LECTURE NOTE

ON

ADVANCED CONCRETE STRUCTURES

COURSE CODE: CE 15031: 3.1.0 (CR 04)

Seventh Semester, B Tech, Civil Engineering

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SYLLABUS

CE 15031: ADVANCED CONCRETE STRUCTURES (3-1-0) CR-04

Module-I (10 Hours)
Introduction to EQ Engineering: Cyclic behavior of concrete and reinforcement, significance of ductility, ductility of beam, design and detailing for ductility, simple problems based on above concept, Computation of earthquake forces on building frame using Seismic Coefficient Method as per IS 1893-2002

Module-II (10 Hours)
Design of Foundations: Combined Footing: Design of Rectangular and Trapezoidal footing, Design of Raft Foundation, Design of Pile Foundation

Module-III (10 Hours)
Retaining walls: Forces acting on retaining wall, Stability requirement,
Design of Cantilever and Counterfort Retaining walls

Module-IV (10 Hours)
Design of Water tanks: Design requirements, Design of tanks on ground and underground

Introduction to Prestressed Concrete: Prestressing methods, Analysis of prestressing systems and losses

Text Book:

1. Advanced Concrete Structure Design by P. C. Verghese, Prentice Hall of India
2. Limit state design- A K Jain, Nem Chand and Brothers

Reference Books:

1. Limit state design of reinforced concrete by B.C. Punmia, AK Jain and A.K. Jain, Laxmi Publishers New Delhi 2007
2. A K Chropra,Dynamics of Structures: Theory and Applications to Earthquake Engineering, Prentice Hall of India

ADVANCED CONCRETE STRUCTURES (CE 15031)

Module-I

Module I Syllabus

***Introduction to EQ Engineering**: Cyclic behavior of concrete and reinforcement, significance of ductility, ductility of beam*

Design and detailing for ductility, Problems based on above concept

Computation of earthquake forces on building frame using Seismic Coefficient Method as per IS 1893-2002

Subject to Revision

Advanced Concrete Structures

INTRODUCTION TO EARTHQUAKE ENGINEERING

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EQ ENGINEERING

Design lateral force →

Horizontal seismic force used to design a structure.

Design seismic base shear (V_B) →

Total design lateral force at base of a structure.

storey shear → (V_i) →

sum of design lateral forces at all levels above the storey under consideration.

Design basis EQ (DBE) →

EQ expected to occur at least once during design life of structure.

max considered EQ (MCE) →

most severe EQ effect as per IS 1893.

Responce spectrum →

- man responce of idealized SDOF system (with a Time period & damping) during EQ.
- man responce is plotted against undamped natural period (T) & various damping values.

Design responce spectrum →

- Average smoothened plot of maximum responce (function of frequency/time period) for a specified damping ratio of EQ excitation at base of a SDOF system.
- Responce can be acceleration/velocity/displacement.

Soft storey →

storey where lateral stiffness is less than 70%.

Weak storey →

(A

(03.12.08) Earthquake [SEISMOLOGY = SEISMOS + LOGOS]
SEISMOS → EQ, LOGOS → science

EQ → A shaking motion of earth surface caused by a sudden stress relieving rupture inside earth crust.

* magnitude of this shaking force is recorded by seismographs

* The most common seismograph is Ritcher scale which uses a non linear scale from one to ten.

EQ Period → ~~[illegible]~~.

* Period during which a part of earth is subjected to EQ shock without any long Pause.

* SEISMOLOGY developed from a need to understand the internal structure and behavior of earth as they relate to EQ phenomenon.

* Seismic waves
  - body waves
    - P wave
    - S wave
      - SV wave
      - SH wave
  - surface waves
    - Rayleigh wave
    - Love wave

* Body waves - travel through interior of earth.
  - → P-waves (primary, compressional, longitudinal waves) involves successive compression & rarefaction of materials through which they pass.
  - → S-waves

P-waves:
* motion of particles in P waves is ~~They travel in~~ parallel to direction of travel.
* Like sound waves, they travel through SOLIDS & FLUIDS.

S-waves:
* secondary, shear or transverse waves.
* cause shearing deformation.
* motion of individual particles is ⊥r to direction of S-wave travel.

* S-wave
  - → SV wave - vertical plain movement
  - → SH wave - Horizontal plain movement.

* P-waves travel faster than S-waves.

* Fluid have no shearing resistance/stiffness & so can't sustain S-waves.

* SURFACE WAVES → Result from interaction betn body waves & the surface of earth called surficial layers of earth.

* They travel along earth surface with amplitude that decreases exponentially with depth.

* Surface waves are more prominant at distances farther from epicenter (source of EQ)

A) (B

STIFFNESS →

* Force required for unit displacement.

Axial displacement, $\Delta = \frac{Fl}{AE}$.

Axial stiffness $\Rightarrow \frac{AE}{l} = \frac{F}{\Delta}$

* Capacity of a structural assembly or individual member to resist load without excessive deflection.

Stiff frame / Redundant frame →

A frame which are having more members than are required for it to be a perfect frame.

Perfect frame →

~~Individual frame~~ structural frame which is stable under loads imposed upon it from any direction and becomes unstable when one of its members is removed / or one of its fixed end becomes hinged.

Flexibility → Displacement caused by a unit force.

$\Delta = \frac{Fl}{AE}$, flexibility $= \frac{\Delta}{F} = \frac{l}{AE}$

Ductility →

capacity to undergo large inelastic deformation without significant loss of strength or stiffness.

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* Important surface waves
  - Rayleigh waves
  - Love waves.
* Rayleigh waves formed by interaction betn P & SV waves with earth surface.
* They create both vertical & horizontal particle motion
* They are similar to wave produced if a stone is thrown to pond.
* Love waves result from interaction betn SH waves with soft surficial layer
* Love waves have no vertical component of motion.

EARTHQUAKE RESISTANT DESIGN 27·11·2008 (Thursday)

* Design of EQ resistant structures should aim at

providing ~~adequate~~ appropriate Dynamic + structural characteristics so that acceptable response level results under design EQ.

* Designer can exercise some degree of control on

magnitude and distribution of stiffness and mass, relative strength of members and their ductility to achieve the desired results.

* Performance criteria in EQ codes requires that the structure should be able to resist

- EQ of minor intensity without damage.
  (structure is expected to resist such frequent minor shocks within its elastic range of stresses.)
- EQ of moderate intensity with minor structural + some non-structural damage.
  (It is believed that with proper design + construction, structural damage due to EQ will be limited to repairable damage)
- major EQ without collapse but severe structural damage is expected.

Seismic Design criteria

EQ	Desired Behavior	controlling parameter
1) minor	No damage to non structural components	Deflection is controlled by providing stiffness.
2) moderate	No significant structural damage. Minor cracks in beams + columns. Response should be elastic.	Yield or permanent damage of members can be avoided by providing strength.
3) major	No collapse of system	Energy can be absorbed by allowing structure to enter into inelastic range (deformation) by providing Ductility

* For satisfactory performance in EQ, structure must have STRENGTH & DUCTILITY.

~~* If each element in a structure have~~

* If elastic strength of each element of a structure > max. load coming to it, then no significant structural damage occurs.

* To achieve it, structure should be designed for lateral forces several times more than the probable EQ force specified in codes.

* ASSUME →

In severe EQ, some resisting elements will be loaded to their full strength.

* If the elements are brittle, they will fail & will not take any load.
* If " " " ductile, they can resist lateral forces nearly upto their full strength & after which they yield throwing only the excess load to remaining elements.

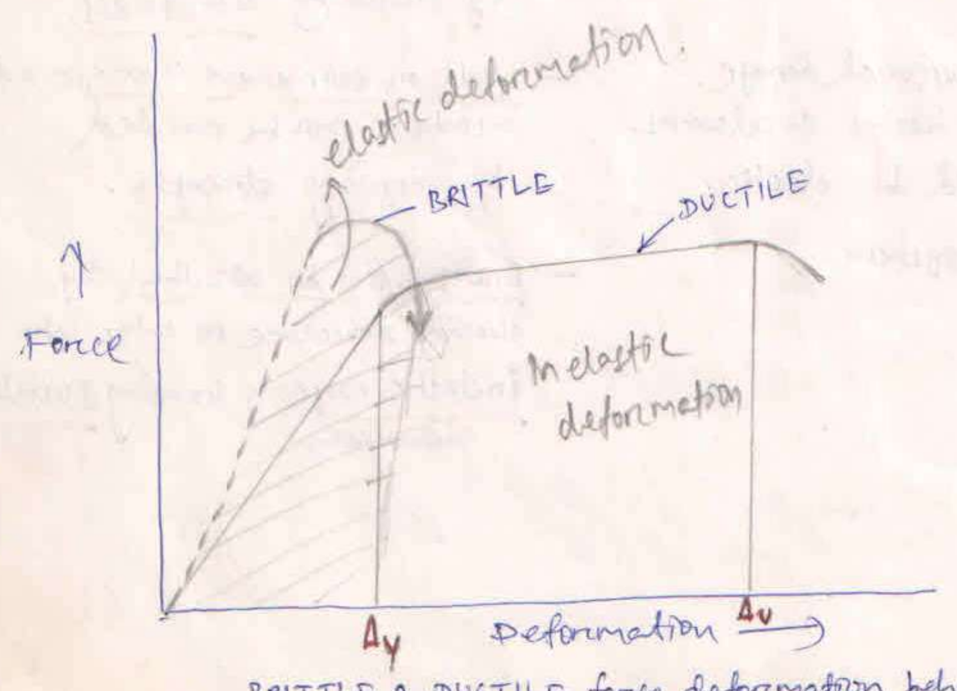
* A ductile material can undergo large strains while resisting load.

* In RCC, Ductility ⇒

Ability to sustain significant inelastic deformations prior to collapse.

Brittle material →

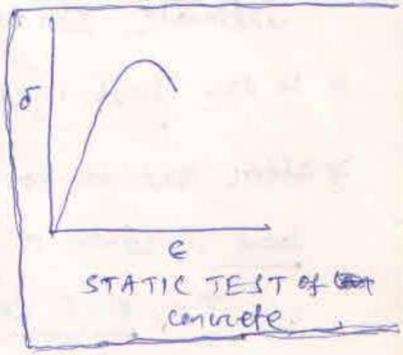
That fails suddenly after attaining the maximum load.



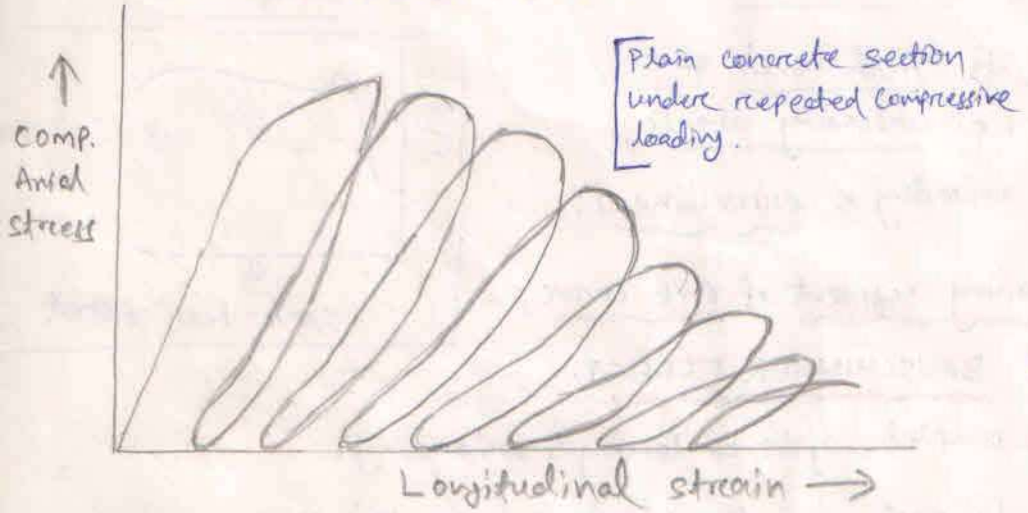
BRITTLE & DUCTILE force deformation behavior.

CYCLIC BEHAVIOR OF CONCRETE (PCC) →

- Plain concrete is a brittle material.
- During first cycle, $\sigma \sim \epsilon$ curve is same as that in static test.
- If specimen is unloaded & reloaded in compression, the curve obtained is as below.



STATIC TEST of ~~con~~ concrete



Plain concrete section under repeated compressive loading.

EQ Design
S - Stiffness
S - Strength
D - Ductility

- It is clear from above figure that

  slope of stress∼strain curve & maximum attainable stress decrease with number of cycles.

- So, stress∼strain relationship for plain concrete subjected to ~~repeated loading~~ repeated compressive load depends on the number of cycles of load.
- In PCC, the decrease in strength & stiffness is due to Crack Formation.
- The comp. strength of concrete depends on rate of loading.
- ~~[illegible]~~ In PCC, compressive strength $\propto$ rate of loading,

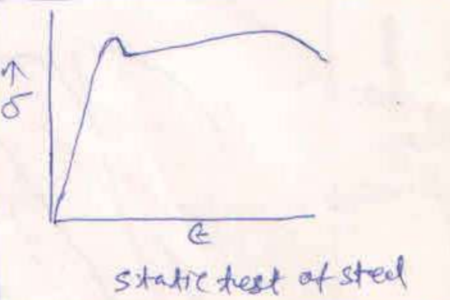
  but, strain at maximum stress decreases.

- PCC can't be subjected to repeated tensile load

  since its tensile strength is zero.

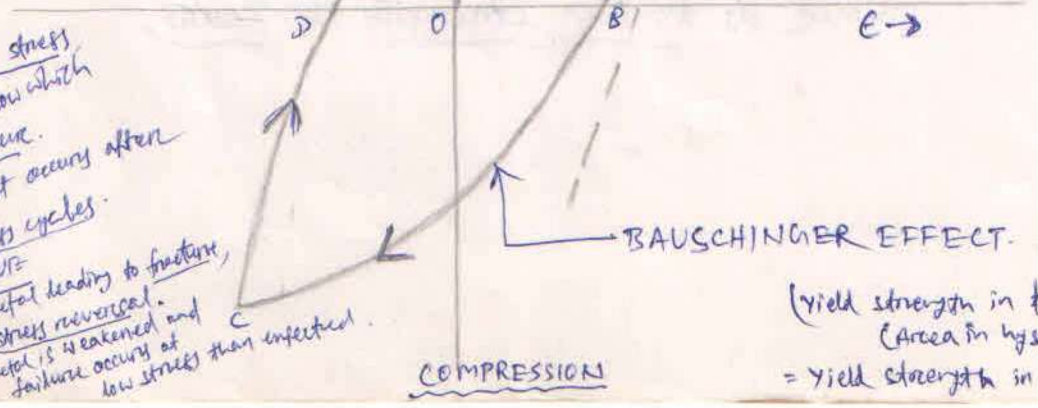
CYCLIC BEHAVIOUR OF REINFORCEMENT →

- Reinforcement has more ductility than PCC.
- Ultimate strain of mild steel is of the order 25%., where ultimate strain of PCC is of the order 0.3%.
- In the first cycle, steel has σ – ε curve similar to that in static test.
- After specimen has reached its yield level & load direction is reversed i.e. unloading starts, the σ~ε curve for unloading is curvilinear.



static test of steel

- The curvature in unloading segment of σ~ε curve, is called BAUSCHINGER EFFECT.
- Figure below shows one complete cycle of loading & unloading.
- σ~ε curve for one complete cycle of loading & unloading is called HYSTERESIS LOOP.
- Area within Hysteresis loop is the energy absorbed by steel in one cycle.
- In next cycles, same path is repeated.
- σ – ε curve for mild steel subjected to repeated reversed loading is independent of number of cycles until steel buckles/fails in fatigue.
- Also, same Hysteresis loop is obtained for a specimen
  - which is first loaded in tension followed by compression or
  - which is first loaded in compression " " tension.
- Yield strength of steel is affected by Rate of loading.



HYSTERESIS BEHAVIOR OF Mild steel reinforcement

AB = Unloading (Tension)
BC = Reloading (Comp)
CD = Unloading (Comp)
DA = Reloading (Tension)

(yield strength in tension) – (Area in hysteresis loop) = Yield strength in compression.

Endurance limit

In fatigue testing, it refers to maximum stress for any material below which fractures will not occur. for steel, this limit occurs after 6 to 10 million stress cycles.

FATIGUE

A phenomenon of metal leading to fracture, due to repeated stress reversal. so metal is weakened and failure occurs at low stress than expected.

4(A)

BAUSCHINGER EFFECT

* It refers to a property of material where material's stress-strain characteristics changes because of microscopic stress distribution of material.

Eg - increase in tensile yield strength at an expense of compressive yield strength..

* It is associated with conditions where
✓ yield strength of metal decreases when direction of strain is changed

* The basic mechanism is related to dislocation of structure in cold worked metal.

→ As deformation occurs, dislocation will accumulate at barriers and produce dislocation pile up.

a mechanism to explain this effect -

① when strain dirn is reversed,
dislocation of opposite sign can be produced from same source that produced slip causing dislocations in initial direction.

→ Dislocation in opposite sign can attract each other.

- since strain hardening is related to increased dislocation density,
reducing no of dislocations reduces strength.

→ conclusion → yield strength for straining in opposite direction is less than it would be if strain had continued in initial direction.

CYCLIC BEHAVIOUR OF RCC →

- * PCC con be subjected to repeated compressive loading cycles & not to repeated tensile loading cycle due to its poor tensile strength.
- * steel can be subjected to repeated reversible tensile & compressive loading cycles & shows stable hysteresis loop.
- * so cyclic behaviour of RCC ~~beam~~ is highly improved due to presence of steel.

Eg → RCC cantilever beam subjected to reversed cyclic loading?

- * steel is present on both faces, because
  - one face – tension, in first half of loading cycle and
  - other face – tension, in next half of loading cycle..

* STIFFNESS → slope of load-deflection curve.

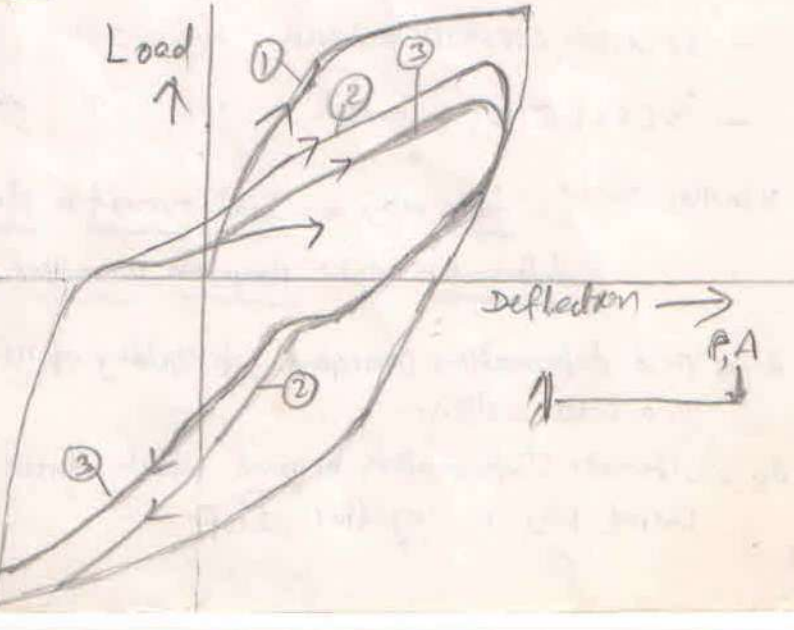
* It is clear from figure that, stiffness of beam $\alpha \frac{1}{\text{no. of cycles}}$ (stiffness decrease with no. of cycles)

⇒ As no of cycles increases, the stiffness of RCC beam decreases.

* This effect of RCC beam & column is called STIFFNESS DEGRADATION effect.

* The non linear behaviour of RCC is affected by
  - → degree of cracking in concrete,
  - → strain hardening & Bauschinger effect of steel,
  - → effectiveness of bond & anchorage betn. concrete & steel.
  - → Presence of high shear.

* Since stiffness degradation starts right after first cycle & progress rapidly in RCC, it is required to improve the capability of RCC to sustain inelastic deformation to avoid collapse.



[Hysteresis behaviour of cantilever beam

[Load-deflection behaviour/curve for a cantilever RCC beam

SIGNIFICANCE OF DUCTILITY → capacity to undergo large inelastic deformation without significant loss of strength or stiffness is called DUCTILITY.

* When a ductile structure is subjected to overloading, it deforms inelastically, and redistributes the excess load to elastic part of structure.
* This concept is utilized in several ways like.
  - → If structure is ductile, it can be expected to absorb unexpected overloads, load reversal, impact & structural movement due to foundation settlement and volume changes.

  Above items are generally ignored in analysis & design, but are assumed to be accepted by the ductility of the structure.

  - → If structure is ductile, its occupants get sufficient warning of the failure thus reducing possibility of loss of life even in collapse.
  - → Limit state design assumes all critical sections in structure will reach their max. capacities at design load for the structure.
    - so all joints & splices must be able to resist forces & deformations corresponding to yielding of reinforcement.

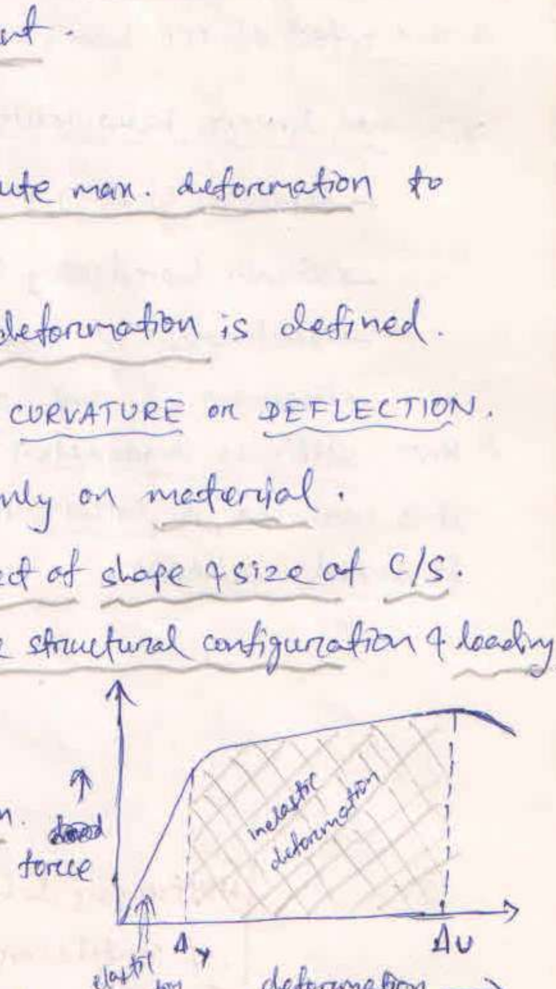
DUCTILITY →

* Structural ductility is defined as ratio of absolute max. deformation to corresponding yield deformation.
* Ductility is understood after method of measuring deformation is defined.
* Deformation is measured w.r.t STRAINS, ROTATIONS, CURVATURE or DEFLECTION.

— STRAIN based ductility definition depends only on material.

— ROTATION, CURVATURE based " " includes effect of shape & size of C/S.

— DEFLECTION based " " includes entire structural configuration & loading.



ductile material

* In this curve, force may be load, moment or stress.

  deformation may be elongation, curvature, rotation/strain.

Δ_y = Yield deformation corresponding to yielding of reinforcement, in a cross section.

Δ_u = Ultimate deformation beyond which force deformation curve has a negative slope

Ductility $\mu = \frac{A_U}{A_Y}$ w.r.t displacement. ——— ①

$= \frac{\phi_U}{\phi_Y}$ w.r.t. curvature ——— ②

$= \frac{\theta_U}{\theta_Y}$ w.r.t. ratation. ——— ③

$= \frac{\epsilon_U}{\epsilon_Y}$ w.r.t strain. ——— ④

DUCTILITY OF BEAM ⟹

- In RCC beam, ductility is defined w.r.t behaviour of individual c/s, or, w.r.t behaviour of entire beam.

- As behaviour of c/s is better to define & calculate, so ductility is defined w.r.t c/s.

⟶ CURVATURE DUCTILITY ⟶

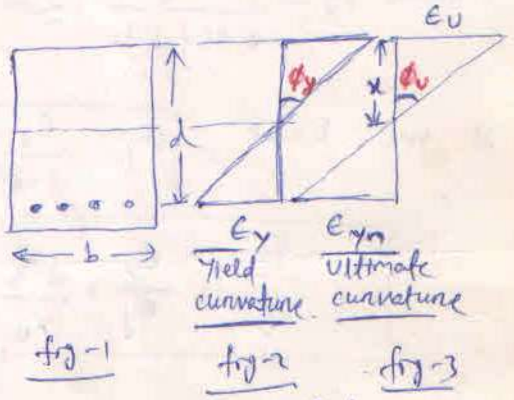
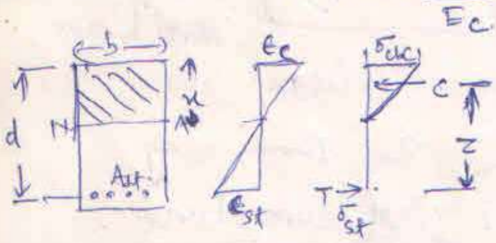


fig-1 fig-2 fig-3

- Let us consider a singly reinforced concrete beam.

modular ratio, $m = \frac{E_s}{E_c}$.

Strain in concrete, $\epsilon_c = \frac{\sigma_{cbc}}{E_c}$, $\epsilon_{st} = \frac{\sigma_{st}}{E_s} = \frac{\sigma_{st}}{mE_c}$.



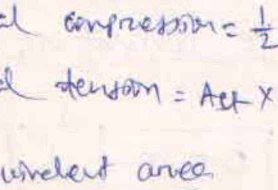
z = lever arm, distance betn CG of comp zone to CG of tension.

x = NA depth

Total compression $= \frac{1}{2}\sigma_{cbc} \times x \times b = \frac{bx}{2} \times \sigma_{cbc}$,

Total tension $= A_{st} \times \sigma_{st}$

Equivalent area



To get position of N.A., take moment of both area about N.A.

$$bx \cdot \frac{x}{2} = mA_{st}(d-x) \Rightarrow \frac{bx^2}{2} - mA_{st}(d-x) = 0.$$

$$\Rightarrow x^2 - \frac{2mA_{st}}{b}(d-x) = 0 \Rightarrow x^2 - \frac{2mA_{st}d}{b} + \frac{2xmA_{st}}{b} = 0.$$

$$\Rightarrow x^2 + 2m\left(\frac{A_{st}}{bd}\right)(xd) - 2m\left(\frac{A_{st}}{bd}\right)d^2 = 0 \Rightarrow x^2 + 2mpxd - 2mpd^2 = 0$$

$$\Rightarrow x = \frac{-2mpd \pm \sqrt{4m^2p^2d^2 + 8mpd^2}}{2} \Rightarrow x = -mpd \pm \sqrt{m^2p^2d^2 + 2mpd^2}$$

$$\Rightarrow \boxed{x = \left(-mp + \sqrt{m^2p^2 + 2mp}\right)d} \text{ ——— ⑤}$$

* Yield curvature can be found using elastic theory,

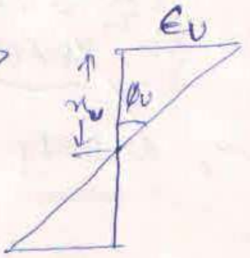
$$\boxed{\phi_y = \frac{\epsilon_y}{d-x}} \text{ - - - - - - - - - - - - - - } ⑥$$

x = is given by eq ⑤ where

ϵ_y = yield strain of tensile steel = $\frac{\sigma_y}{E_s}$.

Tension steel ration $p = \frac{A_{st}}{bd}$.

* Ultimate curvature can be computed as:- → → - →



$$\boxed{\phi_U = \frac{\epsilon_U}{x_U}} \text{ - - - - - - - - } ⑦$$

ϵ_U = ultimate strain at crushing of concrete = 0.0035.

(ultimate curvature).

~~Total~~ $x_U = \frac{0.87 f_y A_{st}}{0.36 f_{ck} b} \leq x_{U,max}$ - - - - - ⑧

* we have $\boxed{\phi_y = \frac{\epsilon_y}{d-x}}$, $\boxed{\phi_U = \frac{\epsilon_U}{x_U}}$

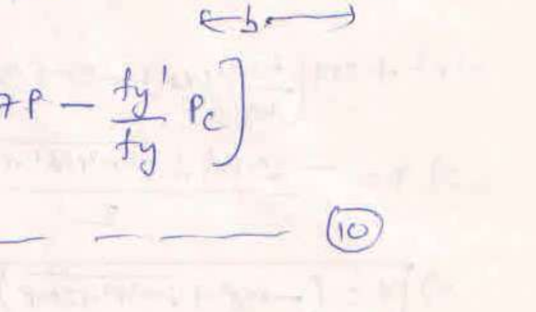
$$\therefore \boxed{\mu = \frac{\phi_U}{\phi_y} = \frac{\epsilon_U}{\epsilon_y} \times \frac{d-x}{x_U}} \Rightarrow \boxed{\mu = \frac{\epsilon_U}{(\epsilon_y/E_s)}\left[\frac{d(1+mp-\sqrt{m^2p^2+2mp})}{x_U}\right]} \text{ - - } ⑨$$

→ Ductility for singly reinforced ~~steel~~ beam.

(9,a)

* Ductility for doubly reinforced beam can be found in the same way.
* Additional comp. steel has relatively little effect on its yield curvature.
* But comp. steel has great effect, to increase high the ultimate curvature

Depth of NA →



$$\boxed{0.36 f_{ck} b x_U + f_y' A_{sc} = 0.87 f_y A_t}$$

$$0.36 f_{ck} b x_U = 0.87 f_y A_t - f_y' A_{sc}$$

$$\Rightarrow x_U = \frac{1}{0.36 f_{ck}}\left[\frac{0.87 f_y A_t}{b} - f_y' \frac{A_{sc}}{b}\right]$$

$$= \frac{1}{0.36 f_{ck}}\left[0.87 f_y \frac{pd}{\cancel{100}} - f_y' p_c d\right] = \frac{f_y d}{0.36 f_{ck}}\left[0.87p - \frac{f_y'}{f_y} p_c\right]$$

$$\Rightarrow \boxed{\frac{x_U}{d} = \left[0.87p - \frac{f_y'}{f_y} p_c\right]\frac{f_y}{0.36 f_{ck}}} \text{ - - - - - - - } ⑩$$

f_y' = stress in compression steel.

$p = \frac{A_t}{bd}$ = tension steel ration

$p_c = \frac{A_{sc}}{bd}$ = comp. steel ration

If, ~~if~~, $f_y' = 0.87 f_y$, then from eq. 10

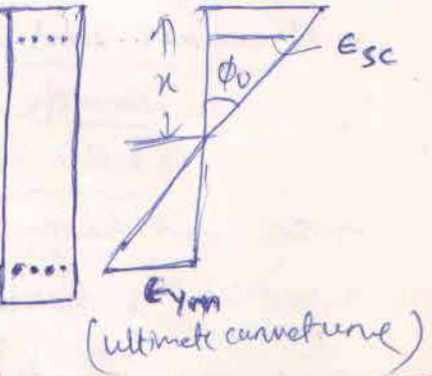
$$\Rightarrow \boxed{\frac{x_u}{d} = (p - p_c)\frac{0.87 f_y}{0.36 f_{ck}}} \leq x_{max} \text{ ------------ } ⑪$$

$$\boxed{\mu \propto \frac{1}{(x_u/d)}}$$

* From figure given →

$$\boxed{\frac{x_u}{d - x_u} = \frac{\epsilon_u}{\epsilon_{ym}}} \Rightarrow \frac{d}{x_u} = \frac{\epsilon_{ym}}{\epsilon_u} + 1 = \frac{\epsilon_{ym} + \epsilon_u}{\epsilon_u}$$

$$\Rightarrow \boxed{\frac{x_u}{d} = \frac{\epsilon_u}{\epsilon_u + \epsilon_{ym}}} \text{ - - - - - - - - } ⑫$$



ϵ_{ym} = max. strain in ~~steel~~ steel = $\mu_s \epsilon_y \Rightarrow \boxed{\mu_s = \frac{\epsilon_{ym}}{\epsilon_y}}$

μ_s = strain ductility.

* Putting eq ⑫ in eq ⑪

$$p - p_c \leq \left(\frac{\epsilon_u}{\epsilon_u + \epsilon_{ym}}\right)\frac{0.36 f_{ck}}{0.87 f_y}.$$

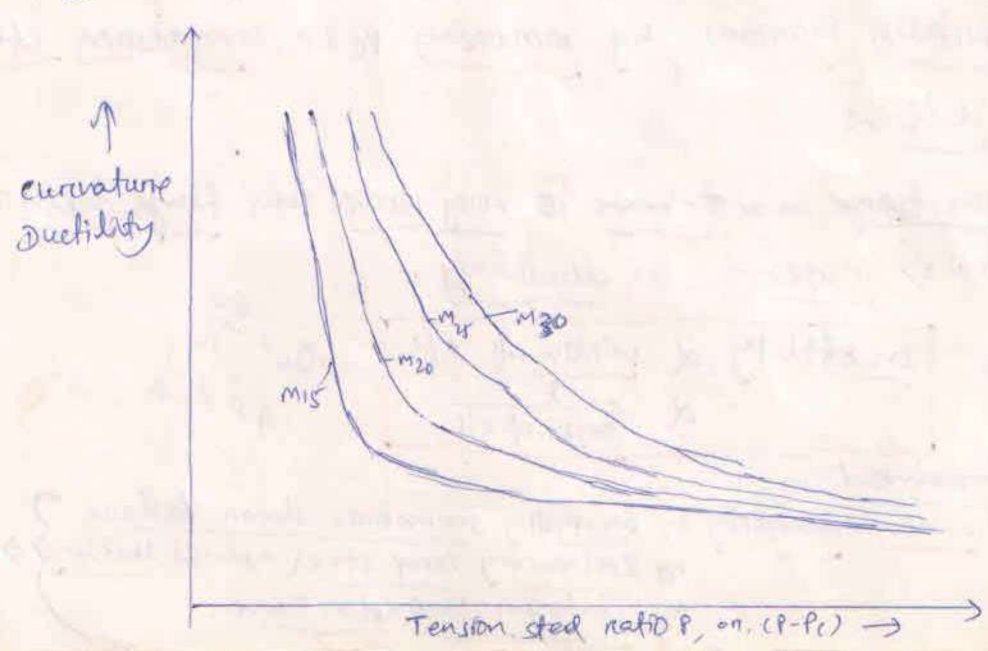
* Ductility can be obtained by using eq. like

$$\boxed{\begin{array}{l} \mu = \dfrac{\epsilon_u}{(\sigma_y/E_s)}\left[\dfrac{1 + mp - \sqrt{m^2p^2 + 2mp}}{(x_u/d)}\right] \text{ - singly RC} \text{ --- } ⑨ \\ \dfrac{x}{d} = (p - p_c)\dfrac{0.87 f_y}{0.36 f_{ck}} \text{ - doubly RC --- } ⑪ \end{array}}$$

OR or

$$\boxed{\begin{array}{l} \mu = \dfrac{\epsilon_u}{\epsilon_y} \times \dfrac{d - x}{x_u} \text{ - singly RC --- } (9(a)) \\ \dfrac{x}{d} = \dfrac{\epsilon_u}{\epsilon_u + \epsilon_{ym}} \text{ - Doubly RC --- } ⑫ \end{array}}$$

* Using eq ⑨ + ⑪ above, ~~it a curve is drawn~~ μ can be calculated + presented in a curve:-



Variables affecting Ductility →

① Tension steel ratio (P) →

→ Ductility of beam c/s increases, as steel ratio P or (P−Pc) decreases. $\rightarrow$ eq (11, 12)

→ If excessive steel is provided at c/s

concrete will crush before steel yields

⇒ brittle failure & $\mu = 1$, $\mu = \frac{\epsilon_u}{\epsilon_y}$, for brittle material $\epsilon_u = \epsilon_y$ so $\mu = \frac{\epsilon_u}{\epsilon_y} = 1$

→ So beam design should be underreinforced.

→ Ductility is directly affected by ultimate conc. strain (ϵ_u), f_{ck} & f_y.

→ ϵ_u, the ultimate strain in concrete is a function of

Characteristic strength of concrete (f_{ck})

Rate of loading.

strengthening effect of stirrups.

→ Ultimate strain of concrete, $\epsilon_u = 0.0035$.

→ From curve it is clear that

- ductility $\propto$ f_{ck}. (obvious from curve at previous page)
- ductility $\propto \frac{1}{f_y}$ (obvious from eq. 9.) (So Fe415 is preferred to Fe500 w.r.t ductility view)
- ductility $\propto \frac{1}{\text{steel ratio (P)}}$

② Compression steel ratio →

* curve at previous page shows

Ductility increases by decreasing (P−Pc) value

⇒ Ductility increases by increasing Pc i.e. compression steel Ratio.

③ Shape of c/s →

* If compression flange in a T-beam is very wide then flange depth at collapse ~~is made~~ is reduced. ⇒ increase in ductility.

∴ $\boxed{\text{Ductility} \propto \text{width of c/s} \propto \frac{1}{\text{Depth of c/s}}}$ $\quad \theta_u = \frac{\epsilon_u}{x}$ $\Rightarrow \mu \propto \frac{1}{d}$

④ Lateral reinforcement →

* Lateral steel improve ductility by preventing premature shear failure
by Restraining comp. steel against buckling
by confining compression zone.
} ⇒ Deformation capacity of RCC is increased.

Design for Ductility →

* It is easy to have a section having adequate strength.

