z-z*
$C\alpha$ = dashpot coefficient for torsional vibration
$K\alpha$ = static spring constant for torsional vibration = $\frac{16}{3} G r_0^3$
α = rotation of the foundation at any time due to the application of a torque $T = T_0 e^{i\omega t}$
The damping ratio $\xi\alpha$ for this mode of vibration has been determined as (Richart, Hall, and Wood, 1970) given below
$\xi_\alpha = \frac{0.5}{1+2B_\alpha}$---------------------- Eq.5.58
Where
$B\alpha$ = the dimensionless mass ratio for torsion at vibration=$\frac{J_{zz}}{\rho r_0^5}$--------------- Eq.5.59

Calculation Procedure for Foundation Response Using Eq. (5.51)

Resonant Frequency

1. Calculate the natural frequency *fn*

$$f_n = \frac{1}{2\pi}\sqrt{\frac{K\alpha}{J_{zz}}}$$-------------------- Eq.5.60

2. Calculate the damping ratio $B\alpha$ using Eq. (5.59) and damping ratio ξ_α by Eq. 5.58
3. For constant force excitation (that is, T_0 = constant), calculate
$f_m = f_n\sqrt{1 - 2\xi_\alpha^2}$-------------------------Eq.5.61

4. For rotating mass type excitation, calculate

$$f_m = \frac{f_n}{\sqrt{1-2\xi_\alpha^2}}\text{-------------------------------------Eq.5.62}$$

Amplitude of Vibration at Resonance

1. For constant force excitation, amplitude of vibration at resonance is

$$\alpha_{(resonance)} = \frac{T_0}{K_\alpha}\frac{1}{2\xi_\alpha\sqrt{1-\xi_\alpha^2}}\text{-----------------------Eq.5.63}$$

2. For rotating mass-type excitation

$$\alpha_{(resonance)} = \frac{m_e e\left({}^{x}/_{2}\right)}{J_{zz}}\frac{1}{2\xi_\alpha\sqrt{1-\xi_\alpha^2}}\text{---------------------Eq.5.64}$$

where m_1 = total rotating mass causing excitation

e = eccentricity of each rotating mass

Amplitude of Vibration at Frequency Other than Resonance

3. For constant force-type excitation

$$\alpha = \frac{{}^{T_0}/_{K_\alpha}}{\sqrt{\left(1-\frac{\omega^2}{\omega_n^2}\right)^2+4(\xi_\alpha)^2\left(\frac{\omega}{\omega_n}\right)^2}}\text{--------------------------} \qquad \text{Eq.5.65}$$

4. For rotating mass-type excitation,

$$\alpha = \frac{\left({}^{m_e e\left({}^{x}/_{2}\right)}/_{J_{zz}}\right)\left({}^{\omega}/_{\omega_n}\right)^2}{\sqrt{\left(1-\frac{\omega^2}{\omega_n^2}\right)^2+4(\xi_\alpha)^2\left(\frac{\omega}{\omega_n}\right)^2}}\text{----------------------------} \qquad \text{Eq.5.66}$$

For constant force excitation, calculate ${}^{\omega}/_{\omega_n}$ and then refer to Figure 5.13 to obtain $\alpha/(T_0/K_\alpha)$.

For rotating mass-type excitation, calculate ${}^{\omega}/_{\omega_n}$ and then refer to Figure 5.14 to obtain $\alpha/[m_1 e(x/2)/J_{zz}]$.

For a rectangular foundation with dimensions $B \times L$, the equivalent radius may be given by

$$r_0 = \sqrt{\frac{BL(B^2+L^2)}{6\pi}}\text{-------------------} \qquad \text{Eq.5.67}$$

The torsional vibration of foundations is uncoupled motion and hence can be treated independently of any vertical motion. Also, Poisson's ratio does not influence the torsional vibration of foundations

5.9 ROCKING VIBRATION OF FOUNDATIONS

A theoretical solution for foundations subjected to rocking vibration was presented by Arnold, Bycroft, and Wartburton (1955) and Bycroft (1956). Rocking mode of vibration for *rigid circular foundations* is shown in Figure 5.18.

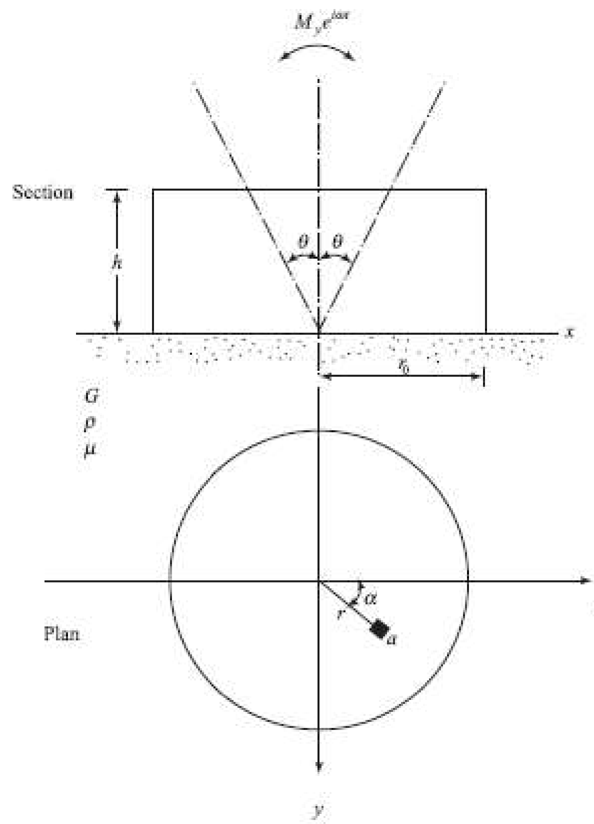


Fig. 5.18: Rocking vibration of a circular rigid foundation

A mass-spring-dashpot model for rigid circular foundations was developed by Hall (1967) in the same manner as Lysmer and Richart (1966) developed for vertical vibration. According to Hall, the equation of motion for a rocking vibration can be given as

$I_0\ddot{\theta} + C_\theta\dot{\theta} + K_\theta\theta = M_y e^{i\omega t}$------------------- Eq.5.68

Where θ = rotation of the vertical axis of the foundation at any time t

I_0 = mass moment of inertia about the y axis (through its base)=$\frac{W_0}{g}\left(\frac{r_0^2}{4}+\frac{h^2}{3}\right)$------- Eq.5.69

where W_0= weight of the foundation

g = acceleration due to gravity

h = height of the foundation

Static spring constant, $K_\theta = \frac{8Gr_0^3}{3(1-\mu)}$------------------- Eq.5.70

Dashpot coefficient, $C_\theta = \frac{0.8r_0^4}{(1-\mu)(1+B_\theta)}\sqrt{G}$--------------------- Eq.5.71

Inertia ratio, $B_\theta = \left(\frac{3(1-\mu)}{8}\right)\frac{I_0}{\rho r_0^5}$-------------------------- Eq.5.72

A. Resonant Frequency

1. Calculate the natural frequency: $f_n = \frac{1}{2\pi}\sqrt{\frac{K_\theta}{I_0}}$------------------------ Eq.5.73

2. Calculate the damping ratio ,

Critical damping coefficient, $C_{c\theta} = 2\sqrt{K_\theta I_0}$--------------------- Eq.5.74

$\xi_\theta = \frac{0.15}{(1+B_\theta)\sqrt{B_\theta}}$-------------------------- Eq.5.75

3. Calculate the resonant frequency:

$f_m = f_n\sqrt{1-2\xi_\theta^2}$ ----(for constant force excitation)----------- Eq.5.76

$f_m = \frac{f_n}{\sqrt{1-2\xi_\theta^2}}$ -----------(for rotating mass-type excitation)--------------- Eq.5.77

B. Amplitude of Vibration at Resonance

Constant force type vibration

$\theta_{resonance} = \frac{M_y}{K_\theta}\frac{1}{2\xi_\theta\sqrt{1-\xi_\theta^2}}$-------------------- Eq.5.78

For rotating type excitation

$\theta_{resonance} = \frac{m_1 e Z'}{I_\theta}\frac{1}{2\xi_\theta\sqrt{1-\xi_\theta^2}}$-------------------- Eq.5.79

Where, m_1 = total rotating mass causing excitation

e = eccentricity of each mass

C. Amplitude of Vibration at Frequencies Other than Resonance

For constant force-type excitation

$\theta = \frac{M_y/K_\theta}{\sqrt{\left(1-\frac{\omega^2}{\omega_n^2}\right)^2+4(\xi_\alpha)^2\left(\frac{\omega}{\omega_n}\right)^2}}$---------------- Eq.5.80

For rotating mass type vibration

$$\theta = \frac{\left({m_1 e z'}/{I_0}\right)\left({\omega}/{\omega_n}\right)^2}{\sqrt{\left(1-\frac{\omega^2}{\omega_n^2}\right)^2+4(\xi_\alpha)^2\left(\frac{\omega}{\omega_n}\right)^2}} \text{---------------------} \qquad \text{Eq.5.81}$$

In the case of rectangular foundation, the preceding relationships can be used by determining the equivalent radius as

$$r_0 = \sqrt[4]{\frac{BL^3}{3\pi}} \text{-------------} \qquad \text{Eq.5.82}$$

Table 5.1: Values of various Lysmer's Analog Parameters

Mode of Vibration	Equivalent Radius	Mass ratio	Damping factor	Spring Constant
Vertical	$r_0 = \sqrt{\frac{BL}{\pi}}$	$B_z = \frac{(1-\mu)m}{4\rho r_0^3}$	$\xi_z = \frac{0.425}{\sqrt{B_z}}$	$K_z = \frac{4Gr_0}{1-\mu}$
Horizontal	$r_0 = \sqrt{\frac{BL}{\pi}}$	$B_x = \frac{7-8\mu}{32(1-\mu)}\frac{m}{\rho r_0^3}$	$\xi_x = \frac{0.288}{\sqrt{B_X}}$	$K_x = \frac{32.4(1-\mu)Gr_0}{7-8\mu}$
Torsional	$r_0 = \sqrt{\frac{BL(B^2+L^2)}{6\pi}}$	$B_\alpha = \frac{J_{zz}}{\rho r_0^5}$	$\xi_\alpha = \frac{0.5}{1+2B_\alpha}$	$K_\alpha = \frac{16}{3}Gr_0^3$
Rocking	$r_0 = \sqrt[4]{\frac{BL^3}{3\pi}}$	$B_\theta = \left(\frac{3(1-\mu)}{8}\right)\frac{I_0}{\rho r_0^5}$	$\xi_\theta = \frac{0.15}{(1+B_\theta)\sqrt{B_\theta}}$	$K_\theta = \frac{8Gr_0^3}{3(1-\mu)}$

Example No.1: A concrete foundation is 2.5 m in diameter. The foundation is supporting a machine. The total weight of the machine and the foundation is 270 kN. The machine imparts a vertical vibrating force $Q = Q_0 \sin\omega t$. Given

Q_0=27 kN (not frequency dependent). The operating frequency is 150 cpm. For the soil supporting the foundation, unit weight = 19.5 kN/m^3, shear modulus = 45000 kPa. , and Poisson' ratio = 0.3. Determine:

a. resonant frequency,
b. the amplitude of vertical vibration at resonant frequency, and
c. the amplitude of vertical vibration at the operating frequency

Sol:
The machine imparts a vertical vibrating force $Q = Q_0 \sin\omega t$ where Q_0=27 kN
The operating frequency = 150 cpm=2.5 Hz
Equivalent radius $r_0 = 1.25\ m$
Total weight of the machine and the foundation =270 kN
Mass ratio

$$B_z = \frac{(1-\mu)m}{4\rho r_0^3} = \frac{(1-\mu)W}{4\gamma r_0^3} = \frac{(1-0.3)\times 270}{4\times 19.5\times 1.25^3} = 1.24$$

Damping factor

$$\xi_z = \frac{0.425}{\sqrt{B_z}} = \frac{0.425}{\sqrt{1.24}} = 0.382$$

Spring constant

$K_z = \frac{4Gr_0}{1-\mu} = \frac{4\times4500\times1.25}{1-0.3}$ =32142.86 kN/m^2

Natural frequency $f_n = \frac{1}{2\pi}\sqrt{\frac{4Gr_0}{(1-\mu)m}} = \frac{1}{2\pi}\sqrt{\frac{4\times4500\times1.25}{(1-0.3)\times{}^{270}/_{9.81}}}$=5.44 Hz

a) Resonance frequency $f_m = f_n\sqrt{1-2\xi^2}$=5.44× $\sqrt{1-2\times0.382^2}$=4.58 Hz

b) Amplitude of vertical vibration at resonant frequency:

$A_{z_{resonance}} = \frac{Q_0(1-\mu)}{4Gr_0} \times \frac{B_z}{0.85\sqrt{B_Z-0.18}} = \frac{27(1-0.3)}{4\times4500\times1.25} \times \frac{1.24}{0.85\sqrt{1.24-0.18}}$=

c) The amplitude of vertical vibration at the operating frequency:

$$A_z = \frac{{}^{Q_0}/_{K_z}}{\sqrt{\left(1-\frac{\omega^2}{\omega_n^2}\right)^2+4(\xi_z)^2\left(\frac{\omega}{\omega_n}\right)^2}} = \frac{{}^{27}/_{32142.86}}{\sqrt{\left(1-\frac{2.5^2}{5.44^2}\right)^2+4(0.382)^2\left(\frac{2.5}{5.44}\right)^2}} =$$

Example No.2:

A radar antenna foundation is shown below. For torsional vibration of the foundation, given

T_0 = 250 kN-m (due to inertia)

T_0 = 83 kN-m (due to wind)

Mass moment of inertia of the tower about the axis z-z = 13 × 106 kg·m^2, and the unit weight of concrete used in the foundation = 24 kN/m^3. Calculate

a) the resonant frequency for torsional mode of vibration; and

b) angular deflection at resonance.

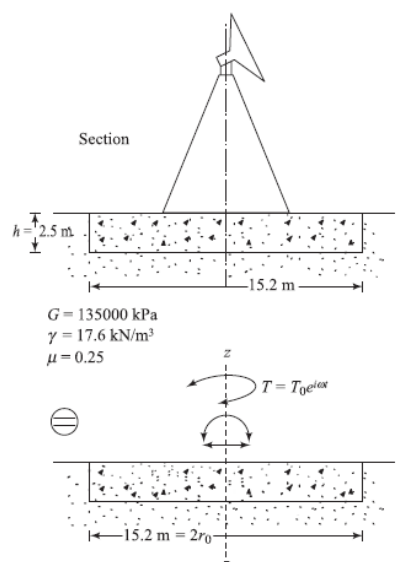


Fig: Foundation for radar of antenna

Solution:

a)

$$J_{zz} = J_{zz(\text{tower})} + J_{zz(\text{foundation})}$$
$$= 13 \times 10^6 + \frac{1}{2}\left[\pi r_0^2 h\left(\frac{24 \times 1000}{9.81}\right)\right] r_0^2$$
$$= 13 \times 10^6 + \frac{1}{2}\left[\pi (7.6)^2 (2.5)\left(\frac{24 \times 1000}{9.81}\right)\right] (7.6)^2$$
$$= 13 \times 10^6 + 32.05 \times 10^6 = 45.05 \times 10^6 \text{ kg·m}^2$$
$$B_\alpha = \frac{J_{zz}}{\rho r_0^5} = \frac{45.05 \times 10^6}{(17.6 \times 10^3/9.81)(7.6)^5} = 0.99$$

Damping factor

$$\frac{0.5}{1+2B_\alpha} = \frac{0.5}{1+(2)(0.99)} = 0.168$$
$$k_\alpha = \frac{16}{3} G r_0^3 = \left(\frac{16}{3}\right) \times 135 \times 10^6 \times (7.6)^3 = 3.16 \times 10^{11} \text{ N-m}$$
$$f_n = \frac{1}{2\pi}\sqrt{\frac{k_\alpha}{J_{zz}}} = \frac{1}{2\pi}\sqrt{\frac{3.16 \times 10^{11}}{45.05 \times 10^6}}$$
$$= 13.33 \text{ Hz}$$

Thus, the damped natural frequency

$$f_m = f_n \sqrt{1 - 2D_\alpha^2} = (13.33)\sqrt{1-(2)(0.168)^2}$$

$$\mathbf{= 12.92 \text{ Hz}}$$

b) Angular frequency at resonance

If the torque due to wind (T_0) is to be treated as a static torque, then

$$\frac{T_0}{\alpha_{\text{static}}} = k_\alpha$$

So

$$\alpha_{\text{static}} = \frac{3}{16 G r_0^3} T_{0(\text{static})} = \left[\frac{83 \times 10^3}{3.16 \times 10^{11}}\right]$$
$$= 0.0263 \times 10^{-5} \text{ rad}$$

Angular deformation due to torque produced by inertia

$$\alpha_{\text{resonance}} = \frac{T_0}{k_\alpha} \frac{1}{2D_\alpha \sqrt{1 - D_\alpha^2}}$$

$$= \left[\frac{250\times10^3}{3.16\times10^{11}}\right]\left[\frac{1}{(2)(0.168)\sqrt{1-(0.168)^2}}\right]$$

$$= 0.24\times10^{-5}\ \text{rad}$$

At resonance, the total angular deflection is

$$\alpha = \alpha_{\text{inertia}} + \alpha_{\text{static}} = (0.24 + 0.0263)\times10^{-5} = \mathbf{0.2663\times10^{-5}\ rad}$$

5.10 DYNAMIC BEARING CAPACITY OF SHALLOW FOUNDATION

During the application of single pulse dynamic loads which may be in vertical or horizontal directions, the foundation may get excessive settlement. Horizontal dynamic loads on foundations are due mostly to earthquakes. These types of loading may induce large permanent deformation in foundations. Isolated column footings, strip footings, mat footings, and even pile foundations all may fail during seismic events. Such failures are generally attributed to liquefaction. However, a number of failures have occurred where field conditions indicate there was only partial saturation or a dense soil and therefore liquefaction alone is a very unlikely explanation. Rather, the reason for the seismic settlements of these foundations seems to be that the bearing capacity was reduced (Richards, Elms and Budhu,1993).

During the analysis of the time dependent motion of a foundation subjected to dynamic loading or estimating the bearing capacity under dynamic conditions several factors need to be considered.

Most important of these factors are

a) Nature of variation of the magnitude of the loading pulse,
b) Duration of the pulse, and
c) Strain-rate response of the soil during deformation

5.10.1 Ultimate Dynamic Bearing Capacity

Bearing Capacity in Sand

The equations for static ultimate bearing capacity evaluation are valid for dense sands where the failure surface in the soil extends to the ground surface. This is referred as the case of general shear failure. For shallow foundations (i.e., ${}^{D_f}/_{B} \leq 1$), if the relative density of granular soils R_D is less than about 70%, *local or punching shear failure* may occur. Hence, for static ultimate bearing capacity calculation, if $0 \leq R_D \leq 0.67$, the values of internal angle of friction, φ should be replaced by the modified friction angle

$$\emptyset' = tan^{-1}[(0.67 + R_D - 0.75R_D^2)tan\emptyset] \text{-------} \qquad \text{Eq.5.83}$$

However, when load is applied rapidly to a foundation to cause failure, the ultimate bearing capacity changes by somewhat. This fact has been shown experimentally by Vesic, Banks, and Woodward (1965), who conducted several laboratory model tests with a 101.6 mm diameter rigid rough model footing placed on the surface of a dense river sand (i.e., $D_f = 0$), both dry and saturated. The rate of loading to cause failure was varied in a range of 2.54×10^{-4} mm/s to over 254 mm/s. Hence, the rate was in the range of static (2.54×10^{-4} mm /s) to impact (254 mm/s) loading conditions. All but the four Based on the experimental results available, the following general conclusions regarding the ultimate dynamic bearing capacity of shallow foundations in sand can be drawn:

1. For a foundation resting on sand and subjected to an acceleration level of $a_{max} \leq 13g$, it is possible for general shear type of failure to occur in soil (Heller, 1964).

2. For a foundation on sand subjected to an acceleration level of $a_{max} > 13g$, the nature of soil failure is by punching (Heller, 1964).
3. The difference in the nature of failure in soil is due to the inertial restrain of the soil involved in failure during the dynamic loading. The restrain has almost a similar effect as the overburden pressure as observed during the dynamic loading which causes the punching shear type failure in soil.
4. The minimum value of the ultimate dynamic bearing capacity of shallow foundations on dense sands obtained between static to impact loading range can be estimated by using a friction angle $\emptyset_{dy}$, such that (Vesic, 1973)

$\emptyset_{dy} = \emptyset - 2^0$------------------ Eq.5.84

The value of $\emptyset_{dy}$ can be subsequently used to find various bearing capacity factors.
However, if the soil strength parameters with proper strain rate are known from laboratory testing, they should be used instead of the approximate equation.
5. The increase of the ultimate bearing capacity at high loading rates is due to the fact that the soil particles in the failure zone do not always follow the path of least resistance. This results in at higher shear strength of soil, which leads to a higher bearing capacity.
6. In the case of foundations resting on loose submerged sands, transient liquefaction effects may exist (Vesic, 1973). This may results in unreliable prediction of ultimate bearing capacity.
7. The rapid increase of the ultimate bearing capacity in dense saturated sand at fast loading rates is due to the development of negative pore water pressure in the soil.

The dynamic bearing capacity problem attracted attention of the investigators in 1960 when the performance of foundations under transient loads became of concern to the engineering profession (Wallace, 1961; Cunny and Sloan, 1961; Fisher, 1962; Johnson and Ireland, 1963; Mckee and shenkman 1962: White, 1964; Chummar, 1965; Triandafilidis, 1965).

All analytical approaches are based on the assumption that soil rupture under transient loads occurs along a static rupture surface. In this section the salient features of the analysis developed by Triandafilidis (1965) and Wallace (1961) for transient vertical load; and by Chummar (1965) for transient horizontal load have been presented.

5.10.2 Triandafilidis's Solution:

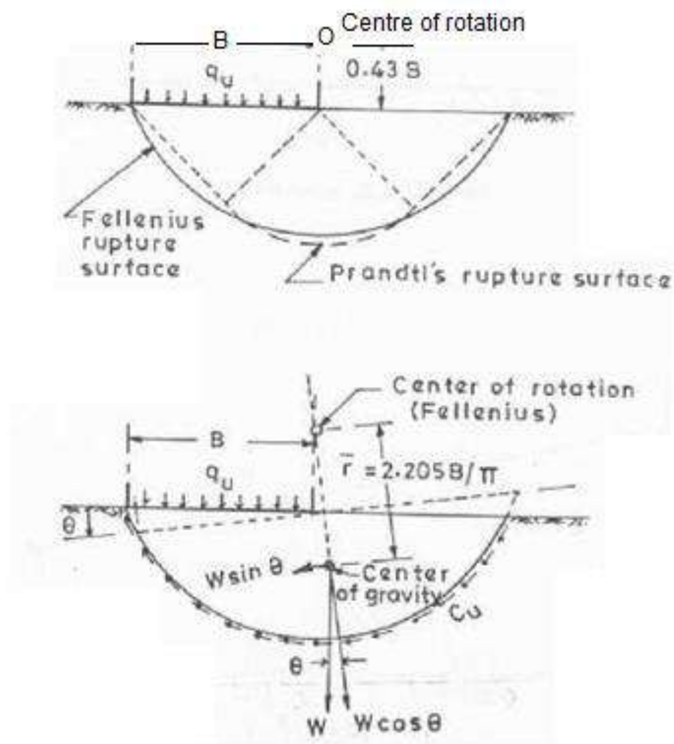


Fig. 5.19: Illustrations of mode of failure and dynamic equilibrium of moving soil mass

Triandafilidis's Analysis

Triandafilidis (1965) has presented a solution for dynamic response of continuous surface footing supporting by saturated cohesive soil (φ=0 condition) and subjected to vertical transient load. The analysis is based on the following assumptions:

(i) The failure surface of soil is cylindrical for evaluation of bearing capacity under static condition (Fig.5.19).

(ii) The saturated cohesive soil (φ= 0) behaves as a rigid plastic material (Fig. 5.20).

(iii) The forcing function is assumed to be an exponentially decaying pulse (Fig.5.21)

(iv) The influence of strain rate on the shear strength is neglected.

(v) The dead weight of the foundation is neglected.

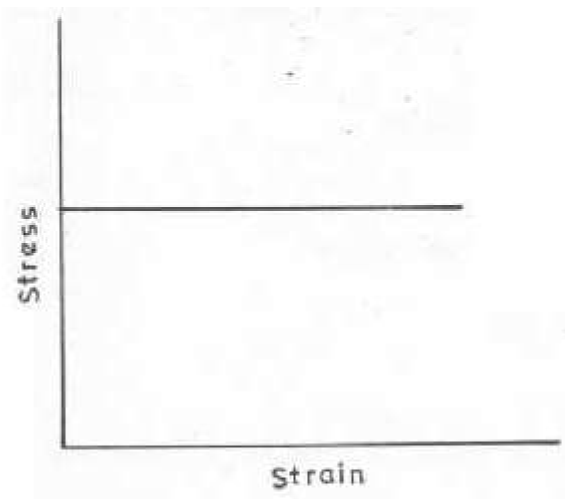


Fig. 5.20 Assumed Stress-strain relationship

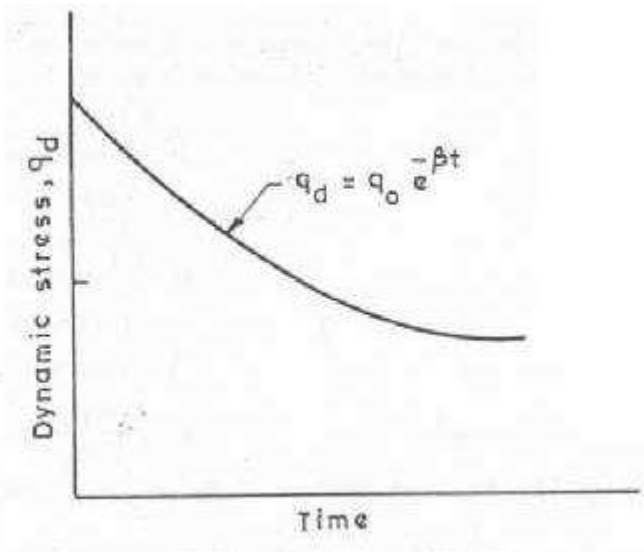


Fig. 5.21 Transient Vertical Load

Analysis

Let the transient stress pulse be expressed in the form

$q_d = q_0 e^{-\beta t} = \lambda q_u e^{-\beta t}$-------- Eq.5.85

Where q_d= Stress at any time t

β= Decaying function

q_u= Static bearing capacity of continuous footing

q_0=instantaneous peak intensity of stress pulse

λ= Over load factor = $\frac{q_0}{q_u}$

The rupture surface is shown in Fig. 5.19 with centre of rotation at point0 located at a height of 0.43 B above the ground surface.

The equation of motion is written by equating the moment of the disturbing and restoring forces taken about the point O. The only disturbing and restoring force is an externally applied dynamic pulse. The restoring forces consist of shearing resistance along the rupture surface, the inertia of the soil mass taken in the motion and the resistance caused by the displacement of centre of gravity of soil mass.

Disturbing moment M_{dp} due to applied dynamic pulse is given as

$M_{dp} = \frac{1}{2} q_d B^2$--------- Eq. 5.86

Where B=Width of the footing

The static bearing capacity of a continuous footing along the failure surface (Fellenius, 1948) is given as

$$q_u = 5.54c_u$$

Where c_u is the un-drained shear strength of soil

Now Resisting moment M_{rs} due to shear strength is taken as

$M_{rs} = \frac{1}{2} q_u B^2$------------- Eq.5.87

Due to the application of pulse, the soil mass is subjected to an acceleration. So the resisting moment M_{ri} due to the rigid body motion of the failed soil mass is given as

$M_{ri} = J_0 \ddot{\theta}$------- Eq.5.88

J_0=Polar mass moment of inertia=$\frac{WB^2}{1.36g}$

W=Weight of the cylindrical soil mass=$0.31\gamma\pi B^2$

Where γ is the unit weight of soil

There fore $M_{ri} = \frac{WB^2}{1.36g}\ddot{\theta}$------- Eq.5.89

The displaced position of the soil mass generates a restoring moment M_{rw} which may be expressed as

$M_{rw} = W\bar{r}sin\theta$-------- Eq.5.90

For small rotation $M_{rw} = W\bar{r}\theta$ where $\bar{r} = \frac{2.205B}{\pi}$

By equating the moments of disturbing forces to those of the restoring forces, the following equation of motion is obtained

$\mathrm{M_{dp}} = \mathrm{M_{rs}} + \mathrm{M_{ri}} + \mathrm{M_{rw}}$------ Eq.5.91

Substituting for moments and rearranging, we get

$\ddot{\theta} + \frac{3g}{\pi B}\theta = \left[\frac{0.68g}{W}\right] q_u\left[\lambda e^{-\beta t} - 1\right]$------ Eq.5.92

Equation (5.92) is a second order, non-homogeneous, linear differential equation with constant coefficients. The natural frequency and the time period of the system are given by

$\omega_n = \sqrt{\frac{3g}{\pi B}}$--------- Eq.5.93

Time period of vibration

$T = \frac{1}{2\pi}\sqrt{\frac{\pi B}{3g}}$-------- Eq.5.94

Solution of Eq. (5.92) gives the following relation

$\frac{W}{0.68gq_u}(\theta) = \frac{T^2}{4\pi^2+\beta^2T^2}\left[\left\{1 - \lambda + \frac{\beta^2T^2}{4\pi^2}\right\}\cos\left(\frac{2\pi t}{T}\right) + \frac{\beta\lambda T}{2\pi}\sin\left(\frac{2\pi t}{T}\right) + \lambda e^{-\beta t} - \frac{\beta^2T^2}{4\pi^2} - 1\right]$------Eq.5.95

The above re1ation can be used to trace the history of motion of the foundation. For determination of the maximum angular deflection θ, Eq. (5.95) can be differentiated with respect to time. Thus

$$\frac{W}{0.68 g q_u}(\dot{\theta}) = \frac{2T\pi}{4\pi^2+\beta^2T^2}\left[\left\{\lambda - 1 - \frac{\beta^2T^2}{4\pi^2}\right\}\sin\left(\frac{2\pi t}{T}\right) + \frac{\beta\lambda T}{2\pi}\cos\left(\frac{2\pi t}{T}\right) - \frac{\beta\lambda T}{2\pi}e^{-\beta t}\right] \text{-------} \quad \text{Eq. 5.96}$$

For obtaining the critical time $t = t_{cr}$ which corresponds to $\theta = \theta_{max}$ the right-hand side of Eq. (5.96) is equated to zero. Since $\frac{2\pi T}{4\pi^2+\beta^2T^2}$ cannot be zero,

$$\left[\left\{\lambda - 1 - \frac{\beta^2T^2}{4\pi^2}\right\}\sin\left(\frac{2\pi t}{T}\right) + \frac{\beta\lambda T}{2\pi}\cos\left(\frac{2\pi t}{T}\right) - \frac{\beta\lambda T}{2\pi}e^{-\beta t}\right] = 0 \text{------} \quad \text{Eq. 5.97}$$

By using small increments of time t in Eq. (5.97), the value of t_{cr} can be obtained. This value of $t = t_{cr}$ can then be substituted in to Eq. (5.96) with known values of λ, β and B to obtain

$\frac{W}{0.68 g q_u}\theta_{max} = K,$ is the dynamic load factor.

Figures 5.22 to 5. 24 give the values of K (s^2) for B = 0.6, 1.5 and 3.0 m, respectively, with λ = 1-5 and β= 0-50 s^{-1}

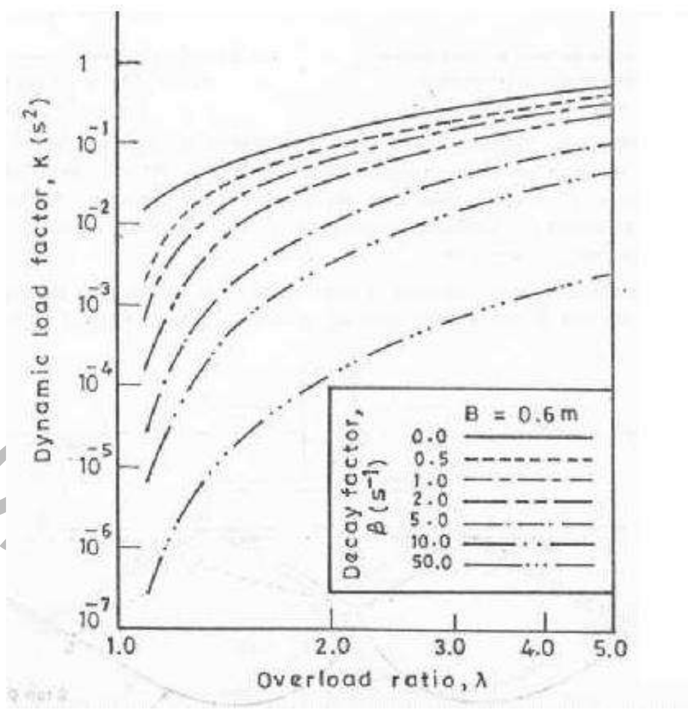


Fig.5.22 Relationship between overload ratio and dynamic load factor for continuous footings 0.6 m wide

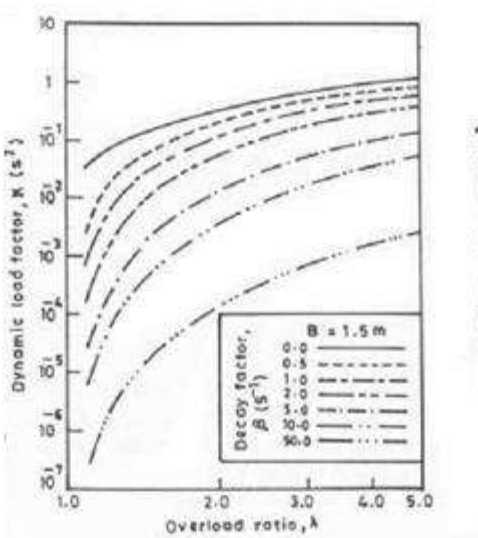


Fig. 5.23Relationship between overload ratio ratio and dynamic load factor for continuous footings footing of width 1.5 m

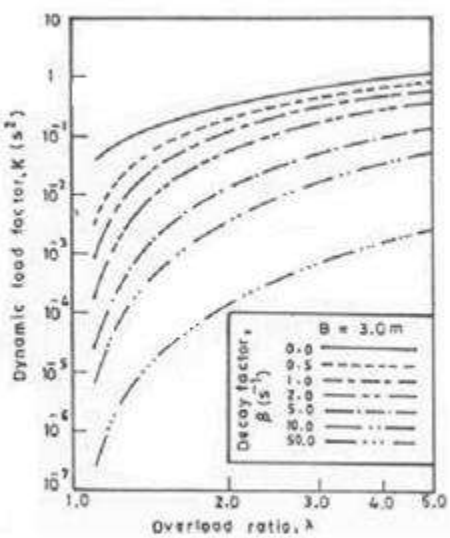


Fig.5.24 Relationship between overload dynamic load factor for continuous wide 3.0 m wide

5.10.3 Chummar's Solution

Chummar (1965) presented as solution for dynamic response of a strip footing supported by cohesive soil and .The analysis is based on the following assumptions:

(i) The failure of the footing occurs with the application of horizontal dynamic load acting at a certain height above the base of the footing.