* It is difficult to have a section having desired strength & ductility.

* To ensure sufficient ductility designer should pay attension to
  - detailing of reinforcement
  - bar cut off
  - splicing & joint details.

* Sufficient ductility can be ensured by following design details :-

(i) • structural lay out should be simple & regular.
- offsets of beams to columns or. of columns from floor to floor, should be avoided.
- change in stiffness should be gradual from floor to floor.

(ii) • Amount of tensile reinforcement should be restricted.
- more compression reinforcement should be provided.
- compression reinforcements should be enclosed by stirrups to prevent them from buckling.

(iii) • In an RCC frame, beams & columns are so designed that inelasticity is confined to beams only and columns should remain elastic.

Reason may be
In columns are in elastic condition & load is exceeding the elastic limit then cols should go to inelastic state & can receive more unseen loads/unpredicted loads of EQ.

- To ensure this,

For the design axial load at a beam-col. joint,

sum of moment capacity of columns >
sum of beam capacities of beams.
(along each principal plane)

$$\boxed{\sum M_{column} > 1.2 \sum M_{beam}}$$

- The flexural resistance can be summed such that

Column moments oppose beam moments. [if not collapse occurs due to huge moment.]

(iv) shear reinforcement should be adequate to ensure that

strength in shear exceeds strength in flexure.

& so prevent a non-ductile shear failure before fully reversible flexural strength of member has been developed.

(v) closed stirrups/spirals should be used to confine concrete at section of max. moment to increase ductility of the member.

[eg] upper & lower ends of columns and

within beam-column joints which donot have beams on all sides.

• if axial load > 0.4 × balanced axial load,

spiral column is preferred.

(vi) Bond failure can be prevented if

splices & bar anchorages are adequate.

(vii) Beam-column connections should be monolithic.

(viii) Reversal of stresses in beams and columns due to

reversal of earthquake force direction must be considered.

by providing adequate reinforcement.

(ix) For ductile design of structures against EQ, requirement is

Strong column & weak beam.

---

③ — V_B = design seismic base shear (function of A_h & W)

② — A_h = design horizontal acceleration coefficient (function of Z, I, R, Sa/g)

W_i = seismic weight of ith floor

h_i = height of floor

Sa/g = structural response factor

T = natural period

Z = zone factor

I = importance factor

R = response reduction factor

① — Ta = fundamental time period (function of 'h')

④ — Q_i = design lateral force in each storey (ith) (function of V_B, W_i, h_i)

Q A single reinforced beam has c/s (300 x 500 mm), reinforced with 3 bars of 16mm, tor steel (HYSD). Find ductility, if M 20 conc. used. Fe 415 - steel

Sol

$$m = \frac{280}{3\sigma_{cbc}} = \frac{280}{3 \times 7} = 13.33.$$

$$p = \frac{A_s}{bd} \cancel{\times 100} = \frac{3 \times \frac{\pi}{4} \times 16^2}{300 \times 465} \times 100 = 0.43\% = \underline{0.0043}$$

$$x = \left(-mp + \sqrt{m^2p^2 + 2mp}\right) d$$

$$\Rightarrow \frac{x}{d} \; \cancel{= -(13.33 \times 0.0043 + \sqrt{13.33^2 \times 0.43^2 + 2 \times 13.33 \times 0.0043}} =$$

$$= -13.33 \times 0.0043 + \sqrt{13.33^2 \times 0.0043^2 + 2(13.33) \times (0.0043)}$$

$$\cancel{\Rightarrow \frac{x}{d} = -0.057 + \sqrt{\quad}}$$

$$\Rightarrow \frac{x}{d} = 0.29.$$

$$x_u = \frac{0.87 f_y A_{st}}{0.36 f_{ck} b} = \frac{0.87 \times 415 \times 603}{0.36 \times 20 \times 300} = 100.8.$$

$$\phi_y = \frac{\epsilon_y}{d-x} = \frac{f_y / E_s}{d-x} = \frac{415 / 2\times10^5}{\cancel{465}\,(1-\frac{x}{d})d} = \frac{415 / 2\times10^5}{(1-0.29)465} = 0.62\times10^{-5}$$

$$\phi_u = \frac{\epsilon_u}{x_u} = \frac{0.0035}{100.8} = 3.47 \times 10^{-5}$$

$$\mu = \frac{\phi_u}{\phi_y} = \frac{3.47\times10^{-5}}{0.62\times10^{-5}} = \underline{5.6.}$$

Computation of EQ forces on building frame using SEISMIC COEFFICIENT METHOD

(Design starts from cl. 6.4 (P-14) of IS:1893-2002)

Q → Height of each floor of a four storey RCC frame building is 3m. The building is located somewhere in Delhi & is used as in institutional building. The soil below foundation consists of hard rock. The total transverse lumped load acting on each floor level of building is 2500 kN. Find the total base shear & show the base shear distribution along the height of the building applying seismic coefficient method.

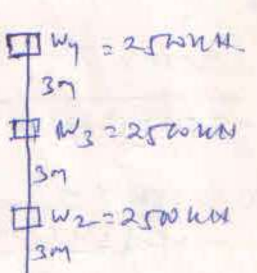
Sol →

* Cl. 7.6.1 of IS: 1893-2002 (P-24)

①* Fundamental time period $\boxed{T_a = 0.075\, h^{0.75}}$

$= 0.075 \times 12^{0.75}$

$\Rightarrow T_a = 0.48$ second



②* Cl. 6.4.2 of IS: 1893-2002 (P-14)

→ Horizontal Seismic coefficient or Seismic Responce coefficient, $\boxed{A_h = \frac{Z}{2} \times \frac{I}{R} \times \frac{S_a}{g}}$.

where

Z = zone factor in Table-2, (P-16), Z = 0.24 (as Delhi is in seismic zone-4)

I - Importance factor, Table-6 (P-18), I = 1.5 (as it is a college)

R - Responce reduction factor, Table-7, (P-23), R = 3 (as it is ordinary RC moment resisting frame)

$\frac{S_a}{g} = \frac{1}{T} = \frac{1}{0.48} = 2.08$ ~~cos~~ (cl. 6.4.5, P-16)

∴ seismic coefficient = $\boxed{A_h} = \frac{0.24}{2} \times \frac{1.5}{3} \times \frac{1}{0.48} = 0.125$

* cl 7.7.1 (P-24), designed base shear distribution

~~$Q_i = V_B \frac{W_i h_i^2}{\sum_{j=1} W_j h_j}$~~

Design base shear distribution $\boxed{Q_i = V_B \frac{W_i h_i^2}{\sum_{j=1}^{n} W_j h_j^2}}$

Q_i = Design lateral force on each floor.

W_i = seismic wt of floor i,

h_i = height of floor i measured from base

n = no. of storey in the building = no. of ~~bases~~ levels where masses are located.

= Q7O2 ,

* cl. 7.5.3 of IS: 1893 - 2002 (P - 24)

Design seismic base shear $\boxed{V_B = A_h W}$

A_h = Design horizontal acceleration coefficient value

W - seismic wt of building

* Design seismic base shear =

③ → $V_B = A_h W = 0.125 \times 4 \times 2500 = 1250$ kN.

④ →

$$Q_1 = V_B \frac{W_i h^2}{\sum_{j=1}^{4} W_j h_j^2} = 1250 \times \frac{2500 \times 3^2}{2500[3^2+6^2+9^2+12^2]} = 41.67 \text{ kN}.$$

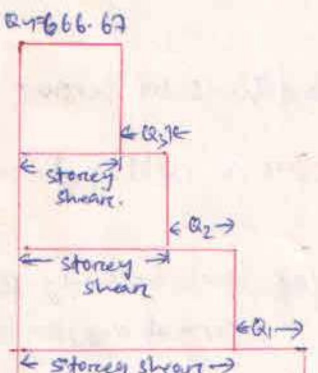
$$Q_2 = 1250 \frac{2500 \times 6^2}{2500[3^2+6^2+9^2+12^2]} = 166.67 \text{ kN}.$$

$$Q_3 = 1250 \times \frac{2500 \times 9^2}{2500[3^2+6^2+9^2+12^2]} = 375 \text{ kN}.$$

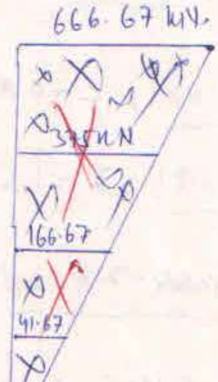
$$Q_4 = 1250 \times \frac{2500 \times 12^2}{2500[3^2+6^2+9^2+12^2]} = 666.67 \text{ kN}.$$

check

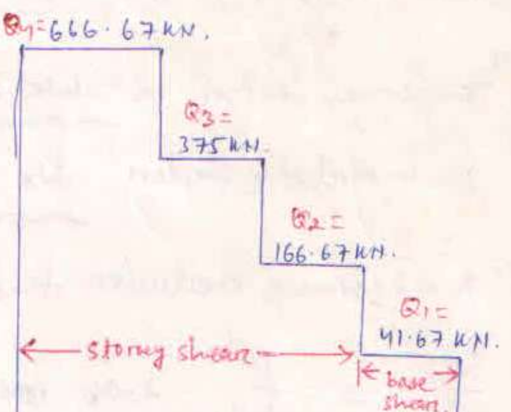
$\boxed{V_B = Q_1 + Q_2 + Q_3 + Q_4} = 41.67 + 166.67 + 375 + 666.67 = 1250$ kN.



Storey shear



storey shear distribution



(Base shear distribution)

Storey shear →

sum of design lateral forces at all levels above the storey under consideration

Height of each floor is given.

The building is located in monghyr (zone 4 =

steel moment resisting power station

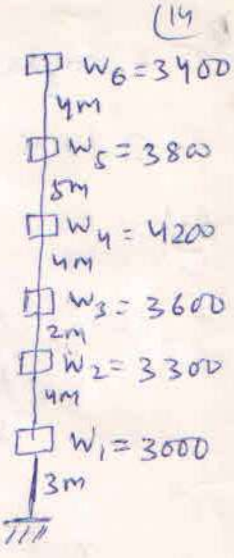
The ~~RCC~~ frame building is used (I = 1.5)

$Ta = 0.085 h^{0.75}$

R = 5, Black cotton soil

The total transverse load on each floor is shown in figure.

Find total base shear & show base shear distribution along height of building.



(P-24) → $Ta = 0.085 h^{0.75} = 0.085 \times 22^{0.75} = \underline{0.86 S}.$

$A_h = \frac{Z}{2} \times \frac{I}{R} \times \frac{Sa}{g} =$

$= 0.12 \times 0.3 \times 1.942 = 0.07$

$\frac{Z}{2} = \frac{0.24}{2} = 0.12$ (P-16, Table-2)

$\frac{I}{R} = \frac{1.5}{5} = 0.3$ P-18, Table-6 I = 1.5

$\frac{Sa}{g} = \frac{1.67}{T} = \frac{1.67}{0.86} = 1.942$ P-23, Table-7 R = 5

P-16 cl. 6.4.5

$V_B = A_h W = 0.07 \times 21300 = 1491 kN.$

$Q_1 = V_B \frac{w_i h_i^2}{\Sigma w_j h_j^2} = 1491 \times \frac{3000 \times 3^2}{406690} = 9.9 kN$

$\Sigma w_j h_j^2$

$= 3000 \times 3^2 + 3300 \times 7^2 + 3600 \times 9^2$

$+ 4200 \times 13^2 + 3800 \times 18^2 + 3400 \times 22^2$

$= 406690$

$Q_2 = 1491 \times \frac{3300 \times 7^2}{406690} = 59.3 kN$

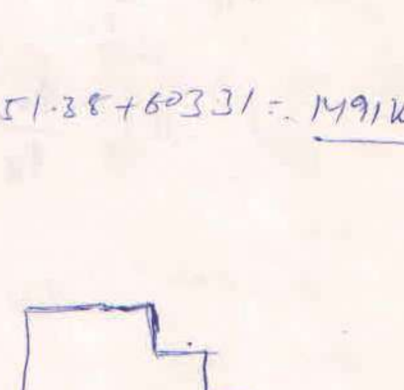
$Q_3 = 1491 \times \frac{3600 \times 9^2}{406690} = 106.9 kN$

$Q_4 = 1491 \times \frac{4200 \times 13^2}{406690} = 260.23$

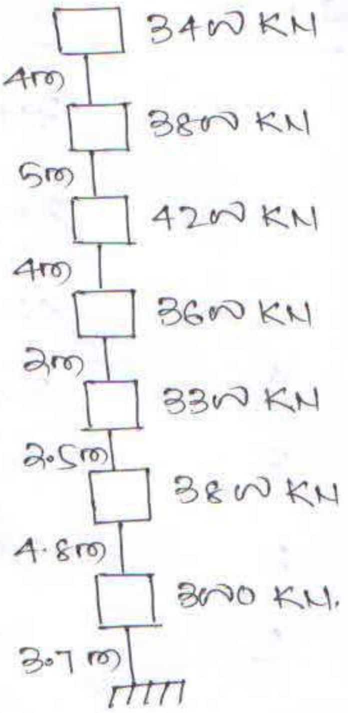
$Q_5 = 1491 \times \frac{3800 \times 18^2}{406690} = 451.38 kN$

$Q_6 = 1491 \times \frac{3400 \times 22^2}{406690} = 603.31$

$Q = 1491 = 9.9 + 59.3 + 106.9 + 260.23 + 451.38 + 603.31 = \underline{1491 kN}$



Q:- Building is located Monghyr
Steel moment resisting frame of power station
Black cotton soil.



Soln:- Zone factor = $\underline{IV}$ $= 0.24 = Z$

Importance factor $= I = 1.5$

Response factor $= R = 5.0$

Zone - $\underline{IV}$ ⇒ Annex E, pg-36

Zone factor ⇒ Table-2, pg-16

Importance factor ⇒ Table-6, pg-18

Response factor ⇒ Table-7, pg-23, sl no- $\underline{IV}$

Height $= h = 26m$.

$T_a = 0.085\, h^{0.75} = 0.085 \times 26^{0.75} = 0.9787$ [pg-24
cl-7.6.1

For black cotton soil

$$\frac{Sa}{g} = 1.67/T = 1.67/0.9787 = 1.706$$

$$A_h = \frac{Z}{2} \times \frac{I}{R} \times \frac{Sa}{g}$$ [pg-14
cl-6.4.2

$$= \frac{0.24}{2} \times \frac{1.5}{5} \times 1.706$$

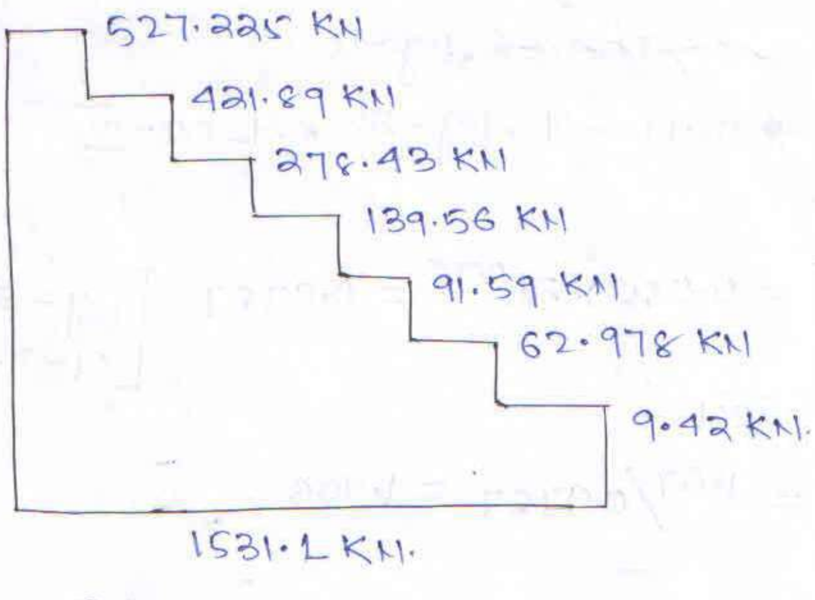
$$= 0.061$$

$V_B = A_h \times W$ [pg-24
cl-7.5.3

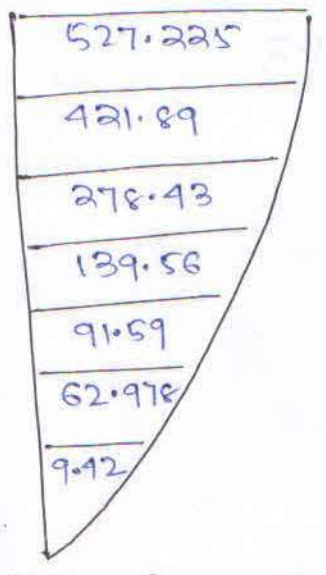
$= 0.061 \times 25100$

$= 1531.1$ KN.

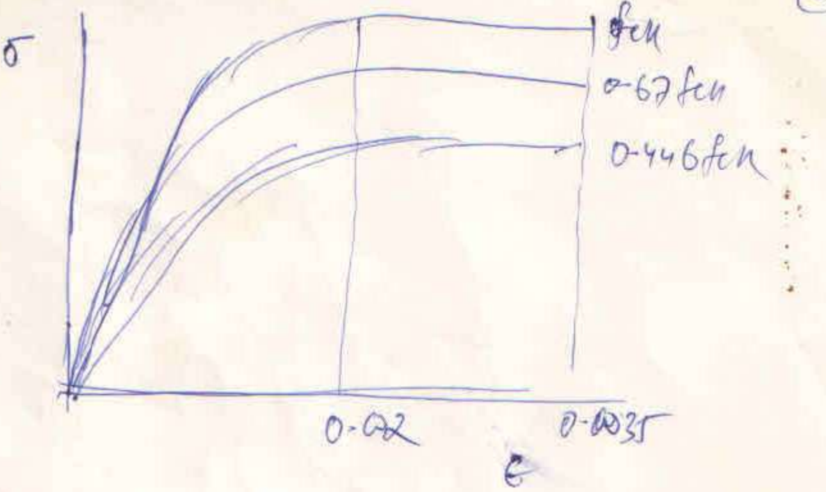
Floor	W_i	h_i	h_i^2	$W_i h_i^2$	Q_i
7	3400	26	676	2298400	527.225
6	3800	22	484	1839200	421.89
5	4200	17	289	1213800	278.43
4	3600	13	169	608400	139.56
3	3300	11	121	399300	91.59
2	3800	8.5	72.25	274550	62.978
1	3000	3.7	13.69	41070	9.42
				$\Sigma = 6674720$	$\Sigma = 1531.1$ KN



(Base shear).



(Storey shear).



$$\frac{d-x_u}{x_u} = \frac{f_{ym}}{f_{ck}} = \frac{0.002+\frac{0.87f_y}{E_s}}{0.0035}$$

$$\frac{d-x_u}{x_u}+1 = \frac{d}{x_u} \approx \frac{0.0055+\frac{0.87f_y}{E_s}}{0.0035}$$

Imposed Load calculation P-17 (cl 7.3) & Table-8 (P-24)

Factors affecting EQ design of buildings

- Natural frequency of structure (T)
- Damping factor of building (Sa/g) avg response acceleration coefficient
- Type of foundation
- Importance of building

ADVANCED CONCRETE STRUCTURES (CE 15031)

Ductile Detailing of RC frames
for
Seismic Forces

Ref: IS 13920: 2016
EQ TIPS:18, 19,20, IITK

Ductile detailing of RC frame for seismic forces: (IS13920:2016) (SMRF with R=5):-

Against EQ necessary codes
- IS 1893 (P1): 2016 - specifies seismic design force
- IS 4326:2013 - specifies general principles & features of EQ design
- IS13920:2016 - Design details for ductility in EQ region of structures

→: IS 456:2000: Design & detailing of RC structures (OMRF structures only)
↳ non ductile system with R = 3

Requirements for ductile detailing:

① Performance criteria of structure against EQ

② cyclic behaviour of concrete, reinforcement (material ductility):
→ → → → → → → → → → → → → → → (structural ductility) → cyclic behaviour of RCC

③ significance of ductility

④ Types of ductility

⑥ Ductility of structures: (SR beam and DR beam)

⑦ Factors affecting ductility

⑧ Design aspects for ductility :→ Estimation of base shear in building with Design

⑨ →→ Ductile detailing of RC frames for seismic forces
- * ductile detailing of beam: cl 6 (P4-P7), IS13920 // EQ Tip 18 (P35-36) - IITK
- * ductile detailing of column: cl 7 (P7-P11), IS13920 // EQ Tip 19 (P37-P38) - IITK
- * ductile detailing of joint: cl 9 (P13-P14), IS13920 // EQ Tip 20 (P39-P40) - IITK

* Increased load in OMRF, lead to increase in main steel
* Increased load in SMRF, lead to increase in transverse steel.

* India has four seismic zones (II, III, IV & V) [zone I is merged to zone II, since 2002]
Around 60% Indian area are under zone III, IV & V.

* Cost of construction goes up slightly in zones II & III with ductile detailing.

* But it is more economical to design frames as ductile frame in zones III to V. than ordinary frames.

* Ductile detailing for zone III, IV & V is compulsory for design against lateral load, and is optional for those in zone II.

* Ductile detailing is applicable only to monolithic structure, not to precast & PSC.

(DP2

Factor that increases ductility / objectives of IS 13920:2016 :-

- use simple & regular structural configuration
- use more redundancy on lateral load resisting system
- Avoid column failure / hinge formation in column by adopting 'weak beam-strong column' principle in design
- Avoid foundation failure
- Avoid brittle failure due to shear, bond, anchorage / comp failure in bending
- Providing special confinement of concrete at critical points by provision lateral resisting members so that concrete can undergo large compressive strain before failure
- using under reinforced (UR) beam sections so that they can undergo large rotation before failure
- The above objectives can be satisfied by adopting methods as per IS 13920
  - → specifications for material ductility cl.5, P3, IS13920
  - → ductile detailing in beam - cl 6, P4, IS13920
  - → ductile detailing of columns and frame members with axial force (P) & moment (M) - cl 7, P7, IS13920
  - → ductile detailing of beam-column joints in RC frames, cl 9, P13, IS13920
  - → Lets study IS 13920:2016 & EQ TIPS 18, 19, 20 - IITK

DUCTILE DETAILING IN BEAM

(DDB1)

Q-1 A RC frame consists of a beams having spans of 6M C/C, M20, Fe415. A typical floor inner beam carries a negative BM of 450KNM and a shear of 325KN at the face of beam-column Joint due to gravity and earthquake loads. Design the beam section for ductility.

Sol → Let the beam C/S = 350×650 MM

- effective cover for tension steel = 50 mm at top face of beam Joint
- " " " compression steel = 45 mm at bottom face

(i) $\frac{\text{width}}{\text{depth}} = \frac{350}{650} = 0.54 > 0.3$ (cl. 6.1.1, IS 13920:2016, P 4)

(ii) and beam width > 200 MM OK (cl 6.1.2, P.4, IS 13920)

clear span ≃ 5.5M

cl 6.1.3, P-4, IS 13920, $\frac{\text{Clear span}}{4} = \frac{5500}{4} = 1375$

depth of beam 650mm < $\frac{\text{Clear span}}{4}$ (OK)

factored BM = 450 × 1.2 = 540 KNM (cl. 6.3.3, P-6, IS 13920)

» Lets choose a doubly reinforced section (With d' = 45 mm & d = 600 mm)

$\Rightarrow \frac{d'}{d} = 0.075$

d' = effective cover for compression steel

» cl. 6.2.1, P-5, IS 13920, minimum longitudinal steel ratio on any face of section ⇒

$\rho_{min} = 0.24 \frac{\sqrt{f_{ck}}}{f_y} = 0.24 \frac{\sqrt{20}}{415} = 0.0026 = 0.26\%$

cl. 6.2.2, P-5, IS 13920, maximum longitudinal steel ratio, $\rho_{max} = 0.025 = 2.5\%$.

Let us provide tension steel 1.5% and compression steel 0.75%.

Total longitudinal steel in section = 2.25% < ρ_{max} (i.e. 2.5%)

because as per cl 6.2.3, P5, IS 13920, at beam column joint (tension steel/-ve steel which is provided at top face & compression steel/+ve steel is at bottom face of beam), bottom steel (+ve steel) be at least 50% of top steel (-ve steel), which justifies, $P_t = 1.5\%$ & $P_c = 0.75\%$ and total steel < 2.5%. (OK)

(DDB2)

$\Rightarrow P_t = 1.5\% = \frac{1.5}{100} \times b d = 0.015 \times 350 \times 600 = 3150\ mm^2$

* Let us provide 6-28ϕ bars at top face of beam in the beam column joint.

$\Rightarrow A_{st}$ provided $= 3695\ mm^2$

$P_c = 0.75\%\ bd = 1575\ mm^2$

* Let us provided 3-28ϕ bars at bottom face of beam in the beam column joint.

Asc provided $= 1847\ mm^2$ (Ok)

shear force

Factored shear force $= 1.2 \times 325 = 390\ kN = V_u$

shear strength of concrete for $P_t = 1.5\%$, $\tau_c = 0.72\ N/mm^2$

(Table 19, P73, IS456)

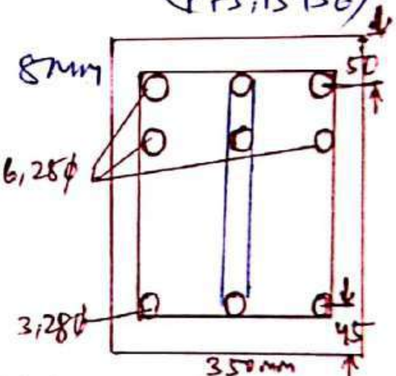
$$\tau_v = \frac{V_u}{bd} = \frac{390 \times 10^3}{350 \times 600} = 1.86 < \tau_{cmax}\ (= 2.8\ N/mm^2)$$ (Table 20 P73, IS456)

(OK)

As per cl 6.3.2, P6, IS13920, minimum link diameter is 8mm

$\Rightarrow$ Let us have 8mm ϕ 4 legged stirrup

$$\text{spacing} = \frac{0.87 f_y A_{sv}}{(\tau_v - \tau_c) b} = \frac{0.87 \times 415 \times 4 \times \frac{\pi}{4} 8^2}{(1.86 - 0.72) 350} = 181\ mm$$



cl. 6.3.5, P7, IS 13920, spacing of links shall not exceed (cl 6.3.5, P7, IS 13920)

$\ngtr \frac{d}{4} = \frac{600}{4} = 150\ mm$

$\ngtr 8\phi = 8 \times 28 = 224\ mm$

$\ngtr 100\ mm$

* Let us provided 8mm ϕ 4 legged stirrups @ 100mm c/c in a length of '2d' = 1200 mm from the face of the beam column joint (cl 6.3.5.2, P7, IS13920)

* cl 6.3.5.1, P7, IS 13920, first link shall be at a distance of 50mm from beam column joint face. (cl 6.3.5.1, P7, IS 13920)

* Remaining spacing = 2d − 50 = 1150 mm

{ A total of 12 no of 4 legged 8mm ϕ stirrups are provided at a distance of 1200mm from face of joint.

DUCTILE DETAILING IN COLUMN

Q Design the column section in a RC multistory building for ductility which is subjected to an axial force of 2500 kN and a bending moment of 650 kNm under gravity and earthquake loads. M20 - Fe415.

Sol → Let column size = 600 mm × 600 mm

factored BM, $M_u = 1.2 \times 650 = \underline{780}$ kNm

factored axial force = $1.2 \times 2500 = \underline{3000}$ kN (cl 6.3.3, P6 IS 13920)

$$\frac{P_u}{f_{ck}bD} = \frac{3000\times10^3}{20\times600\times600} = 0.417$$

$$\frac{M_u}{f_{ck}bD^2} = \frac{780\times10^6}{20\times600\times600^2} = 0.18$$

* using chart 44, P 129, SP 16:1980,

for $\frac{d'}{d} = 0.1$ for columns reinforced on all the four faces of column

$$\frac{p}{f_{ck}} = 0.162 \Rightarrow P_t = 0.162 \times f_{ck} = 0.162\times20 = \underline{3.24\% < 4\%}$$

(cl 26.5.3.1, P48, IS 456)

Required longitudinal steel = 3.24% = $\frac{3.24}{100}\times600\times600 = 11664$ mm^2

Provide $\underline{16 \text{ no. of } 32 \text{ mm}}$ ϕ bars, Ast provided = 12868 mm^2

minimum dimension of column ≮ $20 d_b = 20\times32 = 640$
L beam reinforcement diameter

≮ 300 mm (cl 7.1.1, P-7, IS 13920)

* Aspect ratio of column = $\frac{\text{smaller dimension}}{\text{larger dimension}} = \frac{600}{600} = 1$ ≮ 0.45 (cl 7.1.2, P7, IS 13920)

(so column size provided satisfies the ductility requirement as per IS13920)

Lateral steel:

* cl 7.4.2(a), P9, IS 13920, minimum dia of transverse links (rectangular) for main steel dia of 32 mm = 8mm.

{* maximum spacing of parallel links is 300 mm c/c (cl 7.4.2(b), P-9, IS 13920)
maximum spacing of links = $\frac{\text{Least lateral col. dimension}}{2} = \frac{600}{2} = 300$ mm (cl 7.4.2(d) P-10, IS13920)}

* Because larger transverse reinforcement is required for shear strength consideration, we need to provide special confining reinforcement as follows:

(DDC2

* The special confining links with diameter 8mm complying cl 8.1 (a), IS 13920 and have a maximum spacing as per cl. 8.1(b), P-11, IS 13920

special link spacing $\ngtr \frac{1}{4} \times$ minimum column dimension i.e. $\frac{1}{4} \times 600 = 150$ mm

$\ngtr 6 \times 32 = 192$ mm

$\ngtr 100$ mm

* confining reinforcement area (A_{sh}) for rectangular link cl. 8.1.c(2), P(12), IS13920

$$A_{sh} = 0.18 S_v h \frac{f_{ck}}{f_y}\left(\frac{A_g}{A_k}-1\right) \text{ — — — — } ①$$

h = link longer dimension to its outer face = 600 − 40 − 40 = 520 mm > 300 mm

(cl.7.4.2(c), P9, IS13920)

⇒ cross tie around middle longitudinal bar is provided

so revised h = 520/2 = 260 mm < 300 mm ok

* Area of confined concrete core, $A_K = (600-40-40)(600-40-40) = 520 \times 520$ mm²

Gross area of concrete = $A_g = 600 \times 600$ mm²

Area of one stirrup / link = $\frac{\pi}{4} \times 8^2 = 50$ mm²

⇒ putting in eq ① ⇒ $50 = 0.18 \times S_v \times 260 \left(\frac{600 \times 600}{520 \times 520} - 1\right)\frac{20}{415}$

⇒ $S_v = 67$ mm ≃ 65 mm

* If clear height of column = 3.6M

As per cl. 8.1(a), P11, IS 13920, special confining reinforcement shall be provided over a length lo from face of joint towards mid height of column on either side of beam-column joint not less than

$l_o \nless \frac{1}{6} \times$ column clear span $= \frac{1}{6} \times 3600 = 600$ mm

$\nless 450$ mm

Let us adopt lo = 650 mm toward mid height of column

* for special confining links, let us 8mm diameter links/stirrups @ 65mm c/s in a distance equal to 650mm from face of beam column joint towards column mid height.

(DDC3

* so column clear height = 3600mm

special confining links are provided for 650mm + 650M = 1300mm from joint face towards column mid height

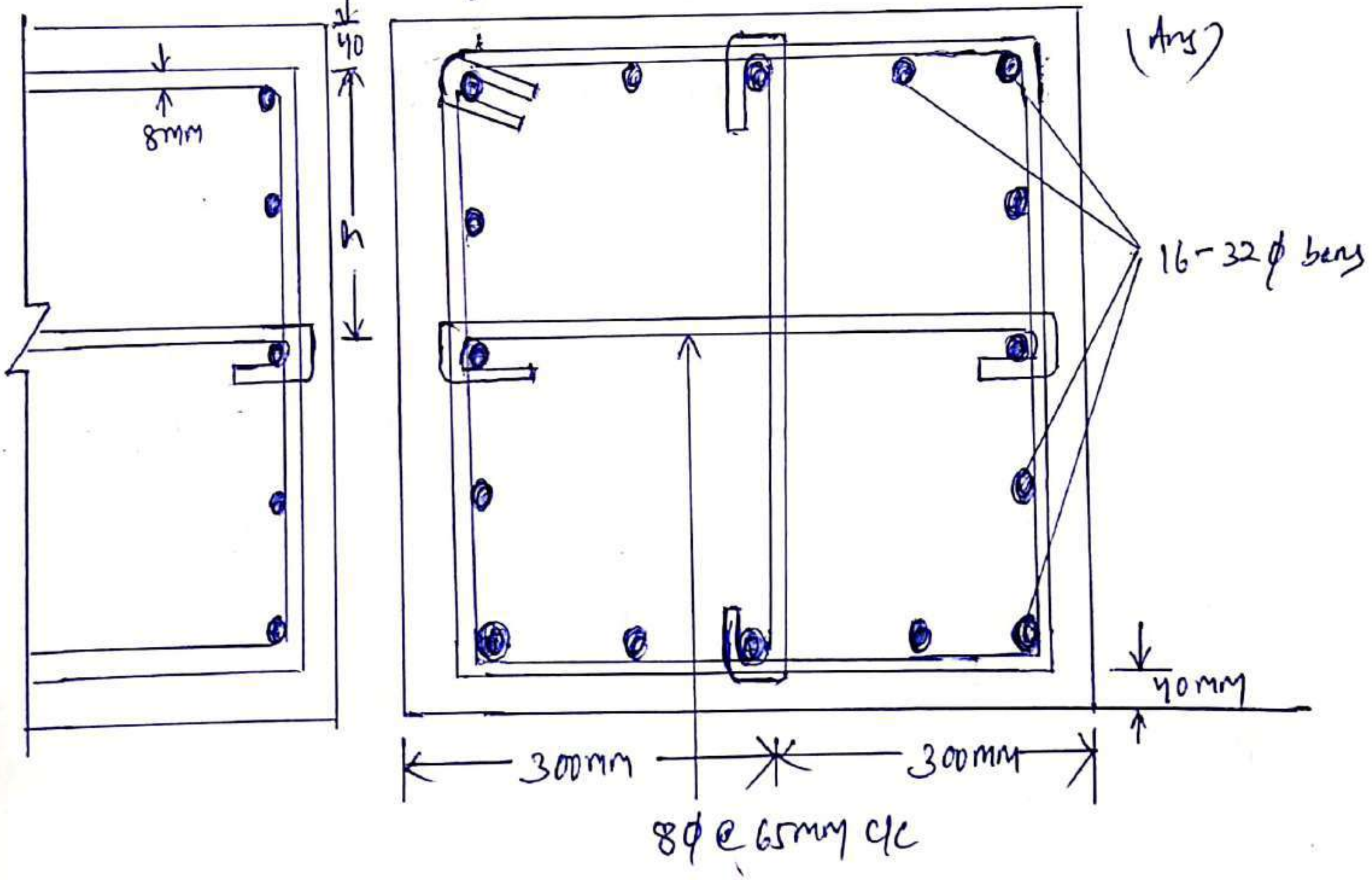
Remaining column length = 3600 – 1300 = 2300mm

* For this region of column, column link spacing/pitch have

→ a minimum spacing (cl 7.4.2(d), P-10, IS 13920

$$= \frac{\text{Least lateral dimension of column}}{2} = \frac{600}{2} = 300\text{mm}$$

→ maximum link spacing < 300mm (cl. 7.4.2(b), P9, IS 13920



(Ref Fig 12, P 12, IS 13920 : 2016) ← Confined joint

(Ref. Fig 10 (B), P 10, IS13920:2016) ← cross tie

DUCTILE DETAILING

Check for STRONG COLUMN-WEAK BEAM Requirement

Q In an eight storey building in chandigarh, the inner beam-column joint in the ground floor roof is considered for its ability testing against earthquake. check the joint whether it satisfies strong column-weak beam requirement in the joint. check also the shear in beam and column for following data:-

- materials M25-Fe415
- clear span of beam to left side & right side of the joint = 4.5m and 4m
- slab thickness = 125mm and slab finish = 50mm thick
- Live load on floor = 2 kN/m²
- wall thickness on beam = 115mm
- At joint, the axial load in column = 900 kN
- Beam size = 230 × 550mm with 1.5% top steel (3-25mm & 2-16mm) and 0.8% bottom steel on either side of joint
- column size = 230 × 650 mm with 3.46% steel (8-25mm and 4-20mm bars)

Sol → (a) check the beam-column joint in bending :-

bending of column about weak axis :-

* for P_t 1.5%, P_c = 0.8%, f_{ck} = 25, f_y = 415, b = 230 mm, d = 550 − 50 = 500 mm

using concepts of doubly reinforced beam section (DR beam section)

⇒ Beam hogging moment capacity = M_u = 255.87 kNm ($M_{u\,hogging}$)

* corresponding balanced section of corresponding DR section with P_t = 1.5% with b = 230mm & d = 500mm,

⇒ Beam sagging moment capacity = 143.75 kNm ($M_{u\,sagging}$)

* To find column capacity (use chart 32, P 117 of SP 16:1980

$$\frac{P_u}{f_{ck}bD} = \frac{1.2 \times 900 \times 1000}{25 \times 230 \times 650} = 0.29 \quad \text{and} \quad \frac{P_t}{f_{ck}} = \frac{3.46}{25} = 0.138$$

$$\Rightarrow \frac{M_u}{f_{ck}bD^2} = 0.225$$

$$\Rightarrow M_u = 0.225 \times 25 \times 230 \times 650^2 \times 10^{-6} = \underline{193.4 \text{ kNm}}$$

(DDJ2

$\Rightarrow \Sigma M_{beam} = 255.87 + 143.75 = 399.6$ kNm   ($\Sigma M_{beam} = M_{u\,sag} + M_{u\,hog}$)

$\Sigma M_{column} = 2 \times 193.4 = 386.8$ kNm

cl 7.2.1, P8, IS13920

$1.4 \times \Sigma M_{beam} = 1.4 \times 399.6 = 559.44$ kNm

so $\Sigma M_c < 1.4 \times \Sigma M_{beam}$

hence, beam column joint is not based on strong column - weak beam proportions

=) column capacity can be increased by increasing column width

(b) Beam shear capacity on left side of joint:

* Beam effective length = 4 + 4.5 + 0.15 + 0.65 = 9.3 m
  (0.15 ↓ wall, 0.65 | col size ll to beam span)

* Dead load on beam = $0.23 \times 0.55 \times 9.3 \times 24 (kN/m^3) = 28.5$ kN/m length

* Live load on beam = $0.23 \times 9.3 \times 2 = 7$ kN/m

{ Factored shear due to gravity = $1.2(28.5 + 7) \times \frac{4.5}{2} = 95.85$ kN (cl 6.3.3 P6, 13920)

{ shear due to plastic hinge formation in beam = $1.4 \times \frac{M_{u\,sag} + M_{u\,hog}}{l_{ab}(left)}$ (cl 6.3.3 (b2), P6)

$= \frac{1.4}{4.5}\left(\frac{255.87 + 143.75}{1}\right) = 124.32$ kN

Total $V_u = 95.85 + 124.32 = 120.80$ kN

$\tau_v = \frac{220 \times 10^3}{230 \times 500} = 1.91 \ N/mm^2$

for 1.5% steel (tensile), $\underline{\tau_c = 0.72}$ for M25 (Table 19, P73, IS 456)

provide 10mm ϕ, 2 legged stirrups @ 150mm c/c.