(ii) The resulting motion in the footing is of a rotatory nature. The failure surface is a logarithmic spiral with its centre on the base corner of the footing, which is also the centre of rotation as shown in (Fig.5.25).

(iii) The rotating soil mass is considered to be a rigid body rotating about a fixed axis.

(iv) The soil exhibits rigid plastic, stress-strain characteristics.

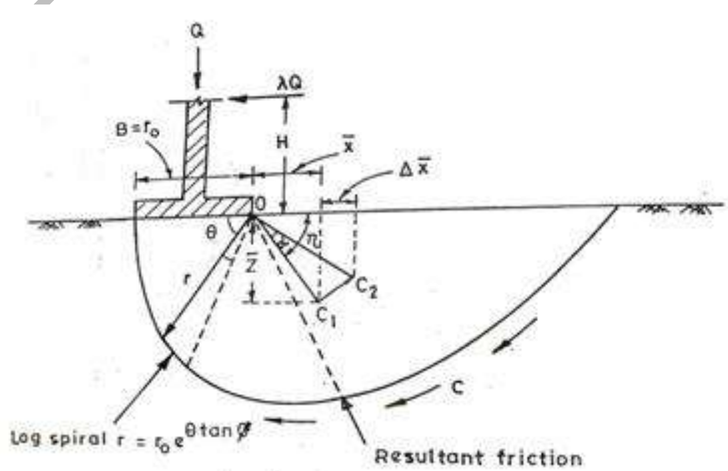


Fig.5.25: Transient horizontal load on a continuous strip footing resting on ground surface

Analysis: The static bearing capacity of the footing is calculated by assuming that the footing fails when acted upon by a vertical static load, which causes rotation of the logarithmic spiral failure. The ultimate static bearing capacity q_u is given by:

$q_u = cN_c + \frac{1}{2}\gamma N_\gamma B$ Eq. 5.98

Where c =Cohesion
B= Footing width and equal to the initial radius of spiral curve.
γ=Unit weight of the soil.

N_c and N_γ are bearing capacity factors for the assumed type of failure.

Now considering moment of the forces about0, the centre of rotation:

Moment due to cohesion c, $M_{RC} = \Psi c B^2$, where $\frac{(e^{2\pi tan\emptyset}-1)}{2tan\emptyset} = \Psi$---------- Eq.(5.99)

Moment due to weight W of soil wedge, $M_{RW=\varepsilon\gamma B^3}$, where $\frac{tan\emptyset(e^{3\pi tan\emptyset}+1)}{9tan^2\emptyset+1} = \varepsilon$-----Eq.(5.100)

$\emptyset$ is the angle of internal friction

Moment due to q_u about point O is given as $q_u\frac{B^2}{2}$

Under equilibrium condition, we get $q_u\frac{B^2}{2}$=$M_{RC} + M_{RW}$, which gives

$q_u = \frac{c}{tan\emptyset}\left(e^{2\pi tan\emptyset} - 1\right) + \frac{2\gamma B tan\emptyset(e^{3\pi tan\emptyset}+1)}{9tan^2\emptyset+1}$--- Eq. (5.101)

Combining Eq.(5.98) and (5.101), yields

$N_\gamma = \frac{4tan\emptyset(e^{3\pi tan\emptyset}+1)}{9tan^2\emptyset+1}$ -------------- Eq. (5.102)

$N_c = \frac{e^{2\pi tan\emptyset}-1}{9tan^2\emptyset+1}$-------------------------------- Eq. (5.103)

By considering a suitable factor of safety F, the static vertical force on the foundation per unit length can be given as

$Q = \frac{B}{F}(cN_c + \frac{1}{2}\gamma B N_\gamma)$----------------------- Eq. (5.104)

The variation of dynamic force in the above analysis is considered as

$Q_{d(\max)} = \lambda Q$-------------------------------- Eq. (5.105)

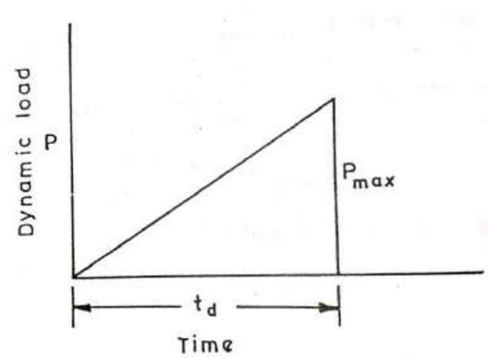


Fig.5.26: Loading factor.

Where, $Q_{d(\max)}$ is the maximum value of horizontal transit load per unit length acting at height H above base of the footing and λ is over load factor.

For considering of the dynamic equilibrium of the foundation with the horizontal transient load, the moment of each of the forces (per unit length) about the centre of the log-spiral needs to be considered:

1. Moment due to the vertical force Q,$M_1 = \frac{1}{2}QB$-------------------- Eq. (5.106)
2. Moment due to the horizontal force Q_d at any time t,

   $M_2 = Q_d H = \frac{Q_{d(max)}Ht}{t_d} = \frac{M_{d(max)}t}{t_d}$------------------------------------- Eq. (5.107)
3. Moment due to the cohesive force acting along the failure surface is given by Eq.(5.88)
4. Moment due to weight of soil mass in the failure wedge is given by Eq. (5.89)
5. Moment of the force due to displacement of the centre of gravity of the failure wedge (as shown in Fig.5.26) from its initial position:

   $M_3 = W\Delta X$ --------------------------- Eq. (5.108)

Where W is the weight of the failure wedge, and given by

$W = \frac{(e^{2\pi tan\emptyset}-1)\gamma B^2}{4tan\emptyset}$------------------------ Eq. (5.109)

$\Delta\bar{X} = Rcos(\eta - \alpha) - Rcos\eta$----------------- Eq. (5.110)

When α is small, Eq.(5.110) can be written as $\Delta\bar{X} = (Rsin\,\eta)\alpha$------------- Eq. (5.111)

But $R = \sqrt{\bar{x}^2} + \bar{z}^2$ and

$\bar{x} = \frac{-4Btan^2\emptyset(e^{3\mu tan\emptyset}+1)}{(9tan^2\emptyset+1)(e^{2\pi tan\emptyset}-1)}$------------------------------------- Eq. (5.112)

$\bar{z} = \frac{4Btan^2\emptyset(e^{3\mu tan\emptyset}+1)}{3(\sqrt{9tan^2\emptyset+1})(e^{2\pi tan\emptyset}-1)}$------------------------------------- Eq. (5.113)

Now Eq. (5.108) becomes, $M_3 = \beta B^3(sin\eta)\alpha$---------------------- Eq. (5.116)

Where $\beta = \frac{(e^{3\mu tan\emptyset}+1)}{3(\sqrt{9tan^2\emptyset+1})}$------------------------------------- Eq. (5.117)

6. Moment due to inertia force of soil wedge

   $M_d = \left(\frac{\partial^2\alpha}{\partial t^2}\right)J$ ------------- Eq. (5.118)

Where J is the mass moment of inertia of the soil wedge about the axis of rotation

$J = \left|\frac{\gamma B^4}{16gtan\emptyset}\right|(e^{2\pi tan\emptyset} - 1)$-------------- Eq. (5.119)

Now substituting for J in Eq. (5.118), we get

$M_d = (\frac{\partial^2\alpha}{\partial t^2})\frac{\mu_c\gamma B^4}{g}$ ------------------------------- Eq. (5.120)

Where $\mu_c = \frac{e^{4\pi tan\emptyset}-1}{16tan\emptyset}$--------------------------- Eq. (5.121)

Moment due to the frictional resistance along the failure surface will be zero as its resultant will pass through the centre of log-spiral. Now for the equation of motion,

$M_1 + M_2 = M_{RC} + M_{RW} + M_3 + M_4$------------------------------- Eq. (5.122)

Substitution of the proper terms for the moments in Eq. (5.122) gives

$$\left(\frac{\partial^2\alpha}{\partial t^2}\right) + K^2\alpha = A\left[\left(\frac{M_{d(max)}t}{t_d}\right) + \frac{1}{2}QB - E\right] \text{----------------------} \quad \text{Eq. (5.123)}$$

Where $K = \sqrt{\frac{g\beta sin\eta}{\mu_C B}}$, $A = \frac{g}{\gamma B^4 \mu_C}$ and $E = \psi c B^2 + \epsilon\gamma B^3$------ Eq. (5.124)

Now general solution to the second order differential Eq. (5.123) can be found as

For $t \leq t_d$

$$\alpha = \frac{A}{K^2}\left(E - \frac{1}{2}QB\right)\cos(Kt) - \frac{A}{K^3}\frac{M_{d(max)}}{t_d}\sin(Kt) + \frac{A}{K^2}\left(\frac{M_{d(max)}}{t_d} + \frac{1}{2}QB - E\right) \text{------} \quad \text{Eq. (5.125)}$$

For $t > t_d$

$$\alpha = [\frac{1}{K}][G_1 K\cos(Kt_d) - G_2\sin(Kt_d)]\cos(\mathrm{Kt}) + (\frac{1}{K})[G_1 K sin(Kt_d)$$
$$-\mathrm{G}_2\cos(Kt_d)]\sin(Kt) + \left(\frac{A}{K^2}\right)(\frac{1}{2}QB - E) \text{--------------} \quad \text{Eq. (5.126)}$$

Where $G_1 = \frac{A}{K^2}\left(E - \frac{1}{2}QB\right)\cos(Kt_d) - \frac{A}{K^3}\frac{M_{d(max)}}{t_d}\sin(Kt_d) + \frac{AM_{d(max)}}{K^2}$------------ Eq. (5.127)

and $G_2 = \frac{A}{K}\left(E - \frac{1}{2}QB\right)\sin(Kt_d) - \frac{A}{K^2}\frac{M_{d(max)}}{t_d}\cos(Kt_d) + \frac{AM_{d(max)}}{K^2 t_d}$-------------- Eq. (5.128)

Example No.2.1

A 2.5m wide continuous surface footing is subjected to a horizontal transient load of duration 0.4s applied at a height of 4.0 m from the base of footing. The properties of the soil are γ=17kN/m^3, c=30kN/m^3 and φ =32°.Determine the value of the maximum horizontal load that can be applied on the footing. Also compute the angular rotation at time equal to 0.6 s.

Sol: Given that

γ=17kN/m^3, c =30kN/m^{3}and φ =32°.H= 4.0 m, t_d=0.4 s

$N_c=\frac{(e^{2\pi tan\emptyset}-1)}{tan\emptyset}$=79.4, $N_\gamma=\frac{4tan\emptyset(e^{3\pi tan\emptyset}+1)}{9tan^2\emptyset+1}$=200

Q=$\frac{B}{2}(cN_c + \frac{1}{2}\gamma B N_\gamma)$=8290 Taking a suitable value of factor of safety as 2.0

i) Determination of various parameters

$\psi=\frac{(e^{2\pi tan\emptyset}-1)}{2tan\emptyset}$=39.7, $\epsilon=\frac{tan\emptyset(e^{3\pi tan\emptyset}+1)}{9tan^2\emptyset+1} = 50$

$\frac{(e^{3\mu tan\emptyset}+1)}{3(\sqrt{9tan^2\emptyset+1})}$=56.6, $\mu_c = \frac{e^{4\pi tan\emptyset}-1}{16tan\emptyset}$= 256

$\bar{x} = \frac{-4Btan^2\emptyset(e^{3\mu tan\emptyset}+1)}{(9tan^2\emptyset+1)(e^{2\pi tan\emptyset}-1)}$ =-2.52 B, $\bar{z} = \frac{4Btan^2\emptyset(e^{3\mu tan\emptyset}+1)}{3(\sqrt{9tan^2\emptyset+1})(e^{2\pi tan\emptyset}-1)}$=2.85 B

And, $sin\eta=\frac{\bar{z}}{\sqrt{\bar{x}^2+\bar{z}^2}}$=0.75

ii) Determination of K, A and E

$K = \sqrt{\frac{g\beta sin\eta}{\mu_C B}}$ = 807, $A = \frac{g}{\gamma B^4\mu_C}$ = 0.0000577, and $E = \psi c B^2 + \epsilon\gamma B^3$= 20700 kN

iii) Determination of $M_{d(max)}$ in terms of λ

$M_{d(max)}=HQ_{d(max)}=HQ\lambda$

= 4x8290λ=33160λ

iv) Determination of λ_{cr} which corresponds to α=0

$$\alpha = \frac{A}{K^2}\left(E - \frac{1}{2}QB\right)\cos(Kt) - \frac{A}{K^3}\frac{M_{d(max)}}{t_d}\sin(Kt) + \frac{A}{K^2}\left(\frac{M_{d(max)}}{t_d} + \frac{1}{2}QB - E\right)$$

For t=t_d equals to 0.4s

$$\alpha = \frac{0.0000577}{0.807^2}\left(20700 - \frac{1}{2}8290 \times 2.5\right)\cos(0.8070 \times 0.4) - \frac{0.0000577}{0.807^3}\frac{33160\lambda}{0.4}\sin(0.8070 \times 0.4) + \frac{0.0000577}{0.807^2}\left(\frac{33160\lambda}{0.4} + \frac{1}{2}8290 \times 2.5 - 20700\right)$$

$=0.9159\cos(0.3228) - 9.10\lambda\sin(0.3228) + 2.94\lambda 0.9181 - 1.834$

$=0.05\lambda - 0.0474$

For $\alpha=0$, $\lambda=0.948=\lambda_{cr}$

v) Determination of $M_{d(max)}$ for $\lambda=\lambda_{cr}$

$M_{d(max)}=33160\lambda_{cr}= 33160\times 0.948=31436$ kNm

vi) Determination of G_1 and G_2

$$G_1 = \frac{A}{K^2}\left(E - \frac{1}{2}QB\right)\cos(Kt_d) - \frac{A}{K^3}\frac{M_{d(max)}}{t_d}\sin(Kt_d) + \frac{AM_{d(max)}}{K^2} = 0.9159$$

$$G_2 = \frac{A}{K}\left(E - \frac{1}{2}QB\right)\sin(Kt_d) - \frac{A}{K^2}\frac{M_{d(max)}}{t_d}\cos(Kt_d) + \frac{AM_{d(max)}}{K^2 t_d} = -4.05$$

vii) Determination of α for t=0.6 s

$$\alpha = [\frac{1}{K}][G_1 K\cos(Kt_d) - G_2\sin(Kt_d)]\cos(Kt) + (\frac{1}{K})[G_1 K\sin(Kt_d) - G_2\cos(Kt_d)]\sin(Kt) + \frac{A}{K^2}\left(\frac{1}{2}QB - E\right) = -8229 \text{ rad}$$

5.10.4 Wallace's Solution

Analysis presented by Trianadafilidis (1965) is based on rotational mode of failure. However, it is possible that a foundation may fail by vertically punching into the soil mass due to the application of vertical transient load. Wallace (1961) presented a procedure for the estimation of the vertical displacement of continuous footing considering punching mode of failure. The analysis is based on the following assumptions:

(i) The failure surface in the soil mass is assumed to be of similar type as suggested by Terzaghi (1943) for the evaluation of static bearing capacity of strip footings. This is shown in Figure 5.27

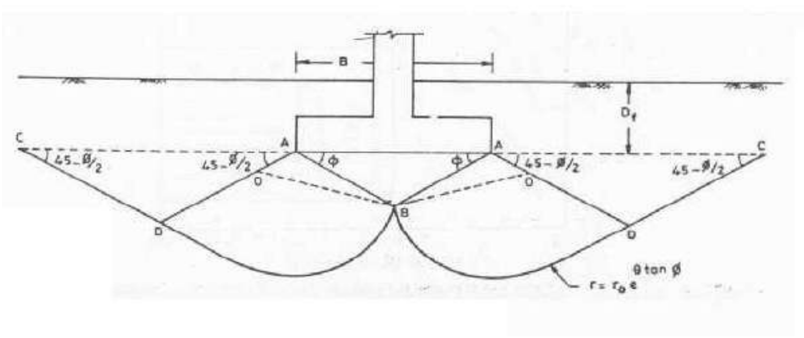


Fig. 5.27: Failure Surface

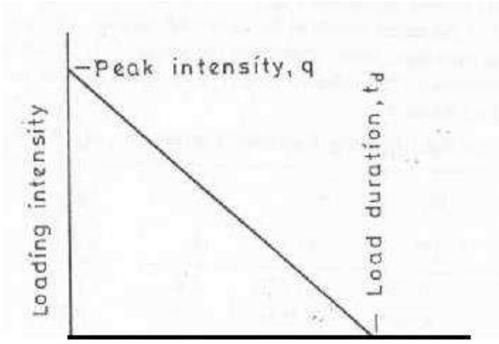


Fig.5.28: Loading Function

(ii) The soil behaves as a rigid plastic material

(iii) The ultimate shear strength is given by

$s = c + \sigma tan\emptyset$--- Eq. (5.129)

Where, s = Ultimate shear strength
c = Cohesion
σ= Normal stress
$\emptyset$= Angle of internal friction

iv) The dynamic load applied to the footing is initially peak triangular force pulse (Fig.5.28).

v)The footing is assumed to be weightless and to impart uniform load to the soil surface

Analysis:

The applied load is assumed to be an initial-peak triangular force which decays to zero at time t_d as shown in Fig.5.28. The peak load q is expressed in pressure units. Since the function is discontinuous at time t_d, two equations are necessary

For $0 \leq t \leq t_d$ Loading function $=qB\left(1-\frac{t}{t_d}\right)$--------- Eq. (5.130)

For $t \geq t_d$ Loading function = 0

In Fig. 5.27, BD is an arc of a logarithmic spiral with its centre at O. It is defined by the Eq. (5.131).