(c) shear capacity of column:

Let storey height = 3.25 m

for column factored shear = $1.4 \times \frac{M_{u\,sag} + M_{u\,hog}}{h_{st}}$ (cl 7.5 b(1), P10, IS 13920)

$= 1.4 \times \frac{399.6}{3.25} = 172$ kN

$\tau_v = \frac{172 \times 10^3}{230 \times 650} = \underline{1.15 \ N/mm^2}$

for 3.46% steel in column, 1.73% p_t on one face of column $\Rightarrow \underline{\tau_c = 0.77 \ N/mm^2}$ (Table 19, P73, IS 456)

As per cl 40.2.2, P72, IS 456, for axial compression, enhanced shear strength = $\boxed{\tau_c \times \left(1 + \frac{3P_u}{A_g f_{ck}}\right)}$ ←

$\Rightarrow$ shear strength = $0.77 \times \left(1 + \frac{3 \times 900 \times 1000}{230 \times 650 \times 25}\right) = 1.43 \ N/mm^2 > \tau_v$ (OK)

ADVANCED CONCRETE STRUCTURES (CE 15031)

Module-II

Module II Syllabus

Design of Foundations (Shallow and Deep Foundations)

Design of Combined Footing: (Shallow Foundations)
(Rectangular and Trapezoidal footing

Design of Raft Foundation,

Design of Pile Foundation (Deep Foundations)

Subject to Revision

Advanced Concrete Structures

DESIGN
OF
COMBINED FOOTING

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COMBINED FOOTING

COMBINED FOOTING (05/02/2010)

- Entire structure may be divided into
  - Sub structure (foundation)
  - Plinth
  - Super Structure.

- Foundation →

  The lowest artificially prepared part below surface of surrounding ground which is in direct contact of sub ~~ground~~ strata that distributes the load to underlying soil.

- Footing →

  * The structural load is spreaded over a large area of stratum to minimise bearing pressure.

  * The enlarged portion of foundation is called Footing.

* All soil compress when loaded & the loaded structure settles.

* Requirements of foundation design! →

- Total settlement of structure < Permissible settlement
- Differential settlement should be eliminated.

* To minimise settlement, structural load may be transmitted to strong soil stratum & load should be spreaded over large area to ~~minimise bearing p~~ minimise bearing pressure.

PLINTH →

middle part of structure above surface of surrounding ground upto floor level immediately above ground level

Super structure →

Part of structure above plinth level i.e. ground floor level.

FOUNDATION

* most building failure is due to faulty foundation design.

* A good foundation remains in position without sliding, bending & overturning.

- To achieve this sub structure, plinth & super structure should act together.

Requirements of foundation

* To distribute total load over large bearing area to prevent foundation movement.
* To load bearing surface at a uniform rate to prevent differential settlement.
* To prevent lateral movement of supporting material.
* To secure a level & firm natural bed.
* To increase stability of structure without overturning/sliding.

Bearing capacity →

The minimum gross pre. intensity at base of foundation at which soil fails in shear.

$$\text{Allowable soil pressure} = \frac{\text{ultimate bearing capacity}}{\text{Factor of safety } (=3)}$$

Foundation classification →

According to Terzaghi, foundation is classified depending upon relation between depth of foundation & width of foundation.

① shallow foundation/open foundation where depth of foundation ≤ the width.
- It is practicable upto a depth of 5m.
- It is convenient above water table.

② Deep foundation when depth > width of foundation
- depth of foundation > 5m.

Shallow foundation

- Plain isolated column footing (square/rectangular/circular in shape)
- Spread footing of wells.
- Combined footing
  - Rectangular combined footing (for two or more columns)
  - Trapezoidal combined footing (for two or more columns)
  - connected/strap/cantilever footing (for two or more columns) (individual isolated col. footing connected by a strap beam.
  - Raft/mat foundation for a structure as a whole.

Deep foundation

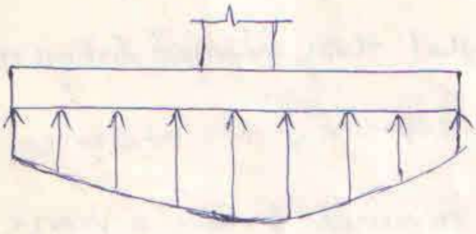
- Pile foundation
- Pier foundation
- Well foundation.

Pressure distribution under footing →

* From studies it is found that

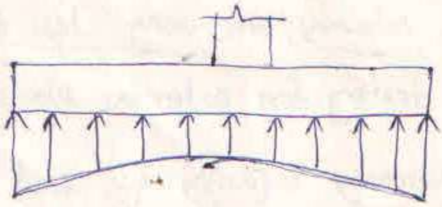
- Pressure distribution is not uniform even when symmetrically loaded.

* Foundation is assumed to act as a rigid body which is in equilibrium under applied forces from structure & stresses in the soil.



(Rigid footing on cohessionless soil)

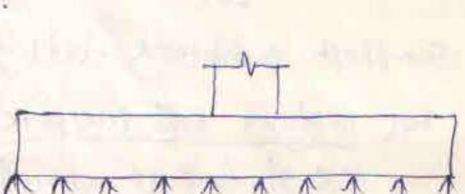
On cohessionless soil, grains at outer edge have no lateral restraint



(Rigid footing on cohessive soil.)

On cohesive soil, edge stresses may be very large as outer edge have lateral restraint.

* The non uniformity in pressure distribution is uncertain in magnitude & their values are variable, depending on type of soil.



(uniform pressure distribution)

The influence of the irregularity in pre. distribution on BM & SF in footing is very small & so pre. distribution is assumed to be uniform. This permits the use of Theory of bending to find stress distribution in soil for a given axial load & moment.

* A foundation is designed to resist applied load & moment so that induced reaction & safe bearing capacity of soil is not exceeded.

* All limit states must be considered to ensure required degree of safety & serviceability.

~~BENDING MOMENT →~~

~~* On compressible soil~~

* Permissible bearing pressure under working loads are based on factor of safety 2.5 to 3 against ultimate bearing capacity & to keep settlement within limit.

* For concentrically loaded footing, required area, $A = \dfrac{D.L + L.L}{\text{Permissible bearing pressure } (q_0)}$

* Code permits 25% to $33\frac{1}{3}$% increase in permissible pressure when Wind load or Earthquake load (E L) are considered.

$$A = \frac{DL+LL+WL}{1.33\, q_0} \quad \text{or} \quad A = \frac{DL+LL+EL}{1.33\, q_0}$$

The area is calculated at level of base of footing.
i.e. wt. of footing & wt. of soil on top of footing must be included.

Proportioning of combined footing ⇒

* On compressible soil footing should be loaded concentrically to avoid tilting
* In combined footing; centroid of footing area should coincide with resultant of column loads.
* A combined footing supports load of two/more adjacent columns.

It is necessary because:-

- when columns are very close to each other so that their indivigual footings overlap.
  eg- footing for columns placed side by side with expansion joints inbetween.
- when bearing capacity of soil is less so it is required to have a more spreaded area for footing & so footings of adjacent columns overlap.
- when ~~external~~ external column is close to property line, it is not possible to provide isolated footing for that column because it may be extended ~~to~~ beyond property line, & so combined footing solves the problem.

* Simplest combined footing is a two column footing.
* For uniform soil pressure distribution, footing is arranged such that CG of footings coincide with CG of applied loads.
* If outer column ~~at~~ near property line carries heavier load, trapezoidal footing is essential.
* Four types of combined footing
  - Combined Rectangular footing
  - Combined Trapezoidal "
  - Strap footing
  - Raft footing.
* Rectangular footing is used when two columns carry equal loads.
* Trapezoidal footing is used a column near property line carries heavier load.
* A strap footing consists of spread footings of two columns connected by a strap beam.
* A Raft footing covers the entire area under a structure & supports all columns & walls.

COMP

COMBINED RECTANGULAR FOOTING →

(1) Proportioning of footing →

w_1 & w_2 = Two column loads

w_3 = wt of footing.

q_0 = Safe bearing capacity.

A = Area of footing , $\boxed{A = \frac{w_1 + w_2 + w_3}{q_0}}$

Suitable values for length (L) & width (B) for footing is choosen such that $\boxed{L \times B = A}$

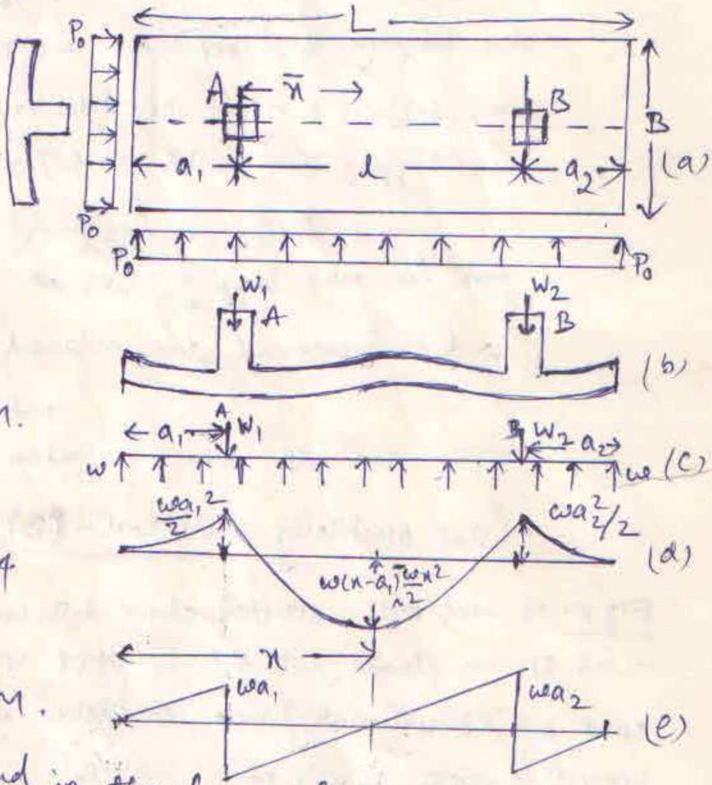
* {For uniform soil pressure distribution, footing is arranged such that CG of footing coincide with CG of column loads.}

$\bar{x}$ = CG distance of col. loads from centre of col. A

a_1 , a_2 = projections beyond CG of columns.

→ $a_1 + \bar{x} = a_2 + l - \bar{x} = \frac{L}{2}$

* Net upward pressure, $P_0 = \frac{W_1 + W_2}{\frac{1}{2}L(B_1 + B_2)}$



(2) Bending Pattern →

* The footing will bend in both longitudinal & transverse direction.
* In longitudinal direction, footing has sagging BM in two cantilever portions, & hogging BM in middle portion.
* In transverse direction, footing has sagging BM.
* Near columns, footing has a tendency to bend in the form of SAUCER.
* Transverse bending will decrease at a distance away from columns.
* But mainly, bending is in longitudinal direction,
* So sections around columns are subjected to heavier bending stresses.

· Design for Longitudinal bending →

* In this footing design, no longitudinal/transverse beams are there.
* So no exact analysis, only approximate analysis can be done.

~~Projection, $b = \frac{1800 - 400}{2} = 700\,mm = 0.7M$.~~

~~width of bending strip, $B_1 = b + 2d = 0.4 + 2 \times 0.3 = 1M$~~

~~width available to right of outer face of~~

* Common Design Practice

- Let us consider footing as a longitudinal beam of width B.
- UDL on beam $w = P_0 B$/unit length. which is supported on two columns with reactions W_1 & W_2.
- BMD & SFD are shown in previous page.

max hogging BM between W_1 & W_2 where SF = 0

max sagging BM at col. Axis $= \frac{w a_1^2}{2}$

max hogging BM $= W_1 (x - a_1) - \frac{w x^2}{2}$ x = distance of section from footing edge.

But footing is designed for sagging BM at outer face of each column. and for max hogging BM ~~at~~

∴ Reinforcement are placed at bottom face for sagging BM and on top face for hogging moment.

This analysis is an assumption that full width B is available for longitudal bending.

But practically full width (B) is not available for whole length of footing in actual bending.

Fig (a) of next page, critical strip $A_1B_1C_1D_1$ for transverse bending.
Total SF on planes A_1D_1 & B_1C_1 PLUS upward soil reaction on strip $A_1B_1C_1D_1$ = Column load W_1
strip A_1B_1BA will bend like a cantilever with UDL $= \frac{1}{2} \times W_1$
upward pressure intensity on $A_1B_1C_1D = P_0' = \frac{W_1}{\text{Area of } A_1B_1C_1D_1}$

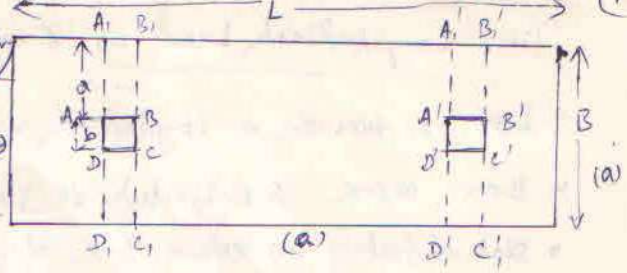
If strip ~~A_1B_1BA~~ A_1B_1BA is designed as cantilever with upward pressure (P_0'), it will be overstrained & yield.

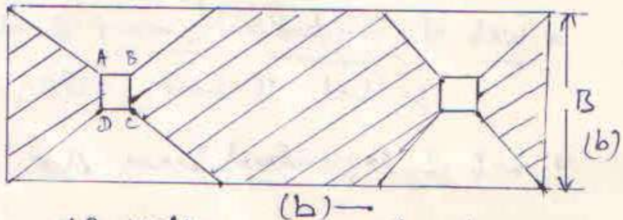
So effective available width for longitudinal bending will be AD only & not full B.
Available width will increase from AD = b at column face to B. [p. 7, fig (b).]

There are 3 ways for designing footing for Longitudinal bending

(i) To design footing for width equal to AD = b at col. face →

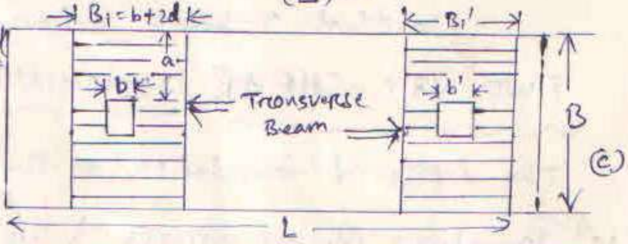
* If footing is designed as Longitudinal bending for width 'b' at col. face, thickness required for compression to resist B.M at face of column will be excessive & uneconomical, specially when cantilever length is more.
* For hogging moment, full width 'L' may be used
* This method is ok, when cantilever projections a_1 & a_2 are small and sagging BM are much less than hogging BM.

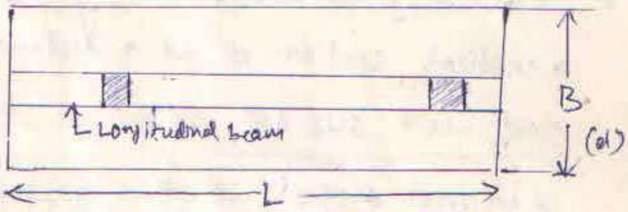




(ii) Transverse Beam →

* Let us have a transverse beam of width B_1 (fig C), to transfer the load W_1 to soil safely.
* shaded portions will bend transversely & remaining Portion will bend longitudinally as a beam using full width B for compression resistance.





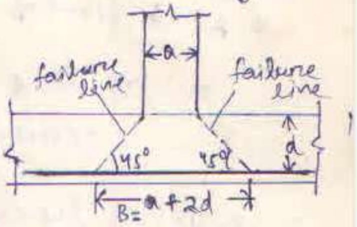
* B_1 width of transverse beam may vary from b to B.

  If $B_1 = b$, thickness required will be more than rest of footing

  If $B_1 = B$, each portion of transverse beam under A & B, will act as individual footing.

  So a intermediate value should be chosen.

* Failure due to shear occurs approximately along 45° lines, when footings are tested for destruction.



∴ width of transverse beam, $B_1 = b + 2d$, & $B_1 < B$.

upward soil pressure, $p_0' = \frac{W_1}{B_1 \times B}$

Thickness for transverse beam should be found for B.M at col. face $= \frac{(P_0' B_1) a^2}{2}$

This thickness > thickness of rest of footing (d)

Even if both are same, transverse steel is found for above B.M. & provided in B_1 width, similar steel for other transverse width B_1' should be provided.

For rest of footing, minimum transverse steel = 0.15% of c/s area of footing
= 0.12% of c/s area of footing } is provided

In longitudinal direction footing is designed for BM & SF as in fig (d) & (e) in page 15).

(iii) Longitudinal beam → (common method)

- Let us provide a longitudinal beam all along the length, joining two columns [fig d, P(7)]
- Beam alone is subjected to BM & SF [fig d, (e) in P(5)]
- Slab of footing on either side of footing is designed as cantilever, for transverse bending.
- Web of longitudinal beam should project below slab, if sagging moment in beam are more, so that T-beam action is available in cantilever portion of beam.
- web of longitudinal beam should project at top of slab, if hogging moment is more, so that T-beam action is available for central portion of beam.

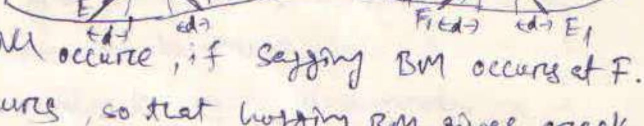
TWO WAY SHEAR / PUNCHING SHEAR →

The depth of foundation on the basis of BM should be tested for ~~punching shear~~ Punching shear, so that Punching shear stress ≯ its permissible value.

- critical section is at a distance d/2 from column face.

ONE WAY SHEAR →

- critical section is at a distance 'd' from column face, in respective cantilever portion
- In central portion, two locations, F & G should be tested for diagonal tension.



- At F, distance d from col. face, tension crack will occure, if sagging BM occurs at F.
- At G (point of contraflexure), tension crack occurs, so that hogging BM gives crack on top face.
  corresponding SF at G can be determined.
- out of three points E, F & G, diagonal tension crack may occur at a point, which is nearer to column, face or, at which SF is more.
- if $\tau_v > \tau_c$, stirrups are provided for required portion where $\tau_v > \tau_c$. But nominal stirrups should be provided throughout the the beam length.

Combined Trapezoidal footing →

* It is required when

- heavily loaded column is nearer the property line
- there is some restriction on total length of footing.

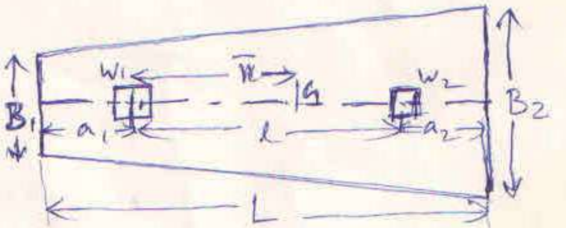
* Footing width B_1 & B_2 are so proportioned that CG of footing coincides with CG of column loads.

* Column loads W_1 & W_2

Spacing betn columns $= l$.

Length of footing $= L$

Cantilever projections $= a_1$ & a_2



$\therefore$ Area of footing $= \dfrac{W_1+W_2+\text{footing weight}}{q_0}$

$$\Rightarrow \frac{(B_1+B_2)L}{2} = \frac{W_1+W_2+W'}{q_0} \Rightarrow B_1+B_2 = \frac{2(W_1+W_2+W')}{Lq_0} \quad \text{—①}$$

Distance $\bar{x}$ of CG of loads $= \dfrac{W_2 l}{W_1+W_2}$

Distance of CG of loads from shorter edge $= a_1 + \dfrac{W_2 l}{W_1+W_2}$

Distance of CG of trapezium from shorter edge $= \dfrac{L}{3}\left[\dfrac{B_1+2B_2}{B_1+B_2}\right]$

equating two, $a_1 + \dfrac{W_2 l}{W_1+W_2} = \dfrac{L}{3}\left[\dfrac{B_1+2B_2}{B_1+B_2}\right]$ —②

* Net upward soil pressure intensity $\boxed{p_0 = \dfrac{W_1+W_2}{\frac{1}{2}L(B_1+B_2)}}$ which is uniform throughout. —③

* Trapezoidal footing design is exactly same as rectangular footing design after knowing p_0, BM & SF.

Design of Rectangular Combined Footing (29/03/2010)

Q - Two columns (loads) A & B carrying loads 500kN & 700kN with size 300×300mm & 400×400mm. C/C spacing of columns is 3.4m. Safe bearing capacity of ~~columns~~ of Soil 150kN/m2. M 20, Fe 250.

A// $W_1 = 500$kN

$W_2 = 700$kN.

Let wt of footing $= W' = 10\%\,(W_1+W_2) = 120$kN.

C/S area of footing $= A = \frac{500+700+120}{150} = 8.8\,m^2$

Let size of footing $= 1.8 \times 5$ m.

* Projections a_1 & a_2 should be such that CG of footing coincides with CG of column loads — — — — (1)

* Distance $\bar{x}$ of CG of column loads from centre of column A

$$\Sigma M_A = 0 \Rightarrow (W_1+W_2)\bar{x} = W_2 \times 3.4 \Rightarrow \bar{x} = \frac{700 \times 3.4}{500+700} \simeq 2m$$

* From statement (1), $a_1 + \bar{x} = \frac{L}{2} \Rightarrow a_1 = \frac{L}{2} - \bar{x} = \frac{5}{2} - 2 = \boxed{0.5m}$

$$a_2 = L - (l + a_1) = 5 - (3.4 + 0.5) = \boxed{1.1m}$$

* Net upward pressure $P_0 = \frac{W_1+W_2}{B \times L} = \frac{500+700}{1.8 \times 5} = 133.33$ kN/m2

Upward Pressure per meter length, $w = P_0 \times B = 133.3 \times 1.8 = 240$ kN/m.

* BMD & SFD ->

SF(A_{left}) $= w \times 0.5 = 240 \times 0.5 = 120$kN

SF(A_{right}) $= -500 + 120 = -380$kN/m width

SF(B_{right}) $= -w \times 1.1 = -240 \times 1.1 = -264$kN/m width

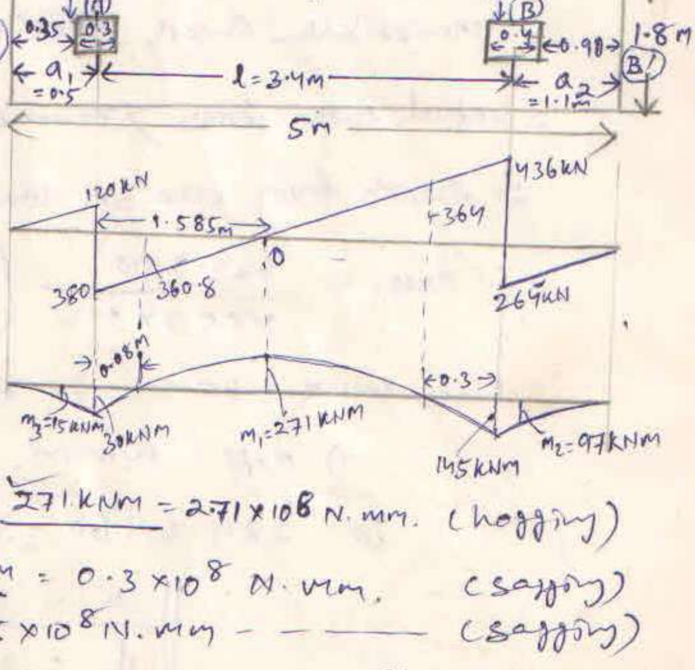
SF(B_{left}) $= 700 - 264 = 436$ kN/m width

SF is zero at x from A' $\Rightarrow wx - 500 = 0$

$\Rightarrow (P_0 \times \text{width})x - 5W = 0 \Rightarrow$ ~~[illegible]~~

$\Rightarrow (133.8 \times 1.8)x - 5W = 0 \Rightarrow x = \frac{5W}{240} = 2.085$ m.

or. SF = 0 at $(2.085 - 0.5) = 1.585$m from A)



* max hogging BM occurs here at 1.585m from A

$$BM_{max} = -240 \times \frac{2.085^2}{2} + 500 \times 1.585 \Rightarrow m_1 = 271\,kNm = 2.71 \times 10^8\ N.mm. \ (\text{hogging})$$

* Sagging BM at $A = w \times \frac{0.5^2}{2} = 240 \times \frac{0.5^2}{2} = 30$ kNm $= 0.3 \times 10^8$ N.mm. (Sagging)

* Sagging BM at $B = 240 \times \frac{1.1^2}{2} = 145$ kNm $= 1.45 \times 10^8$ N.mm — — — (Sagging)

* BM at left outer face of col A $= 240 \times \frac{0.35^2}{2} \Rightarrow m_3 = 15$ kNm $= 0.15 \times 10^8$ N.mm. (Sagging)

* BM at Right outer face of col. B $= 240 \times \frac{0.9^2}{2} = 97$ kNm $\Rightarrow m_2 = 0.97 \times 10^8$ N.mm (Sagging)

Location of ~~left~~ Point of contraflexure (x from centre of col. A)

$$\frac{240 \times (x+0.5)^2}{2} - 500x = 0 \Rightarrow \underline{x = 0.08\,m} \text{ and } 3.08\,m$$ (other value is absurd)

~~Location of Right Point of contraflexure~~

* Ist Point of contraflexure occurs at 0.08m from A.

* 2nd point of " " " 3.08m from A or $(3.4-3.08) \simeq$ 0.3m from B.

SF at Ist PC (0.08m from A) $= 240 \times 0.58 - 500 \times 0.08 = \underline{-360.8\,kN}$

SF at 2nd PC (0.3m from B) $= 700 - 240 \times 1.4 = \underline{364\,kN}$

Effective depth (from max. BM)

max BM = $M_1 = 2.71 \times 10^8$ Nmm.

equating MR with M_1

$$0.1487 f_{ck} b d^2 = 2.71 \times 10^8$$

$$\Rightarrow 0.148 \times 20 \times 1800 \times d^2 = 2.71 \times 10^8 \Rightarrow \underline{d = 225.5\,mm}$$

Punching shear consideration →

critical plain at d/2 around column B.

width B_o of critical plain = 400 + 113 = 513mm.

Punching shear force at col. B $= 700 - (P_o)(0.513)^2 \simeq$

$= 700 - 133.33 \times 0.513^2 = \underline{664.9\,kN}$

$$\tau = \frac{664.9 \times 10^3}{4 \times B_o \times d} = \frac{664.9 \times 10^3}{4 \times 513 \times 225.5} = \underline{1.44\,m}.$$

Permissible shear stress $= k_s \tau_c = 1 \times 0.25\sqrt{20} = \underline{1.12\,N/mm^2}$ (Cl 31.6.3.1 P-58)

∴ Actual shear stress > Permissible shear stress.

So Depth from max BM consideration is inadequate.

$$\therefore d_{req.} = \frac{664.9 \times 10^3}{4 \times 513 \times 1.12} \left(\boxed{\frac{V}{4 \times B_o \times \tau_c}}\right) = 289.3\,mm.$$

Let cover = 60mm to centre of steel

$\Rightarrow$ deff = 60mm.

$D = 289.3 + 60 = 349.3 \simeq \underline{360\,mm}$

$$\therefore \left\| \begin{aligned} D &= 360\,mm \\ d &= 300\,mm \end{aligned} \right.$$

Design of Bending tension in longitudinal direction

(1) Reinforcement for hogging BM, M_1,

$$M_1 = 0.87 f_y A_{st}(d - 0.42 x_u)$$

$$= 0.87 f_y A_{st}\left(d - \frac{0.42 \times 0.87 f_y A_{st}}{0.36 f_{ck} b}\right)$$

$$= 0.87 \times 250 \times A_{st}\left(300 - \frac{0.42 \times 0.87 \times 250\, A_{st}}{0.36 \times 20 \times 1800}\right)$$

$$\Rightarrow 271 \times 10^6 = 217.5 A_{st}(300 - 0.007 A_{st})$$

$$\Rightarrow 1.52 A_{st}^2 - 65250 A_{st} + 271 \times 10^6 = 0.$$

$$A_{st} = \frac{+65250 \pm \sqrt{42.57 \times 10^8 - 16.5 \times 10^8}}{3.04} = \frac{65250 \pm 51058.8}{3.04}$$

$\therefore A_{st} = 38259.5$ mm or 4668.16 mm^2

$\therefore A_{st} = 4668.16$ mm^2

using 16mm ϕ, no of bars $= \frac{4668.16}{201} = 23.22 \simeq 24$ nos

so provide 24 bars uniformly distributed over width 1.8m over top face.

* At point of contraflexure near B, SF = 364kN.

Let n = no of 16mm ϕ bars required at PC

$$\frac{M_1}{V} + L_0 \geq L_d \quad (\text{cl. } 26.2.3.3 \quad P\text{-}44)$$

$$M_1 = M_R = 0.87 f_y A_{st}\left(d - 0.42 \times \frac{0.87 f_y A_{st}}{0.36 f_{ck} b}\right)$$

$$= 0.87 \times 250 \times n \times 201\left(300 - \frac{0.42 \times 0.87 \times 250 \times 201 \times n}{0.36 \times 20 \times 1800}\right)$$

$$= 43717.5n(300 - 1.42n) = 1.3 \times 10^7 n - 0.006 \times 10^7 n^2$$

$V = 364$ kN $= 364 \times 10^3$ N

$L_0 = d$ or 12ϕ which is more $= 300$ mm

$$L_d = \frac{0.87 f_y \phi}{4 \tau_{bd}} = \frac{0.87 \times 250 \times 16}{4 \times 1.2} = 725$$

$$\frac{1.3\times10^7 n - 0.006\times10^7 n^2}{364\times10^3} + 300 = 725$$

$$\Rightarrow 1.3\times10^7 n - 0.006\times10^7 n^2 = 15.47\times10^7$$

$$\Rightarrow 0.006n^2 - 1.3n + 15.47 = 0$$

$$\Rightarrow n = \frac{1.3 \pm \sqrt{1.69 - 0.37}}{0.012} = \frac{1.3 \pm 1.15}{0.012} = 12.5 \text{ or } \cancel{204}$$

$$n = 13 \text{ nos.}$$

* All 24 bars are taken up to the outer faces of both the columns.

* Out of these 13 bars are stopped 4 bars are taken straight up to either edge of footing. These bars will serve as anchorage for stirrups.

② Reinforcement for sagging BM M_2 (near column B) →

$$M_2 = 0.87f_y A_{st}\left(d - 0.42\frac{0.87f_y A_{st}}{0.36f_{ck}b}\right)$$

$$97\times10^6 = 0.87\times250\times A_{st}\left(300 - \frac{0.42\times0.87\times250\times A_{st}}{0.36\times20\times1800}\right)$$

$$\Rightarrow 97\times10^6 = 217.5 A_{st}(300 - 0.007A_{st})$$

$$= 65250A_{st} - 1.52A_{st}^2 \Rightarrow 1.52A_{st}^2 - 65250A_{st} + 97\times10^6 = 0$$

$$\therefore A_{st} = \frac{65250 \pm \sqrt{65250^2 - 4\times1.52\times97\times10^6}}{2\times1.52} = \cancel{\frac{65250 \pm 64796.5}{3.04}}$$

$$= \frac{65250 \pm 60562.4}{3.04} = 41385.6 \text{ or } 1541.97\text{ mm}^2$$

$$\therefore A_{st_2} = 1541.97\text{ mm}^2$$

using 12mm ϕ, no of bars = 13.64 ≃ 14 nos.

→ No of bars should be sufficient from development length point of view

→ At point of contraflexure (PC), near the inner face of B

To satisfy criteria for development length.

$$\frac{M_1}{V} + L_0 \geq L_d$$

if n = no of bars required to satisfy the above criteria

$$M_1 = 0.87f_y A_{st}\left(d - 0.42\frac{0.87f_y A_{st}}{0.36f_{ck}b}\right) = 0.87\times250\times n\times\frac{\pi}{4}\times12^2\left(300 - \frac{0.42\times0.87\times250\times n\times\frac{\pi}{4}\times12^2}{0.36\times20\times1800}\right)$$

$$= 24577.5n(300 - 0.8n) \Rightarrow M_1 = 7373250n - 19662n^2$$

$SF = V = 364$ KN. at PC.

$L_0 = d$ or 12ϕ which is more $= 300$ mm.

$$L_d = \frac{0.87 f_y \phi}{4 \tau_{bd}} = \frac{0.87 \times 250 \times 12}{4 \times 1.2} = 543.75$$

$$\frac{7373250n - 19662n^2}{364 \times 10^3} + 300 = 543.75$$

$$\Rightarrow 7373250n - 19662n^2 = 88725000$$

$$\Rightarrow 19662n^2 - 7373250n + 88725000 = 0$$

$$n = \frac{7373250 \pm \sqrt{(7373250)^2 - 4 \times 88725000 \times 19662}}{2 \times 19662}$$

$$= 12.4 \simeq \boxed{13 \text{ nos}}$$

so 14 nos of 12mmϕ is sufficient.

These bars should be extended by $l_0 = 300$ mm (d) beyond point of contraflexure.

* After this 300mm from PC in inward direction, curtail (50%) 7 nos of bars and continue with 7 bars of 12mmϕ through out the length to serve as an anchor bars for stirrups.

③ Reinforcement for sagging BM M_3 (near col A) →

$$M_3 = 0.87 f_y A_{st} \left(d - 0.42 \frac{0.87 f_y A_{st}}{0.36 f_{ck} b}\right)$$

$$\Rightarrow 15 \times 10^6 = 0.87 \times 250 \times A_{st} \left(300 - \frac{0.42 \times 0.87 \times 250 A_{st}}{0.36 \times 20 \times 1800}\right)$$

$$= 217.5 A_{st} (300 - 0.007 A_{st}) = 65250 A_{st} - 1.52 A_{st}^2$$

$$\Rightarrow 1.52 A_{st}^2 - 65250 A_{st} + 15 \times 10^6 = 0, \quad A_{st_3} = \frac{65250 \pm \sqrt{(65250)^2 - 4 \times 1.52 \times 15 \times 10^6}}{2 \times 1.52}$$

$$\Rightarrow A_{st_3} = \frac{65250 \pm 64547.37}{3.04} \Rightarrow A_{st} = 231.13 \text{mm}^2 \text{ or } 4269.5$$

$$\therefore A_{st_3} = 231.13 \text{mm}^2$$

minimum steel $= 0.15\%$ c/s area $= 0.15 \times 360 \times \frac{1800}{100} = 972 \text{mm}^2$

No of 12mmϕ bars $= 8.6 \simeq 9$ nos.

* No of bars should be sufficient from development length point of view
* At point of contraflexure (PC), near inner face of col. A, criteria for development length

$$\frac{M_1}{V} + L_0 \geq L_d$$

$$M_1 = 0.87 \times 250 \times n \times \frac{\pi}{4} \times 12^2 \left(300 - \frac{0.42 \times 0.87 \times 250 \times n \times \frac{\pi}{4} \times 12^2}{0.36 \times 20 \times 1800}\right) = 7373250n - 19662n^2$$

$V = $ SF at PC near A $= 360.8$ KN

$L_0 = 12\phi$ or d which is more $= 300$, $L_d = \frac{\phi \times 0.87 f_y}{4 \tau_{bd}} = 543.75$

$$\frac{7373250n - 19662n^2}{360.8 \times 10^3} + 300 = 543.75$$

$$\Rightarrow 7373250n - 19662n^2 = 87945000 \Rightarrow 19662n^2 - 7373250n + 87945000 = 0$$

$$\therefore n = \frac{7373250 \pm \sqrt{(7373250)^2 - 4 \times 19662 \times 87945000}}{2 \times 19662}$$

$$= \frac{7373250 \pm 6888259.374}{2 \times 19662} \Rightarrow n = 12.33 \simeq 13 \text{ nos.}$$

* To make no of reinforcement at both columns equal

* Let us have 14 no. of reinforcement ($\phi = 12mm$) are sufficient.