$r = r_0 e^{\theta tan\emptyset}$-------- Eq. (5.131)

Where r_0 = distance OD see Fig. (5.27)

$\emptyset$= Angle of internal friction

The static bearing capacity q_u for such a failure surface is given by

$q_u = cN_c + qN_q + \frac{1}{2}\gamma BN_\gamma$-------- Eq. (5.134)

where, c = Cohesion
$q = \gamma D_f$
γ= Unit weight of soil

D_f = Depth of footing
N_c, N_q, N_γ = Bearing capacity factors

The bearing capacity factors depend on $\emptyset$ and K, K being 2 (Distance OA)/H, Fig. 5.27. The value of K locates the centre of the spiral which is the centre of rotation. Obviously the correct value of K is that which yields the minimum value of the bearing capacity. It is obtained by trial and error for each set of problem parameters. The values of N_c, N_q, N_γ for various values of $\emptyset$ and K are given in Column 3, 4 and 5 of Table 5.2.

Any acceleration of the soil mass ACDBA due to the downward movement of the footing will cause inertial forces which will resist such movement. The inertial forces are directly proportional to the acceleration of each individual soil mass and thereby dependent on displacements. The effective total inertial force is obtained by combining the inertial forces on each separate mass using energy considerations.

The inertial force is given by $I_f = N_I \gamma B \frac{d^2\Delta}{dt^2}$------------ Eq. (5.135)

Where, Δ= Displacement at any time t

N_I=Coefficient of dynamic inertial shear resistance.

The coefficient N_I depends on $\emptyset$ and K, and its values are listed in column no. 6 of Table 5.2

Displacement of the soil mass within the failure surface due to downward movement of the footing will increase the restoring moment about the point 0, and the increase in moment will be proportional to the displacement provided the rotation is not excessive. It is expressed as

$R_F = N_R B \gamma \Delta$--------- Eq. (5.136)

The coefficient N_R also depends on $\emptyset$ and K. Its values are listed in column no. 7 of Table 5.2. The differential equations are established by equating the four vertical forces to zero. There must be separate equations for before and after time t_d

For $0 \le t \le t_d$

$N_I \gamma B \frac{d^2\Delta}{dt^2} + N_R \gamma B \Delta + q_u B - qB\left(1 - \frac{t}{t_d}\right) = 0$------- Eq. (5.137)

Or

For $t \ge t_d$

$N_I \gamma B \frac{d^2\Delta}{dt^2} + N_R \gamma B \Delta + q_u B = 0$----------- Eq. (5.138)

Or, $\frac{d^2\Delta}{dt^2} + \frac{N_R}{N_I B}\Delta = -\frac{q_u}{N_I \gamma B}$ ------------------ Eq. (5.139)

The solution of the differential equations will yield equations of footing displacement versus time. The forms of the particular solutions of Eq. 5.140 are found to be

$\Delta = C_1 \cos(K't) + C_2 \sin(K't) + \left(\frac{q - q_u}{N_R \gamma}\right) - \left(\frac{q}{N_R \gamma t_d}\right) t$---------- Eq. (5.140)

And $\Delta = C_3 \cos(K't) + C_4 \sin(K't) - \left(\frac{q_u}{N_R \gamma}\right)$--------- Eq. (5.141)

In which $K' = \sqrt{\frac{2N_R}{N_I R}}$ and $C_1, C_2, C_3\, and\, C_4$ are coefficient of integration. The C_1, C_2 are evaluated by the initial conditions. The coefficients C_3 and C_4 are obtained by conditions of displacement and velocity at t_d as defined by Eq. (5.140). After finding the solution and substitution of the coefficients yield non-dimensional Eqs. 5.140 & 141 as

For $0 \leq t \leq t_d$

$$\left(\frac{N_R\gamma}{q_u}\right)\Delta = \left(\frac{q}{q_u} - 1\right)\left[1 - \cos(K't)\right] + \frac{\frac{q}{q_u}}{t_d K'}\left[\sin(K't) - K't\right] \text{----------} \qquad \text{Eq. (5.142)}$$

For $t \geq t_d$

$$\left(\frac{N_R\gamma}{q_u}\right)\Delta = \left[\left(1 - \frac{q}{q_u}\right) + \frac{\frac{q}{q_u}}{t_d K'} sin(K' t_d)\right]\cos(K' t_d) + \left[\frac{\frac{q}{q_u}}{t_d K'}\left(1 - \cos(K' t_d)\right)\right]\sin(K' t_d) - - - - - \text{Eq. (5.143)}$$

The coefficients $N_\gamma, N_c, N_q, N_I\, and\, N_R$ are dependent only on values of $\emptyset$ and K. Using magnitudes of $\emptyset$ from 0° to 45° and of K for the region where the ultimate static shear resistance could be a minimum, these coefficients were evaluated. The values obtained are given, in Table 5.2 for every fifth degree. The maximum displacement from Eq. 5.142 and 5.143 is the predicated permanent footing displacement, since downward motion ceases at the time of the maximum displacement.

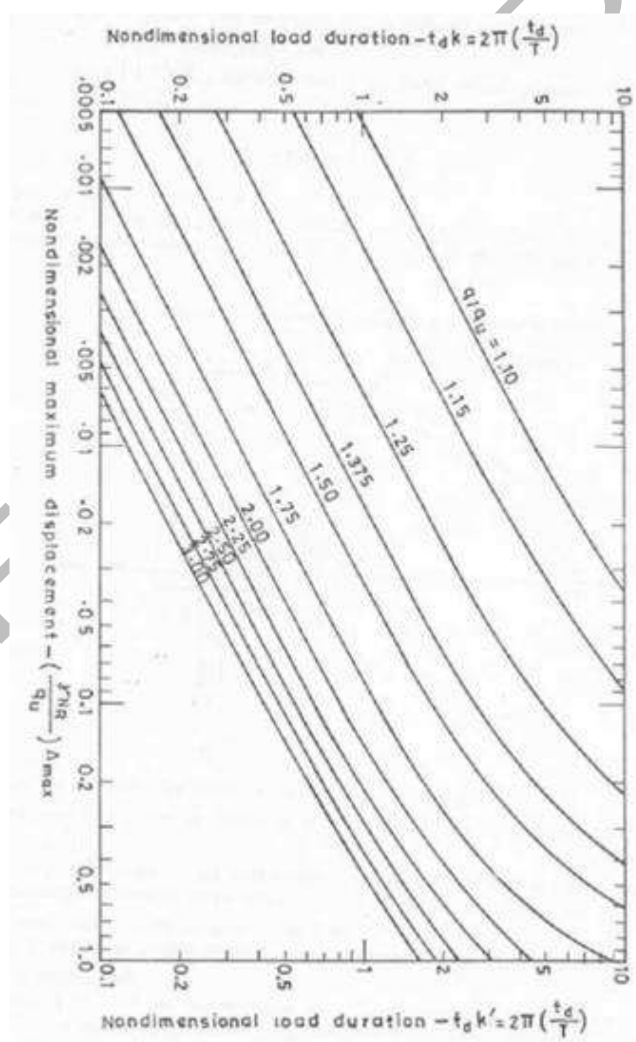


Fig. 5.29: Non-dimensional maximum displacement

Table 5.2: Bearing Capacity Factors

Φ(dgree)	K	N_γ	N_c	N_q	N_I	N_R	$\sqrt{\frac{N_R}{N_I}}$
0	-0.05	0.000	5.7277	1.00	0.0633	2.0125	5.6366
	0.00	0.000	5.7124	1.00	0.0631	1.9723	5.5887
	+0.05	0.000	5.7258	1.00	0.0633	1.9433	5.5394
5	-0.65	0.0454	79.6255	7,9664	0.03755	8.9076	4.8709
	-0.60	0.1445	29.8163	3.6986	0.2280	6.4362	5.3126
	-0.55	0.1481	18.9958	2.6619	0.1579	5.0332	5.6460
	-0.50	0.1553	14.3469	2.2552	0.1213	4.1699	5.8636
	-0.45	0.1655	11.8179	2.0339	0.1011	3.6088	5.9750
	-0.40	0.1786	10.2699	1.8985	0.0897	3.2299	6.0020
	-0.35	0.1945	9.2580	1.8100	0.0833	2.9674	5.9698
	-0.30	0.2131	8.5723	1.7500	0.0799	2.7828	5.9005
	-0.25	0.2344	8.1007	1.7087	0.0786	2.6523	5.8108
	-0.20	0.2585	7.7778	1.6805	0.0785	2.5604	5.7116
	-0.15	0.2855	7.5629	1.6617	0.0793	2.4969	5.6099
	-0.10	0.3154	7.4291	1.6500	0.0809	2.4547	5.5096
	-0.05	0.3483	7.3580	1.6437	0.0829	0.4288	5.4128
	0.00	0.3843	7.3366	1.6419	0.0853	2.4155	5.3205
	0.05	0.4233	7.3553	1.6435	0.0881	2.4122	5.2330
10	-0.60	0.5700	53.9491	10.5127	0.1120	5.7922	7.1922
	-0.55	0.5588	28.9945	6.1125	0.0935	4.8411	7.1948
	-0.50	0.5645	20.5266	4.6194	0.0833	4.2238	7.1228
	-0.45	0.5832	16.3539	3.8837	0.0779	3.8095	6.9932
	-0.40	0.6127	13.9337	3.4569	0.0757	3.5264	6.8293
	-0.35	0.6521	12.4031	3.1870	0.0755	3.3323	6.6445
	-0.30	0.7008	11.3881	3.0080	0.0767	3.2008	6.4587
	-0.25	0.7586	10.7004	2.8868	0.0790	3.1147	6.2781
	-0.20	0.8253	10.2345	2.8046	0.0821	3.0625	6.1071
	-0.15	0.9012	9.9267	2.7503	0.0858	3.0360	5.9474
	-0.10	0.9863	9.7361	2.7167	0.0901	3.0294	5.7994
	-0.05	1.0807	9.6352	2.6990	0.0948	3.0386	5.6676
	0.00	1.1848	9.6049	2.6936	0.0999	3.0604	5.5360
	+0.05	1.2986	9.6313	2.6983	0.1053	3.0923	3.4187
15	-0.55	1.5462	46.5473	13.4724	0.0707	5.2677	8.6324
	-0.50	1.5198	30.2759	9.1124	0.0696	4.7177	8.2310
	-0.45	1.5342	23.2038	7.2175	0.0707	4.3564	7.8481
	-0.40	1.5806	19.3483	6.1844	0.0734	4.1189	7.4903
	-0.35	1.6540	16.9964	5.5542	0.0773	3.9669	7.1622
	-0.30	1.7520	15.4722	5.1458	0.0823	3.8766	6.8645
	-0.25	1.8730	14.4550	4.8732	0.0881	3.8322	6.5961
	-0.20	2.0166	13.7730	4.6905	0.0947	3.8232	6.3542

	-0.15	2.1825	13.3257	4.5706	0.1020	3.8418	6.1361
Φ(dgree)	K	N_γ	N_c	N_q	N_I	N_R	$\sqrt{\frac{N_R}{N_I}}$
15	-0.10	2.3710	13.0501	4.4968	0.1101	3.8825	5.9388
	-0.05	2.5823	12.9048	4.4579	0.1183	3.9413	5.7596
	0.00	2.8168	12.8613	4.4462	0.1282	4.0149	5.5961
	+0.05	3.0750	12.8991	4.4563	0.1383	4.1008	5.4463
20	-0.50	3.6745	46.2884	17.8477	0.0673	5.6658	9.1768
	-0.45	3.6419	33.8986	13.3381	0.0728	5.3067	8.5380
	-0.40	3.6943	27.6099	11.0492	0.0796	5.0886	7.9941
	-0.35	3.8151	23.9213	9.7067	0.0877	4.9684	7.5267
	-0.30	3.9952	21.5875	8.8572	0.0970	4.9199	7.1214
	-0.25	4.2298	20.0542	8.2992	0.1076	4.9258	6.7672
	-0.20	4.5161	19.0369	7.9289	0.1194	4.9764	6.4552
	-0.15	5.8533	18.3742	7.6877	0.1325	5.0582	6.1783
	-0.10	5.2413	17.9678	7.5398	0.1470	5.1704	5.9309
	-0.05	5.0864	17.7542	7.4620	0.1629	5.3068	5.7084
	0.00	6.1717	17.6903	7.4368	0.1802	5.4638	5.5072
	+0.05	6.7161	17.4757	7.4589	0.1989	5.6486	5.3243
25	-0.50	8.5665	73.8778	35.4499	0.0732	7.2346	9.9384
	-0.45	8.3599	51.2706	24.9079	0.0835	6.8363	9.0503
	-0.40	8.3728	40.7056	19.9814	0.0954	6.6214	8.3291
	-0.35	8.5541	34.7663	17.2119	0.1094	6.5339	7.7297
	-0.30	8.8760	31.1015	15.5029	0.1254	6.5404	7.2223
	-0.25	9.3230	28.7315	14.3977	0.1437	6.6199	6.7864
	-0.20	9.8871	27.1750	13.6720	0.1646	6.7584	6.4075
	-0.15	10.5646	26.1681	13.2024	0.1882	6.9462	6.0748
	-0.10	11.3542	25.5533	12.9157	0.2148	7.1761	5.7803
	-0.05	12.2569	25.2309	12.7654	0.2445	7.4429	5.5178
	0.00	13.2745	25.1345	12.7205	0.2775	7.7423	5.2825
	+0.05	14.4095	25.2180	12.7594	0.3139	8.0710	5.0704
30	-0.45	19.3095	80.8644	47.6872	0.1064	9.3123	9.3540
	-0.40	19.1315	62.4470	37.0539	0.1267	9.0899	8.4705
	-0.35	19.3718	52.5548	31.3426	0.1506	9.0494	7.7518
	-0.30	19.9400	46.6067	27.9084	0.1787	9.1446	7.1533
	-0.25	20.1887	42.8208	25.7226	0.2116	9.3473	6.6458
	-0.20	21.9566	40.3597	24.3017	0.2500	9.6392	6.2095
	-0.15	23.3512	38.7778	23.3884	0.2944	10.0081	5.8303
	-0.10	24.9984	37.8159	22.8330	0.3456	10.4452	5.4979
	-0.05	26.8993	37.3127	22.5425	0.4041	10.9441	5.2044
	0.00	29.0580	37.1624	22.4558	0.4706	11.4998	4.9436
	+0.05	31.4810	37.2926	22.5309	0.5457	12.1084	4.7107

35	-0.45	46.2942	134.3023	95.0397	0.1527	13.4981	9.4021
Φ(dgree)	K	N_γ	N_c	N_q	N_I	N_R	$\sqrt{\frac{N_R}{N_I}}$
35	-0.40	45.4427	100.66099	71.4837	0.1887	13.2639	8.3844
	-0.35	45.6687	83.4477	59.4308	0.2323	13.3114	7.5703
	-0.30	46.7356	73.3676	52.3727	0.2849	13.5708	6.9017
	-0.25	48.5145	67.0529	47.9511	0.3481	14.0015	6.3419
	-0.20	50.9356	62.9887	45.1052	0.4237	14.5786	5.8661
	-0.15	53.9640	60.3926	43.2874	0.5133	15.2895	5.4569
	-0.10	57.8568	58.8199	42.1864	0.6191	16.1127	5.1018
	-0.05	61.8051	57.9989	41.6113	0.7428	17.0515	4.7911
	0.00	66.6196	57.7539	41.4398	0.8868	18.0970	4.5175
	+0.05	72.0773	57.9662	41.5884	1.0526	19.2451	4.2753
40	-0.40	115.7097	172.8231	146.0161	0.3229	20.8738	8.0404
	-0.35	115.5504	141.1002	119.3973	0.4107	21.1138	7.1701
	-0.30	117.6386	123.0124	104.2199	0.5195	21.7125	6.4650
	-0.25	121.5875	111.8576	94.8599	0.6536	22.6077	5.8817
	-0.20	127.1879	104.7472	88.8935	0.8175	23.7619	5.3914
	-0.15	134.3346	100.2323	85.1051	1.0168	25.1570	4.9741
	-0.10	142.9868	97.5069	82.8181	1.2572	26.7775	4.6152
	-0.05	153.1451	96.0866	81.6263	1.5450	28.6173	4.3038
	0.00	164.839	95.6630	81.2709	1.8870	30.6724	4.0317
	+0.05	178.1176	96.0303	81.5791	2.2904	32.9409	3.7924
45	-0.40	327.6781	322.2748	323.2752	0.6576	36.2961	7.4295
	-0.35	325.4943	259.1345	260.1349	0.8611	37.0113	6.5559
	-0.30	329.9752	224.0769	225.0772	1.1194	38.3965	5.8568
	-0.25	339.8627	202.7837	203.7840	1.4447	40.3468	5.2846
	-0.20	354.4804	189.3358	190.3361	1.8515	42.8070	4.8083
	-0.15	373.4971	180.8450	181.8452	2.3565	45.7496	4.4062
	-0.10	393.7473	175.7358	176.7361	2.9784	49.1634	4.0628
	-0.05	424.2605	173.0775	174.0778	3.7386	53.0475	3.7669
	0.00	456.1177	172.2851	173.2853	4.6607	57.4067	3.5096
	+0.05	492.4763	172.9729	173.9732	5.7709	62.2499	3.2843