* These bars should be extended by $l_0 = d = 300mm$ beyond PC.

* After this 300mm from PC, in inward direction, curtail 50% i.e. 7nos. of bars and continue with 7nos. of bars of 12mm ϕ through out the length to serve as an anchor bars for stirrups.

~~check for one way shear~~

Transverse Reinforcement $\Rightarrow$

Near column. A $\rightarrow$

* The footing will bend transversely near each column face.

* Projection beyond column face (A) $= \frac{1800 - 300}{2} = 750mm. \Rightarrow a = 750mm.$

* width of bending strip $= b + 2d = 300 + 2 \times 300 = 900mm. \Rightarrow B_1 = 900mm.$

* width available to left of outer face of column A = 350mm. ∴ 350mm > d

∴ width of bending strip $= B_1 = 900mm$ (possible)

Net upward pressure $p_0' = \frac{\text{cal. load A}}{B \times B_1} = \frac{500}{1.8 \times 0.9} = 308.64 \, KN/m^2$

max. BM at face of col. A $= p_0' \frac{a^2}{2} = 308.64 \times \frac{0.75^2}{2} = 86.8 \, KNm.$

$$\therefore 86.8 \times 10^6 = 0.1487 f_{ck} bd^2 \Rightarrow d = \sqrt{\frac{86.8 \times 10^6}{0.1487 \times 20 \times 1000}} = 170.84mm.$$

But our provided d = 300mm which is sufficient enough.

So transverse beam is of same thickness as of rest of the footing.

But Transverse reinforcement is provided above longitudinal steel.

So available, d = 300 − 12 = 288 mm.

$$M_{max} = 0.87 f_y A_{st} \left(d - 0.42 \frac{0.87 f_y A_{st}}{0.36 f_{ck} b}\right) = 0.87 \times 250 \times A_{st} \left(288 - \frac{0.42 \times 0.87 \times 250 \times A_{st}}{0.36 \times 20 \times 1000}\right)$$

$$\Rightarrow 86.8 \times 10^6 = 217.5 A_{st} (288 - 0.013 A_{st}) = 62640 A_{st} - 2.83 A_{st}^2$$

$$\therefore 2.83 A_{st}^2 - 62640 A_{st} + 86.8 \times 10^6 = 0 \Rightarrow A_{st} = \frac{62640 \pm \sqrt{62640^2 - 4 \times 2.83 \times 86.8 \times 10^6}}{2 \times 2.83}$$

$\therefore A_{st} = 1485.4\,mm^2$

using 12mm ϕ, spacing, $S = \frac{1000 \times 113}{1485.4} = 76.07 \simeq \underline{75\,mm}$

For these transverse reinforcement, development length $= \frac{\phi \times 0.87 f_y}{4 \tau_{bd}} = \frac{12 \times 0.87 \times 250}{4 \times 1.2} = \underline{543.8\,mm}$

$\therefore$ development length $\simeq$ 550 mm. (required)

Projection beyond face of col = 750mm

Let clear cover for reinforcement on sides = 50mm.

$\therefore$ length of bar available = 750 − 50 + anchorage value of hook > required L_d (ok)

Near column B

Projection, $b = \frac{1.8 - 0.4}{2} = 0.7\,m$.

width of bending strip, $B_1 = b + 2d = 0.4 + 2 \times 0.3 = \underline{1m}$.

width available to right of outer face of col B = 0.9m > d, so ok.

$p_0' = \frac{\text{col load B}}{B \times B_1} = \frac{700 \times 10^3}{1.8 \times 1} = 388.9\,kN/m^2$

BM_{max} at face of col. B $= 388.9 \times \frac{0.7^2}{2} = 95.3\,kNm.$ $\left(\frac{p_0' b^2}{2}\right)$

$\therefore 95.3 \times 10^6 = 0.1487 f_{ck} b d^2 \Rightarrow d = \sqrt{\frac{95.3 \times 10^6}{0.1487 \times 20 \times 1000}} = 179\,mm.$

But provided d = 300mm so ok

So transverse beam is of same thickness as rest of footing.

Available d here = 300 − 12 = 288 mm.

$M_{max} = 0.87 f_y A_{st} \left(d - 0.42 \frac{0.87 f_y A_{st}}{0.36 f_{ck} b}\right) = 0.87 \times 250 A_{st} \left(288 - \frac{0.42 \times 0.87 \times 250 A_{st}}{0.36 \times 20 \times 1000}\right)$

$\Rightarrow 95.3 \times 10^6 = 62640 A_{st} - 2.83 A_{st}^2 \Rightarrow 2.83 A_{st}^2 - 62640 A_{st} + 95.3 \times 10^6 = 0$

$A_{st} = \frac{62640 \pm \sqrt{(62640)^2 - 4 \times 2.83 \times 95.3 \times 10^6}}{2 \times 2.83} = 1643.4\,mm^2$

using 12mm ϕ, spacing $= \frac{1000 \times 113}{1643.4} = 68.76\,mm. \simeq \underline{65\,mm\ c/c}.$

$\therefore$ Provide transverse steel of 12mmϕ @ 65mm c/c for the strip of 1m.

Required $L_d = \frac{\phi 12 \times 0.87 \times 250}{4 \times 1.2} = 543.8 \simeq 550\,mm.$ < projection $b = 0.7m$ (ok)

For the rest of footing provide transverse steel @ 0.15% c/s area (minimum A_{st})

$A_{st\,min} = \frac{0.15}{100} \times$ width × Depth $= \frac{0.15}{100} \times 1000 \times 360 = \underline{540\,mm^2}$

using 12mm ϕ, spacing $= \frac{1000 \times 113}{540} = 209.3\,mm \simeq \underline{200\,mm\ c/c}.$

check for one way shear :-

Near column B

(a) * In cantilever portion (in zone BB'),

test for one way shear (diagonal tension) is at a distance (d) from col. face of B

i.e. from centre of column B; distance = 0.2 + 0.3 = 0.5m.

Shear force, $V = -w \times (1.1 - 0.5) = -240 \times 0.6 = -144$ kN.

$$\tau_v = \frac{V}{bd} = \frac{144 \times 10^3}{1800 \times 300} = 0.27 \text{ N/mm}^2$$

Permissible shear stress $= k\tau_c$ (for solid slab) (P-84, Cl. B.5.2.1.1)

As slab depth (D) > 300 mm, k = 1

Assuming balanced section $P_{t\,lim} = 1.75\%$.

∴ Permissible τ_c = ~~0.47 N/mm2 (P-84, Table-23)~~

= 0.71 N/mm2 (P-73) T-19) IS 456

$> \tau_v$ (safe at footing near col. B.)

(b) Shear force near col. A < col B, so safe, i.e. $\tau_c > \tau_v$.

(c) Diagonal tension between Col. A & B, near col. B

crack may occur at bottom face of footing (i.e. for sagging BM) at a distance d = 0.3m or

at top of footing (i.e. for hogging B.M) near point of contraflexure at a distance ~~0.3m~~ of 0.3m from centre of col B which ever is nearer.

* SF at point of contraflexure = 364 kN which is more.

$$\tau_v = \frac{364 \times 10^3}{bd} = \frac{364 \times 10^3}{1800 \times 300} = 0.674 \text{ N/mm}^2$$

∴ $\tau_v \not< \tau_c$ (0.71 N/mm²) ⇒ stirrups not required

$$M_u = 0.87 f_y A_{st} (d - 0.42 x_u) \qquad \left[P_t = \frac{100 A_{st}}{bd}\right]$$

$$= \frac{0.87 f_y\, bd P_t}{100}\left(1 - 0.42\frac{x_u}{d}\right)d$$

$$= \frac{0.87}{100} P_t f_y\, bd^2 \left(1 - 0.42\frac{x_u}{d}\right)$$

$$\Rightarrow P_t f_y = \frac{100}{0.87}\,\frac{M_u}{bd^2\left(1 - 0.42\frac{x_u}{d}\right)}$$

$$= \frac{100}{0.87}\,\frac{0.36 f_{ck} b x_u (d - 0.42 x_u)}{bd^2\left(1 - 0.42\frac{x_u}{d}\right)}$$

$$\Rightarrow \frac{P_t f_y}{f_{ck}} = \frac{100 \times 0.36}{0.87}\,\frac{bd^2 \frac{x_u}{d}\left(1 - 0.42\frac{x_u}{d}\right)}{bd^2\left(1 - 0.42\frac{x_u}{d}\right)}$$

$$= 41.4 \frac{x_u}{d} \Rightarrow \frac{P_t f_y}{f_{ck}} = 41.4\frac{x_u}{d}$$

for limiting moment, $P_{t\,lim}$, & $x_{u\,max}$

for M 20, Fe 250, $\frac{x_{u\,max}}{d} = 0.53$

$$\therefore P_{t\,lim} = \frac{41.4 \times 0.53 \times 20}{250} = 1.75\%$$

$$\boxed{\frac{x_{u\,max}}{d} = \frac{0.0035}{0.0035 + 0.002 + \frac{0.87 f_y}{E_s}}}$$

using 12 mm φ 8 legged stirrups, $A_{sv} = 8 \times 113 = 904$ mm²

$$\times\ \boxed{\text{Spacing} = \frac{0.87 f_y A_{sv} d}{V_{us}}}$$

Cl 40.4 P-72

$(V_{us} = V - \tau_c bd)$ for vertical stirrups

So minimum shear steel provided (Cl. 26.5.1.6, P = 48), $S_v = \frac{0.87 f_y}{0.4}\frac{A_{sv}}{b} = \frac{0.87 \times 250 \times 904}{0.4 \times 1800} \approx 270$ mm

SF for $\tau_c = 0.71$ N/mm² ⇒ SF = $0.71 \times bd = 0.71 \times 1800 \times 300 = 383.4$ kN.

This SF is at a distance ~~1.0815 m of~~ ~~436~~ $\frac{1.815}{436} \times 383.4 = 1.60$ m from '0' [1.815 = 5 − 0.5 − 1.585 − 1.1]

i.e. 0.215 from centre of B

from 0 to 1.6m toward right of 0, minimum stirrups of 12 mm φ 8 legged @ 270 mm c/c is provided.

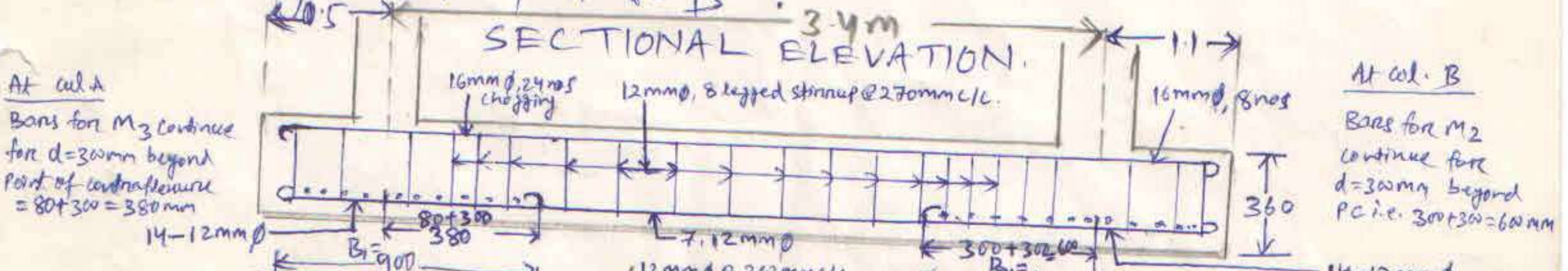
* Near column A, tension crack may occur in hogging portion at inner face of col, since point of contraflexure occurs under col. A.

SF at inner face of col. A = $\frac{380}{1.585} \times 1.435$ = 344 kN.

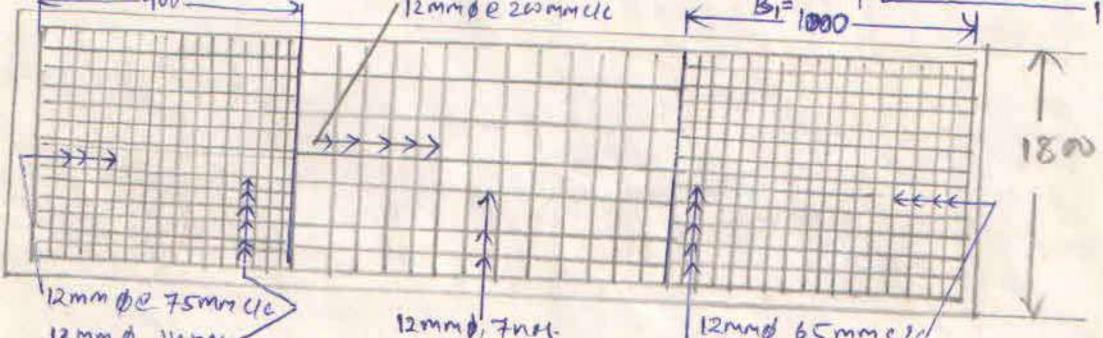
$$\tau_v = \frac{344 \times 10^3}{1800 \times 300} = 0.64\ N/mm^2$$

$$\tau_c = 0.71\ N/mm^2 > \tau_v .$$

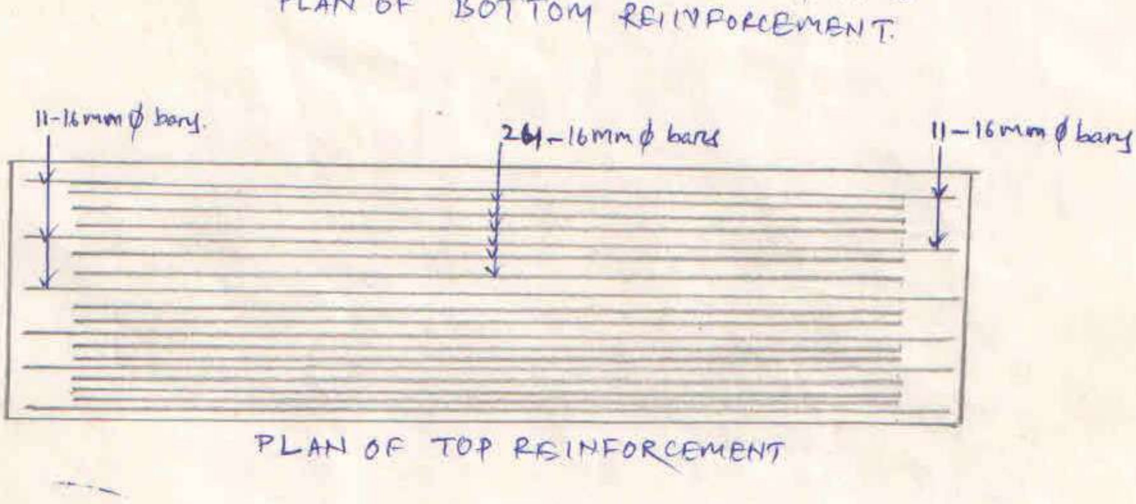
∴ provide minimum stirrups of 12mm φ @ 8 legged stirrups @ 270mm c/c all along ~~the~~ from A to B.



SECTIONAL ELEVATION.



PLAN OF BOTTOM REINFORCEMENT.



PLAN OF TOP REINFORCEMENT

Advanced Concrete Structures

DESIGN
OF
RAFT FOUNDATION

Dr. S. K. Panigrahi
Associate Professor
Department of Civil Engineering
VSSUT, Burla

RAFT
FOUNDATION

Raft foundation

09.01.07

* A combined footing supports load of two/more adjacent columns.

* combined footing is provided where

- → columns are very close to each other so that their footings overlap.
- → when bearing capacity of soil is less, which needs more area under individual footing.
- → when end column is very near to property line, so that the footing cant spread in that direction.

* combined footing may be < rectangular / trapezoidal.

* The aim is to get uniform pressure distribution under this footing.

* The diff types of combined footings →

- Rectangular combined footing
- Trapezoidal " "
- strap footing
- Raft/mat footing.

Raft /mat

① * If load transmitted by columns are so heavy / allowable soil pressure are so small that individual footing would cover more than about one half of the area,

then it is better to provide a continuous footing under all columns & ~~beams~~ walls. Such footing is called Raft/mat foundation.

② * Raft foundation are used to reduce the settlement of structure located above highly compressible deposite i.e. controls differential settlement.

③ Next page bottom

* Large volume of excavation is required for Raft foundation.

- For above reason nearly double the settlement is allowed for raft foundation than that in normal footings.

* If wt of excavated soil ≤ wt of structure + Raft and CG of excavation & structure coincides,

settlement is negligible & pressure distribution is uniform.

* A Raft may be rectangular / circular and may be with / without an opening.

* If columns are equally spaced & loads aren't heavy, raft is designed as of uniform thickness.

* If columns are equally spaced & loads are equal, pressure on soil uniform, otherwise moment of loads about centre of base is taken & pressure distribution by $\boxed{\sigma = \frac{P}{A} \pm \frac{M_x}{I_x} y \pm \frac{M_y}{I_y} x.}$

This is for rigid member but raft is not a rigid member.

Solution will have error if eccentricity is very large.

* Raft may be ribbed where column spacing is irregular / for economy in using a relatively thin slab over most of the area.

or, raft may be thickened at column locations for economy & depth should be sufficient to resist shear.

* A ribbed raft foundation consists of a slab acted upon by upward soil pressure at its underside & supported by beams from column to column at its top which balance the upward pressure with downward col. load.

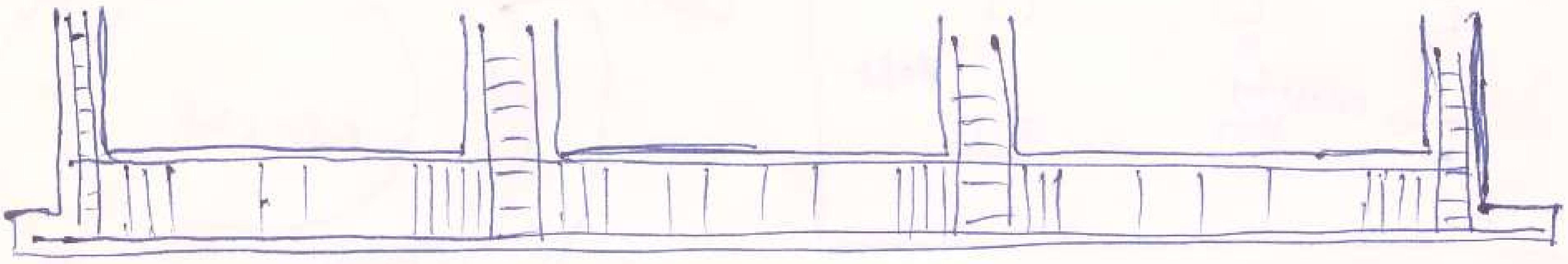
* It's similar to floor slab resting on beams & columns.

* Portion betn beams is designed as oneway / two way slabs.

* { The weight of the raft is not considered in structural design becoz it is assumed to be carried directly by subsoil.

3 * When hard soil is not available within 1.5 to 2.5m, a raft foundation is adopted.

* The raft is composed of RC beam with a relatively thin slab below it.



$$\boxed{\begin{aligned}\frac{x_{u\,max}}{d} &= 0.53 \text{ — Fe250}\\ &= 0.48 \text{ — Fe415}\\ &= 0.46 \text{ — Fe500}\end{aligned}}$$

* $\frac{x_u}{d} < \frac{x_{u\,max}}{d} \Rightarrow M_u = 0.87 f_y A_{st} (d - 0.42 x_u)$ — UR sec

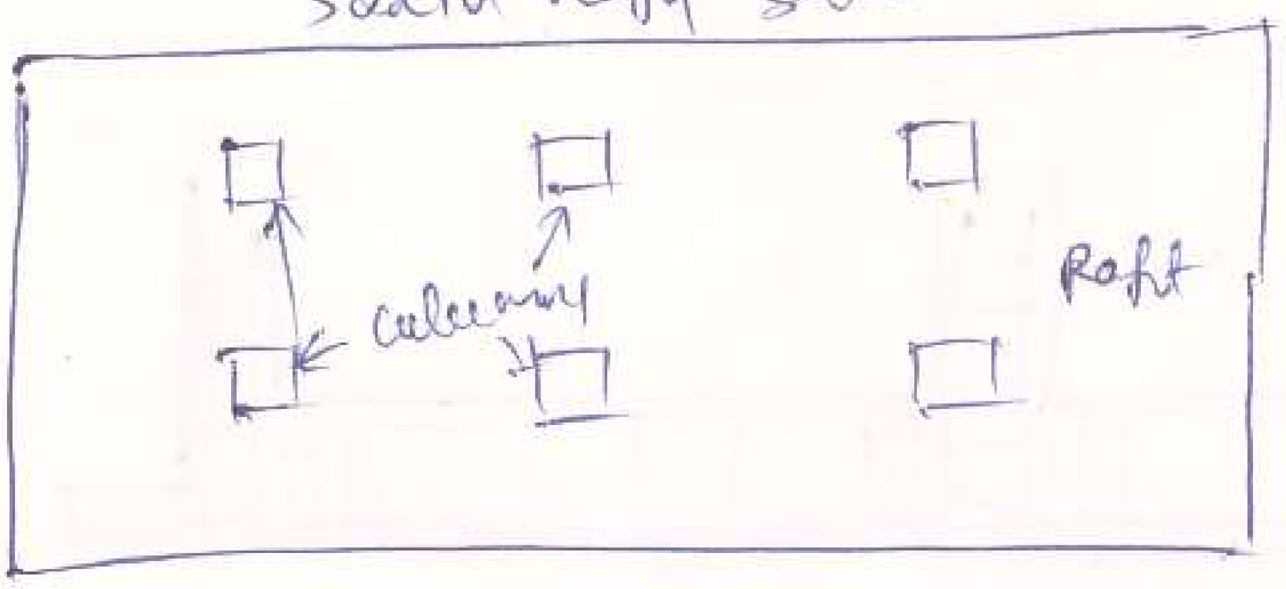
* $\frac{x_u}{d} = \frac{x_{u\,max}}{d} \Rightarrow M_u = 0.36 f_{ck} b x_{u\,max} (d - 0.42 x_{u\,max})$ — balanced condition.

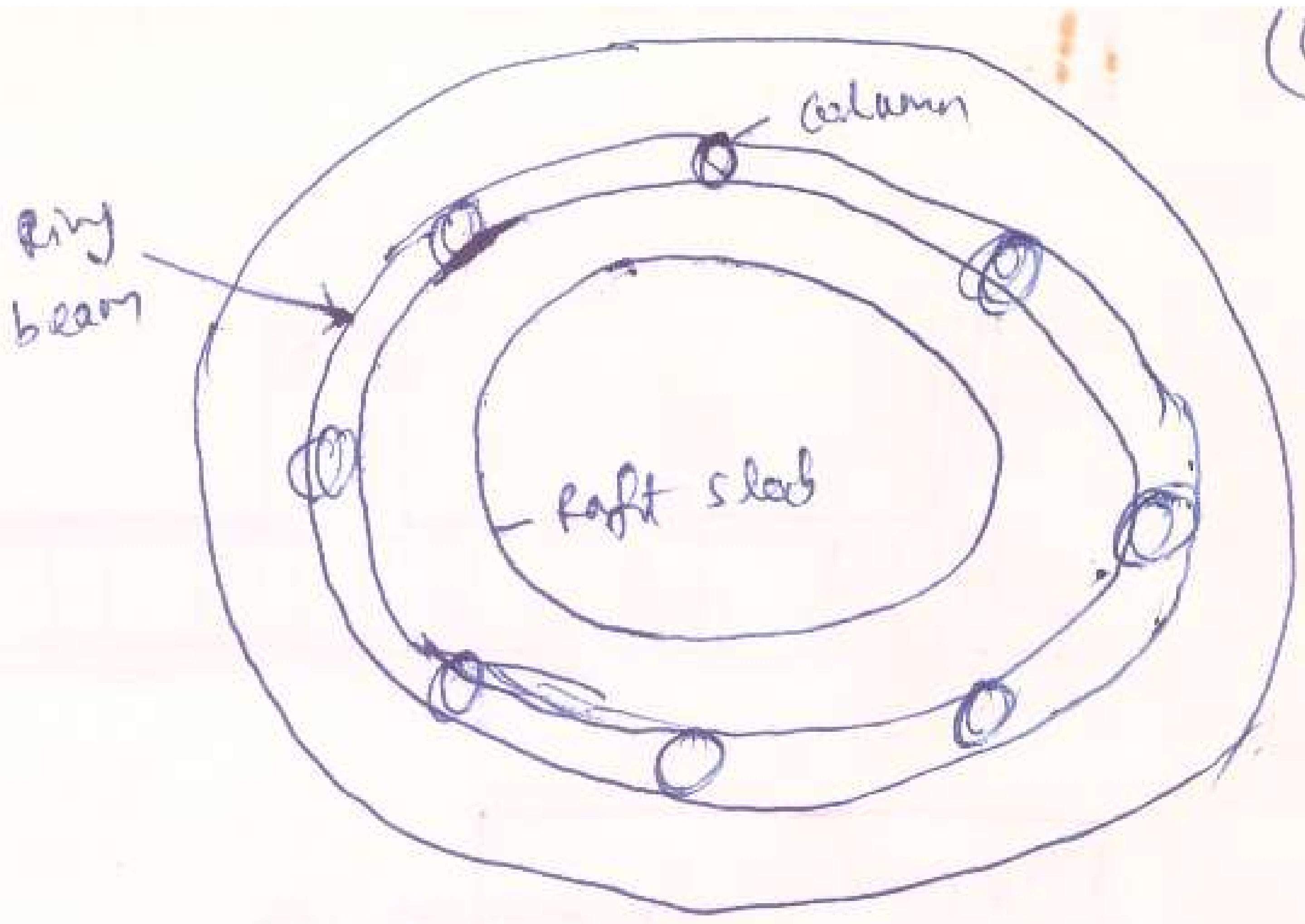
$$\boxed{\begin{aligned}M_u &= 0.149 f_{ck} bd^2 \text{ — Fe 250}\\ &= 0.138 f_{ck} bd^2 \text{ — Fe 415}\\ &= 0.133 f_{ck} bd^2 \text{ — Fe 500.}\end{aligned}}$$

$$\boxed{\frac{x_{u\,max}}{d} = \frac{0.87 f_y A_{st}}{0.36 f_{ck} b d}}$$

* $\frac{x_u}{d} > \frac{x_{u\,max}}{d} \Rightarrow M_u = 0.36 f_{ck} b x_u (d - 0.42 x_u)$

Solid raft slab





Q. Design a raft foundation supporting the columns of a building. Load on each column is 400 kN. M 20 & Fe 415. Safe bearing capacity of soil = 120 kN/m2.

given
beam size = 350×350 mm
c/c spacing of col = 3m

Sol → Total load on columns = 12 × 400 = 4800 kN.

App. wt of foundation (10%) = 480 kN

Total load = 5280 kN.

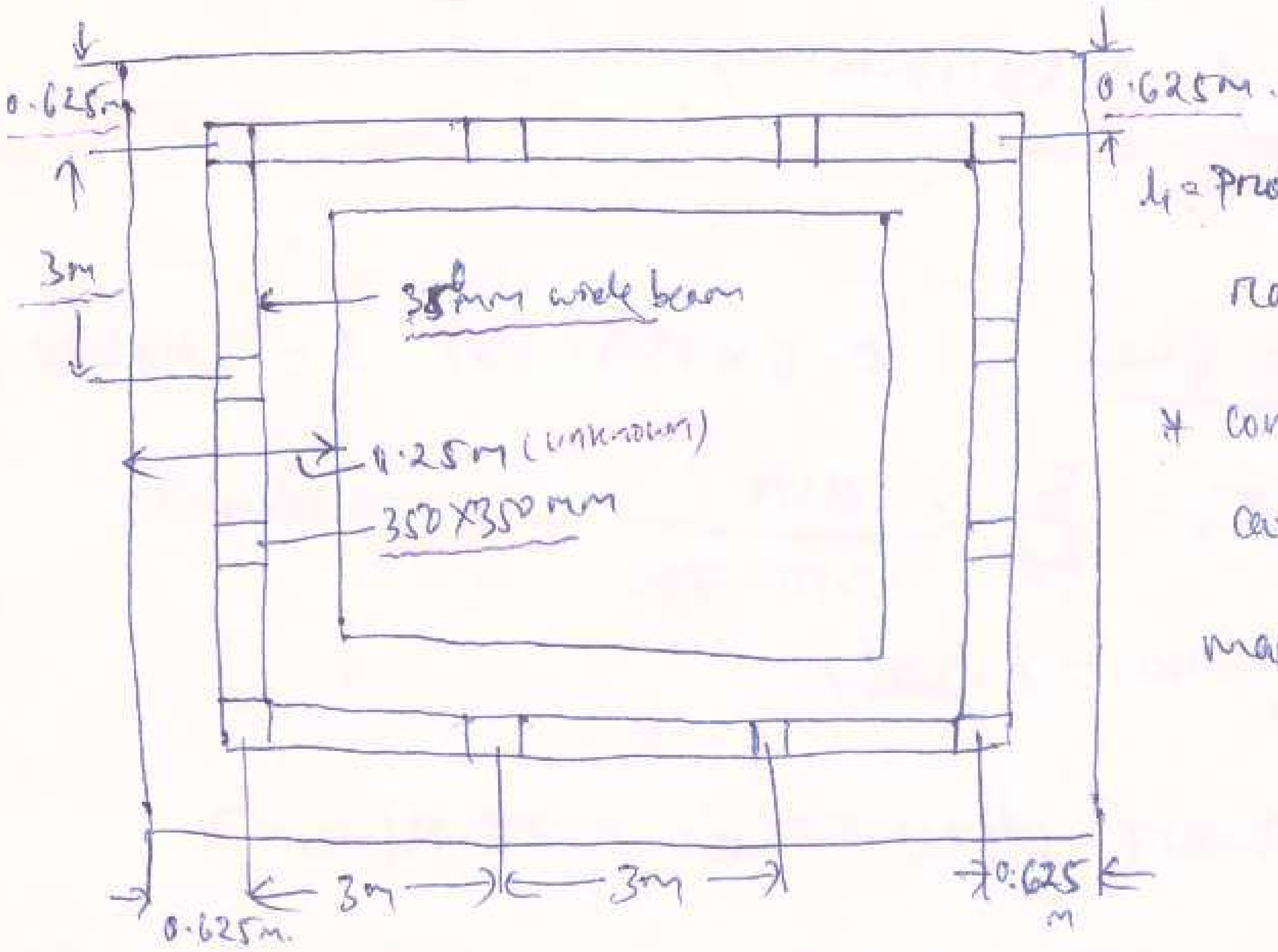
Area of raft foundation = $\frac{5280}{120} = 44 m^2$.

Total length of raft slab = ~~(12 × 3) = 36m~~ (along along length all the round)
= 9×4 = 36m

width required for raft slab = $\frac{44}{36} = 1.22 m$.

Provide a width of 1.25m for raft slab.

w = Net upward pre. intensity on raft slab = $\frac{4800}{36 \times 1.25} = 106.67 kN/m^2$



Design of Raft slab

l_1 = Projection of Raft slab from face of raft beam = $\frac{1.25 - 0.35}{2} = 0.45m$.

* consider 1m wide strip of raft slab cantilevering from face of beam.

max. BM = $\frac{wl_1^2}{2} = \frac{106.67 \times 0.45^2}{2}$

= 10.8 kN-m.

$\sigma_{cbc} = 7 N/mm^2$, $\sigma_{st} = 230 N/mm^2$, m = 13.

equating MR to BM ⟹ $0.91 bd^2 = 0.91 \times 1000 \times d^2 = 10.8 \times 10^6$

⟹ d = 108.9mm.

Provide an effective cover 60mm

overall depth = 108.9 + 60 = 169 ≃ 170 mm

∴ D = 170
d = 170 - 60 = 110 mm.

$A_{st} = \frac{10.8\times10^6}{230\times0.9\times110} = 401\ mm^2$.

Spacing of 10mm ϕ bars = $\frac{79\times1000}{401} = 197$ mm

∴ provide 10mm ϕ bars @ 190mm c/c (along 0.45m projection length)

Design of continuous Raft beam =>

w_1 = upward load transmitted to beam/m = 106.67 × 1.25 = 133.3 kN/m

$BM_{max} = \frac{w_1 l^2}{10} = \frac{133.33\times3^2}{10} = 120$ kNm.

$0.9\,bd^2 = 0.9\times350\,d^2 = 120\times10^6 \Rightarrow d = 726$ mm.

Provide 16mm dia bars.

~~D~~ D = 726+60 = 786 ≈ 790mm.

$A_{st} = \frac{120\times10^6}{230\times0.9\times730} = 798\ mm^2$.

Provide 4 bars 16mm ϕ (804 mm²).

Design of shear

max shear = S = 0.6wl = 0.6 × 133.33 × 3 = 240kN.

nominal shear stress = $\tau_v = \frac{S}{bd} = \frac{240}{350\times730} = 0.94\ N/mm^2$.

steel % age = $\frac{804}{350\times730}\times100 = 0.32\%$.

Permissible nominal shear stress = $\tau_c = 0.25\ N/mm^2$

Shear resistance of concrete = $S_c = \tau_c bd = 0.25\times350\times730 = 63612.5$ N.

max SF = S = 240×10^3 N.

Net SF for which shear rein. required = $V_s = 240\times10^3 - 63612.5$
$= 176387.5$ N.

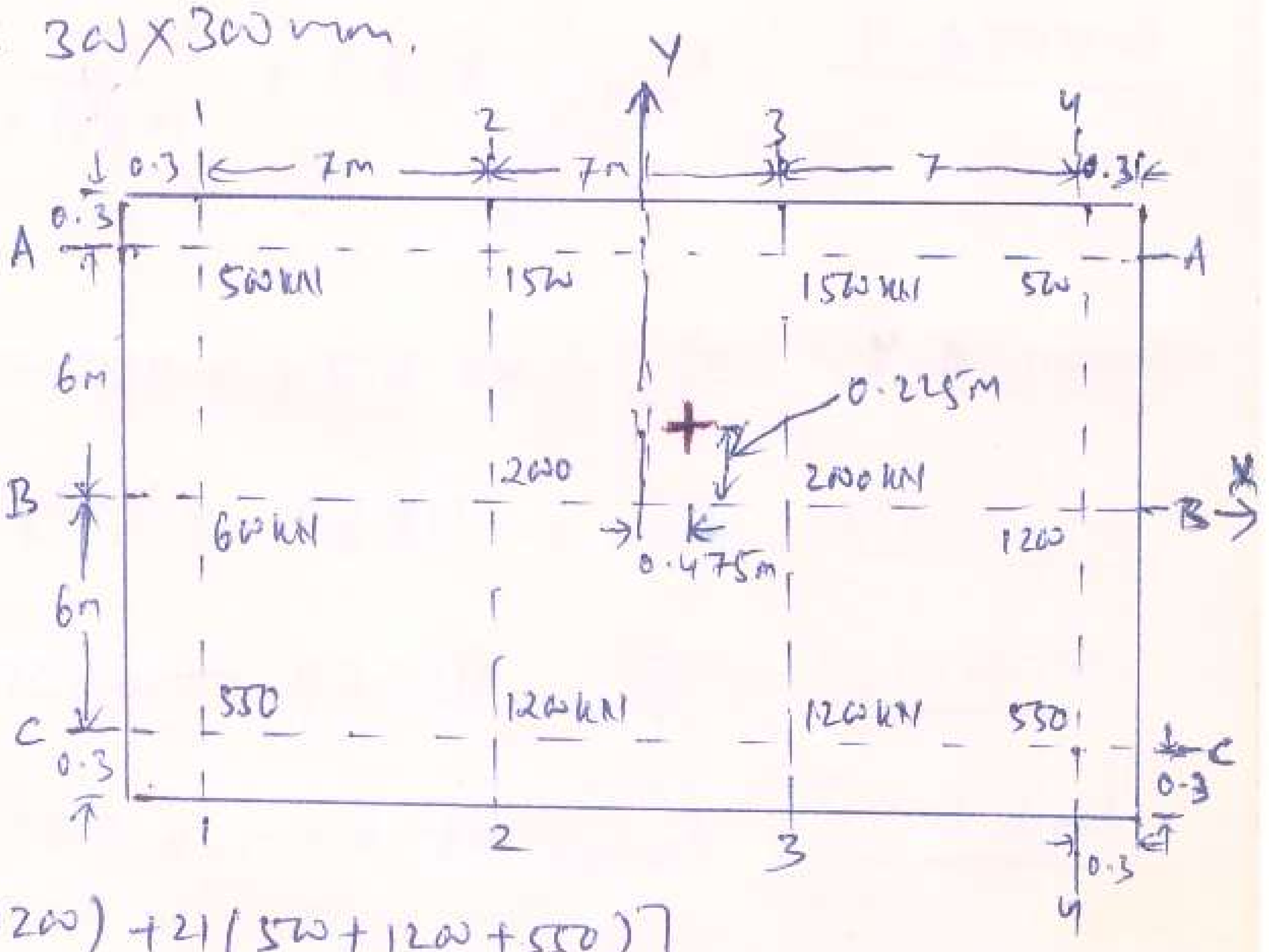
Spacing of 4 legged 10mm ϕ stirrups.

$= \frac{4\times79\times140\times730}{V_s} = 182$ mm

∴ provide 4 legged 10mm ϕ stirrups @ 180mm c/c.

|| size of footing given ||

Q. Design a raft foundation for the given layout. Net bearing capacity of soil is 65 kN/m² & column size is 300 X 300 mm.