5.11 Seismic Bearing Capacity and Settlement in Granular Soil

The shallow foundations may fail during seismic events. Published studies relating to the bearing capacity of shallow foundations in such instances are rare. In 1993, however, Richards et al. developed a seismic bearing capacity theory to find seismic bearing capacity of granular soil. Figure 5.30 shows a failure surface in soil assumed for the subsequent analysis, under static conditions. Similarly, Figure 5.31 shows the assumed failure under earthquake conditions. Note that, in the two figures,

α_A, α_{AE} = inclination angles for active pressure conditions

And

α_P, α_{PE}= inclination angles for passive pressure conditions

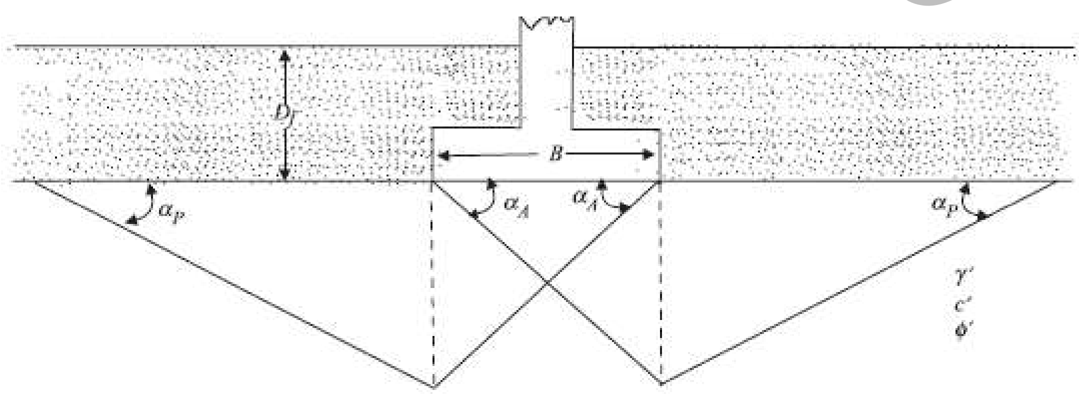


Fig.5.30: Assumed failure surface in soil for static bearing capacity analysis

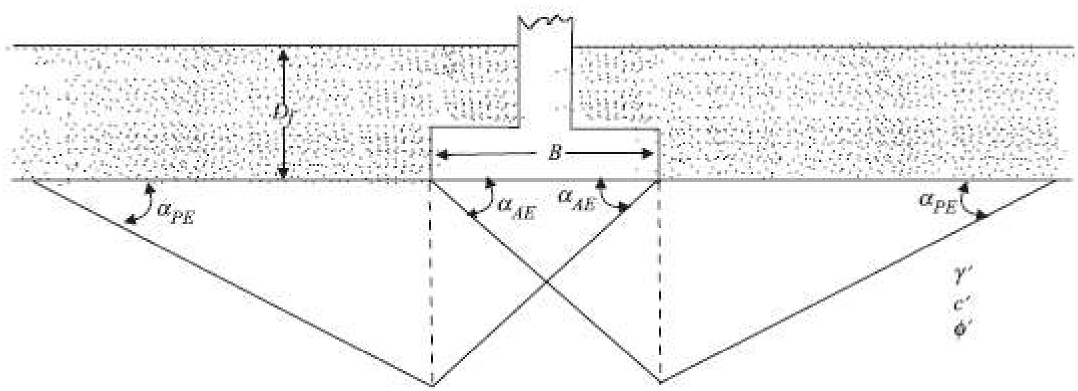


Fig.5.31: Assumed failure surface in soil for seismic bearing capacity analysis

According to this theory, the ultimate bearing capacities for *continuous foundations* in granular soil are

$q_u = qN_q + \frac{1}{2}\gamma B N_\gamma$, For static condition--------------------- Eq. (5.144)

$q_{uE} = qN_{qE} + \frac{1}{2}\gamma B N_{\gamma E}$, For Earthquake conditions------------ Eq. (5.145)

Where, N_q, N_γ, N_{qE}, $N_{\gamma E}$ are bearing capacity factors

Again, $N_q, N_\gamma = f(\emptyset')$

And $N_{qE}, N_{\gamma E} = f(\emptyset', tan\theta)$

Where $tan\theta = \frac{k_h}{1-k_v}$---------------- Eq. (5.146)

k_h= Horizontal coefficient of earthquake acceleration

k_v= Vertical coefficient of earthquake acceleration

Using the failure surface shown in Figure 5.31, Richards, Elms and Budhu (1993) provided the values of bearing capacity factors, N_q, and N_γ,. They are given in Table 5.3

Table 5.3: Bearing capacity factors

ϕ(deg)	N_q	N_γ
0	1.0	0
10	2.4	1.4
20	5.9	6.4
30	16.5	23.8
40	59.0	112.0

The variations of N_q and N_γ with $\emptyset'$ are shown in Figure 5.32. Figure 5.33 shows the variations of ${N_{\gamma E}}/{N_\gamma}$ and ${N_{qE}}/{N_q}$ with $tan\theta$ and the soil angle $\emptyset'$ based on this analysis.

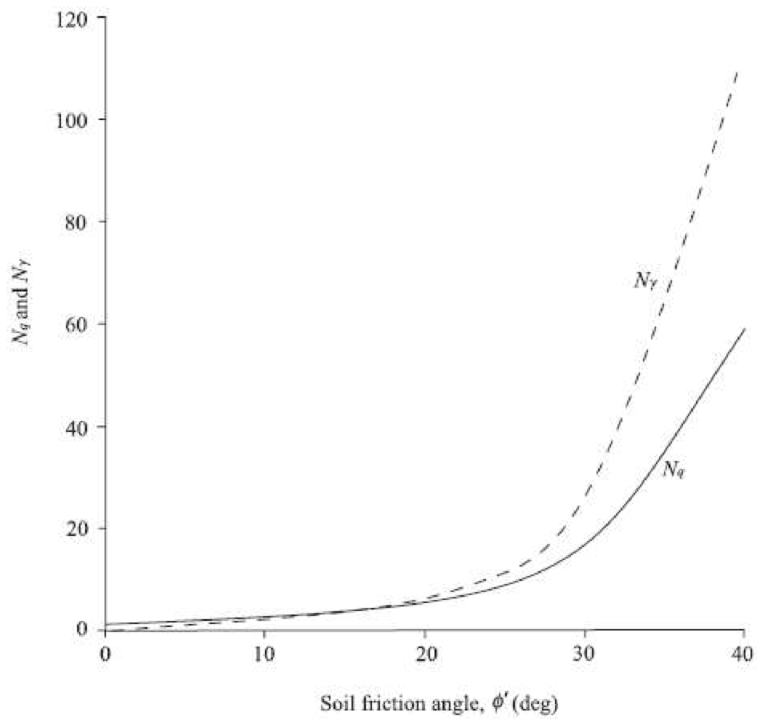


Fig. 5.32: Variation of N_q and N_γ based on failure surface assumed in Figure 5.30

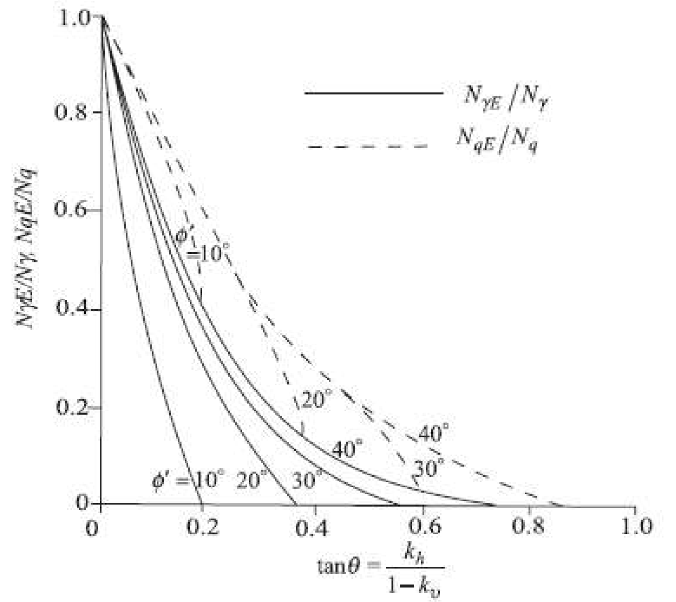


Fig. 5.33: Variation of $^{N_{\gamma E}}/_{N_{\gamma}}$ and $^{N_{qE}}/_{N_q}$ with $tan\theta$

Under static conditions, bearing capacity failure can lead to a substantial sudden downward movement of the foundation. However, bearing capacity related settlement in an earthquake is important and it takes place when the ratio $tan\theta = {}^{k_h}/_{(1-k_v)}$ reaches the critical value. The critical value can be expressed as $\left[\frac{k_h}{(1-k_v)}\right] critical$ becomes equal to k_h^* when k_v=0

Table 5.4: Variation of α_{AE} with k_h^* and soil friction angle $\emptyset'$
(Compiled from Richards, Elms and Budhu, 1993)

	$\tan\alpha_{AE}$				
k_h^*	$\phi' = 20°$	$\phi' = 25°$	$\phi' = 30°$	$\phi' = 35°$	$\phi' = 40°$
0.05	1.10	1.24	1.39	1.57	1.75
0.10	0.97	1.13	1.26	1.44	1.63
0.15	0.82	1.00	1.15	1.32	1.48
0.20	0.71	0.87	1.02	1.18	1.35
0.25	0.56	0.74	0.92	1.06	1.23
0.30		0.61	0.77	0.94	1.10
0.35		0.47	0.66	0.84	0.98
0.40		0.32	0.55	0.73	0.88
0.45			0.42	0.63	0.79
0.50			0.27	0.50	0.68
0.55				0.44	0.60
0.60				0.32	0.50

Figure 5.34 shows the variation of k_h^* (for $k_v = 0$) with the factor of safety (FS) applied to the ultimate static bearing capacity [Eq. 5.144], with $\emptyset'$, and with $^{D_f}/_B$ (for $\emptyset'$ = 30^0 and 40^0).

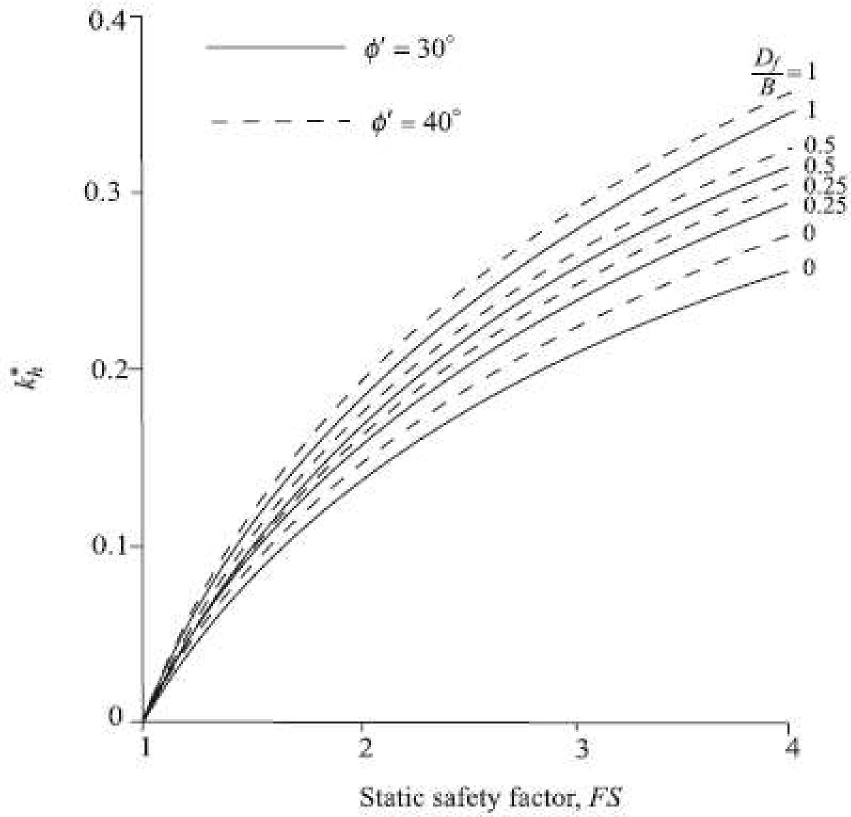


Fig.5.34: Critical acceleration k_h^* for k_v=0

The settlement of a strip foundation due to an earthquake using a sliding block approach can be estimated (Richards, Elms and Budhu, 1993) as

$S_{Eq}(m) = 0.174\frac{V^2}{Ag}\left[\frac{k_h^*}{A}\right]^{-4} tan\alpha_{AE}$---------------------- Eq. (5.147)

Where, V = peak velocity for the design earthquake (m/sec)
A = acceleration coefficient for the design earthquake
g = acceleration due to gravity (9.81 m/s^2)
The values of k_h^* and α_{AE} can be obtained from Figure 5.34 and Table 5.4, respectively. This approach can be used to design a footing based on limiting seismic settlements.

Problem No.1
A rectangular foundation has a length L of 2.5 m. It is supported by medium dense sand with a unit weight of 17 kN/m^3. The sand has an angle of friction of 36º. The foundation may be subjected to a dynamic load of 735 kN increasing at a moderated rate. Using a factor of safety equal to 2, determine the width of the foundation. Use D_f = 0.8 m.

APPENDIX **(Foundation for Various Types Machines)**

DESIGN OF RECIPROCATING MACHINE FOUNDATION

Reciprocating Machine should be placed over suitable Vibration Absorber to reduce the magnitude of displacement due to dynamic unbalanced force produced due the Machine.

Design for Vibration absorber: The following steps are required

1. Make a trial design without absorber
2. Depending on requirement of minimum foundation size, for the machine selection of block foundation size is made.
3. Determine stiffness due to vertical vibration $K_z=K_1$
4. Determine the natural frequency of soil, considering total mass due to machine and foundation, $w_{nl1}=\sqrt{\frac{K_1}{m_1+m_2}}$
5. Find mass ratio $n=\frac{m_2}{m_1}$
6. Determine frequency ratio $a_1=\frac{w_{nl1}}{w}$, w is the operating frequency
7. Compute displacement magnitude without absorber as , Z_{max}
   $Z_{max}=\frac{F_0}{(m_1+m_2)(w_{nl1}^2-w^2)}$
8. Calculate efficiency of absorber $\eta=\frac{Z_1}{Z_{max}}$, z_1 is the permissible displacement
9. Again efficiency can be expressed as
   $\eta=\frac{a_2^2(1+n)(a_1^2-1)}{[1-(1+n)(a_1^2+a_2^2-a_1^2a_2^2)]}$ where a_2 is the frequency ratio, $a_2=\frac{w_{nl2}}{w}$
   w_{nl2} is the natural frequency of absorber.
10. Find stiffness of absorber $K_2=m_2w_{nl2}^2$
11. Suitable absorber may be selected from K_2
12. Find $Z_2=\frac{[(1+n)a_1^2+na_2^2-1]F_0}{m_2\omega^2[1-(1+n)(a_1^2+a_2^2-a_1^2a_2^2)]}$, Than check for maximum force of resistance
13. Find magnitude of force resistance $F_0=K_2Z_2$

Q.1 Determine the stiffness of the absorber to be kept between a reciprocating machine and foundation to bring the vibration amplitude to less than 0.02 mm. The weight of the machine is 18 kN. It produces an unbalanced force of 4 kN, when operated at speed of 600rpm. Shear modulus of foundation soil G=20MN/m^2 and μ= 0.35.