Sol → Total vertical column load =

∴ $P = 1500 + 600 + 550 + 1500 + 2000 + 1200 + 1500 + 2000 + 1200$
$500 + 1200 + 550 = 13300$ kN.

∴ $P = 13300$ kN.

Eccentricity along x-direction is obtained by taking moment of col. loads about grid 1-1.

$$\bar{x} = \frac{[7(1500+2000+1200)+14(1500+2000+1200)+21(500+1200+550)]}{13300} = 10.975$$

$$e_x = 10.975 - 10.5 = 0.475\,m.$$

Eccentricity along y-direction is obtained by taking moment of col. loads about grid C-C.

$$\bar{y} = \frac{6(600+1200+2000+1200)+12(500+1500+1500+500)}{13300} = 6.225$$

$$e_y = 6.225 - 6 = 0.225\,m.$$

$$I_x = \frac{21.6 \times 12.6^3}{12} = 3600.68\,m^4$$

$$I_y = \frac{12.6 \times 21.6^3}{12} = 10581.58\,m^4$$

$$A = 12.6 \times 21.6 = 272.16\,m^2$$

$$M_x = P e_y = 13300 \times 0.225 = 2992.5\,kNm$$

$$M_y = P e_x = 13300 \times 0.475 = 6305.0\,kNm.$$

$$\frac{P}{A} = \frac{13300}{272.16} = 48.87\,kN/m^2$$

Soil pressure →

Soil pressure at different points, $\sigma = \frac{P}{A} \pm \frac{M_y}{I_y} x \pm \frac{M_x}{I_x} y$.

Corner A-4 $\quad \sigma_{A-4} = 48.87 + \frac{6305}{10581.58} \times 10.8 + \frac{2992.5}{3600.68} \times 6.3 = 48.87 + 6.435 + 5.236$

$= \underset{\sim\sim}{60.541} < 65\ kN/m^2$

Corner C-4 $\quad \sigma_{C-4} = 48.87 + 6.435 - 5.236 = \underset{\sim\sim}{50.069}\ kN/m^2$

Corner A-1 $\quad \sigma_{A-1} = 48.87 - 6.435 + 5.236 = 47.671\ kN/m^2$

Corner C-1 $\quad \sigma_{C-1} = 48.87 - 6.435 - 5.236 = 37.119\ kN/m^2$

Grid B-4 $\quad \sigma_{B-4} = 48.87 + 6.435 + 0 = \underset{\sim\sim}{55.305}\ kN/m^2$

Grid B-1 $\quad \sigma_{B-1} = 48.87 - 6.435 + 0 = 42.435\ kN/m^2$

✻ In X-direction, raft is divided in 3 strips, i.e. 3 equivalent beams.

(i) Beam A-A with 3.3 m width & soil pressure $\underline{60.541\ kN/m^2}$

(ii) Beam BB with 6 m width & soil pre. of $\frac{1}{2}(60.541 + 55.305) = \underline{57.92\ kN/m^2}$

(iii) Beam C-C with 3.3 m width & soil pre. of $\frac{1}{2}(55.305 + 50.069) = 52.69\ kN/m^2$

Bending moment →

✻ Bending moment is obtained by using coefficient $\frac{1}{10}$ & span (l) as centre to centre of column spacing/distance. $\quad +M = -M = \boxed{M = \frac{wl^2}{10}} \longrightarrow$ in x-direction.

For strip A-A → $\quad$ max. moment $= 60.541 \times \frac{7^2}{10} = \underline{296.6}\ kNm/m$ (Max M_x)

strip B-B → $\quad$ max moment $= 57.92 \times \frac{7^2}{10} = 283.8\ kNm/m$.

strip C-C → $\quad$ max. moment $= 52.69 \times \frac{7^2}{10} = 258.2\ kNm/m$.

✻ For any strip in Y-direction coefficient $= \frac{1}{8}$, $\boxed{BM = \frac{wl^2}{8}} \to$ as there is only a 2 span equivalent beam.

For strip 4-4 → $\quad$ max moment $= \frac{60.54 \times 6^2}{8} = \underset{\sim\sim}{272.4}\ kNm/m$. & [max pressure $= \underline{60.54}\ kN/m^2$]

and so on in Y direction.

This is the max. moment in y-direction.

Depth of the slab

* The depth of the raft is governed by 2-way shear at one of the exterior columns.

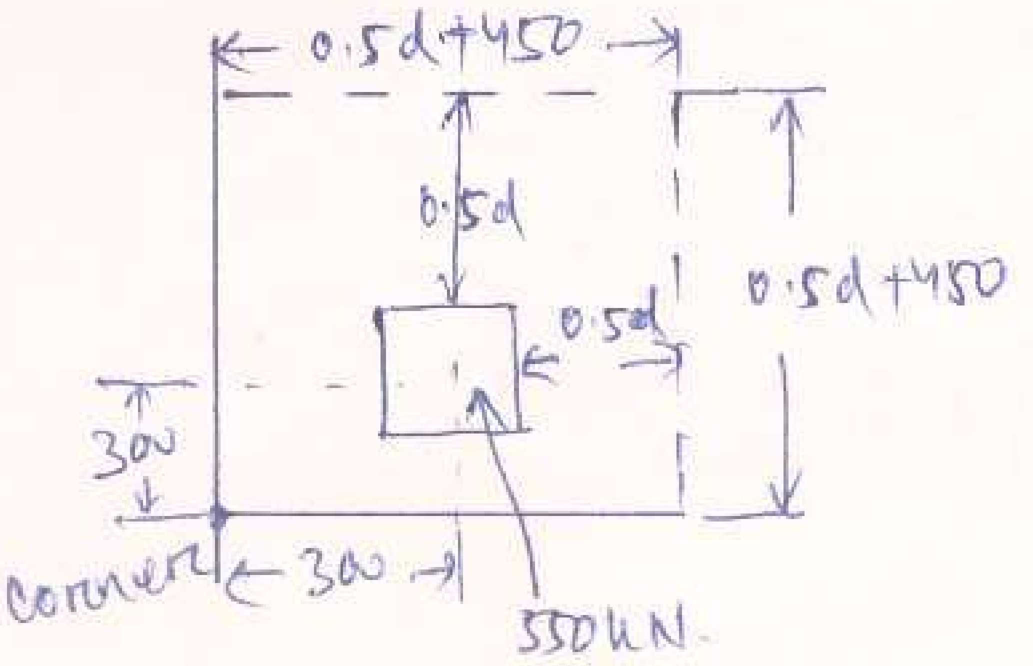
* If critical ~~section~~ shear location is not obvious, it may be necessary to check all possible locations.

* As per IS 456-2000, Cl. 31.6.3.1 P(59)

31.6.3.1 (P-58)

Shear strength of concrete, $\tau_c' = \tau_c = 0.25\sqrt{f_{ck}} = 0.25\sqrt{20} \doteq 0.97$ N/mm2.

for a corner column let C-1 →



Perimeter of critical section

$$b_o = 2(0.5d+450) = d+900$$

col load

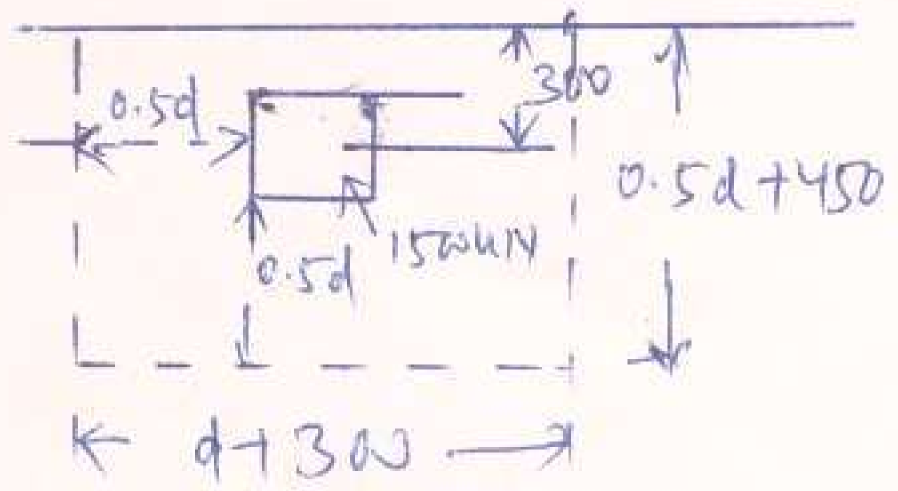
$$\tau_v = \frac{V_u}{b_o d} = \frac{1.5\times550\times1000}{(d+900)d} = \tau_c = 0.97. \quad = 0.25\sqrt{f_{ck}} = 0.25\sqrt{20} = 1.12$$

$$\Rightarrow 825\times1000 = 0.97d^2 + 0.97\times900d$$

$$\Rightarrow d^2 + 900d - 850515 = 0$$

$$\Rightarrow d = \frac{-900 \pm \sqrt{810000 + 4\times850575}}{2} \Rightarrow d = \cancel{576} \xrightarrow{} 817\text{mm}$$

for a side column let A-2 →



$$b_o = 2[(0.5d+450)] + (d+300) = 2d+1200$$

$$\tau_v = \frac{V_u}{b_o d} = \frac{1.5\times1500\times1000}{(2d+1200)d} = \tau_c = \cancel{0.97} \to 1.12$$

$$\Rightarrow d^2 + 600d - 1159800 = 0 \quad \Rightarrow d = \cancel{817}\text{mm} \to 1.669\text{ m}$$

Let $d_{eff} = 820$ & $D = 860$mm.

Steel provision

In long direction → $M = 0.87 f_y A_{st}(d - 0.42x_u) = 0.87 f_y A_{st}\left(d - 0.42\times\frac{0.87 f_y A_{st}}{0.36 f_{ck} b}\right)$

$$\Rightarrow M = 0.87 f_y A_{st}\left(d - \frac{f_y A_{st}}{f_{ck} b}\right)$$

⇒ Maximum moment in x-direction = 296.6 kNm/m.

$$\therefore 296.6\times10^6 = 0.87\times415 A_{st}\left(820 - \frac{415\times A_{st}}{20\times1000}\right) \Rightarrow A_{st} = 1040 \text{ mm}^2/\text{m}.$$

IS 456-2000, Cl. 26.5.2.1, minimum reinforcement $= 0.12\% = \frac{0.12}{100}\times860\times1000$
$= 1030 < 1040$ mm²/m.

* minimum steel will govern in remaining part.

* Provide 20mm ϕ bars @ 300mm c/c ($A_{st} = 1050$mm²) at top & bottom in both direction.

$$\frac{(131.25 \times 0.5 \times 4.571) - 150}{4.571} = 262.53.$$

65.62

328.125 kN/m.

$$\left[\frac{\left(\underset{w}{131.25} \times 0.5 \times \underset{(w)}{4.571}\right)}{4.571}\right] = 65.625 \text{ kN/m width}$$

|| Size of footing is not given !!!

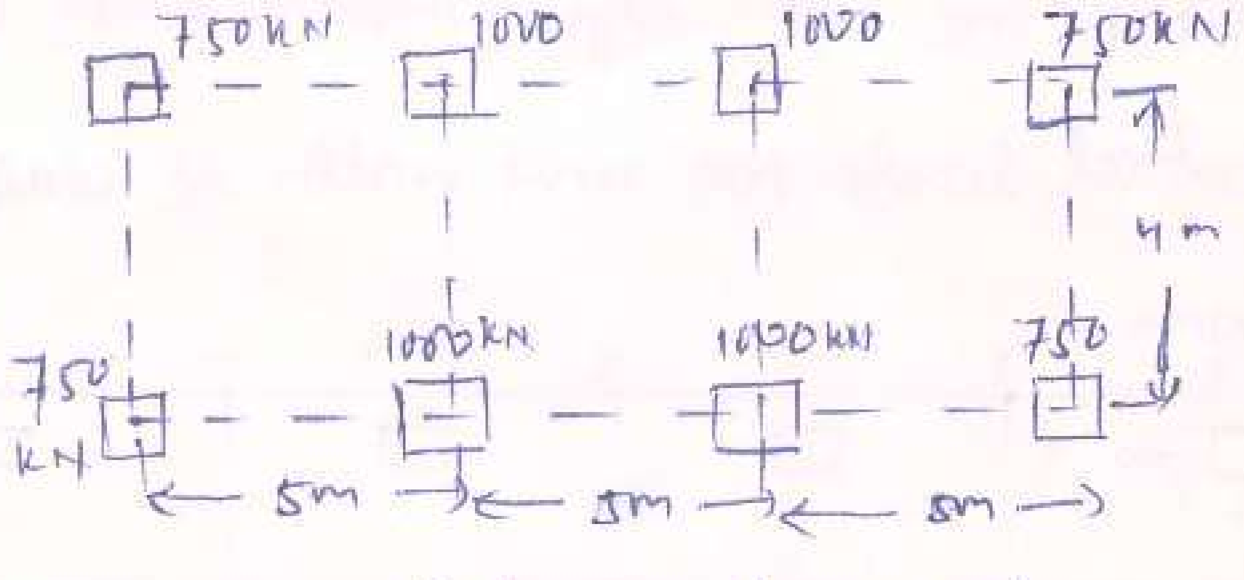
(10

Q Design a mat foundation for columns shown. The column size is 400×400mm. consider sp. wt of soil $\gamma = 20 kN/m^3$, Angle of repose = 30° Allowable bearing capacity of soil, $q_0 = 100 \, kN/m^2$, M-20, Fe415.

sol → Depth of foundation →

$$h = \frac{q_0}{\gamma}\left(\frac{1-\sin\phi}{1+\sin\phi}\right)^2 = \frac{100}{20}\left(\frac{1-\sin 30}{1+\sin 30}\right)^2$$

$\Rightarrow h = 0.56m.$



(columns layout)

The base of foundation is located at a ~~distance of~~ depth of 1m below which soil is not subjected to seasonal vol. changes by alternate wetting & drying.

Footing Dimension →

$$\text{Area of footing, } L \times B = \frac{\text{total col loads + self wt of mat } (\approx 10\% \text{ of col load})}{\text{allowable bearing capacity}}$$

$$\text{Area of footing} = \frac{(4\times750 + 4\times1000) + 0.1(4\times750 + 4\times1000)}{100} = 77m^2$$

Let footing dimension is 16m × 5m.

The CG of footing coincides with the total load from columns.

Thickness of footing →

- It is governed either by
  - maximum moment
  - maximum one way shear.

- maximum moment & max. one way shear at critical section can be determined by analysis of mat foundation idealised as longitudinal & transverse strips.

- width of strip are determined such that upward soil pressure balance the downward load from columns.

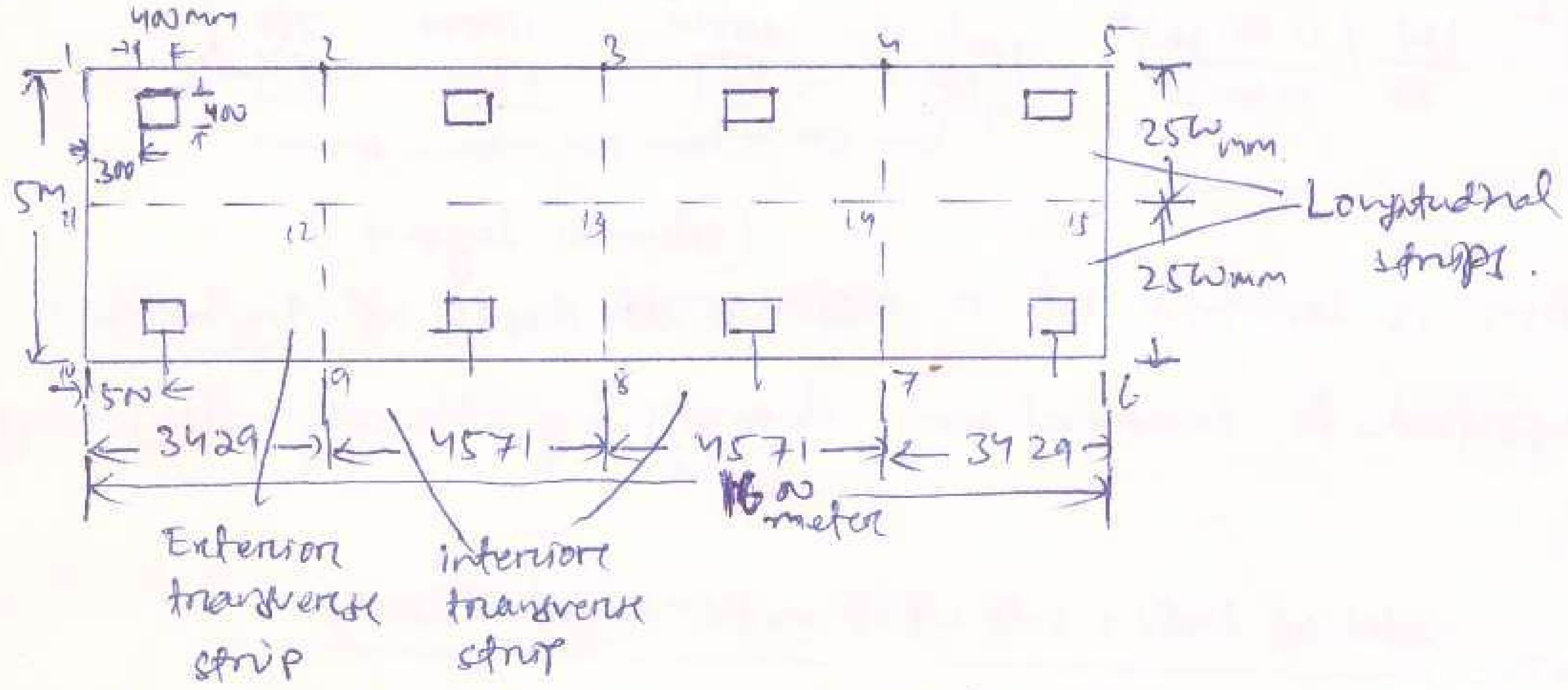
- The two longitudinal strips are identical as identical equal loads on them from columns.

- The width of transverse edge & interior strips are found as! →

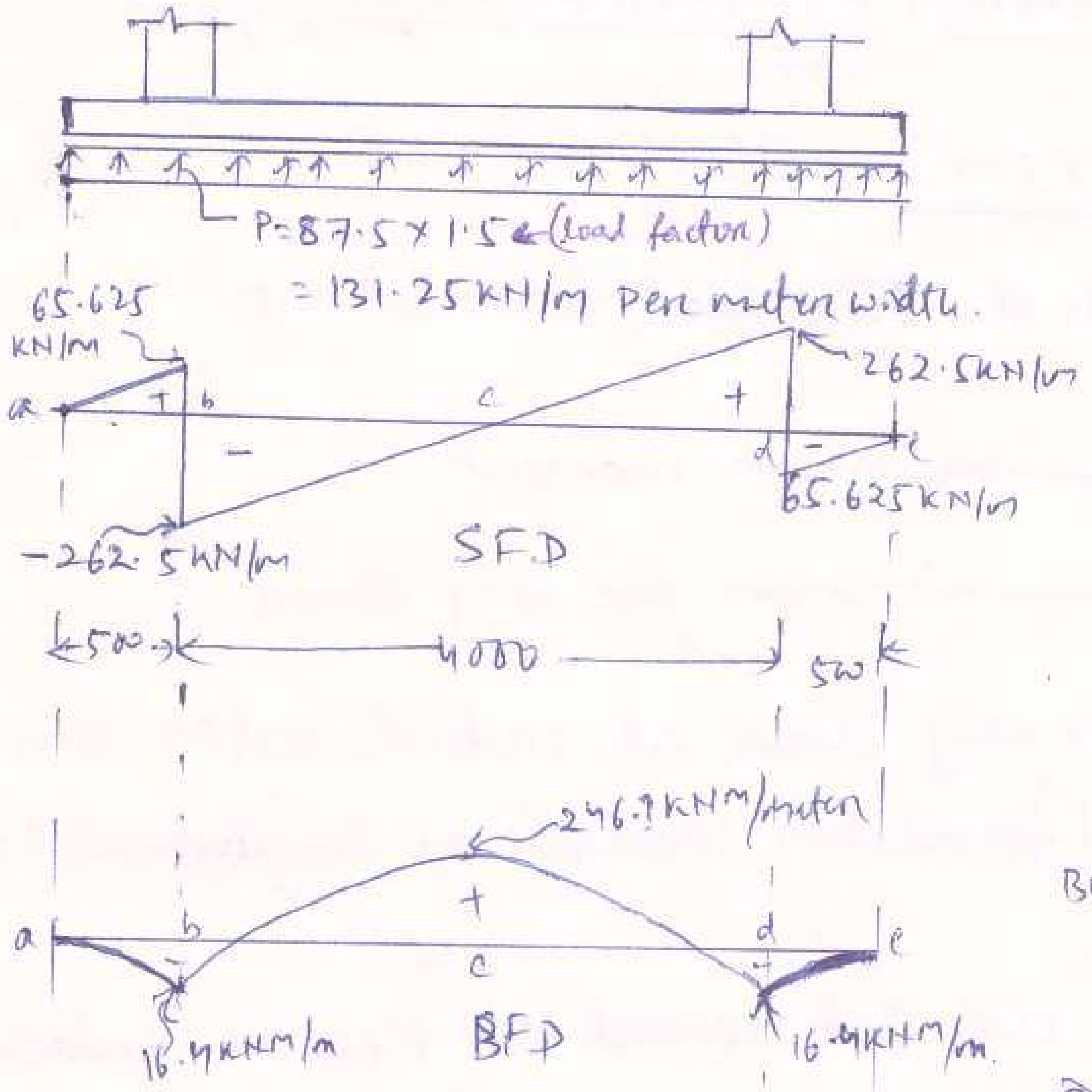
$$b = \frac{\text{load from col. on these strip}}{\text{length of strip} \times \text{net upward soil pre.}} \quad \left[\text{net upward soil pre.}(P) = \frac{4\times750 + 4\times1000}{16\times5} \right. \Rightarrow P = 87.5 \, kN/m^2$$

width of edge transverse strip $= \frac{2\times750}{5\times87.5} = 3.429m$, width of ~~edge~~ intermediate transverse strip $= \frac{2\times1000}{5\times87.5} = 4.571 \, m.$

- The analysis for longitudinal & transverse strips are made for ultimate bending moment & shear force.
- The BM & SF are expressed as per unit width of strips.
- BM & SF for both edge & intermediate transverse strips are identical because of identical loads per unit width of strips.



(Longitudinal & transverse strips)



[Intermediate transverse strip
2389 or 3478.]

$$\text{SF at b(left)} = \frac{131.25 \times 0.5 \times 4.571}{4.571} = 65.625 \text{ KN/m}$$

$$\text{SF at b(right)} = \frac{(131.25 \times 0.5 \times 4.571) - (\overset{\text{load factor}}{1.5} \times \underset{\text{column load}}{1000})}{4.571}$$

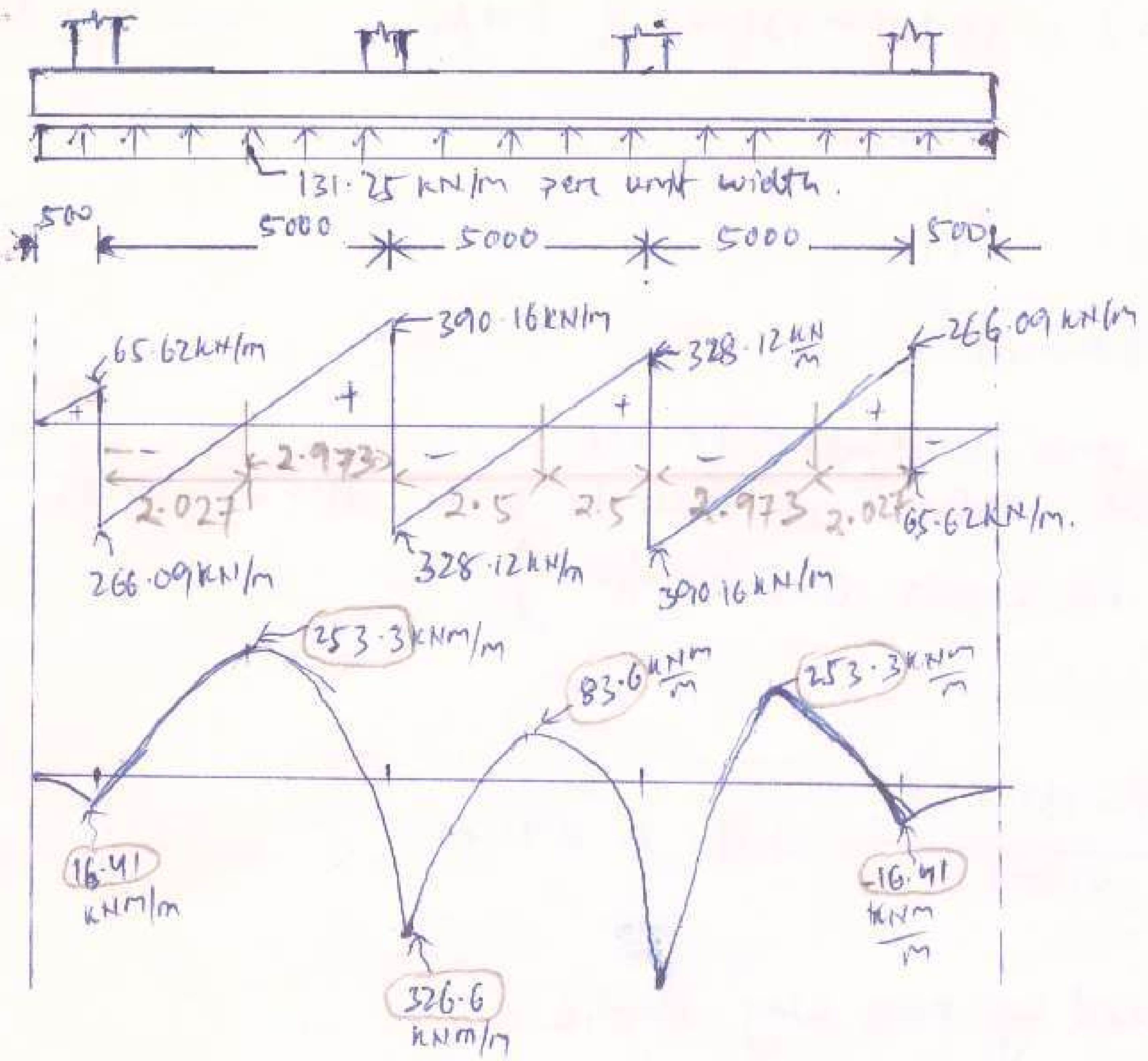
$$= -262.53 \text{ KN/m width}$$

$$\text{BM at b(left)} = \left(\frac{131.25 \times 0.5 \times \overset{\text{span}}{4.571}}{4.571}\right) \times \frac{\overset{\text{span}}{0.5}}{2} = -16.4 \text{ KNm/m}$$

~~BM at c = $\frac{131.25 \times 2.5 \times 4.571 \times \frac{2.5}{2} - \overset{(1000 \times 1.5)}{1500} \times 2}{4.571}$ = 246.1 kNm/m width~~

$$\text{BM at c} = \frac{131.25 \times 2.5 \times 4.571 \times \frac{2.5}{2} - \overset{(1000 \times 1.5)}{1500} \times 2}{4.571}$$

$$= 246.1 \text{ KNm/m width}$$



BMD for longitudinal strip 12345. 11,12,13,14,15

Thickness of footing based on moment

deff can be calculated by considering singly reinforced balanced section :-

$$M_{u\,max} = 0.36 f_{ck}\, b d \frac{x_{u\,max}}{d}\left(1-0.42\frac{x_{u\,max}}{d}\right)\times d = 0.36 f_{ck}\, b d^2 \frac{x_{u\,max}}{d}\left(1-0.42\frac{x_{u\,max}}{d}\right)$$

$$\Rightarrow bd^2 = \frac{M_{u\,max}}{0.36 f_{ck}\frac{x_{u\,max}}{d}\left(1-0.42\frac{x_{u\,max}}{d}\right)}$$

$M_{u\,max}$ = max. ultimate moment in longitudinal & transverse strips = 326.6 kNm/m

$\frac{x_{u\,max}}{d} = 0.48 \rightarrow$ Fe415 , b = 1m

$$1000\times d^2 = \frac{326.6\times10^6}{0.36\times20\times0.48\,(1-0.42\times0.48)} \Rightarrow d = 343.9\text{mm}$$

Thickness of footing based on one way shear →

can be calculated by considering shear to be resisted without shear reinforcement

$$V_{u\,max} = \tau_{uc}\, b d \Rightarrow \boxed{d = \frac{V_{u\,max}}{\tau_{uc}\, b_o}}$$

$V_{u\,max}$ = max. ultimate one way shear at critical section in exterior span at a distance 'd' from face of interior col. in longitudinal strip

$\therefore V_{umax} = \frac{390.16}{2.973} \times (2.973 - 0.2 - d) = 363.9 - 131.23d$ kN/m.

$b_0 = 1000$ mm

$\tau_{vc} = k\,\tau_c \rightarrow$ for solid slab

$k = 1$ as total depth of raft ≥ 300mm

τ_c = shear strength of concrete given in table-19 (P-73)
(Let it be minimum as flexural reinforcement shall be small due to large depth of foundation slab on shear consideration)

$= 0.36$ N/mm^2

$\therefore d \times 1000 = \frac{V_{umax}}{\tau_{vc}\, b_0} = \frac{363.9 - 131.23d}{0.36 \times 1000} \Rightarrow d = 0.741$ m.

$\therefore$ Thickness of footing is governed by one way shear.

$d = 741$mm $\simeq 750$mm

$D = \underset{(cover)}{750 + 40} + \underset{(\phi/2)}{10} = 800$ mm. (Let taken dia. of bar, $\phi = 20$mm)

Design for moment (both longitudinal & transverse) →

* Bottom & top reinforcements for transverse moments are placed above the bottom reinforcement & below the top reinforcement for longitudinal moments respectively because

transverse moments < longitudinal moments.

moment per unit width (kNm/m)	deff	Ast
End span longitudinal moment = 253.3kNm/m	$d = D - 40 - \frac{20}{2}$ $= 800 - 40 - 10$ $= 750$mm.	Ast = 961 mm^2/m $\nless$ 0.12% bD = 0.0012 × 1000 × 800 = 961 mm^2/m. Provide 16mm ϕ @ 200mm c/c (Ast = 1005 mm^2)
interior span longitudinal moment = 83.6 kNm/m	$d = D - 40 - \frac{20}{2}$ $= 750$ mm.	Ast = 316 mm^2/m $\nless$ 0.12% bD = 960mm^2. Provide 16 mm ϕ @ 200mm c/c (Ast = 1005 mm^2/m)
Interior support longitudinal moment = 326.6kNm/m	d = 750 mm	Ast = 1250mm^2/m $\nless$ 0.12% bD = 1250 mm^2 Provide 16mm ϕ @ 150mm c/c (Ast = 1340mm^2)
End support longitudinal moment = 16.41kNm/m	d = 750mm	Ast = 163 mm^2/m $\nless$ 0.12% bD = 960mm^2/m Provide same steel as interior support 16 mm ϕ @ 150mm c/c.

moment/unit width	deff	Ast
Transverse support moment = 16.4 KNm/m	$d = D - 40 - 20 - \frac{20}{2}$ $= 800 - 40 - 20 - 10$ $= 730mm$	$A_{st} = 61 mm^2/m \nless 0.12\% bD = 960mm^2/m$ Provide 16mm ϕ @ 200mm C/C $A_{st} = 1005 mm^2/m$)
Transverse span moment = 246.1 KNm/m	d = 730mm	$A_{st} = 961 mm^2/m \nless 0.12\% bD = 960mm^2/m$. Provide = 16mm ϕ @ 200mm C/C ($A_{st} = 1005 mm^2/m$)

check for shear

The footing is adequate for one way shear as depth of footing is based on minimum value of shear strength of concrete.

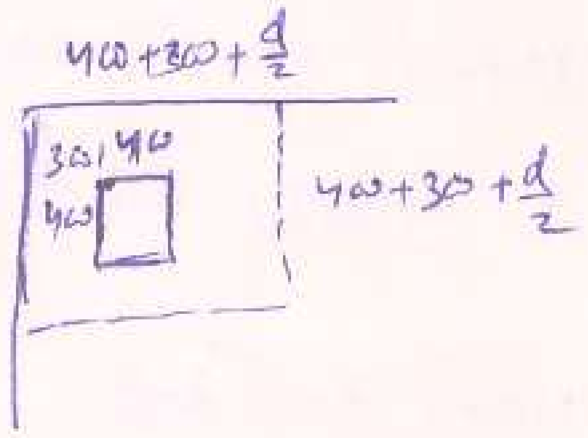
For adequacy of footing in 2 way shear :-

$k_s \tau_{vc} \geqslant \frac{V_{uc}}{b_o d}$, $k_s = 0.5 + \beta_c \leq 1.0 \Rightarrow 0.5 + \frac{0.4}{0.4} \leq 1 \Rightarrow k_s = 1$ (P-59)

$\tau_{uc} = 0.25 \sqrt{f_{ck}} = 0.25 \sqrt{20} = 1.118 N/mm^2$

V_{uc} = ultimate shear strength based on punching of col. on perimeter of critical section which is at a distance of d/2 from face of column.

Punching of col- c_1



$V_{uc1} = 1.5 \times 750 - 131.25 (0.4 + 0.3 + \frac{0.75}{2})^2 \doteq 971.91 KN$.

$b_o = 2(400 + 300 + \frac{760}{2}) = 2160 mm$

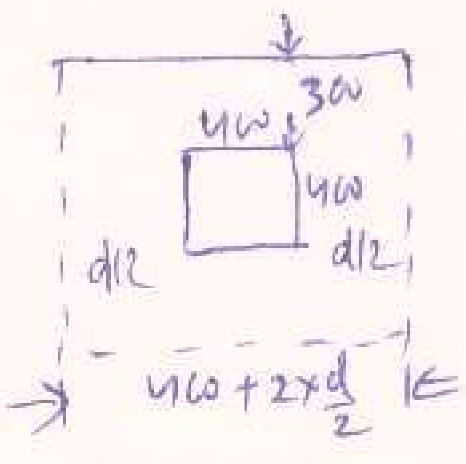
$\frac{V_{uc}}{b_o d} = \frac{971.91 \times 10^3}{2160 \times 750} = 0.592 < k_s \tau_{uc}$
N/mm2

Punching of col- c_2 ⇒

$V_{uc2} = (1.5 \times 1000) - 131.05 (0.4 + 0.75)(0.4 + 0.3 + \frac{0.75}{2}) = 1335.57 KN$.

$b_o = (400 + 760) + 2(400 + 300 + \frac{750}{2}) = 3320 mm$

$\frac{V_{uc2}}{b_o d} = \frac{1335.57 \times 10^3}{3320 * 750} = 0.53 < k_s \tau_{uc}$. (Safe)



Foundatn
- Shallow
- Deep

Shallow

- ✓ Isolated column footing
  - Stepped
  - Projections.
- ✓ Combined footing
  - Rectangular
  - Trapezoidal.
- ✓ Continuous footing
  - → a single continuous RCC slab is provided for 2, 3 or more columns in raw.
  - → Better for EQ prone region
  - → decreases differential settlement
- ✓ Strap / cantilever footing →
  - → Two/more individual footings connected by a beam called a strap.
  - → used where distance betn columns is so great that combined footing if provided becomes narrow & carries more BM
  - → The strap does not contact with soil.
  - → strap is very stiff & so transfers equal pressure ~~to~~ both footings.
- ✓ Grillage foundatn
  - Transfers heavy steel column load to soil of low bearing capacity.

Raft foundatn

- Raft/mat is a combined footing that covers entire area below a structure & supports all the columns.

Deep foundatn
- Pile
- well.