Sol: Mass of the block foundation of dimension 4x3m^2 with 1.5 m height

$m_1=\frac{24\times4\times3\times1.5}{9.81}=44$ kg

Mass of machine $m_2 = \frac{18}{9.81} = 1.83$ kg,

Equivalent radius of circular footing $r = \sqrt{\frac{4\times3}{\pi}}$=1.95 m

Stiffness in vertical direction, $K_z = \frac{4Gr}{1-\mu}$=240000 kN/m

Natural frequency of vibration, $\omega_{nl1} = \sqrt{\frac{240000}{44+1.83}}$=72.36 r/sec

Operating frequency of machine, $\omega = \frac{2\pi\times600}{60}$= 62.83 r/sec

Maximum amplitude of vertical vibration (Without absorber)

$Z_{max} = \frac{F_0}{(m_1+m_2)(\omega_{nl1}^2-\omega^2)}$=0.068 mm

Efficiency of absorber= $\frac{Z_1}{Z_{max}}$=$\frac{0.02}{0.068}$=0.333

Frequency ratio, $a_1 = \frac{\omega_{nl1}}{w}$ =1.15, Mass ratio, $\eta = \frac{m_1}{m_2}$=0.0415

Now efficiency can also be expressed as η=$\frac{a_2^2(1+n)(a_1^2-1)}{[1-(1+n)(a_1^2+a_2^2-a_1^2a_2^2)]}$

Hence, 0.333=$\frac{a_2^2(1+0.0415)(1.15^2-1)}{[1-(1+0.0415)(1.15^2+a_2^2-1.15^2a_2^2)]}$

So a_2=1.31

But $a_2 = \frac{\omega_{nl2}}{\omega}$, hence ω_{nl2}=81.97 r/s

$Z_2=\frac{[(1+n)a_1^2+na_2^2-1]F_0}{m_2\omega^2[1-(1+n)(a_1^2+a_2^2-a_1^2a_2^2)]}=\frac{[(1+0.0415)1.15^2+0.0415\times1.31^2-1]\times4}{1.83\times62.83^2[1-(1+0.0415)(1.15^2+1.31^2-1.15^2\times1.31^2)]}= -58.47$ mm

Resisting force of absorber $F_0 = K_2Z_2$=12298x0.05847=719 kN

MACHINE FOUNDATION ON PILES

Piles divided into two groups: End bearing and Friction Piles

Longitudinal Vibration of Short Elastic Bar:

i) **End Bearing Pile**: *Mass of Pile is negligible*

Wave equation: $\frac{\partial^2 u}{\partial t^2}=v_c^2\,\frac{\partial^2 u}{\partial x^2}$-------- Eq.(1)

Where u is the displacement function and v_c is shear wave velocity

Solution to above second order differential equation is expressed as

$u(x,t)$=U(x)($A_1\sin\omega_n t+A_2\cos\omega_n t$)--- Eq.(2)

Where A_1 and A_2 are two constants, ω_n is the natural frequency and U(x) is amplitude of displacement along the length of the bar and is independent of time "t"

Eq. 1 can be rewritten by putting value from Eq.2 as

$\frac{\partial^2 U(x)}{\partial x^2}+\frac{\rho}{E}\omega_n^2 U(x)=0$----- Eq.(3)

Now general solution to the above Eq.3 can be given as

U(x) $=B_1\sin(\frac{\omega_n x}{v_c})$ $+B_2\cos(\frac{\omega_n x}{v_c})$----- Eq. (4)

Using end conditions a) Fixed

At x=0, U(x) =0 which gives B_2=0

At x=L, $\frac{dU(x)}{dx}=0$, gives $\frac{B_1}{v_c}\omega_n cos\,\frac{\omega_n L}{v_S}$=0,

Hence, $cos\,\frac{\omega_n L}{v_c}=0$, or $\frac{\omega_n L}{v_c}=\frac{(2n-1)\pi}{2}$

Which gives, $\omega_n=\frac{1}{2}(2n-1)\pi\frac{v_c}{L}$, for n=1 $\omega_n=\frac{\pi}{2}\frac{v_c}{L}$

So natural frequency $f_n=\frac{\omega_n}{2\pi}=\frac{1}{4L}\sqrt{\frac{E_p}{\rho_p}}$--------------- Eq.(5)

Where E_p and ρ_P are Young's modulus and density of pile material

Now mass of pie is considered as "m"

The general solution is again taken as U(x) $=B_1\sin(\frac{\omega_n x}{v_c})$ $+B_2\cos(\frac{\omega_n x}{v_c})$

From end condition, we get B_2=0 which gives U(x) $=B_1\sin(\frac{\omega_n x}{v_c})$

At x=L, the inertia force of mass m is acting on the soil column and this can be expressed as

$F=-m\frac{\partial^2 u}{\partial t^2}$------------------ Eq.(6)

Strain in the pile is expressed as $\epsilon=\frac{\partial u}{\partial x}=\frac{F}{AE}$------------ Eq.(7)

Where E and A are Young's modulus and area of cross section of pile respectively.

$\frac{F}{AE}=\frac{\partial u}{\partial x}=\frac{\partial U}{\partial x}(A_1 sin\omega_n t+A_2 cos\omega_n t)$-------- Eq.(8)

Substituting for, $\frac{\partial U}{\partial x}$, above Expression becomes

$\frac{B_1\omega_n}{v_c}\cos\left(\frac{\omega_n x}{v_c}\right)(A_1 sin\omega_n t+A_2 cos\omega_n t)=\frac{F}{AE}$------- Eq.(9)

Now, $F=-m\frac{\partial^2 u}{\partial t^2}=\ -m[B_1\sin(\frac{\omega_n x}{v_c})]\ \frac{\partial^2}{\partial t^2}(A_1 sin\omega_n t+A_2 cos\omega_n t)$------ Eq.(10)

Or, $F=m\omega_n^2 B_1\sin(\frac{\omega_n x}{v_c})(A_1 sin\omega_n t+A_2 cos\omega_n t)$----------- Eq.(11)

From Eq.(9) and (10), we get

$\frac{AE}{v_c}\cos\left(\frac{\omega_n x}{v_c}\right)=m\omega_n\sin(\frac{\omega_n x}{v_c})$------ Eq.(12)

At x=L , $AE = m\omega_n v_c \tan(\frac{\omega_n L}{v_c})$---------- Eq.(13)

Or, $A\rho v_c^2 = m\omega_n v_c \tan(\frac{\omega_n L}{v_c})$-------------- Eq.(14)

as $v_c = \sqrt{\frac{E}{\rho}}$

Now consider a non-dimensional parameter,

$\frac{AL\gamma}{W} = \frac{\omega_n L}{v_c}\tan(\frac{\omega_n L}{v_c})$--------------- Eq. (15)

which may be expressed as $\beta = \alpha tan\alpha$

Where $\beta = \frac{AL\gamma}{W}$ and $\alpha = \frac{\omega_n L}{v_c}$

The above relation can be placed in tabular form as Table1

T1: Coefficients for natural frequency of piles

β	0.1	0.3	0.5	0.7	1.0	2.0	4.0	10.0
α	0.32	0.53	0.66	0.75	0.86	1.08	1.27	1.43

When pile mass is negligible as compared to mass of on pile cap, we get

$\frac{AL\gamma}{W} = (\frac{\omega_n L}{v_c})^2$, and hence $\omega_n = \sqrt{\frac{AEg}{LW}}$------------------- Eq.(16)

ii) **Friction Pile:**

Assumptions:

1. Pile is vertical and circular in cross section, if not equivalent circular radius is taken
2. The pile is floating the soil foundation, (Not restricted)
3. The pile is perfectly connected to soil.
4. The soil above the pile tip behaves as an infinitesimal thin independent linearly elastic layer.
5. The dynamic stiffness and damping of pile material can be described interms of a complex stiffness matrix as proposed by Novak and EI-Sharnouby 1983.

Stiffness $K = K_1 + iK_2$-------------------------- Eq. (17)

K_1 is real part of stiffness $=R_{ek}$

And K_2 is imaginary part$=I_{mk}$

Let vertical stiffness $K_z = R_{ek}$ --------------------- Eq.(18)

And Damping Coefficient $C_z=\frac{I_{mk}}{\omega_n}$--------------- Eq.(19)

Now force acting on pile Q=KZ------------------- Eq.(20)

Or $Q = (K_z + i\omega_n C_z)Z$ or $Q = \left(K_z Z + \dot{Z} C_z\right)$ - Eq.(21)

Stiffness and Damping are rewritten as

$K_z = (\frac{AE}{R})f_{z1}$ and $C_z = (\frac{EA}{\sqrt{\frac{G}{\rho}}})f_{z2}$ -------- Eq.(22)

Where f_{z1} and f_{z2} are two factors can be taken from Fig. 1 (a),(b), proposed by Novak et al.

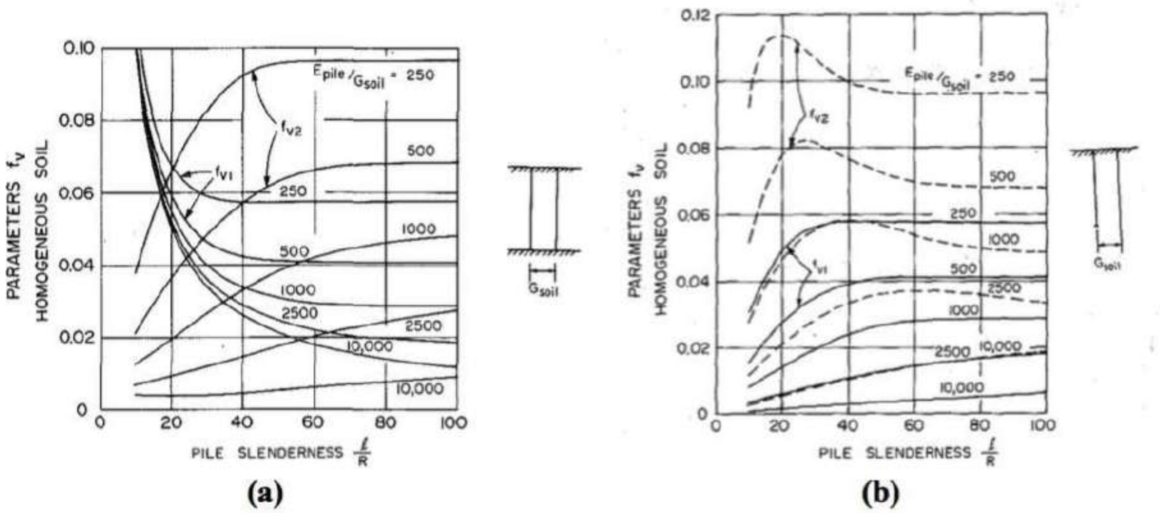


Fig. 1: Stiffness and Damping Parameters of Vertical Response for: a) End Bearing Piles and b) Floating Piles (reprinted from Novak and El Sharnouby 1983, © ASCE)

Stiffness and Damping of Pile Group

$K_z(g) = \frac{\sum K_z}{\sum \alpha_r}andC_z(g) = \frac{\sum C_z}{\sum \alpha_r}$ ---------------------- Eq.(23)

Where α_r= Interaction factor and is obtained from the table T:3.6 page 49 (Hand book of Machine Foundation by Srinivasulu &Vaidyanatham)

S/D=Pile Spacing/Diameter of Pile	3.0	45	6.0	∞
α_r	0.35	0.58	0.63	1.0

Stiffness and Damping Coefficient of Pile Cap:

$K_z(cap) = G_s D_f S_1$--------------------------------------- Eq.(24)

$C_z(cap) = D_f r_0 S_2 \sqrt{G_s \rho_s}$------------------------------- Eq.(25)

Where r_0 is the equivalent radius of pile cap and D_f is the depth of pile cap from ground surface

S_1 and S_2 are constants and may be taken as 2.7 and 6.7 respectively.

Now total stiffness and Damping Coefficient can be obtained as

$K_z(T) = K_z(g) + K_z(cap)$------------------------------- Eq.(26)

$C_z(T) = C_z(g) + C_z(cap)$------------------------------- Eq.(27)

Damping Ratio in Vertical Direction

$D_z = \frac{C_z(T)}{2\sqrt{K_z(T)m}}$ -- Eq.(28)

$\omega_n = \sqrt{\frac{K_z(T)}{m}}$-- Eq.(29)

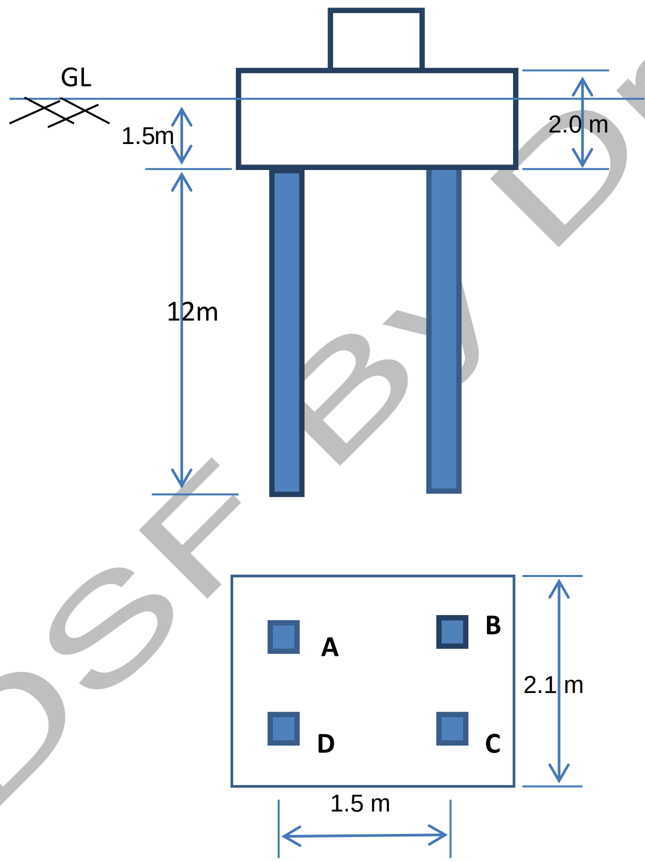
Amplitude of Displacement: For constant force of excitation

$A_z = \frac{Q_0}{K_z}\frac{1}{D_z\sqrt{(1-D_Z^2\ )}}$--------------------------------------- Eq.(30)

Amplitude of Displacement: For rotating mass type of excitation

$A_z = \frac{m_e e}{M}\frac{1}{D_z\sqrt{(1-D_Z^2)}}$--------------------------------------- Eq.(31)

Q.1 A group of four piles having dimension 0.3x0.3 m^2 is supported a machine foundation as shown in Fig. Determine total stiffness and damping coefficient, given E_p=2.1x10^7 kN/m^2, unit weight of soil18.9 kN/m^2, Poisson's ratio 0.5 and shear modulus G_s=28120 kN/m^2.



Solution: Equivalent radius of pile cap $r_0=\sqrt{\frac{(1.5X0.6)^2}{\pi}}$= 1.18 m

Equivalent radius of pile R= $\sqrt{\frac{(0.3X0.3)^2}{\pi}}$= 0.17 m

Vertical stiffness of pile, $K_z = (\frac{AE}{R})f_{z1}$= $\frac{2.1x10^7 \times 0.3^2}{0.17} \times f_{z1}, f_{z1}$is found from Fig 1(b) as 0.035

= 389.1x10^{3}kN/m^2

Now$\frac{E}{G} = \frac{2.1\times10^7}{28120}$= 746.8= say 750

$\frac{L}{R} = \frac{12}{0.17}$=70.58, From Fig. 1(b)$f_{z2}$=0.06

Hence Damping Coefficient is calculated as $C_z = (\frac{EA}{\sqrt{\frac{G}{\rho}}})f_{z2} = \frac{2.1\times10^7\times0.3^2}{\sqrt{\frac{28120\times9.81}{18.9}}}$X0.06=938.7 kN.s/m

Calculation of Interaction factor:

$\frac{S}{D} = \frac{1.5}{0.34}$=4.41 for pile A to B, A to D

A	A	B	C	D
	1	0.54	0.48	0.54
B	0.54	1.0	0.54	0.48
C	0.48	0.54	1.0	0.54
D	0.54	0.48	0.54	10
$\sum \alpha_r$	2.56	2.56	2.56	2.56

If the sum of interaction factors is not same, than take average value of $\sum \alpha_r$

K$_{zr}$(g)=$\frac{\sum K_z}{\sum \alpha_r} = \frac{4\times389100}{2.56}$=607968.75kN/m

$C_Z(g) = \frac{\sum C_z}{\sum \alpha_r}$=1446.78 kNs/m

Now Stiffness and Damping Coefficient for Pile Cap

$$K_z(cap) = G_s D_f S_1 = 28120 \times 1.5 \times 2.7 = 113886\ kN/m$$

And Damping Coefficient for Pile Cap

$C_z(cap) = D_f r_0 S_2 \sqrt{G_s \rho_s}$=2720 kNs/m

Now total Stiffness and Damping Coefficient are calculated as

$K_z(T) = K_z(g) + K_Z(cap)$=607968.75+113886=721854.75 kN/m

$C_z(T) = C_z(g) + C_Z(cap)$=1446.78+2720=4166.78 kNs/m

Problem 1 A machine is supported by four pre-stressed concrete piles driven into a bed rock. The length of each pile is 8o ft long and is 12x12 in^2 in cross section. The weight of the machine and foundation is 300x10^{3}lbs, unit weight 150 lb/ft^3. The Young's modulus is 3.5x10^{6}lb/in^2. Determine the natural frequency of pile foundation system.

FRAMED FOUNDATION FOR MACHINE

In the case of a frame foundation, it is necessary to check the frequencies and amplitudes of vibration and also to design the members of frame from structural considerations. The methods for carrying out dynamic analysis may be divided into two categories:

(a) Two-.dimensional analysis

(b) Three-dimensional analysis

The two-dimensional analysis (Also known as Simplified Method) is based on the following assumptions*:*

Each transverse frame that consists of two columns and a beam perpendicular to the main shaft of the machine is considered separately. The stiffness of the equivalent spring is calculated as the combined stiffness of the beam and the column acting together and the mass is determined by the mass of total loads acting on the cross frame.

Following Assumptions are made.

i) Frame columns are fixed at their lower ends into the rigid base slabs.

ii) The difference in vertical deformation of individual frame column is neglected

iii) Torsional resistance of longitudinal beam is insignificant compare to the deformation resistance of transverse beam. Therefore, the effect of longitudinal beam on vertical vibration of transverse frame can be neglected.

iv) The natural frequencies of individual cross frame are practically of same order.

v) The effect of elasticity of soil is neglected.

vi) The connection of transverse beam with column is also neglected.