Advanced Concrete Structure

Design of Pile Foundation

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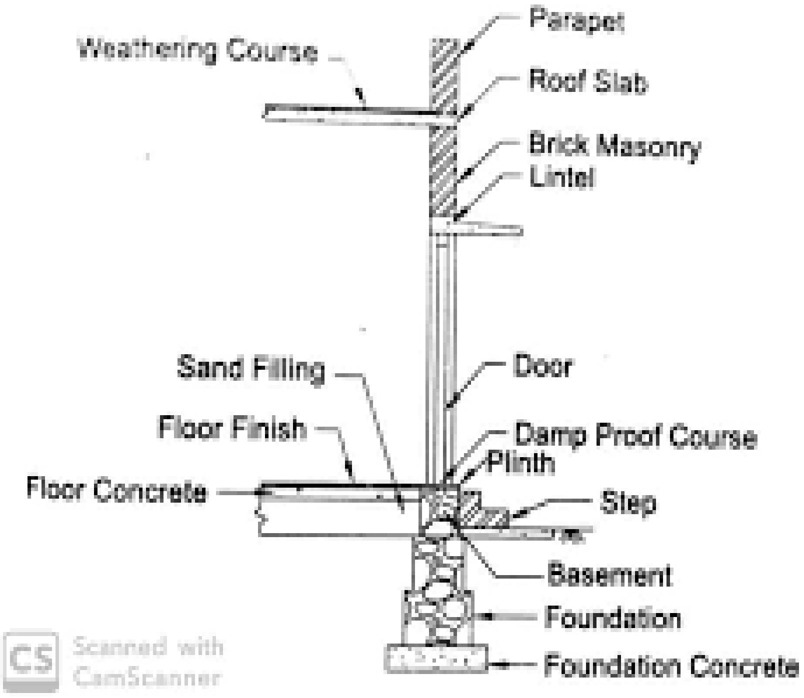
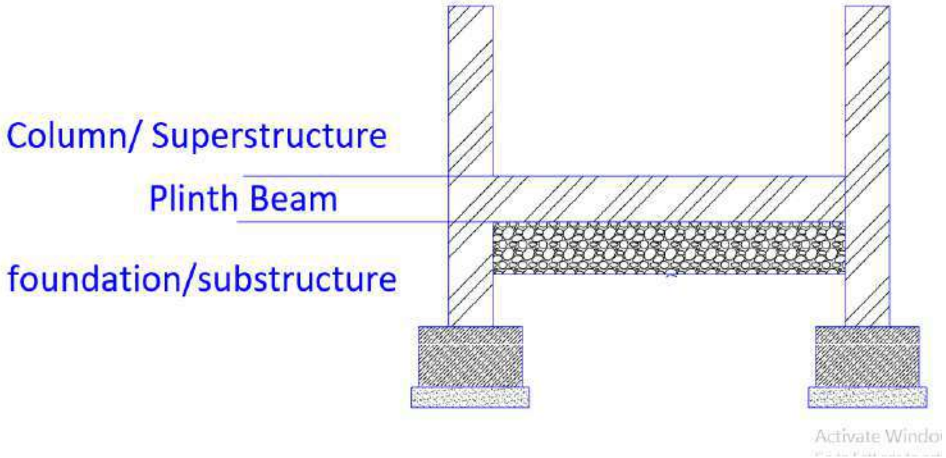
PILE DESIGN THEORY

TYPES OF FOUNDATIONS:

- Every structure is broadly classified into **three parts**.
    1. **Substructure**
    2. **Plinth**
    3. **Superstructure**
- Part of structure build **below the ground level** is called **sub-structure** and that of **above the ground level** is called **super structure**
- Plinth is not only the base of a structure, it's a platform that supports a pedestal, column, or structure.
- A part that **separates super structure from sub structure** and from which **floor starts**
- The function of **footing** (shallow foundation) or a **foundation** (can be shallow / deep) is to safely and effectively transmit the load from the column to the soil





- The permissible pressure that the soil can take without any failure is called as safe bearing capacity (SBC) of soil
- Foundation is defined as that part of the structure that transfers the **load** from the structure constructed on it as well as its self-weight over a large area of soil in such a way that the amount does not exceed the **ultimate bearing capacity of the soil** and the **settlement** of the whole structure remains within a **tolerable limit**.

- Objectives of a foundation:

  - Distribute the weight of the structure over a **large area of soil**.
  - Avoid **unequal settlement**.
  - Prevent the **lateral movement** of the structure.
  - Increase **structural stability**.

- There are **different types of soil**, and hence the soil bearing capacity is different too for each type of soil. Depending on the **soil profile**, **size**, and **load of the structure**, there are different kinds of foundation.

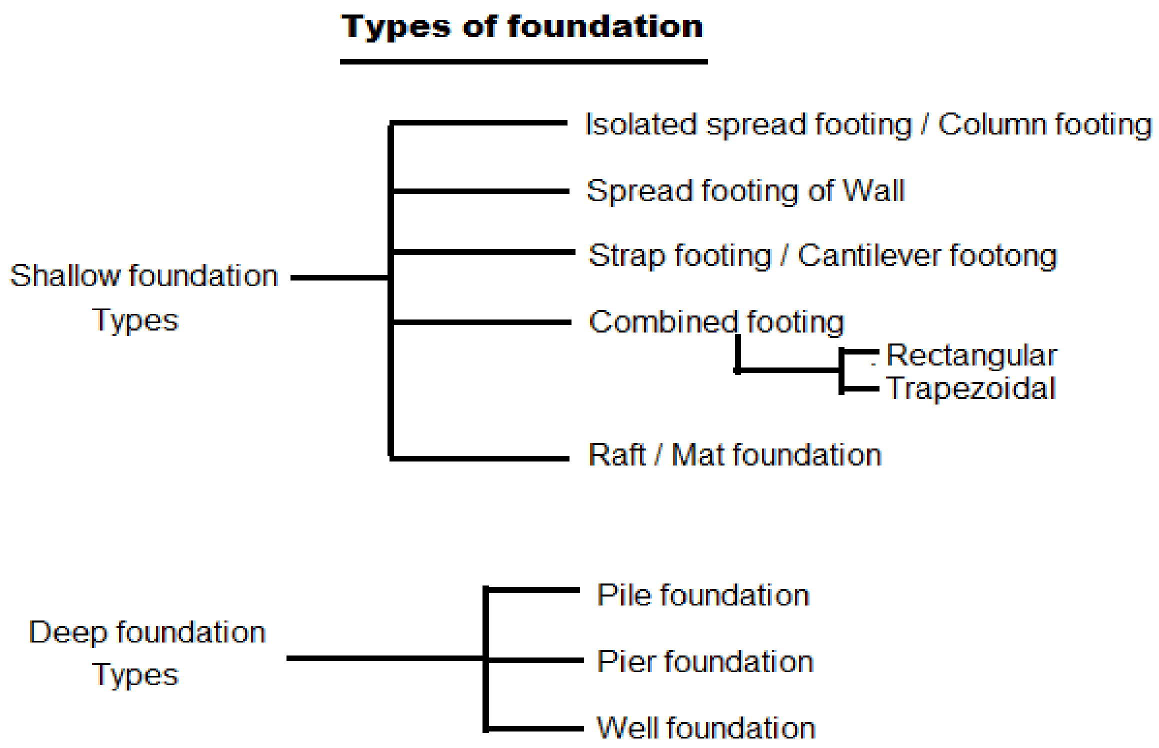
- All foundations are divided into **three** categories: According to **Terzaghi**, depth to width relationship is the basis of foundation classification.
  - **Shallow** foundations: **if Depth ≤ Width** of foundation
  - **Deep** foundations: **if Depth > Width** of foundation
  - **Special** foundation: Foundations built for **transmission line towers, chimneys** etc

- The terms Shallow and Deep Foundation refer to the depth of the soil at which it is placed

Difference between Foundation and Footing:

- Foundation is a structure which transfers the loads from the superstructure to the ground, while footing is the foundation which is in contact with the earth.

- A foundation can be shallow and deep, while a **footing** is a type of a **shallow foundation**. so, all footings are foundations but all foundations cannot be footings.

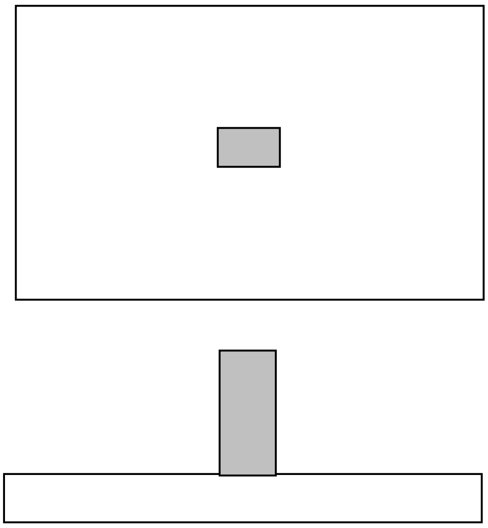


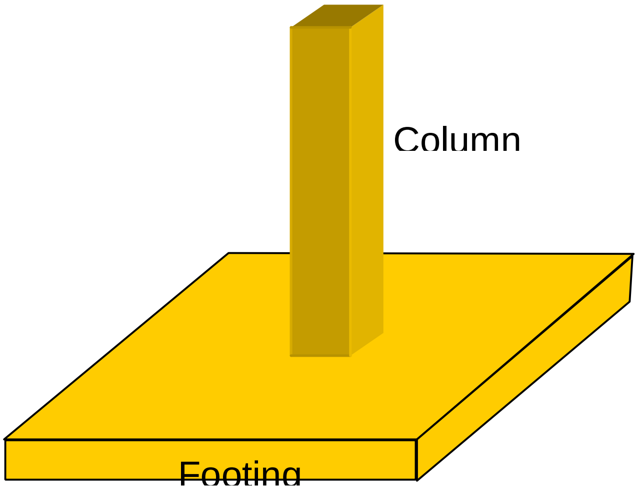
SHALLOW FOUNDATIONS

- They are usually located no more than 6 ft below the lowest finished floor.
- A shallow foundation system generally used when
  - The soil closes the ground surface has sufficient bearing capacity
  - Underlying weaker strata do not result in excessive settlement.
- The shallow foundations are commonly used most economical foundation systems
- **Types of spread footing: (either for Column or for Wall**)

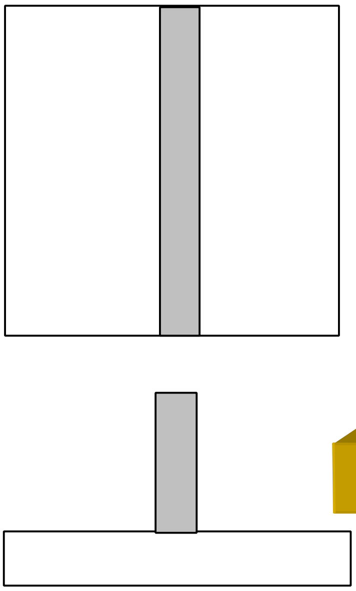
a) Single pad footing.
b) Stepped footing for a column.
c) Sloped footing for a column.
d) Wall footing without step.
e) Stepped footing for walls.
f) Grillage foundation.

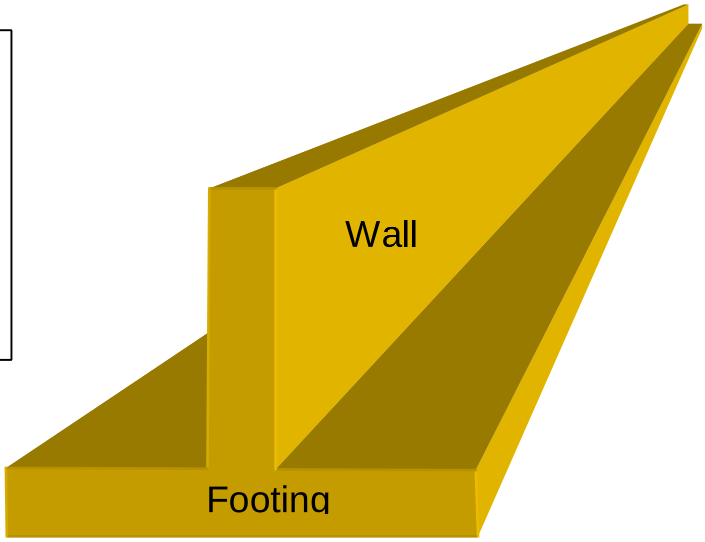
(a) **Isolated spread footings** under individual columns which can be square, rectangular or circular.





(b) **Wall footing** is a continuous slab strip along the length of wall





(c) **Combined footings** support two or more columns. These can be rectangular or trapezoidal in plan.

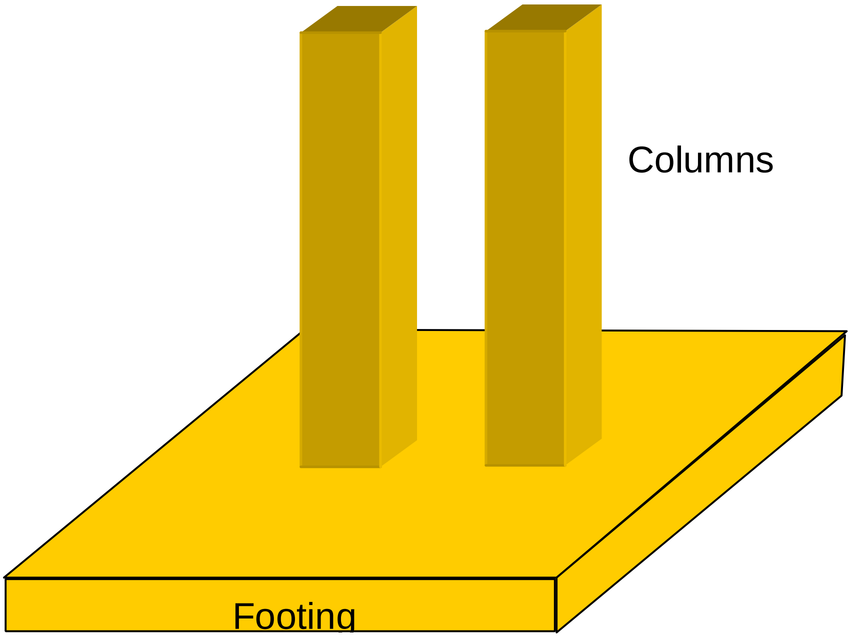
➢ A combined footing is necessary in following **three reasons**:

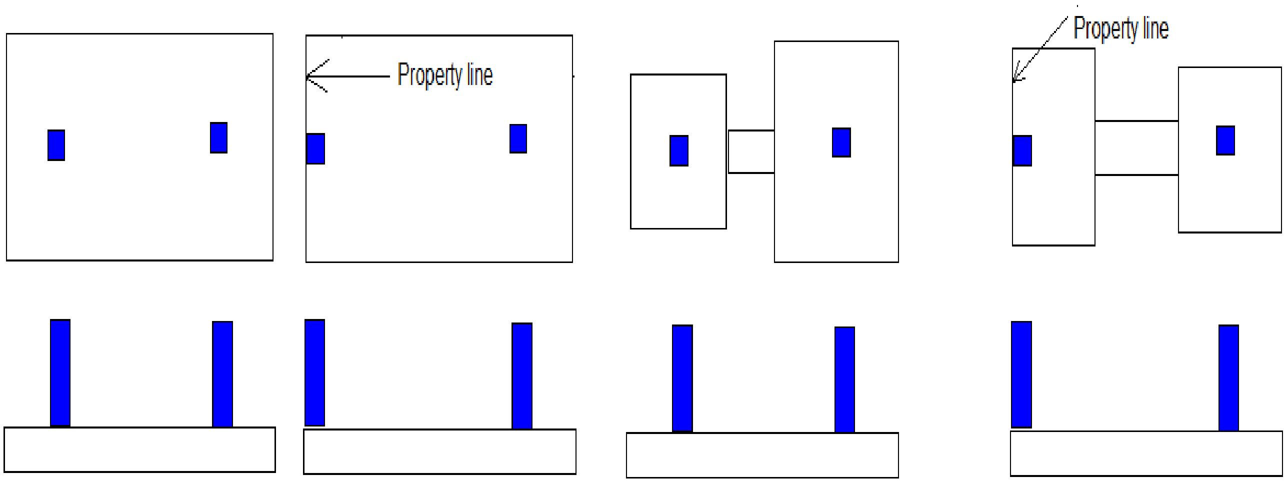
- Columns are placed **very close to each other** so that their individual footings overlap each other
- When **bearing capacity of soil is less** so it is required to have a more spread area for footing and so footing of adjacent column may overlap
- When external column is **close to property line**, it is not possible to provide isolated footing for that column because it may be extended beyond the property line and so combined footing solves the problem

- The **essential condition** to satisfy in **combined footing** is that, **centroid of footing area should coincide with resultant of column loads** so that **soil pressure distribution is uniform under soil.**
- **Types of combined footing**:

  - Combined footing (Rectangular):
  - Combined footing (Trapezoidal):

    If outer column near property line carries a heavier load
  - Strap footing
  - Raft / mat foundation





Combined footing

Strap footing

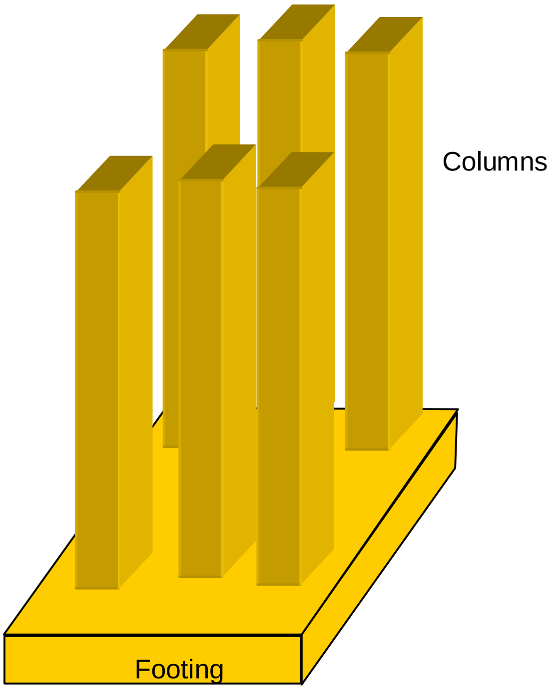
Combined footing

(d) Strap or Cantilever Footing

- Strap footings are similar to combined footings.
- Reasons for considering or choosing strap footing are identical to the combined one.
- In *strap footing*, the foundation under the columns is built individually and connected by a **strap beam**.
- Generally, when the **edge of the footing cannot be extended beyond the property line**, the **exterior footing is connected by a strap beam with interior footing**.

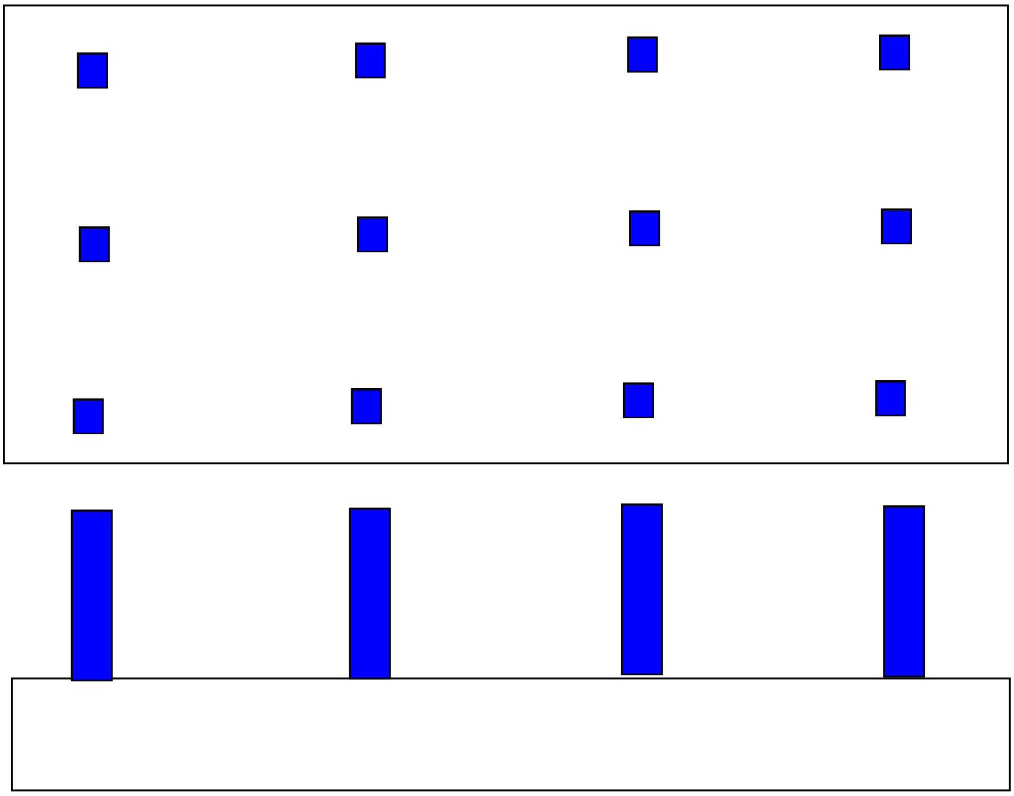
(e) Raft / mat foundation:

- This is a large continuous footing supporting all the columns of the structure.
- This is used when soil conditions are poor but piles are not used.
- Raft foundation is provided
  - When **load** transmitted by **columns** are so **heavy** or **allowable soil pressure** are so small that individual footings if provided would **cover more than about half** of the area, then it is better to provide a continuous footing called raft foundation under all columns and walls
  - Raft foundations are used to reduce settlement of structure located above heavy compressible deposits i.e. they control differential settlement
- **Types of raft foundation**:
  - **Solid raft** (A continuous slab covering all the columns)
  - **Ribbed raft** (mat with a central hollow region when all the columns are connected by a continuous beam which gets supported on the raft slab



Raft foundation

Mat or Raft



DEEP FOUNDATION

1. PILE FOUNDATION

- A **pile** is a **slender column** provided with a **cap** to receive the **column load** and transfer it to **undelaying soil layer / layers**.
- **Pile foundation** is a common type of deep foundation.
- Pile is a slender member with a small cross-sectional area compared to its length.
- It is used to transmit foundation loads to a deeper soil or rock strata when the bearing capacity of soil near the surface is relatively low.
- Pile transmits load either by skin friction or bearing.
- Piles are also used to resist structures against uplift and provide structural stability against lateral and overturning forces.
- They are used to reduce cost, and when as per soil condition considerations, it is desirable to transmit loads to soil strata which are beyond the reach of shallow foundations.
- Pile foundations are economical when

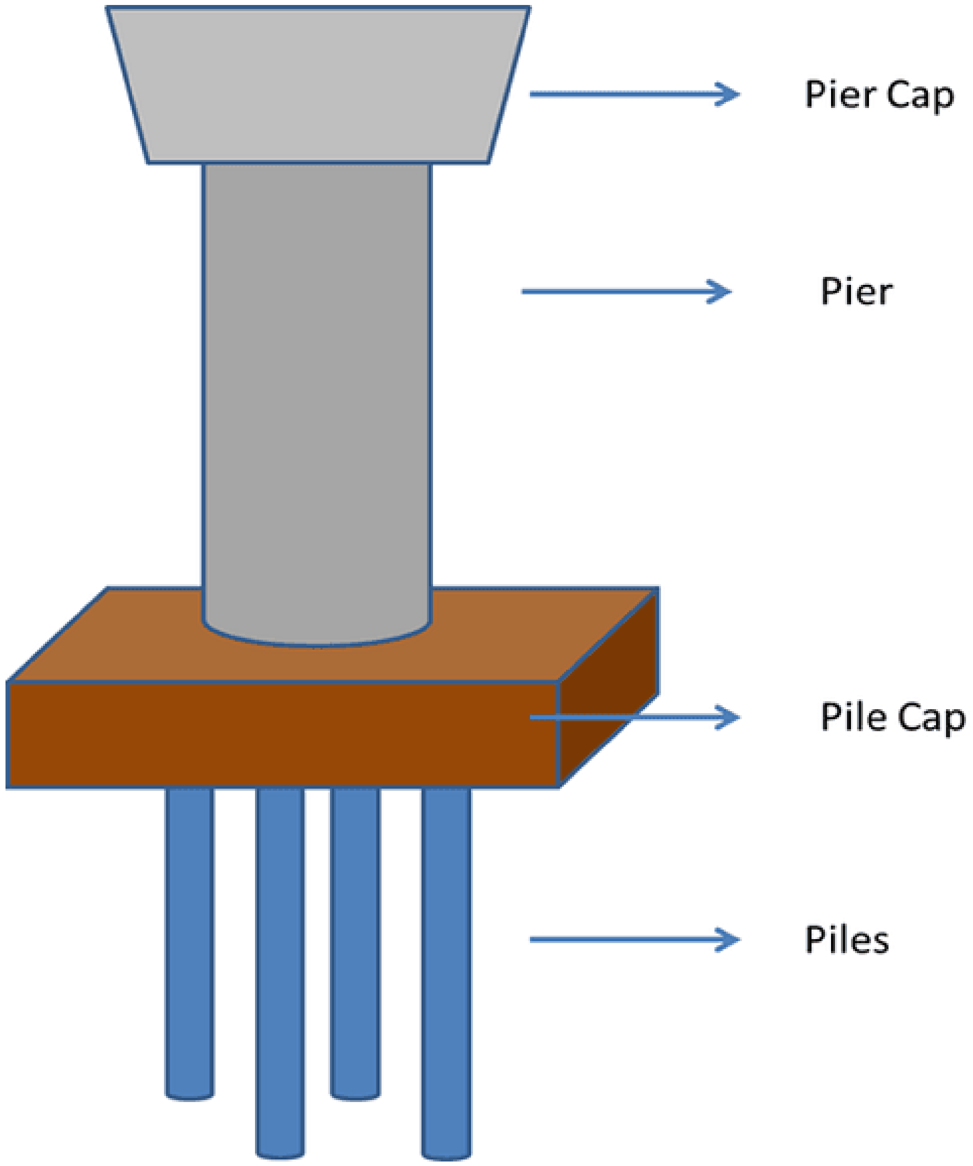
Soil with higher **bearing capacity** is at a greater depth.
When the foundation is subjected to a **heavily concentrated load**
The foundation is subjected to **strong uplift force**
Lateral forces are relatively pre dominant
When there are chances of construction of **irrigation canals** in the nearby area.
Expansive soil like **black cotton soil** are present at the site
In **marshy places** where soil is wet soil/ soft soil/ water logged/ low laying area
When the **topsoil laye**r is **compressible** in nature.
In the case of bridges, when the **scouring** is **more** in the **river bed**.
When it is very expensive to provide **raft** or **grillage**.

2. PIER FOUNDATION

- Pier is a deep foundation structure above ground level that transmits a more massive load, which cannot be carried by shallow foundations.
- It is usually shallower than piles.
- Pier foundation is a cylindrical structural member that transfer heavy load from superstructure to the soil by end bearing.
- Unlike piles, it can only transfer load by **end bearing** only and by **not skin friction**.

Difference between Pile and Pier foundation

Pile	Pier
Piles are always below the ground level	Piers are always above the ground
Larger in length and smaller in diameter	Smaller in length and larger in diameter
Adopted when there is no hard bearing strata of soil available at reasonable depth	Adopted when there is hard bearing strata of soil available at reasonable depth but other types of foundation construction is not economical
Piles are driven through overburden soil into load bearing strata	Pier is drilled by drilling machine
Transfers full load through both bearing and friction action only	Transfers full load through bearing action only
Constructed at greater depth	Constructed at shallower depth
Resist greater intensity of load	Resist smaller intensity of load



PIER foundation with PILE

3.WELL / CAISSON FOUNDATION

- Caisson foundation is a watertight retaining structure used as a bridge pier, construction of the dam, etc.
- It is generally used in structures that require foundation beneath a river or similar water bodies.
- The reason for choosing the caisson is that it can be floated to the desired location and then sunk into place.
- Caisson foundation is a ready-made hollow cylinder depressed into the soil up to the desired level and then filled with concrete, which ultimately converts to a foundation.
- It is mostly used as bridge piers.
- Caissons are sensitive to construction procedures and lack construction expertise.

- There are several types of caisson foundations.

1. Box Caissons.
2. Floating Caissons.
3. Pneumatic Caissons.
4. Open Caissons.
5. Sheeted Caissons.
6. Excavated Caissons.

CAISSON Foundation

DETAILS OF PILE AND PILE CAP

Classification of Pile foundation:

1. Based on Function or Use:

a) **End Bearing Piles:**

These are the pile used to transfer loads through water or soft soil to a suitable bearing stratum.

b) **Friction Piles:**

This type of pile utilizes the frictional resistance force between the pile surface and adjacent soil to transfer the superstructure load.

c) **Combined end bearing and friction pile:**

This pile transfers the super-imposed load both through side friction as well as end bearing. Such piles are more common, especially when the end bearing piles pass through granular soils.

d) **Compactor Piles:**

These are used to compact loose granular soil thus increasing their bearing capacity.

e) **Batter pile:**

A pile driven at an angle with the vertical to resist a lateral force

f) **Sheet Piles:**

Used as impervious cut-off to reduce seepage and uplift under hydraulic structures.

They are rarely used to furnish vertical support but are used to function as retaining wall

g) **Anchor pile:**

It provides anchorage against horizontal pull from sheet piling

Anchor piles can transfer both **compressive** and **tensile** forces as well as **bending moments** to the ground, making them ideal as anchors for offshore moorings, basements, and tunnels, etc. Moored floating offshore structures impose a variety of load conditions on the anchor system.

h) **Tension/uplift pile:**

It anchors down the structures subjected to uplift due to hydro static pressure, seismic activity or due to overturning moment

2. Based on Materials:

a) Timber Piles
b) Concrete Piles
c) Steel Piles
d) Composite Piles

3. Based on construction process:

a) Bored Piling:

Bored piles are installed by auguring into the ground forming a hole into which concrete can be poured, thereby casting the pile in position.

b) Driven Piling:

Driven piles are driven or hammered into the ground with the use of vibration

c) Screw Piling

Screw piles are wound into the ground, much like a **screw** is wound into wood. This is an efficient means of installation and coupled with their mechanism of dispersing load, provides effective in-ground performance in a range of soils, including earthquake zones with liquefaction potential

d) Mini Piling

Mini piling is a variation on piling that uses a narrower diameter. This makes them light and inexpensive whilst still being able to support considerably heavy loads. For the most common type of mini piling a hollow steel shaft is screwed or drilled into the ground

e) Sheet Piling

Sheet pile walls are retaining walls constructed to retain earth, water or any other filling materials. These walls are thinner in section compared to masonry walls. Sheet pile walls are generally used for following: Water front structures, i.e. in building wharfs, quays and piers.

4. Classification of Piles based on the effect of Installation:

a) **Displacement** pile:(eg: **Driven** Cast in Situ concrete pile and Driven Precast concrete pile)

b) Non- Displacement pile: (eg: **Bored** Cast in Situ concrete pile, Bored Precast concrete pile)

5. Classification of Concrete piles:

a) Driven cast in-situ (CIS) piles (IS 2911-P1-S1-2010)

b) Bore cast in-situ (CIS) piles (IS 2911-P1-S2-2010)

c) Driven precast (PC) piles (IS 2911-P1-S3-2010)

d) Precast (PC) pile in pre bore hole (IS 2911-P1-S4-2010)

Method of pile installation (Driven / Bored)

DIFFERENCE BETWEEN **DRIVEN PILE FOUNDATION** AND **BORED PILE FOUNDATION** LIES IN THE METHOD OF CONSTRUCTION

Driven pile foundation

- A driven pile is formed off-site under **factory-controlled conditions**. Driven Piles are made from preformed material having a predetermined shape and size that can be physically inspected prior to and during installation, which is installed by impact hammering, vibrating or pushing into the earth.
- Driven pile foundations are **longer** than **bored piles**.
- A **driven** pile is driven straight in and transfer the load through **friction and/or bearing**
- Driven piles are sometimes referred to as **displacement piles** because in the process of driving the pile into the ground, soil is moved radially as the pile shaft enters the ground.
- These may include timber, steel, or precast concrete piles.

Advantages: Driven Pile: -

a) Driven pile is the most favoured for works **over water** such as piles in wharf structures or jetties.

b) Driven piles may conveniently be used in places where it is advisable **not to drill holes** for fear of meeting ground water under pressure.

c) A pile driven into granular soil, compacts the adjacent soil mass and as a result the **bearing capacity of the pile is increased**.

d) Piles can be precast to the **required specifications**.
e) Piles of **any size, length and shape** can be made in advance and so progress of the work will be rapid.
f) The work is **neat and clean**. The supervision of work at the site is **minimum**.
g) The **storage space** required is very much **less**.

Bored pile foundation:

- A bored pile is cast-in-place concrete piles, meaning the pile is cast on the construction site.
- In this process, a void is formed by boring or excavation before pile is introduced into the ground. Piles can be produced by casting concrete in the void. Boring piles are considered as non-displacement piles.
- Bored pile foundations, also known as **replacement piles**, are typically poured in place and provide support for structures, transferring their load to layers of soil or rock that **have sufficient bearing capacity** and **suitable settlement characteristics**.

Advantages: Bored Pile: -

a) Piles of variable lengths can be extended through soft, compressible, or swelling soils into **suitable bearing material**.
b) **Vibration is relatively low**, **reducing disturbance** of adjacent piles or structures.
c) **Large excavations** and subsequent backfill are **minimized.**
d) Piles can be extended to depths below frost penetration and seasonal moisture variation.
e) Less disruption to adjacent soil occurs.
f) For many design situations, bored piles offer higher capacities with potentially better economics than driven piles.

ULTIMATE BEARING CAPACITY OF A PILE:

- Like footing, piles are too designed for **Soil** considerations and **Structural** considerations.
- Pile transmit load to ground either, by **Skin friction** with granular / sandy soil, by **Cohesion** with clayey soil, or by **Compression** at pile tip when it reaches a hard stratum.
- Usually, **combination** of **upward skin friction** along pile and **vertical compressive force** at the pile tip is used to calculate the bearing capacity of pile.
- The **ultimate bearing capacity** of a pile is the **maximum load** which it can carry without failure or excessive settlement of the ground.

The **bearing capacity** of a **pile** depends primarily on **3 factors** as given below,

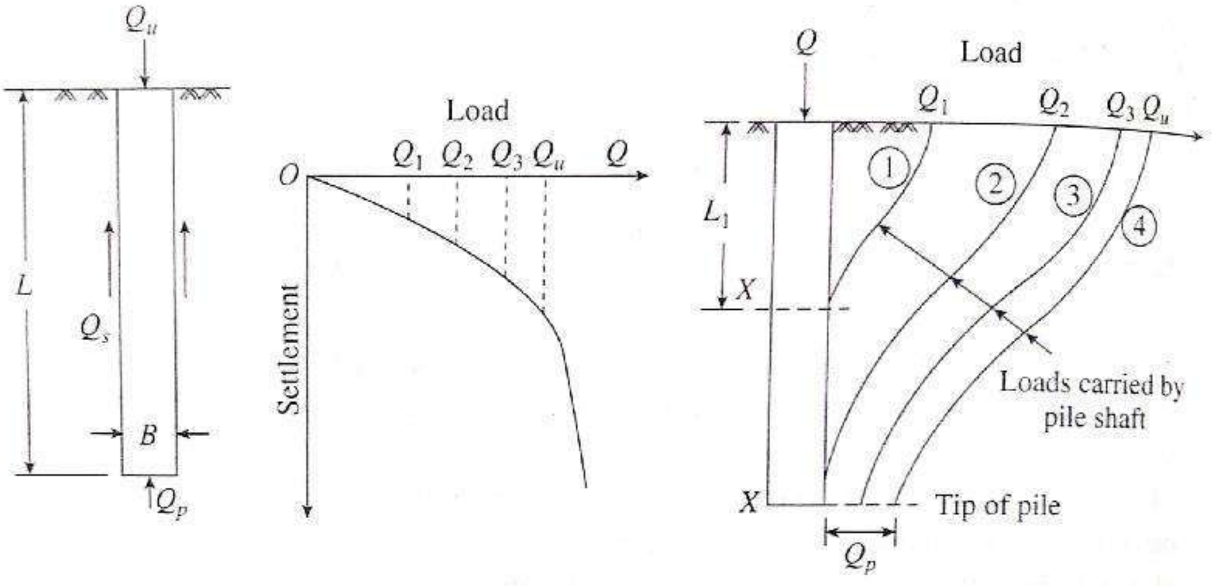
- **Method** of pile installation **(Driven / Bored)**
- **Type of soil** through which pile is embedded (soil shear strength parameter)
- **Pile dimension** (cross section & length of pile)

BEHAVIOUR OF PILE UNDER LOAD:

Let us consider a pile loaded gradually by increasing the load at top (Fig-a below). The load settlement curve for the pile under load is in Fig. b. Under increasing load, the behaviour of pile is as follows:

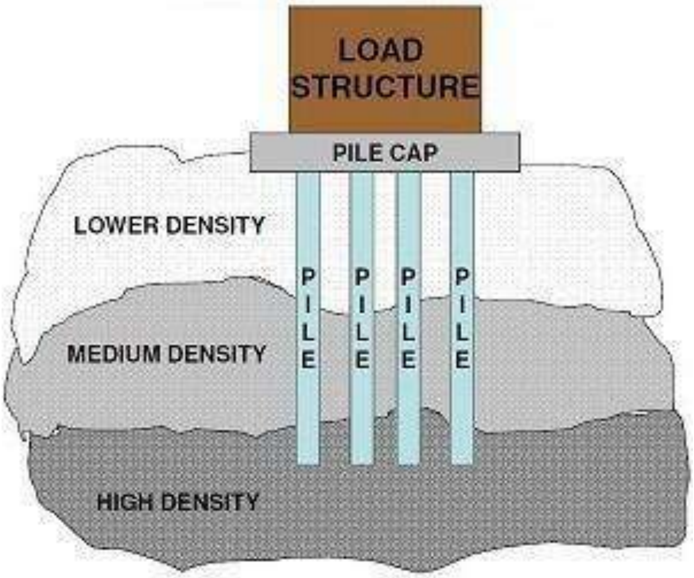
- On application of initial axial load Q1 on top of pile, at distance L1 from top, this load reduces to zero, i.e. load Q1 is resisted by skin friction alone.

- When load increases to Q2, total load is resisted by skin friction along entire pile length.
- When load exceeds Q2, part of load is resisted by hard stratum base soil by compressive force and remaining by skin friction.
- Skin friction attains its ultimate value Qs at such load level and any further load increase will increase compressive load at pile tip.
- On further increase in load, compressive load at pile tip reaches its ultimate value Qp and pile finally fails in Punching shear.
- Hence the ultimate bearing capacity of pile, **Qu= Qp + Qs** (Qp= compressive force at pile tip and Qs= Upward skin friction along pile length



(a) (b) (c)

FIG. 15.35 Behaviour of piles (a) Single pile (b) Load–Settlement curve (c) Load transfer mechanism



PILE FOUNDATION

Estimation of Pile Capacity (general):

The ultimate load carrying capacity of a pile can be determined by the following methods:

1. Static formula (Common formula)
2. Dynamic formula (Useful for driven piles in cohesionless soil)
3. Static in situ test or Pile load test
    - Good for cohesionless soil
    - More reliable than other two
    - Expensive and time consuming,
    - As per IS 2911 (P4): 1985, 0.5% to 2% of total piles are to be tested

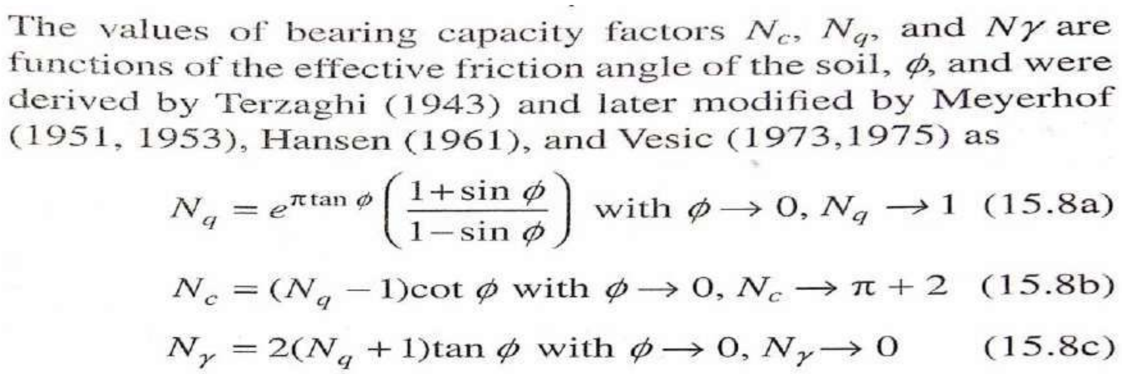
Estimation of Pile Capacity (Cohesionless soil):

As per cl B1, Page 9, IS 2911 (P1-S1): 2010

Q_n = End bearing capacity of pile + Skin friction resistance (+ve)

$$Qn = Ap(0.5D\gamma N\gamma + PDNq) + \sum KiPDi \tan \delta iAsi$$

Where,
Q_n = Ultimate load capacity of driven cast in site concrete pile in cohesionless soil
A_p = Cross sectional area of pile cap in m^2
A_{si} = Surface area of pile shaft in the ith layer
D = Diameter of pile shaft in m
γ= Effective soil unit weight at pile tip in kN/m^3
N_γ and N_q are bearing capacity factors depending on angle of internal friction of soil **(ø)** at pile tip,
P_D = maximum effective over burden pressure at pile tip in kN/m^2 (at critical depth)
for Ø (30^0), P_D = 15xD
for Ø (40^0), P_D = 20xD
P_{Di} = effective over burden pressure at pile tip in ith soil layer in kN/m^2
K_i = coefficient of earth pressure in ith soil layer depends on nature of soil strata, type of pile, spacing of pile and pile construction method
Ki = 1 to 2 for driven pile in dense sand with **Ø** varying from 30^0 to 40^0 (IS 2911)
δi = angle of wall friction between pile and soil in ith layer
δ = Ø (around pile shaft)
∑ is summation is done for soil layers 1 to n in which pile is installed and contributes to positive skin friction



The values of bearing capacity factors N_c, N_q, and $N\gamma$ are functions of the effective friction angle of the soil, ϕ, and were derived by Terzaghi (1943) and later modified by Meyerhof (1951, 1953), Hansen (1961), and Vesic (1973,1975) as

$$N_q = e^{\pi \tan \phi}\left(\frac{1+\sin \phi}{1-\sin \phi}\right) \text{ with } \phi \to 0, N_q \to 1 \quad (15.8a)$$

$$N_c = (N_q - 1)\cot \phi \text{ with } \phi \to 0, N_c \to \pi + 2 \quad (15.8b)$$

$$N_\gamma = 2(N_q + 1)\tan \phi \text{ with } \phi \to 0, N_\gamma \to 0 \quad (15.8c)$$

Estimation of Pile Capacity (Cohessive soil):

As per cl B2, Page 9, IS 2911 (P1-S1): 2010

$$Q_n = \text{End bearing capacity of pile} + \text{Skin friction resistance (+ve)}$$

$$Qn = ApNcCp + \sum \alpha iCiAsi$$

A_p = Cross sectional area of pile in m^2
Nc= Bearing capacity factor (may be 9)
Cp= average cohesion at pile tip in kN/m^2
αi = adhesion factor for ith layer
Ci= average cohesion for ith layer in kN/m^2
A_{si} = Surface area of pile shaft in the ith layer

Negative Skin Friction (NSF):

- It is **down ward drag of pile** by surrounding soil.
- Pile capacity should be reduced to compensate for the downward drag due to NSF.
- It can be compensated by providing **friction reducing material** like bitumen coating or sleeves around pile
- When pile is installed in a fill of loose sand deposit or soil that undergo high consolidation or pile piles are driven through a stratum of soft clay into firm soil, there will be NSF
- Reconsolidation of remoulded clay layer around any driven pile and lowering of water table in clays initiating significant settlement may too result NSF

PILE GROUP:

- Piles are usually installed in groups
- It is economical to use few high-capacity deep piles under column than a large number of low-capacity short piles
- A single pile foundation cannot take moment
- A two-pile group foundation can take moment only in one direction
- A minimum of three piles is required under a column to resist column load and moment in two directions
- Top of piles are connected by a pile cap which helps the piles to act as a single unit.
- Pile group capacity is considerably less than the sum of individual pile capacities
- Because the zone of soil that is stressed by entire group extends to a greater width and depth than zone breadth of the single pile.
- Group action of pile group foundation can cause excessive displacement and failure

Structural Design of pile:

- When a pile is wholly embedded in soil having an undrained shear strength greater than 0.01 N/mm^2, the axial pile capacity is not governed by strength of long column.
- Hence pile is designed as a short column.

- Minimum concrete grade for PILE is M25 (IS2911(P1-S1)
- Generally, Soil Design governs pile design
- Minimum steel is provided in pile (mild steel can be used in pile)
- Minimum steel area (any type/grade) in pile = 0.4% of cross-sectional area of pile shaft
- Clear cover to main steel in pile shaft > 50 mm (generally)
- Clear cover to main steel in pile shaft > 50 mm (corrosive environment)
- Minimum six steel rebars for circular pile and minimum diameter is 12 mm
- **Precast piles** are <u>Square/ Hexagonal</u> and **bored piles** are <u>Circular</u>

##############################

Design of RC PILE

Q-1→
Design a precast pile for an axial load of 275 KN and determine the vertical carrying capacity of the pile as per IS 2911 (P1)(S-1)
The properties of soil and pile data is as follows:

- Diameter of pile = 400mm = D
- Depth of top of pile cap below ground level = 500mm = D_{tPC}
- Thickness of pile cap = 1.5M = D_{PC}
- materials of pile = M 25 and Fe 415
- clear cover to reinforcement = 75mm

- pile is placed in submerged medium dense sandy soil with

angle of internal friction = 32° = ϕ

density of soil = 18 KN/m3

submerged density of soil = 10 KN/m³ = γ_{sub}

angle of wall friction between pile concrete & soil = $\delta = 0.75\phi = 24°$

sol ⇒ : soil design: →

cross sectional area of pile at the toe, $A_p = \frac{\pi}{4} \times 0.4^2 = 0.126\,m^2$

* Let us assume that water table is up to the ground level

* Effective unit weight of soil at pile toe, $\gamma = \gamma_{sub} = 10\,KN/m3$

* Bearing capacity factor, $N_q = e^{\pi \tan\phi}\left(\frac{1+\sin\phi}{1-\sin\phi}\right)$

$$= e^{\pi \tan 32}\left(\frac{1+\sin 32}{1+\sin 32}\right) = 7.12 \times 3.25 = 23.18$$

* Bearing capacity factor, $N_\gamma = 2(N_q+1)\tan\phi = 2(23.18+1)\tan 32 = 30.22$

As per Annex B, notes 5, P9, IS 2911 (P1/S1)-2010
as $\phi = 32°$ & sandy soil, critical depth may be 17.5 times diameter of pile
⇒ critical depth = $17.5 \times 0.4 = 7m = D_c$

* Effective overburden pressure at pile tip, $P_D = \gamma_{sub}(D_{tPC} + D_{PC} + D_C)$

$$\Rightarrow P_D = 10(0.5 + 1.5 + 7) \Rightarrow P_D = 90\,KN/m^2$$

* Effective overburden pressure at bottom of pile cap = $P_{D1} = \gamma_{sub}(D_{tPC} + D_{PC})$
$\Rightarrow P_{D1} = 10(0.5 + 1.5) = 20\,KN/m^2$

DP2

* Effective overburden pressure at pile toe $= P_{D_2} = P_D = 90\ kN/m^2$

$$\text{So} \begin{cases} P_{D1} = 20\ kN/m^2 \\ P_{D2} = P_D = 90\ kN/m^2 \end{cases}$$

* surface area of pile stem in the first layer = π x diameter x length of pile

$$\Rightarrow A_{S1} = \pi D L = \pi \times 0.4 \times L \Rightarrow \underline{A_{S1} = 1.257L}$$

cl 6.8.2, P5, IS 2911(P1-S1)-2010,

the minimum factor of safety on static formula = 2.5

⇒ ultimate load carrying capacity of pile = Qu = factor of safety x axial load

$$\Rightarrow \underline{Q_U} = 2.5 \times 275 = \underline{687.5}\ kN$$

* Annex B, cl B1, P9, IS 2911(P1-S1)-2010

$$\underline{Q_U} = A_p(0.5 D\gamma N_\gamma + P_D N_q) + \sum_{i=1}^{n} K_i P_{Di} \tan \delta_i A_{Si} \text{ ——— ①}$$

/ $A_p = 0.126 m^2$ / $D = 400 mm = 0.4m$ / $\gamma = 10 kN/m^3$ / $N_\gamma = 30.22$ /

/ $P_D = 90 kN/m^2$ / $N_q = 23.18$ / $P_{Di} = \frac{1}{2}(P_{D1} + P_{D2}) = 55\ kN/m^2$ /

/ $\delta_i = 24^0$ / $A_{Si} = 1.257L$ / $Q_U = 687.5 kN$

/ $K_i = 1.5$ as $\phi = 32^0$, cl B1, Note-3, P9, IS 2911(P1-S1)-2010

$$\Rightarrow 687.5 = 0.126(0.5 \times 0.4 \times 10 \times 30.22 + 90 \times 23.18) + [1.5 \times 55 \times \tan 24 \times 1.257L]$$

$\Rightarrow L = 8.69\ m \simeq \underline{L = 9m}$ (length of pile)

DP3

structural design :→

P_{u_1} = factored load = 1.5 × 275 = 412.5 KN

$L/D = \frac{9}{0.4} = 22.5 < 30$

As Per cl. 6.12.1 (a), P 6, IS 2911 (P1/S3) : 2010 for precast driven piles :

Area of main longitudinal reinforcement shall not be less than ⇒

⇒ minimum percentage of longitudinal reinforcement = 1.25% of pile c/s area

if $\frac{L}{D} < 30$

⇒ $A_{st,min} = 1.25\% \times \frac{\pi}{4} \times 400^2 = \frac{1.25}{100} \times \frac{\pi}{4} \times 400 \times 400 = 1570 \text{ mm}^2$

* provide 8 nos, 16mm ϕ bars ⇒ Ast provided = 1608 mm²

→ provided diameter of main steel > min diameter & no. of bars provided > minimum no. required

* Axial load carrying capacity of a short column

cl 6.12.3, P 6, IS 2911 P1/S3)

$\Rightarrow P_u = 0.4 f_{ck} A_c + 0.67 f_y A_{sc}$ (cl 39.3, P 71, IS 456, 2000) ② ✱

$\Rightarrow P_u = \left[0.4 \times 25 \left(\frac{\pi}{4} \times 400^2 - 1608\right) + 0.67 \times 415 \times 1608\right] \times 10^{-3} = 1687 \text{ KN} > 412.5 \text{ KN}$

⇒ $P_u > P_{u_1}$

⇒ Pile strength as a short column > applied factored load

* provide 8mm ties with ~~[illegible]~~ 45mm cover to the centre of main steel (cl. 6.12.3, P6, IS 2911, (P1/S3)

Total length of one tie bar, S = 4 (400 − 2×45) = 1240 mm

* volume of one tie bar, $V = \frac{\pi}{4} \times 8^2 \times 1240 = 62329 \text{ mm}^3$

* The minimum volume of ties in one end zone of 3D length = 3×400 = 1200 mm

(cl. 6.5.1, P4, IS 2911 (P1/S3) : 2010)

⇒ minimum tie volume in the end zone of 3D length

$= 0.6\% \times \text{pile c/s} \times 3D = \frac{0.6}{100} \times \frac{\pi}{4} \times 400^2 \times 3 \times 400 = 904,778 \text{ mm}^2$

* Number of ties in the end zone (3D = 1200mm) = $\frac{904778}{62329}$ = 15 number

* spacing of ties in end zone = $\frac{1200}{15-1}$ = 85 mm

* spacing of ties in mid zone = 3 × spacing @ end zone = 3×85 = 255 mm > spacing min. cl. 6.11.4, P6 (P1-S1) - IS 2911

* Additional stiffner rings of 16mm ϕ be provided along length of cage @ every 1.5 m c/c (cl. 6.11.4 (P6)-(P1-S1) - IS 2911)

DP4

* In this problem pile is submerged i.e. wholly embedded in soil.

cl 6.5.1, (P14 on P13, in IS2911 (P1-S1 or P1-S2): 2010

→ for good soil as in present case, if undrained shear strength $\not< 0.01 \, N/mm^2$ the load carrying capacity of long pile is not limited to strength of long column. so for such soil, pile strength may be found by short column strength, as done here.

* As per SP: 34 (1987) - P 75, cl 6.9.1 (b) for precast pile

|| The volume of lateral reinforcement $\not<$ 0.6% of gross volume at each end of pile for a distance (3× pile diameter)
(i.e. minimum steel)

$\not<$ 0.2% of gross volume in the body of pile (central part) for a distance (L-12D) on both side w.r.t pile centre towards pile end.

* cover of concrete over all reinforcement including ties $\not<$ 40mm.

(Ans)

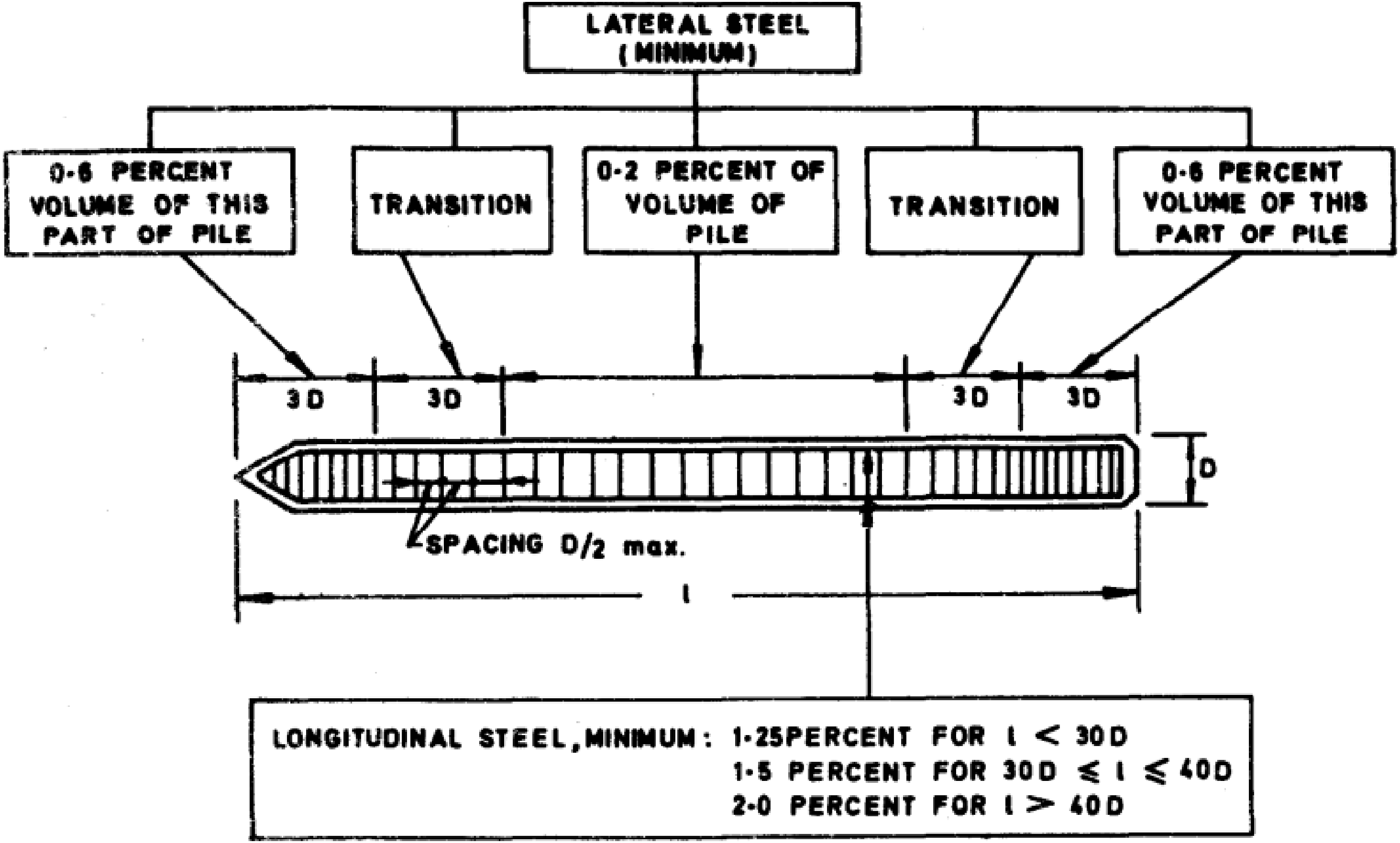
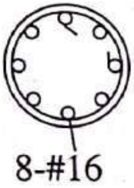
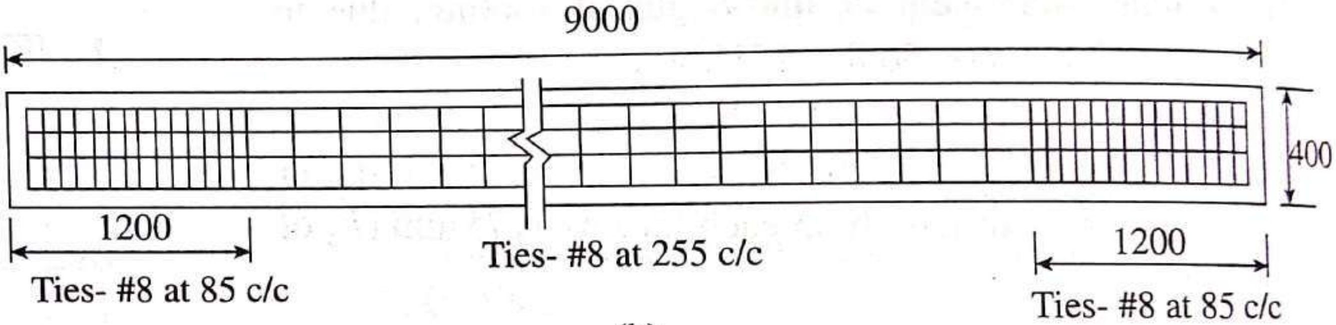


FIG. 6.7 MINIMUM STEEL REQUIREMENTS OF PRECAST CONCRETE PILE


8-#16

(a)


9000
400
1200
Ties- #8 at 85 c/c
Ties- #8 at 255 c/c
1200
Ties- #8 at 85 c/c

(b)

Detailing in a Precast Concrete Pile of 9m length

(a) Pile Cross Section

(b) Pile Longitudinal Section

Design of pile cap for bored cast in situ pile

(DP 5) by STRUT & TIE MODEL

code applicable : IS 2911 (P1-S2) - 2010

Q-2 A RC column of size 500mm x 500mm is supported on four bored cast in situ piles of 300 mm diameter each. The column carries a load of 1000 KN, a moment of 300 KNm in the x-x direction, and a shear force 50KN on top of the pile. The materials of concrete & steel are M25 & Fe415. Design the pile cap assuming that the piles are capable of resisting the reaction from the pile cap.

sol-)

→ size of pile cap :-

cl 6.12.5, P6, IS 2911(P1-S2)-2010,

clear overhang of pile cap beyond outermost pile in group = 150 mm minimum

cl 6.6.1, P4, IS 2911(P1-S2)-2010

minimum spacing between piles = 2.5 x diameter of pile

Let us take c/c spacing of pile = 3 x diameter of pile = 3 x 300 = 900 mm (c/c)

Let us take projection of pile cap beyond outermost pile = 150 mm

Length of pile cap = spacing (c/c) + 2 x $\frac{\text{pile diameter}}{2}$ + 2 x overhang/projection

= 900 + 300 + 2x150 = 1500 mm

[Based on experience, Recommended thickness of pile cap in general by Reynold & steedman

[If diameter of pile < 550 mm, pile cap thickness = (2 x pile diameter) + 100

If diameter of pile > 550 mm, pile cap thickness = $\frac{1}{3}$[(8 x pile diameter) - 600]

As here pile diameter < 550 mm, thickness of pile cap = 2 x pile diameter + 100 (let)

= 2x300 + 100

=) thickness of pile cap = 700 mm

→ Forces on pile :-

* weight of the pile cap = length x breadth x thickness x concrete density

= 1.5m x 1.5m x 0.7 x 25 = 39.38 KN ≅ 40 KN

* column load = 1000 KN

* Total vertical load on four piles = 1000 + 40 = 1040 KN = P

=) P = 1040 KN

* The shear force on top of pile = 50KN

* This will cause a moment, $M_S = SF \times$ pile cap' thickness

$$\Rightarrow M_S = 50 \times 0.7 = 35 \text{ KNm}$$

* Total bending moment = moment carried by column + moment due to SF on pile

$$\Rightarrow M_t = 300 + 35 = 335 \text{ KNm}$$

* This BM M_t will cause equal & opposite forces on pair of piles

$\Rightarrow$ Axial load on a pair of piles due to M_t

$$\Rightarrow \Delta p = \frac{M_t}{\text{spacing of pile}} = \frac{335 \times 10^3}{900} = 372.2 \text{KN}$$

* maximum working load on each pile at forwarded end (piles 1st and 4th)

$$\Rightarrow P_p = \frac{P}{\text{no of piles}} + \frac{\Delta P}{2} = \frac{1040}{4} + \frac{372.2}{2} \Rightarrow \underline{P_p = 446.1 \text{ KN}}$$

* maximum working load on piles 2nd and 3rd

$$P_p' = \frac{P}{\text{no of piles}} - \frac{\Delta P}{2} = \frac{1040}{4} - \frac{372.2}{2} \Rightarrow \underline{P_p' = 73.9 \text{ KN}}$$

* maximum factored load on pile = $\underline{\underline{P_{PU}}} = 1.5 \times 446.1 \simeq \underline{670 \text{KN}}$

→ Tension in the tie →

Cl 6.12.6, P-6, IS 2911 (P1-S2):2010-

min clear cover for main steel in pile cap slab = 60mm

Let us take a clear cover to main steel = 75 mm for 20mm diameter bars.

deff in pile cap slab = $700 - 75 - \frac{20}{2} = \underline{615 \text{ mm}}$

In Δ ABC, BC is the diagonal joining pile reactions P_{PU} in two diagonal piles

BC is a horizontal line joining two diagonally opposite pile centres

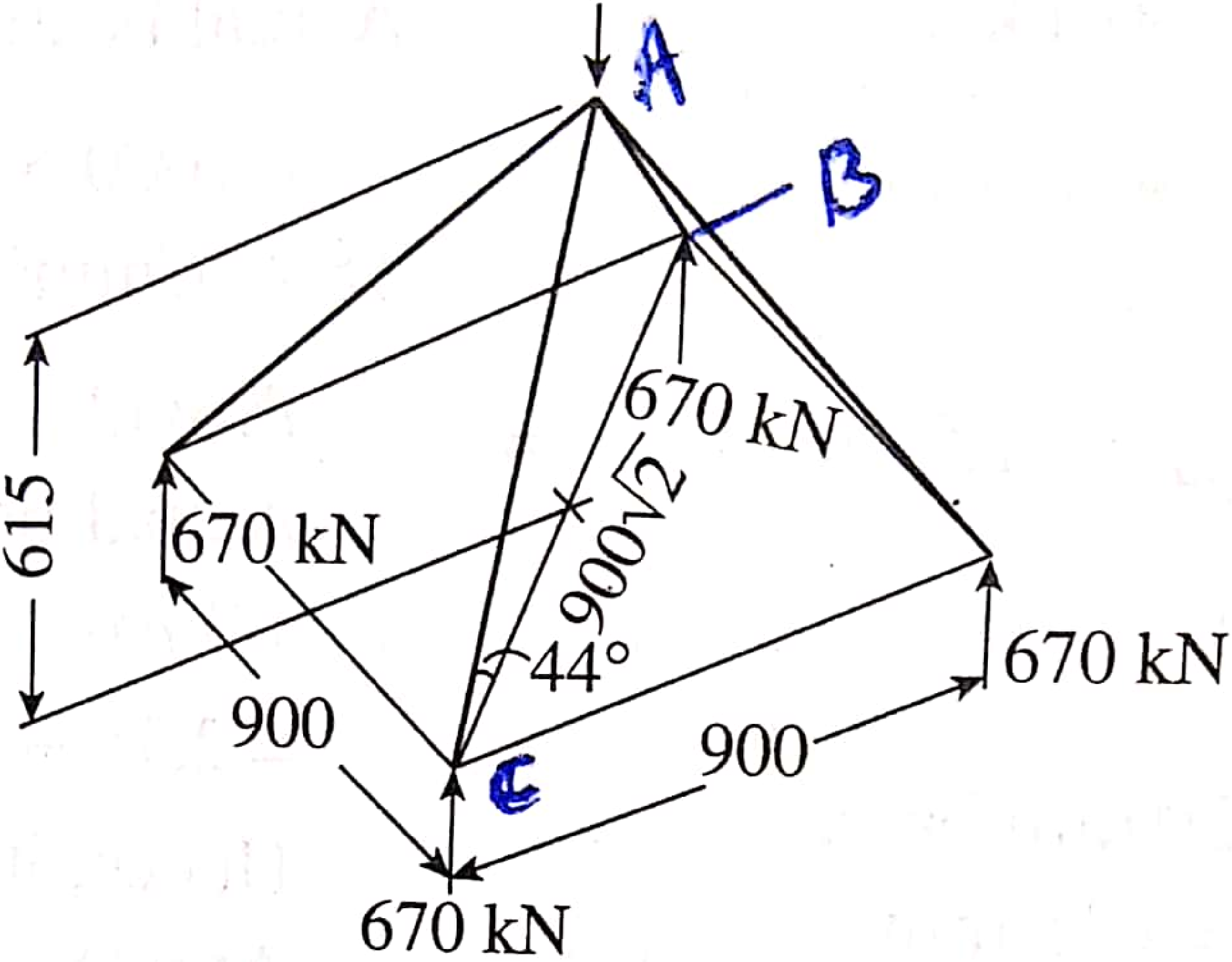
point A corresponds to column point on pilecap & AC is an inclined line which is not horizontal

~~[illegible] pile cap length is [illegible]~~ if pile spacing is 900 mm, diagonal BC = $900\sqrt{2}$ mm

perpendicular from A on BC, bisects it because four piles make a square = $\frac{900\sqrt{2}}{2} = 450\sqrt{2}$

so perpendicular length = effective depth of pile cap = 615 mm

if θ = angle of diagonal compression strut makes with the diagonal of bottom square of piles os



Strut and Tie forces

$\tan\theta = \dfrac{615}{450\sqrt{2}} \Rightarrow \theta = 44^{\circ}$

So now, $\tan 44 = \dfrac{\text{vertical ppu}}{\text{force in horizontal diagonal BC}}$

$\Rightarrow$ Force in horizontal diagonal BC $= \dfrac{670}{\tan 44^{\circ}} = 693\ \text{kN}$

$\therefore$ Force in tension in tie $= \dfrac{\text{force in horizontal diagonal}}{\sqrt{2}}$

$\Rightarrow T = \dfrac{693}{\sqrt{2}} = 490\ \text{kN}$

Required $A_{st} = \dfrac{T}{0.87 f_y} = \dfrac{490\times10^{3}}{0.87\times415} = 1357\ \text{mm}^2$ per tie

$\Rightarrow$ Provide five numbers of 20mm diameter rebars connecting the piles at bottom under each tie within a width of (1.5 × pile diameter = 1.5×300 = 450) 450 mm.

Provided steel, A_{st} provided = 1570 mm² for each tie

The area of elevation of pile cap = 1500 × d_{eff} = 1500 × 615

* considering the pile cap as a wide beam of length = 1500mm and cross section = 1500 mm × 615 mm

Required minimum steel (longitudinal) $\Rightarrow \dfrac{A_{st}}{bd} = \dfrac{0.85}{f_y}$ ((Cl 26.5.1.1) IS 456, P47)

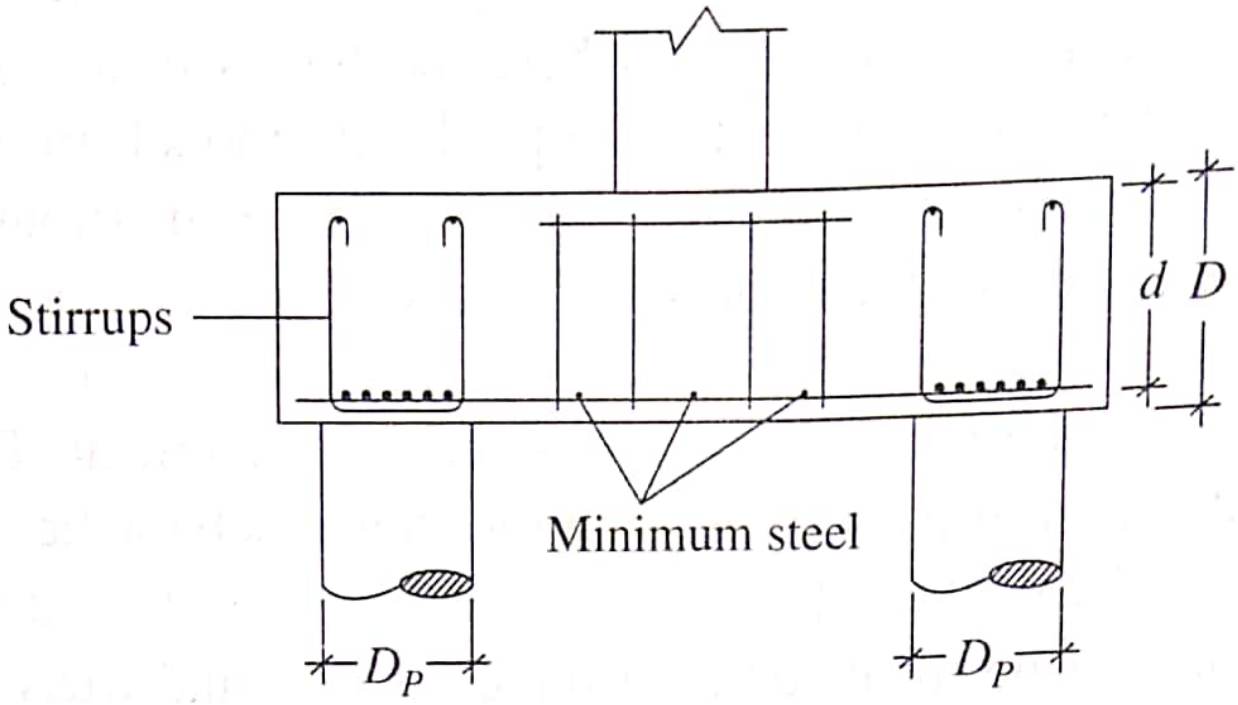
$\Rightarrow A_{st\,min} = \dfrac{0.85}{f_y} bd = \dfrac{0.85}{415} \times 1500 \times 615 = 1890\ \text{mm}^2$

* Longitudinal steel provided in wide beam = 2 × steel for one tie = 2×1570 = 3140 mm²

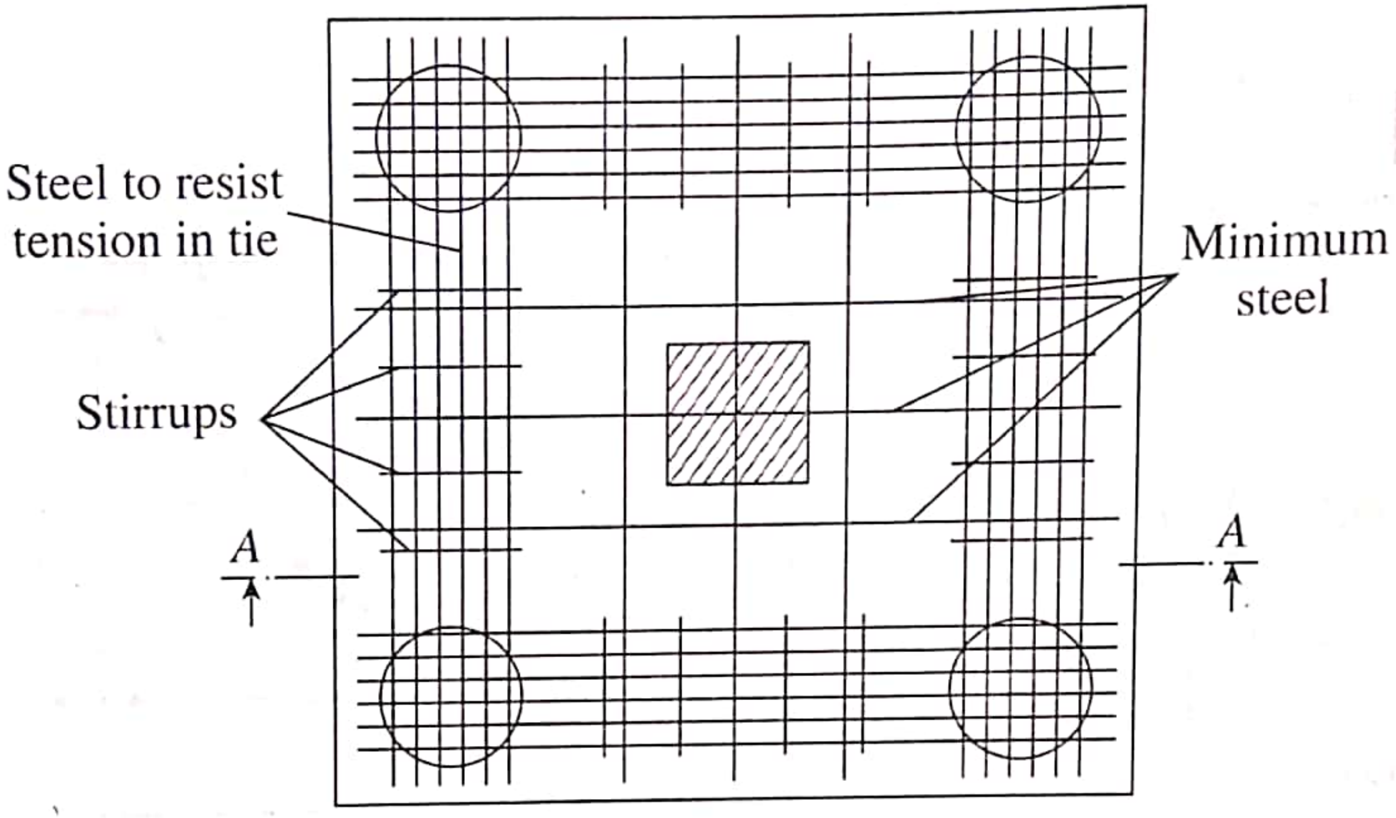
Because in the wide beam of pile cap, ~~along~~ in the c/s two piles exists, for each pile one tie in beam length & for 2 piles, 2 ties along length

* However provide 16mm ϕ rebars @160mm spacing, both ways, in the remaining portion of the pile cap, i.e. in the central region of pile cap, to control cracking. under four piles, four ties, for each tie, five longitudinal bars are laid.

* As per strut tie model method, one way shear crack is not required as column load is transferred as tension in tie steel.



Section A–A



Plan

Detailing in pile cap as per strut and tie model

→ **Check for bearing resistance** → $\begin{cases} \beta = \text{aspect ratio coefficient}, 0 \leq \beta \leq 1 \\ \alpha = \text{Confinement Coefficient} \leq 1 \end{cases}$ (DP8

① At column :→ Let A_1 = bearing resistance for column area

A_2 = bearing resistance for pile cap area

* $\sqrt{\frac{A_2}{A_1}} = \sqrt{\frac{1.5^2}{0.5^2}} = 3$

[by Adebar and Zhou (1993/1996)

[Canadian Code CSA A.23.3 : 2004

$\alpha = \frac{1}{3}\left[\sqrt{\frac{A_2}{A_1}} - 1\right]$

↳ Increase in bearing area of concentrated loads, is better to enhance shear strength of deep pile cap.