Determination of Vertical Frequency:

For obtaining vertical frequency, each transverse frame that consists of two columns and a beam perpendicular to main shaft of the machine is considered separately as shown in Fig. 1

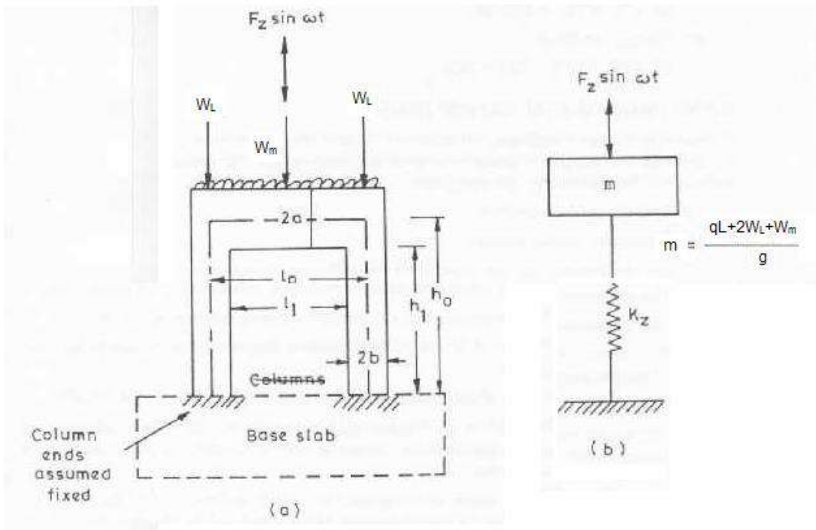


Fig. 1 a) Typical Transverse Frame b) Idealised Model

The loads acting on this frame are

(I) Dead load of the machine and bearing, W_m

(ii) Load transferred to the columns by longitudinal beams, W_1

(iii) Uniformly distributed load due to self weight of cross beam, q per unit length

(Iv) Unbalanced vertical force due to machine operation, $F_z \sin(\omega t)$

The mass-spring system used as model for the frame is shown in Fig.1 (b). The stiffness of equivalent spring (K) is computed as the combined stiffness of the beam and columns acting together. It is given by

$K_z = \frac{W}{\delta_{st}}$------ Eq.1

Where W = Total load on the frame=W_m+2W_L+qL

L= Effective span

δ_{st}= Total vertical deflection at the vcentre of the beam due to bending action of beam and axial compression in column.

So, $\delta_{st} = \delta_1 + \delta_2 + \delta_3 + \delta_4$-------- Eq.2

δ_1=Vertical deflection of beam due to load W_m

δ_2= Vertical deflection of beam due to distributed load q

δ_3=Vertical deflection of beam due to shear

δ_4=Axial compression in column

Now the magnitude of each deflection components can be obtained as

$\delta_1 = \frac{W_m L^3}{96 E I_b} \frac{2K+1}{K+2}$,

$\delta_2 = \frac{q L^4}{384 E I_b} \frac{5K+2}{K+2}$

$\delta_3 = \frac{3}{5 E A_b}(W_m + \frac{qL}{2})$

$\delta_4 = \frac{h}{E A_c}(W_L + \frac{W_m L}{2})$

Where

A_b=Cross sectional area of beam, A_c= Cross sectional area of column

I_c=Moment of inertia of beam about the axis of bending

E= Young's modulus of concrete, K=Relative stiffness factor=$\frac{I_b}{I_c} \times \frac{h}{L}$

L= Effective span of frame, h= Effective height of frame

The values of L and h are taken as

$L = l_0 - 2\alpha b$, $h = h_0 - 2\alpha a$

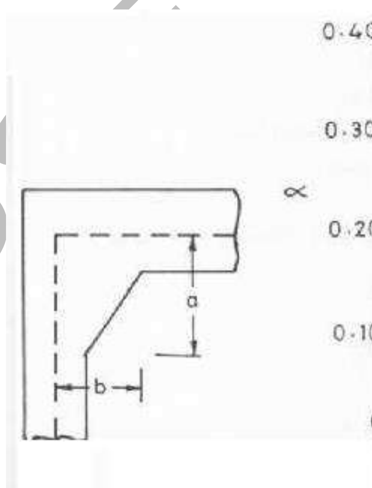
l_0 = Centre to centre distance between columns (Fig. 1a)

h_0 = Height of the column from the top of the base slab to the centre of the frame beam (Fig. 1a)

a = One-half of the depth of the beam for a frame without haunches (Fig. 1a) or the distance as shown in Fig. 2 for a frame with haunches

b = One-half of the column width for a frame without haunches (Fig. 1a) or the distance as shown in Fig. 2 for a frame with haunches.

Knowing the values of h_0, l_0 and b, α can be obtained from Fig. 3.



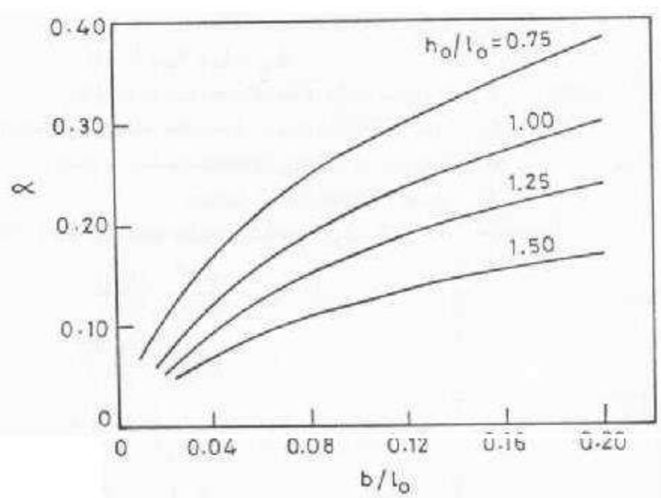


Fig. 2: values of a and b for a frame with haunches.

Fig.3: α versus $\frac{b}{l_0}$

The natural frequency of a transverse frame in vertical vibrations is given by

$$\omega_{nz} = \sqrt{\frac{K_z g}{W}} \text{-------} \qquad \text{Eq.3}$$

Average vertical natural frequency of the T. G. Foundation is taken as:

$$\omega_{nza} = \frac{\omega_{nz1} + \omega_{nz2} + --+\omega_{nzn}}{n} \text{-------} \qquad \text{Eq.4}$$

ω_{nz1} , ω_{nz2}, etc =Vertical frequencies of individual transverse frames

Vertical Vibration Amplitude:

$$A_Z = \frac{P_Z}{\sum K_Z \sqrt{(1-\frac{\omega^2}{\omega_{nza}^2})^2 + (2D\frac{\omega}{\omega_{nza}})^2}} \text{------------} \qquad \text{Eq.5}$$

A_Z=Average vertical amplitude of vibration og foundation

P_Z=Total vertical imbalance force

$\sum K_Z$= Sum of the stiffness of the individual frames

ω_{nza}= Average value of natural frequency

D= Damping ratio

For under-tuned foundation, i.e. $\omega < \omega_{nz}$ or $\omega_n = \omega_{nza}$

The maximum amplitude of vibration can be expressed as

$$A_{Z(max)} = \frac{P_Z}{\sum K_Z} \times \frac{1}{2D} \text{--------------} \qquad \text{Eq.6}$$

Horizontal Vibration Analysis:

The following assumptions are made during analysis of horizontal vibration of frame foundation

i) Columns are fixed into the rigid base slab at lower ends.

ii) The deck slab is rigid in its own plane.

iii) The resistance offered by the column in axial compression is large as compare to their resistance in bending.

iv) Torsional vibration of deck slab is neglected.

v) Elastic resistance of the soil at the base can be neglected.

The spring stiffness is provided by the columns due to their bending action and for any transverse frame is given by

$$K_{xi} = \frac{12EI_c}{h^3}\left(\frac{6K+1}{3K+2}\right) \text{----------} \qquad \text{Eq.7}$$

K_{xi}=Lateral stiffness of an individual transverse frame.

The natural frequency of frame foundation is given by

$$\omega_{nxa} = \sqrt{\frac{\sum(K_x)g}{W_T}} \text{ ---------------} \qquad \text{Eq.8}$$

Where W_T is the total weight of deck slab and machine

The average horizontal amplitude of vibration of the foundation may be expressed as

$$A_x = \frac{P_x}{\sum K_{nx}\sqrt{(1-\frac{\omega^2}{\omega_{nx}^2})^2+(2D\frac{\omega}{\omega_{nx}})^2}} \text{--------} \qquad \text{Eq.9}$$

For under-tuned foundation, i.e. $\omega < \omega_{nz}$ or $\omega_n = \omega_{nza}$

The maximum amplitude of vibration can be expressed as

$$A_{x,max} = \frac{P_x}{\sum K_x} \times \frac{1}{2D} \text{--------} \qquad \text{Eq.10}$$

Two Degree of Freedom (Amplitude Method) Analysis:

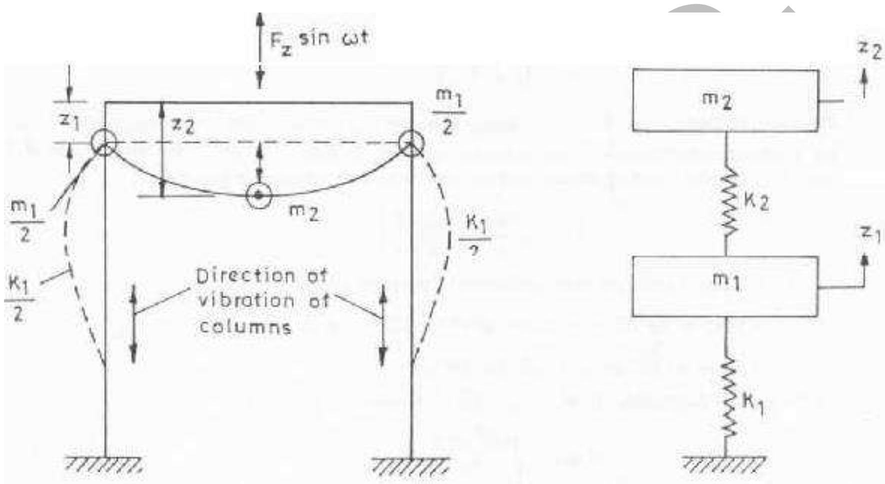


Fig. 4(a) Section nof cross frame (b) Mathematical model

Vertical Vibration Analysis:

For the vertical frequency a two-degree-spring-mass system shown in Fig. 4(b) is adopted. Mass m_1 lumped over the columns is given as

$$m_1 = \frac{W_1+W_2+0.33W_3+0.25W_4}{g} \text{---------} \qquad \text{Eq.11}$$

Mass m_2 acting at the centre of the cross beam is given as

$$m_1 = \frac{W_2+0.45W_4}{g} \text{---} \qquad \text{Eq.12}$$

W_1=Dead load of the machine and bearing

W_2= Load transferred to column by lognitudinal beams.

W_3=Weight of two columns contituting the transverse frame

W_4=Weight of the tranverse beam

The stiffness K_1 of both the column of a transverse frame is given by

$K_1 = \frac{2EA_c}{h}$--------------- Eq.13

The stiffness K_2 of the frame is given as

$K_2 = \frac{1}{\Delta z}$---------- Eq.14

$\Delta z = \frac{l(1+2K)}{96EI_b(2+K)} + \frac{3l}{8GA_b}$------- Eq.15

G = Shear modulus of beam material

E = Young's modulus of columns material

Ac = Cross-sectional area of a column

h = Effective height of the column

l= Effective span of the beam

A_b= Cross-sectional area of the beam

I_b= Moment of inertia of the beam

The equations of motion for m_1 as free vibration will be expressed as

$m_1\ddot{Z}_1 + K_1Z_1 + K_2(Z_1 - Z_2) = 0$------ Eq.16

Similarly equation of motion for m_2 is given as

$m_2\ddot{Z}_2 + K_2(Z_2 - Z_1) = 0$------ Eq.17

The solution of above equations are given as

$Z_1 = A_1 sin\omega_{nt}$-------- Eq.18

$Z_2 = A_2 sin\omega_{nt}$-------- Eq.19

Now substituting these Eqs into Eq. and on simplification we get

$\omega_n^4 - (1+\eta)(\omega_{nl1}^2 + \omega_{nl2}^2) + (1+\eta)\omega_{nl1}^2\omega_{nl2}^2 = 0$----- Eq.20

$\omega_{nl1} = \sqrt{\frac{K_1}{m_1+m_2}}$-------------- Eq.21

$\omega_{nl2} = \sqrt{\frac{K_2}{m_2}}$------------ Eq.22

$\eta = \frac{m_1}{m_2}$--------- Eq.23

Now the two natural frequencies of the system for forced vibration condition can be obtained by considering the equations motion as

$m_1\ddot{Z}_1 + K_1Z_1 + K_2(Z_1 - Z_2) = 0$-------- Eq.24

$$m_2\ddot{Z}_2 + K_2(Z_2 - Z_1) = F_z sin\omega t \text{-------} \qquad \text{Eq.25}$$

By solving these equations the amplitude of vertical vibration can be obtained as

$$A_{Z1} = \frac{F_z\omega_{nl2}^2}{m_1\left[\omega^4-(1+\eta)(\omega_{nl1}^2+\omega_{nl2}^2)\omega^2+(1+\eta)\omega_{nl1}^2\omega_{nl2}^2\right]} \text{-------} \qquad \text{Eq.26}$$

$$A_{Z2} = \frac{F_z\left[(1+\eta)\omega_{nl}^2+\eta\omega_{nl2}^2-\omega^2\right]}{m_2\left[\omega^4-(1+\eta)(\omega_{nl1}^2+\omega_{nl2}^2)\omega^2+(1+\eta)\omega_{nl1}^2\omega_{nl2}^2\right]} \text{-------} \qquad \text{Eq.27}$$

Problem No. 1

Plan of deck slab with loading position is shown. A reinforced concrete frame with vertical loads at bvarious points are also shown. The details of these loads are

1 and 2 = 5t each, 3,4,5 and 6 = 2t each

$E_c = 3x10^6$ t/m^2 and unit weight of concrete = 2.24t/m^3.

Calculate the natural frequency of horizontal vibration in the longitudinal direction by treating the frame vas single degree freedom system.

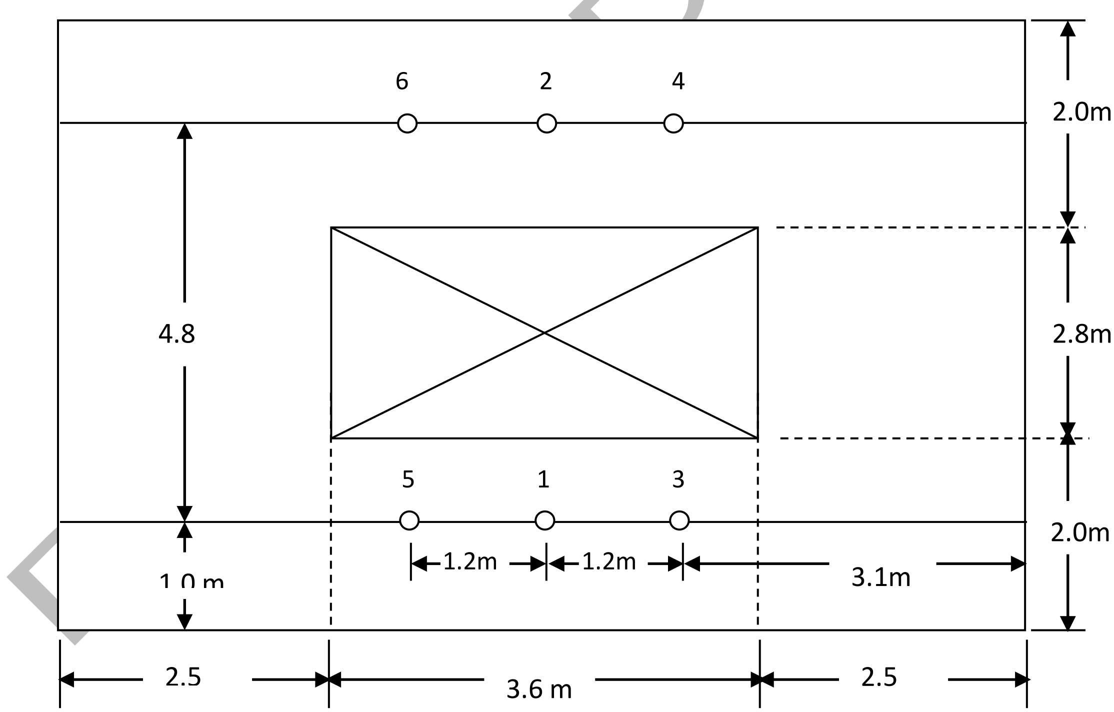


Fig. (a)Deck slab with loading position

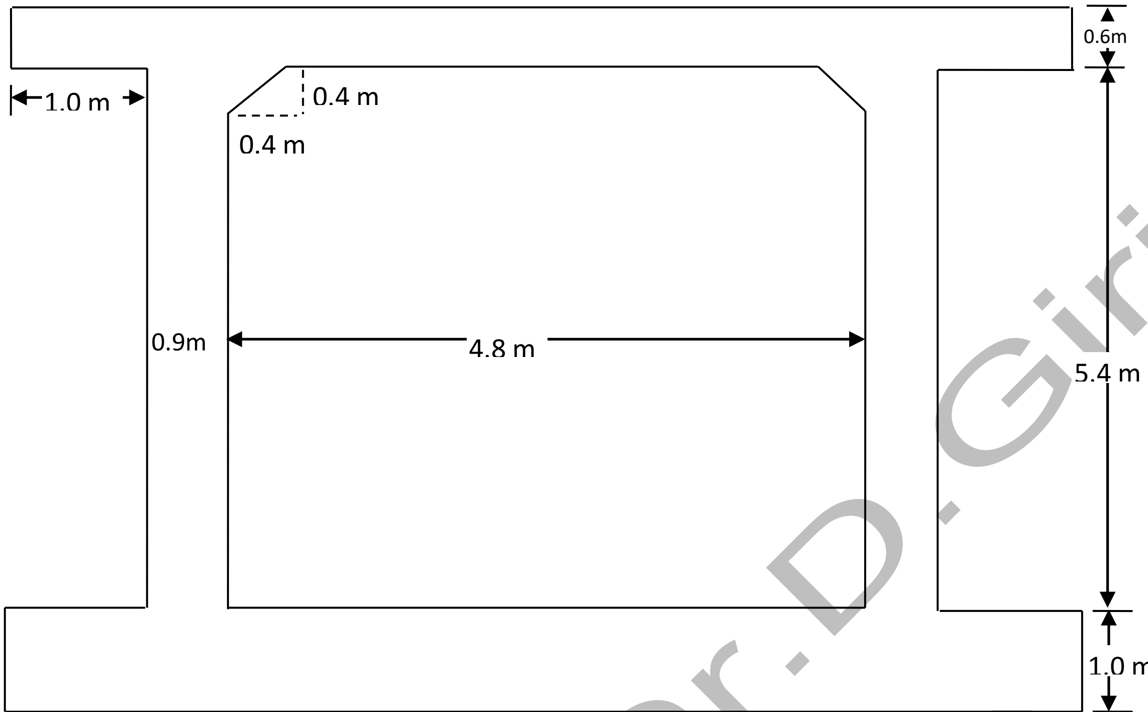


Fig. (b) longitudinal Section

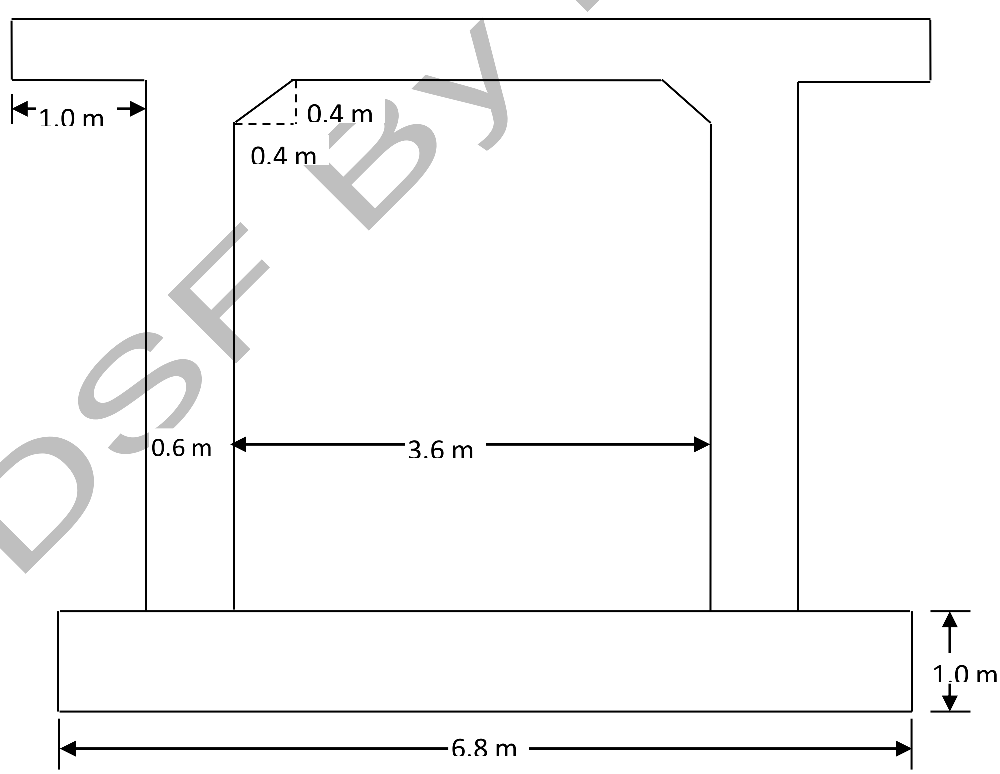


Fig (c) Transverse Section

PRACTICE PROBLEMS

Q.1 An unknown weight W is attached to the end of an unknown spring k and natural frequency of the system was found to be 90 cpm. If 1 kg weight is added to W, the natural frequency reduced to 75 cpm. Determine the unknown weight W and spring constant k

Q.2 A spring and dashpot are attached to a body weighing 140 N. The spring constant is 3.0 kN/m. The dashpot has a resistance of 0.75 N at a velocity of 0.06 m/s. Determine the following for free vibration:whether the system is over damped, under damped or critically damped.

Q.3 A counter rotating eccentric mass exciter is used to produce forced oscillation of a spring supported mass. By varying the speed of rotation, resonant amplitude of 5 mm was recorded. When the speed of rotation was increased considerably beyond the resonant frequency, the amplitude appeared to approach a constant value of 0.6 mm. Determine the damping factor of the system.

Q.4 An SDF system is excited by a sinusoidal force. At resonance the amplitude of displacement was measured to be 2 mm. At an exciting frequency of one-tenth of the natural frequency of the system, the displacement amplitude was measured to be 0.2 mm. Estimate the damping ratio of the system.

Q.5 A body weighing 600 N is suspended from a spring which deflects 12 mm under the load. It is subjected to a damping effect adjusted to a value 0.2 times that required for critical damping. Find the natural frequency of the un-damped and damped vibrations, and in the latter case, determine the ratio of successive amplitudes.

Q.6 In a cyclic plate load test on a plate of 0.60 m × 0.60 m size settles 0.65 mm under a pressure of 20 kN/m^2. On unloading observed plate settlement was 0.60 mm. Determine the value of coefficient of elastic uniform compression of the soil.

Q.7 A mass attached to a spring of 5 N/mm has a viscous damping device. When the mass was displaced and released, the period of vibration was found to be 2 s and ratio of the consecutive amplitudes was 10/3. Determine the damping factor and natural frequency of the system. Determine also the amplitude of motion when a force of 3 sin 4t N acts on the system.

Q.8 A machine of weight 17.5 kN and operating frequency 400 rpm has to be installed on ground which has properties G= 40 MN/m^2 , γ_s=20 kN/m^3 and μ=0.3. The machine contains a unbalanced rotating parts which produce an eccentric moment of 18 Nm in vertical direction. The permissible amplitude of vibration for the system is 0.2 mm and the equivalent diameter of the foundation required to install the machine is 1.6 m. Design the foundation.

Q.9 Determine the stiffness of the absorber to be kept between a reciprocating machine and foundation to bring the vibration amplitude to less than 0.02 mm. The weight of the machine is

25 kN. It produces an unbalanced force of 4 Kn in the vertical direction, when operated at speed of 750 rpm. Shear modulus of foundation soil G=2.5x10^4kN/m^2 and Poisson's ratio 0.3.

Q.10 **A** foundation is subjected to a constant force type vibration. Given that the total weight of machine and foundation block is 1500 N. Unit of foundation soil γ= 15 kN/m^3, Shear modulus and poison's ratio are 15 MN/m^2 and 0.4 respectively. The amplitude of vibrating force F_0 = 1500N.Operating frequency of machine is 80 cpm. The size of foundation block is 10m long and 3 m wide. Determine

i) The resonance frequency and check for type of mode of vibration
ii) Amplitude of vibration at resonance.

Q.11 A machine is supported by four pre-stressed concrete piles driven into a bed rock. The length of each pile is 80 ft and they are 12x12in^2 in cross section. The weight of the machine and foundation is 300x10^3 lbs, Unit of concrete is 150 lb/ft^3. Young's modulus 3.5x10^6 lb/ft^3. Determine the natural frequency of pile foundation system.

Q.12 Following are the field standard penetration test number (N) in a deposit of sand. Ground water table is encountered at a depth of 3m below the ground surface. Soil properties of sand are, dry unit weight 18.5kN/m^3 and saturated unit weight 20.6 kN/m^3. Determine for an earth quake magnitude of 7.5, whether liquefaction will occur? Assume ground acceleration as a_{max}= 0.15g.

Depth(m)	1.5	3.0	4.5	6.0	7.5	9.0	10.5
N	6	8	10	14	16	20	20

Q.13 A horizontal piston type compressor is placed on a block type foundation as shown In Fig.1
The operating frequency is 600 cpm. The amplitude of the horizontal unbalanced force of compression is 30kN and it produces a rocking motion of the foundation about point O. The mass moment of inertia of the compressor assembly about the axis BOB' is 16x10^5kg.m^2. Determine

i) The resonance frequency
ii) The amplitude of rocking vibration at resonance.

Q.14 A concrete bock foundation of a machine has the following dimension, L=4m, B=3m and height H=1.5 m. The foundation is subjected to a sinusoidal horizontal force from the machine having amplitude of 10 kN at a height of 2.0 m from the base of the foundation as shown in Fig.2.

The soil supporting the foundation is sandy clay with G=30,000 kN/m^2, μ=0.25and unit weight γ=17 kN/m^3. Determine

i) The resonance frequency for sliding and rocking mode of vibration of the foundation (Independent mode analysis)

ii) Total horizontal displacement at the top of the foundation block.

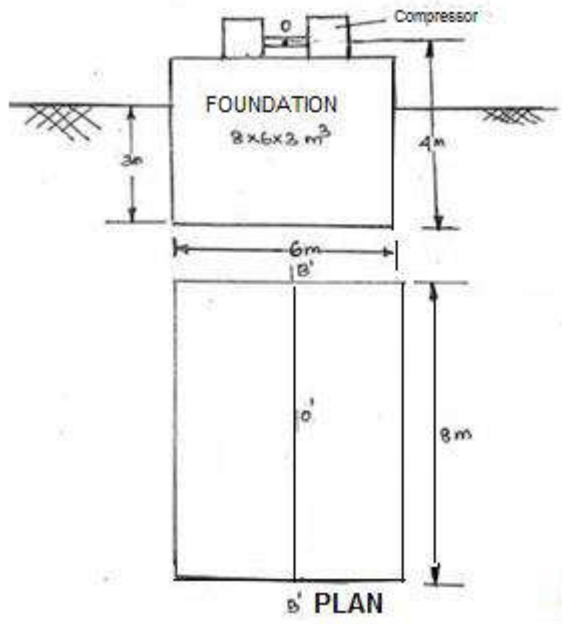


Fig.1

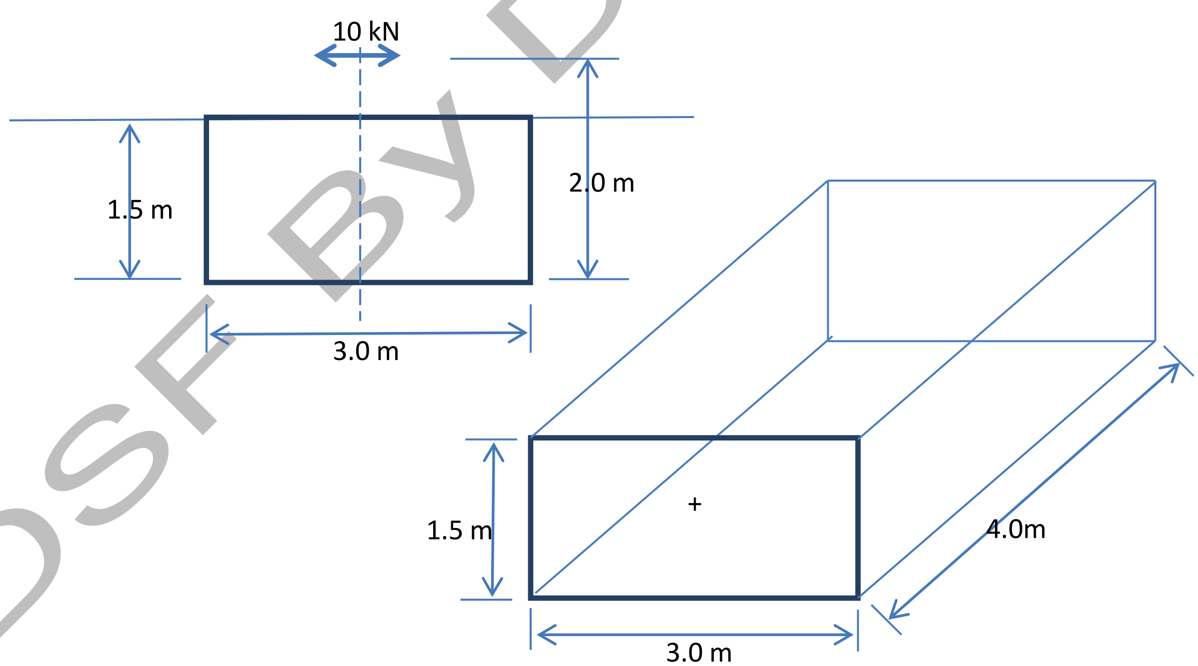


Fig. 2 Dimension of Block Foundation

Q.15 A machine is supported by four pre-stressed concrete piles driven into a bed rock. The length of each pile is 8o ft long and is 12x12 in^2 in cross section. The weight of the machine and foundation is 300x10^{3}lbs, unit weight 150 lb/ft^3. The Young's modulus is 3.5x10^{6}lb/in^2. Determine the natural frequency of pile foundation system.

MULTIPLE CHOICE QUESTIONS

Q.1 A spring and dashpot are attached to a body weighing 200 N. The spring constant is 2.0 kN/m. The dashpot has a resistance of 0.65 N at a velocity of 0.05 m/s. If the system is set to a motion, the system is:

(a) Over damped (b) Under damped (c) Critically damped (d) None of above

Q.2 A counter rotating eccentric mass exciter is used to produce forced oscillation of a spring supported mass. By varying the speed of rotation, resonant amplitude of 6 mm was recorded. When the speed of rotation was increased considerably beyond the resonant frequency, the amplitude appeared to approach a constant value of 0.6 mm. The damping factor (in %) of the system is:

(a) 2 (b) 8 (c) 9 (d) 5

Q.3 An SDF system is excited by a sinusoidal force. At resonance the amplitude of displacement was measured to be 4 mm. At an exciting frequency of one-tenth of the natural frequency of the system, the displacement amplitude was measured to be 0.3 mm. The damping factor of the system (in %) is:

(a) 3.75 (b) 3.05 (c) 1.25 (d) 5.81

Q.4 A body weighing 600 N is suspended from a spring which deflects 10 mm under the load. It is subjected to a damping effect adjusted to a value 0.4 times that required for critical damping. The un-damped natural frequency of the vibrations (in radian per second) is:

(a) 36.33 (b) 28.86 (c) 31.32 (d) 41.81

Q.5 The damped natural frequency of the vibrations (in radian per second) of the system in Q 4 is:

(a) 33.33 (b) 35.16 (c) 28.70 (d) 20.11

Q.6 The ratio of successive peak amplitudes of the vibrations for the system of Q 4 is:

(a) 7.33 (b) 12.34 (c) 18.25 (d) 16.58

Q.7 A mass attached to a spring of 9 N/mm has a viscous damping device. When the mass was displaced and released, ratio of the consecutive amplitudes was 11/6. The damping factor of the system (in %) is:

(a) 7.20 (b) 5.67 (c) 3.93 (d) 9.64

Q.8 As per Indian standard code IS 2974, IV, the permissible amplitude for Rotating machine speed >1500 rpm is

a) 0.4mm b) 0.2mm c) 0.6mm d) 0.8 mm

Q.9 If ω and ω_n are operating and natural frequency of system respectively, which one is correct for the design condition of dynamically loaded foundation

a) $0.5w \leq w_n \leq 1.5w$ b) $0.5w \geq w_n \geq 1.5w$ c) $1.5w \geq w_n \leq 0.25w$ d) None of above

Q.10 An unknown weight W is attached to the end of an unknown spring k and natural frequency of the system was found to be 92cpm. If 1 kg weight is added to W, the natural frequency reduced to 80 cpm. Find the unknown weight

a)5.97kg b)4.07 kg c) 3.11 kg d)3.97 kg

Q.11 The value of equivalent radius of circular footing for rocking mode of vibration of a rectangular foundation of size 4mx6m is

(a) 2.8 m (b) 3.1m (c) 3.5m (d) None of above

Q.12 Damping factor for sliding mode of vibration is given as

(a) $\frac{0.245}{B_x}$ (b) $\frac{0.425}{B_x}$ (c) $\frac{0.2875}{B_x}$ (d) $\frac{0.215}{B_x}$

Q.13 Spring constant for torsional mode of excitation is expressed as

(a) $\frac{4Gr_0}{1-\mu}$ (b) $\frac{32(1-\mu)Gr_0}{7-8\mu}$ (c) $\frac{16}{3}Gr_0^3$ (d) None of above

Q.14 Modified mass ratio as per Lysmer Analysis for a block type foundation of size 4x6 m^2 subjected to vertical mode of vibration due to total weight of 150 kg resting on foundation soil having unit weight and poison's ratio as 17.2 kN/m^3 and 0.25 respectively is

(a) 0.764 (b) 0.205 (c) 1.204 (d) 0.415

Q.15 The magnification factor for mode of vibration in Q 4 is:

(a) 1.208 (b) 1.176 (c) 0.987 (d) 2.316

Q.16 According to the Richart, the maximum operating frequency up to which no noticeable amplitude of vibration can be identified by person

(a) 500 rpm (b) 750 rpm (c) 1000 rpm (d) 2000 rpm

Q.17 According the IS 5249, the relation between Coefficient of uniform elastic compression for various mode of excitation can be given as

(a) C_U=1.73 C_τ, C_φ=2C_U (b)C_U=2C_τ, C_φ=1.75C_U (c)C_U=2.25C_τ, C_φ=2.2C_U (d)None of above

Q.18 The value of coefficient of elastic uniform compression C_u of soil obtained from block vibration test of contact area of 10 m^2 is 12 kN/m^2. The of C_u for a base area of foundation of 12 m^2 is

(a)15 kN/m^2 (b) 10.65 kN/m^2 (c) 12 kN/m^2 (d) None of above

Q.19 Coefficients for natural frequency of end bearing pile of length 12m resting on soil having shear velocity of 350 m/s is 0.43. The natural frequency of vibration is

(a) 14.56 rpm (b) 18.98 rpm (c) 12.54 rpm (d) None of above

Q.20 The efficiency of absorber η used in foundation for reciprocating engine when maximum displacement is 0.034 mm and the permissible displacement of 0.02 mm is

a) 65.2% (b) 58.82% (c) 76.3% (d) None of above