* ⇒ $\alpha = \frac{1}{3}(3-1) = 0.67$

* $\beta = 0.33\left(\frac{hs}{bs} - 1\right)$

hs = length of compressive strut

bs = width of compressive strut

$\left(\frac{hs}{bs}\right)$ = Aspect ratio of compressive strut

⇒ $\beta = 0.33\left(\frac{2d}{b_E} - 1\right)$

⇒ $\beta = 0.33\left(\frac{2 \times 0.615}{0.5} - 1\right) = 0.48$ $\frac{hs}{bs} = \frac{2d}{b_{col}}$ (aspect ratio at column end)

* Bearing stress, $\sigma_b = 0.3\sigma_{ck} + 3.5\alpha\beta\sqrt{\sigma_{ck}}$
at column

$= 0.3 \times 25 + 3.5 \times 0.67 \times 0.48 \times \sqrt{25} = \underline{13.13 \text{ N/mm}^2}$

* Cl. 34.4, P – 66, IS 456 : 2000,

As per LSD, the permissible bear stress on concrete = $\underline{0.45\sigma_{ck}}$

Permissible bearing stress = $0.45 \times 25 = \underline{\underline{11.25 \text{ N/mm}^2}}$

* Actual bearing stress under column =

$$\frac{\text{column load}}{\text{column area}} + \frac{\text{moment in column}}{\text{M.I}} \times y$$

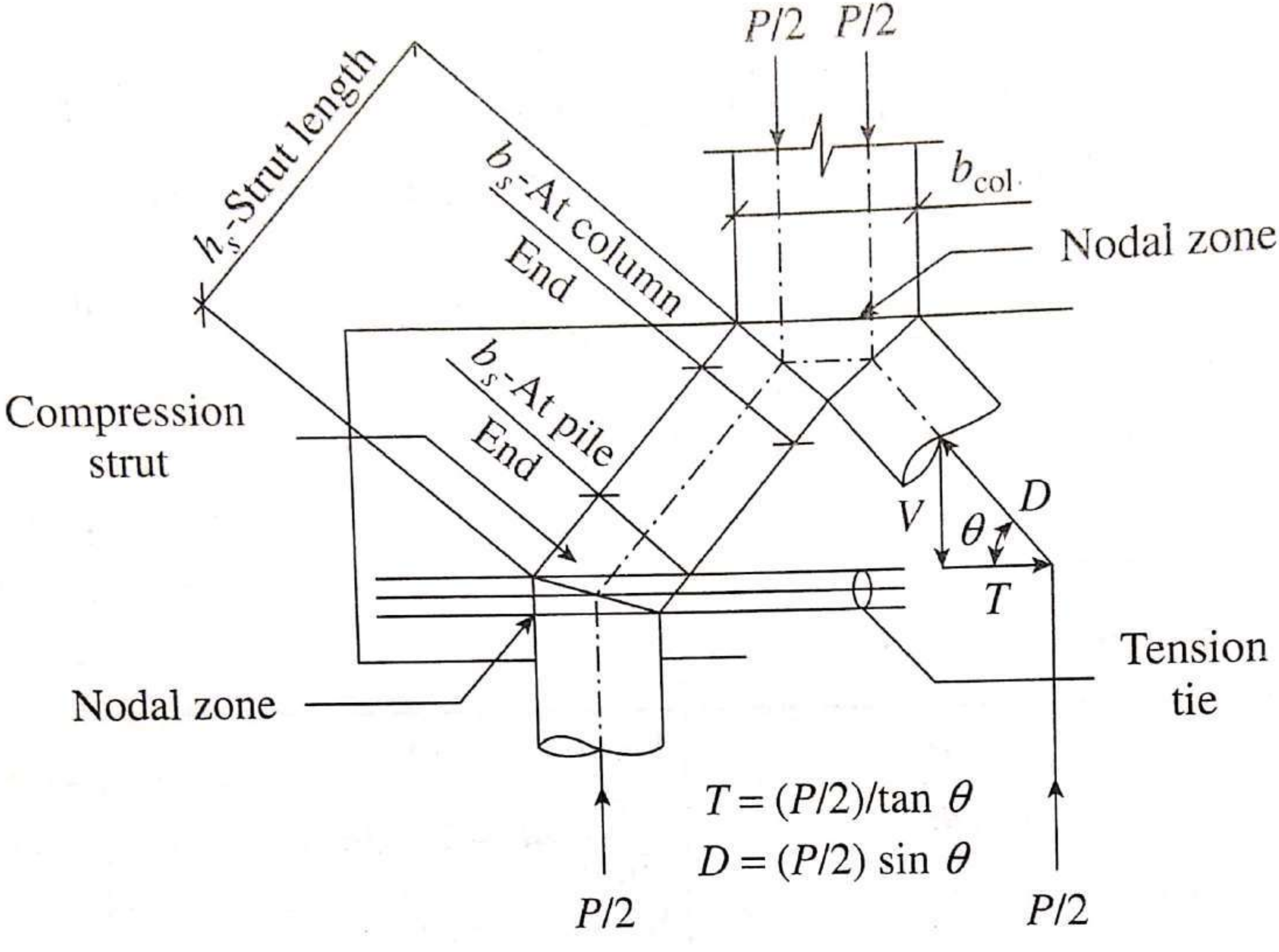
$$= \frac{1000 \times 10^3}{500^2} + \frac{300 \times 10^6}{500 \times 500^3/12} \times \left(\frac{500}{2}\right) = 4 + 14.4 = 18.4 \text{ N/mm}^2$$

* provide a pedestal of size (width 600 mm and depth 200 mm)

Actual bearing stress $= \frac{1000 \times 10^3}{600^2} + \frac{300 \times 10^6}{600 \times 600^3/12}\left(\frac{600}{2}\right) = 2.78 + 8.33 = 11.1 \text{ N/mm}^2$
< Permissible stress.

Hence it is ok now by ~~changing column~~ providing pedestal,

~~* At Pile = $\sqrt{\frac{A_2}{A_1}}$~~ i.e. the existing column section of 500 mm base size is enhanced at its base to 600 mm for 200 mm depth only to enhance bearing resistance. (Ok)


P/2
P/2
h_s-Strut length
b_s-At column End
b_{col}
Nodal zone
b_s-At pile End
Compression strut
D
V
θ
T
Tension tie
Nodal zone
$T = (P/2)/\tan\theta$
$D = (P/2)\sin\theta$
P/2
P/2

② At pile :-

$\sqrt{\frac{A_2}{A_1}} = \sqrt{\frac{\pi \times 300^2}{\pi \times 150^2}} = 2 < 4$

A_2 = pile base of 600 mm dia
A_1 = Pile diameter of 300 mm

So confinement coefficient $\alpha = \frac{1}{3}\left(\sqrt{\frac{A_2}{A_1}} - 1\right) = \frac{1}{3}(2-1) = 0.33 < 1$

Aspect ratio coefficient, $\beta = 0.33\left(\frac{d}{D_p} - 1\right)$, here at pile $\frac{h_s}{b_s} = \frac{d}{D_p}$

$\Rightarrow \beta = 0.33\left(\frac{615}{300} - 1\right) = 0.347$ as one compressive strut exists at pile

Bearing stress, $\sigma_b = 0.3\sigma_{ck} + 3.5\,\alpha\beta\sqrt{\sigma_{ck}}$

$= 0.3 \times 25 + 3.5 \times 0.33 \times 0.347\sqrt{25} = 9.5\ \text{N/mm}^2$

Actual bearing stress $= \frac{P_{PU}}{A_P} = \frac{670 \times 10^3}{\frac{\pi \times 300^2}{4}} = 9.48\ \text{N/mm}^2 < 9.5\ \text{N/mm}^2$ (ok)

IS456, permissible bearing strength $= 0.45 \times 25 = 11.25\ \text{N/mm}^2$

Development length

- As per SP:16, P 184, Table 65,

for M25 & Fe415 and 20mm diameter steel
development length required = 806 mm.

- Available length beyond centre line of pile = (D/2) + projection of pile cap + clear cover
= 150 + 150 − 75 = 225 mm

- Required length beyond edge of pile cap with 90° bend = 806 − 225 − Anchorage value for 90° bend (SP-16, Table 67, P186)
= 806 − 225 − 160 = 421 mm.

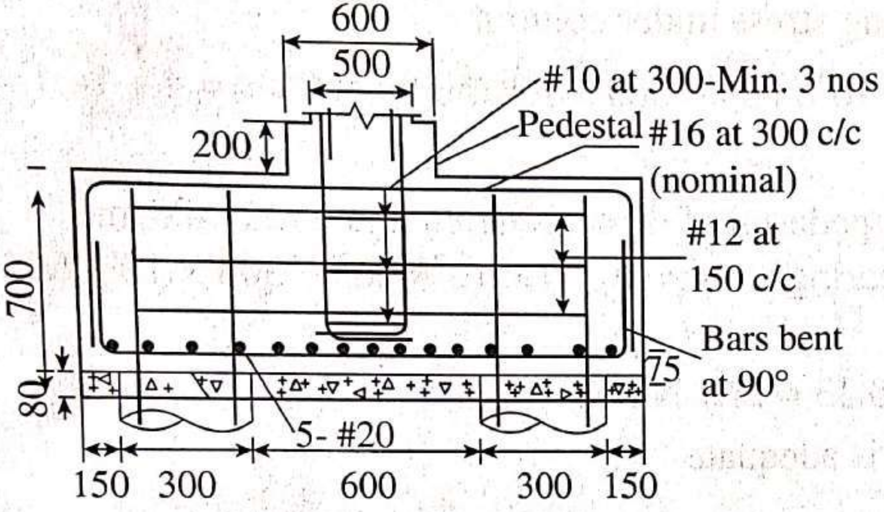
- So provide, $421 + 4\phi = 421 + 4 \times 20 = 501$ mm after 90° bend (check detailing)

- provide horizontal bursting steel around outer pile rods of 12mm ϕ @ 150mm c/c.

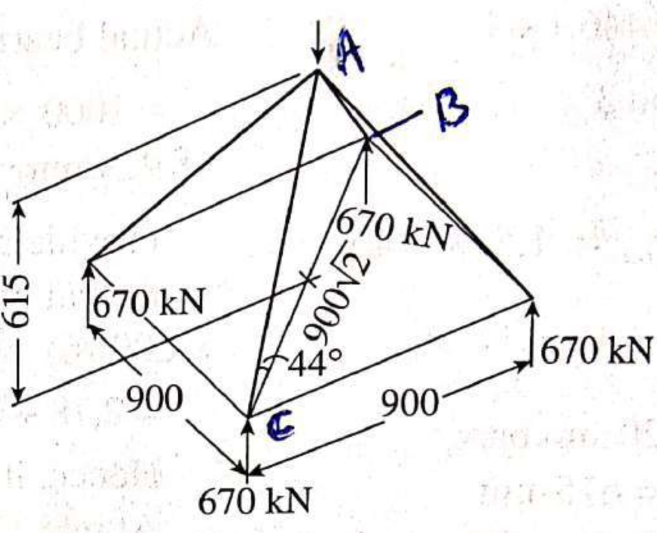
- provide dowels from piles into pile cap, A_{dowel} = (0.5 %) (cross sectional area of pile)
(cl 34.4.3 of IS 456, P 66)
$= \frac{0.50}{100} \times \frac{\pi \times 300^2}{4} = 353\ \text{mm}^2$

provide a minimum four vertical dowels of 16mm ϕ with 12mm ϕ tie @ 250mm c/c extending into cap and pile for a length of development length (501mm)

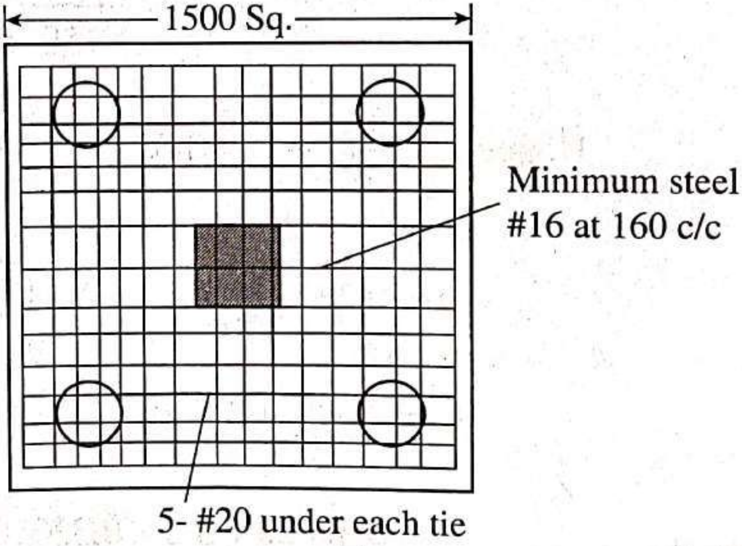
(Ans)


600
500
#10 at 300-Min. 3 nos
200
Pedestal #16 at 300 c/c
(nominal)
#12 at
150 c/c
700
Bars bent
at 90°
75
80
5- #20
150
300
600
300
150

(a)


A
B
615
670 kN
670 kN
900$\sqrt{2}$
670 kN
44°
670 kN
900
900
C
670 kN

(b)


1500 Sq.
Minimum steel
#16 at 160 c/c
5- #20 under each tie

Detailing in a RCC Pile Cap of 1.5m x 1.5m

a) Sectional elevation at the Pile cap
b) Strut and Tie forces
c) Plan of the Pile Cap detailing

Design of pile cap for bored cast in situ pile as per sectional method

IS 2911 (P1-S2) : 2010

Q - A RC column of size 500mm × 500mm is supported on four bored cast in situ piles of 300 mm diameter each. The column carries a load of 1000 kN, a moment of 300 kNm in the x-x direction, a shear force of 50kN on top of the pile, materials M25, Fe415. Design the pile cap assuming that the piles are capable of resisting the reaction from the pile cap. Adopt sectional method of pile cap design.

Sol →

After proceeding as per previous problem,

* maximum factored load on pile, $P_{PU} = 670 kN$

* pile cap size = 1.5 m × 1.5 m × 700 mm with 150mm projection of pile cap beyond outermost pile.

* spacing of the pile = 900mm c/c

* maximum BM at the face of column = P_{PU} (at pile centre) × distance to col face (cl. 34.2.1, P64, IS456)

$\Rightarrow BM_{max} = (670 \times 2)$ (distance from pile centre to column centre $- \frac{1}{2}$ column size)

$$= \underset{\overset{\uparrow}{2\ piles}}{2} \times 670 (0.45 - 0.25) = 268 \; kNm$$

$BM_{max} = 0.138 f_{ck} b d^2 \Rightarrow d^2 = \frac{BM_{max}}{0.138 f_{ck} b} = \frac{268 \times 10^6}{0.138 \times 25 \times 1500} \Rightarrow \underline{d_{eff} = 228 mm}$

deff required for pile cap = 228 mm

But $\underline{\text{provided } d_{eff}} = 700 - \text{clear cover} - \frac{\phi}{2} = 700 - 75 - 10 = \underline{615 mm}$.

So $\underline{\text{provided } d_{eff}}$ (= 615 mm) $\underline{>}$ $\underline{\text{required } d_{eff}}$ (OK)

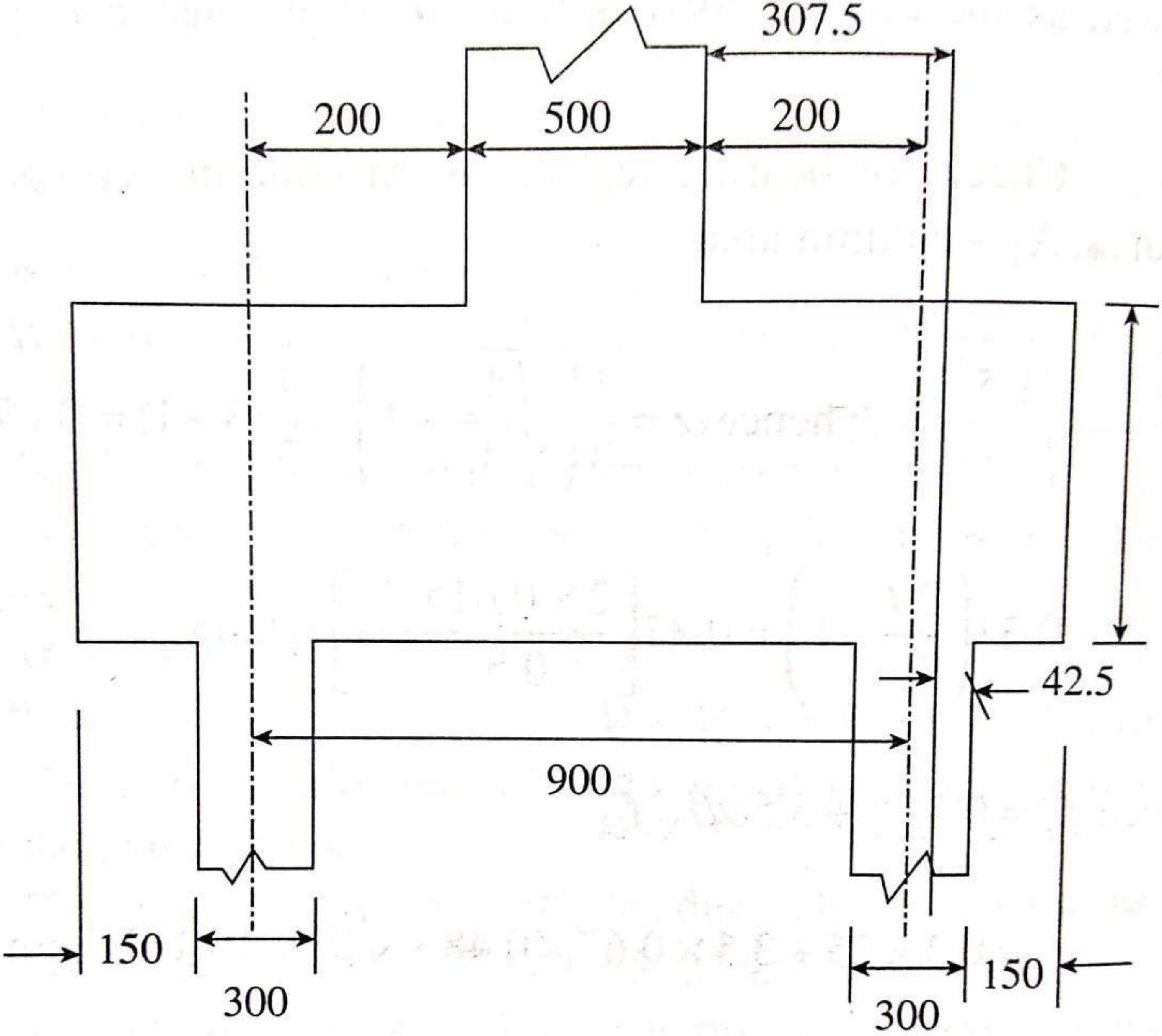
for a singly reinforced beam with $f_{ck} = 25 N/mm^2$ & $f_y = 415 N/mm^2$

$M_{lim} = 0.87 f_y A_{st} (d - 0.42 x_u) = 0.87 \times 415 A_{st} (615 - 0.42 \times 0.479 \times 615)$

$\Rightarrow 268 \times 10^6 = 0.87 \times 415 A_{st} (615 - 123.73) \Rightarrow \underline{A_{st} = 1511 mm^2}$

cl. 26.5.1.1, P47, IS456, $\frac{A_{st\,min}}{bd} = \frac{0.85}{f_y} = \frac{0.85}{415} = 0.002 \Rightarrow A_{st\,min} = 0.002 \times 1500 \times 615$

$\Rightarrow A_{st\,min} = 1890 mm^2 > 1511 mm^2$ (required A_{st})


307.5
200
500
200
700
42.5
900
150
300
300
150

DP 11

For a beam, minimum A_{st} ⇒ Cl 26.5.1.1 ⇒ $\frac{A_{st\,min}}{bd} = \frac{0.85}{f_y} = \frac{0.85}{415} = 0.002$

⇒ for a singly reinforced beam with $f_y = 415\ N/mm^2$, $A_{st\,min} = 0.2\% \times c/s$ area

* So provided steel = 1890 mm² ⇒ Lets provide 10nos, 16mm ϕ rebars in pilecap

* spacing = $\frac{1500-75-75-2\times\frac{16}{2}}{9} = 148\ mm < 180\ mm$, i.e. max spacing for Fe415 (As per Table 15, P46, IS456)

* provided steel = 10 nos of 16mm ϕ bars = $\frac{\pi}{4} \times 16^2 \times 10 = 2010\ mm^2 = \frac{2010}{1500\times615} \times 100 = 0.218\%$

So A_{st} provided = 2010 mm² (10nos, 16 mm ϕ bars) > $A_{st\,min}$ (i.e. 1890 mm²)

* In strut and tie model, A_{st} provided = 3140 mm² = 0.34% of c/s of pile cap

* In sectional method, A_{st} provided = 2010 mm² = 0.22% of c/s of pile cap.

* check for one way shear at 0.5d from face of column = 0.5 × 615 = 307.5 mm

i.e. at 307.5 mm from col face

i.e. at (200+150 − 307.5 = 42.5 mm) from centre line of pile.

* max load on pile = 670 kN & pile diameter = 300 mm

shear force at 42.5 mm from centre of pile by interpolation

at 300/2 = 150 mm from pile centre SF = 670 kN

at 42.5 mm from pile centre, SF = $\frac{670}{150} \times 42.5 = 190\ kN$

Table 19, P73, IS 456, τ_c for M25, $P_t = 0.22\%$, $\tau_c = 0.34\ N/mm^2$

$\tau_v = \frac{190\times10^3}{1500\times615} = 0.21\ N/mm^2 < \tau_c$ (Safe for one way shear)

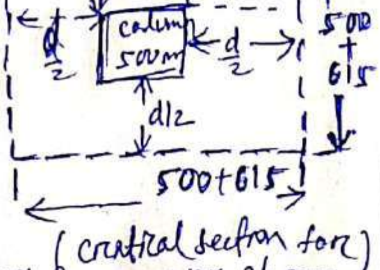
Check for punching shear :—

* critical section is at 0.5d around face of column = 0.5 × 615 = 307.5 mm

Load on column = 1000 kN

factored load = 1.5 × 1000 = 1500 kN

$\tau_v = \frac{V_u}{b_o d} = \frac{1500\times1000}{4(500+615)\times615} = 0.547\ N/mm^2$



(critical section for 2-way shear)

Punching shear strength for concrete, τ

$= K_s \tau_c = (0.5 + \beta_c)\, 0.25\sqrt{f_{ck}} = \left[\left(0.5 + \frac{0.5}{0.5}\right)\right] 0.25\sqrt{25}$

but $K_s \le 1$

$= 1 \times 0.25 \times 5 = 1.25\ N/mm^2$

DP12

As per cl 31.6.3.1, punching shear strength $= K_s \, 0.25\sqrt{f_{ck}}$
P 58-59 (IS456)

$$K_s = 0.5 + \beta_c \le 1$$

$$= 0.5 + \frac{0.5}{0.5} \le 1$$

$$= 1.5 \le 1$$

$$\text{so } K_s = 1$$

∴ Punching shear strength $= 1 \times 0.25\sqrt{25} = 1.25 \ N/mm^2$

$\tau_v = 0.547 \ N/mm^2$ (safe for punching shear)

Development length :-

As per SP : 16, P 184, Table 65,

for M25, Fe415, and 16mm ø rebars

development length required = 645 mm

Available length beyond centre line of pile $= \frac{D}{2}$ + projection of pile cap + clear cover

$= 150 + 150 - 75 = 225$ mm

Required length beyond edge of pile cap with 90° bend

= 645 − 225 − Anchorage value for 90° bend (SP 16, Table 67, P186)

= 645 − 225 − 128 = = 292 mm

So provide 292 + 4ø = 292 + 4×16 = 292 + 64 = 358 mm length after bend.

(Anchorage length for 16 mm ø bars = 128 mm.
(Table 67, P186, SP 16)

provide Bursting steel & dowels for piles same as previous problem.

(Ans)

ADVANCED CONCRETE STRUCTURES (CE 15031)

Module-III

Module III Syllabus

Retaining walls: Forces acting on retaining wall, Stability requirement,

Design of Cantilever Retaining walls

Design of Counterfort Retaining walls

Subject to Revision

Advanced Concrete Structures

DESIGN
OF
CANTILEVER RETAINING WALL

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CANTILEVER RETAINING WALL

cantilever RW

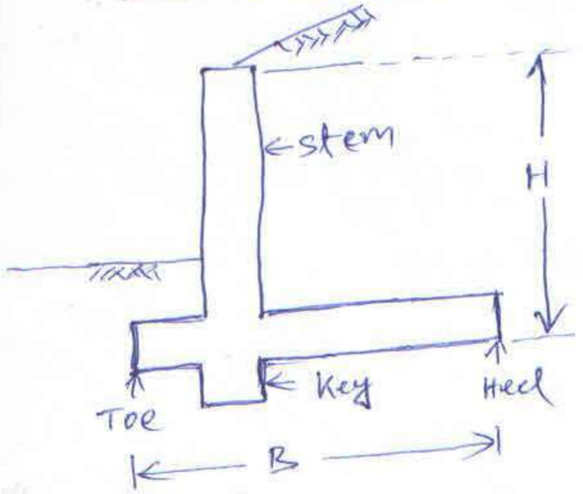
Angle of Internal friction (ϕ)

- measure of ability of rock/soil to withstand shear stress.
- Angle betn normal force (N) & resultant force (R) attained when failure occurs due to shearing stress.

Angle of repose =)

- steepest angle at which a ~~sloping~~ sloping surface formed of loose material is stable.
- max angle at which material can be piled up without slumping.

Retaining wall section :—



$B = 0.4H$ granular soil no surcharge
$B = 0.5H$ " " with "
$B = 0.6H$ cohesive soil no surcharge
$B = 0.7H$ " " with "

→ Factor of safety against overturning
=1.5 (granular soil)
=2 (cohesive soil)

Retaining walls (RW)

* Structure used to retain earth/other loose material which would not be able to stand vertically by itself.

* The retained material exerts a push on structure & tends to overturn/slide it.

Types

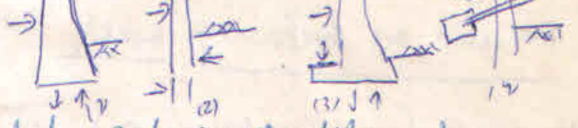
i) The stability of wall against overturning & sliding is maintained by wt of the wall, & wt of the earth on base of wall.

* The walls are subjected to vertical loads & lateral thrust from retained earth.

Gravity RW →

* made of brick masonry & plain cone.
* stability is maintained by wt.
* generally made up to 3m height

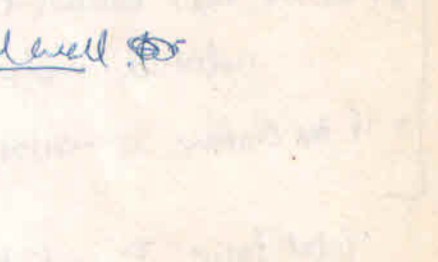
Gravity wall -(1)
Sliding wall -(2)
Cantilever wall (-(3)
Anchored wall -(4)



Cantilever RW →

* consists of vertical wall, heel slab, toe slab act as cantilever beams.
* stability is maintained by weight of retaining wall, wt of earth on wall base
* It is preferred when height of wall is 3m – 8m.

Counterfort RW →



* when height of wall > 8m.
* It is economical to tie vertical wall with heel slab by counterforts at some spacing.

* Counterforts act as tension member to support vertical wall & reduces BM

* " supports heel slab & reduces BM in it.
* counterforts are spaced at 1/3 of height of wall.
* Stability is maintained by wt of earth on base & self wt.

Retained material side - heel
Free side — — — Toe.

Buttress RW →

* vertical transverse wall is tied to toe of RW at some spacing.

* it acts as a compression member to support the vertical wall & reduces BM in it.

* It provides support to ~~bed~~ toe slab & reduces BM in it.

* it is located on the side of vertical wall opposite to the retained material

* spacing — $\frac{1}{3}$ the height of RW.

* Though compression buttress is more economical than ~~tension~~ counterfort, still counterfort is more widely used than buttress because counterfort is hidden beneath the retained material, where as buttress is exposed.

* Buttress is exposed & occupies more space in front of the wall which could be utilized more efficiently.

Forces on Retaining wall

plastic equilibrium — active state (R.W moves away from backfill) / — passive state (RW moves towards backfill)

↳ if every point in soil is on the verge of failure.

* main force acting on RW is due to the retained material.

Earth pressure by retained material, $P = K\gamma h$.

k = coefficient depends on physical property of soil.

γ = density of Retained material

h = depth of section below earth surface

* For Active earth pressure, $P = K_a \gamma h$, $K_a = \cos\theta\left[\frac{\cos\theta - \sqrt{\cos^2\theta - \cos^2\phi}}{\cos\theta + \sqrt{\cos^2\theta - \cos^2\phi}}\right]$

* Pressure acts parallel to top surface of retained material.

* If top surface is horizontal, $\theta = 0$

ϕ = angle of repose of soil / internal friction. friction.

θ = angle of surcharge; i.e. slope of retained earth with horizontal.

(Surcharge is the portion of backfill lying above horizontal plane at the elevation of top of a wall)

Total force $P_a = \frac{1}{2} K_a \gamma h^2$ per unit length of wall.

horizontal component of this force, $P_h = P_a \cos\theta$.

vertical 〃 〃 〃 〃, $P_v = P_a \sin\theta$

* Passive pressure, $P = K_p \gamma h$

Total force, $P_p = \frac{1}{2} K_p \gamma h^2$, $K_p = \cos\theta\left[\frac{\cos\theta + \sqrt{\cos^2\theta - \cos^2\phi}}{\cos\theta - \sqrt{\cos^2\theta - \cos^2\phi}}\right]$

* Passive force is quite high than active force.

* Passive force contribution in design of RW is neglected which is on conservative side.

because passive force calculation is generally more in error than active pre. calculation.

Stability requirement

① * Resisting moment (RM) must be more than overturning moment (OM)

* cl. 20.2 (P-33 of IS 456) → RM ≮ [1.2 (OM due to DL) + ~~OM~~ 1.4 (OM due to imposed load)]

* The factor of safety against overturning is computed by neglecting vertical component of active earth pressure.

W = sum of all vertical loads made up of wt of backfill on inner slab + wt of wall & base slab + wt of front fill.

* In usual cases of retaining wall, no overturning moment due to dead load exists.

$$FS = \frac{\text{resisting moment}}{\text{overturning moment}} \Rightarrow 1 = \frac{0.9 W x_1}{1.4 P_h (H/3)}$$

$$\Rightarrow \boxed{1.55 = \frac{W x_1}{P_h (H/3)}}$$

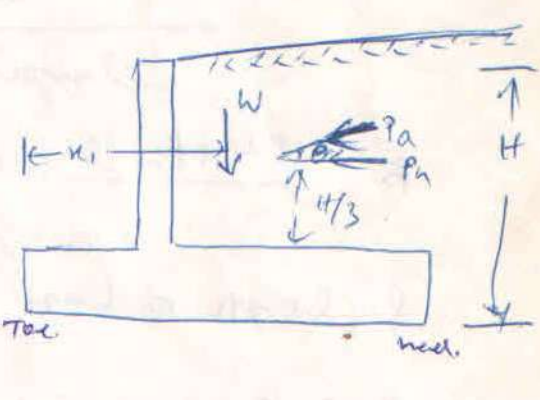
x_1 = CG of vertical loads from toe

H = depth of bottom of base slab below earth surface

F.S against overturning = 1.55.

* earth pre = imposed load.

when earth is wet, it weight will increase & angle of repose decreases, increasing earth pressure.



② Frictional resistance →

Cl. 20.2, (P-33) - IS 456

* not less than F.S = 1.4 against sliding.

$$FS = \frac{\text{resisting force}}{\text{sliding force}} \Rightarrow 1.4 = \frac{0.9\mu W}{P_h} \Rightarrow \boxed{1.55 = \frac{\mu W}{P_h}}$$

μ = coefficient of friction betn soil & footing.

$$F.S = \frac{\text{Resisting force}}{\text{sliding force}} ≮ 1.4 \Rightarrow \text{Resisting force} = 1.4 \times \text{sliding force}$$

$$\Rightarrow 0.9 (\mu W) = 1.4 \times P_h$$

$$\Rightarrow \frac{\mu W}{P_h} = \frac{1.4}{0.9} = 1.55.$$

∴ Factor of safety against sliding = 1.55.

Proportioning of cantilever RW →

* min. depth of foundation below ground level = 1m.

* Total height of RW = desired diff in elevation + depth below ground level

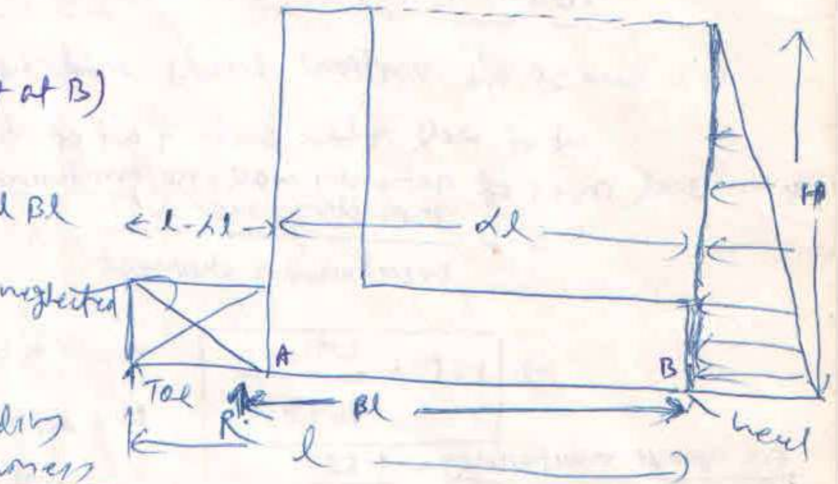
* Length of slab varies from 0.6 − 0.8 of total wall height.

Assume → Level of soil retained is horizontal, $\theta = 0$.

→ Ave. unit wt = γ for concrete & earth.

→ neglecting concrete in toe for equilibrium.

① $R = W = \gamma H \alpha l$



(ii) $P_a \frac{H}{3} + W \times 0.5\, \alpha l = R \beta l$ (moment at B)

$$K_a \frac{\gamma H^2}{2} \cdot \frac{H}{3} + \gamma H \alpha l \times 0.5\, \alpha l = \gamma H \alpha l \beta l$$

$$\Rightarrow \boxed{\frac{l}{H} = \sqrt{\frac{K_a}{3\alpha(2\beta - \alpha)}}}$$

$$\alpha = \frac{\text{length of heel including wall thickness}}{\text{length of base.}}$$

$$\beta = \frac{\text{C.G. of line of action of vertical loads \& horizontal earth force}}{\text{length of base.}}$$

l = length of base.

* variable β is selected based on soil type & desired pres. distribution.

β is assumed betn 0.5 & 0.7.

Also $\boxed{\frac{l}{H} = \sqrt{\frac{K_a \cos\theta}{(1-m)(1+3m)}}}$ ↳ Krishna & Jain

$$\boxed{m = \frac{\text{length of toe}}{\text{length of base}} = 1 - \frac{4}{9q} \text{ if } \theta = 0}$$

$$= 1 - \frac{3}{8q} \text{ if } \theta \neq 0.$$

$$\boxed{q = \frac{\gamma h}{P_s}}$$

P_s = bearing capacity of soil

h = depth of top of heel slab below earth surface.

* m varies betn 0.35 − 0.65,

* $\boxed{\alpha \simeq 1 - m}$

Thickness of footing

* base thickness is usually 10% of total height with a minimum 30cm.
* exact thickness is governed by BM & SF consideration.

Thickness of vertical wall

* Thickness ~~at~~ top of wall $\nless$ 15 cm.
* Base thickness of vertical wall is determined ~~by~~ as required for BM & SF. may be about 15% of wall height.

* Design of heel →

* Normally resultant frc. due to downward wt of earthfill & upward bearing pressure ~~is downward~~ causing tension on top ~~of~~ face of heel.
* Reinforcement is designed for this position as cantilever beam of unit width.

Design of toe →

* Normally wt of earth above toe is neglected.
* It is designed for upward acting bearing pressure as a cantilever beam. Tension — bottom face & steel is designed for this position.

Following retaining wall proportions are usually followed :-

Top width of stem = 200 mm.

Bottom width of stem : to be computed,

width of base slab = b

b = 0.5H to 0.6H for wall without surcharge.

= 0.7H for surcharged wall.

Toe projection = $\frac{H}{6}$ or $\frac{b}{3}$. H = overall height of RW.

* Bottom thickness of stem is determined from B.M. considerations.
* usually thickness of base slab = bottom thickness of stem.

---

* cover to reinforcement in stem — 30mm to 40mm.
* max pressure at base < Bearing capacity of soil.
* cover to reinforcement at base = 50 to 60mm.

Shear reinforcement → Not required for retaining wall.

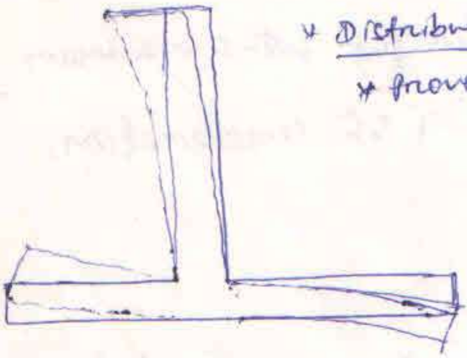
* Resistance of concrete is sufficient to resist SF imposed.

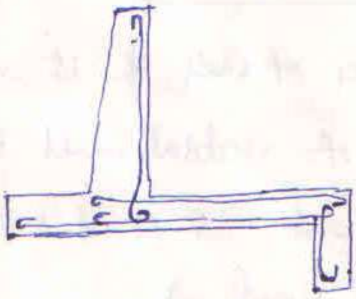
* Distribution steel → (horizontal)

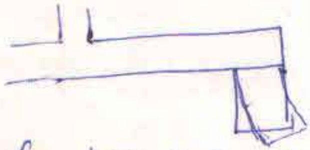
* provided in both stem & base slab.

≮ 0.15% of gross conc. area (M/S bars)

≮ 0.12% of " " " (HYSD)







* reinforcement for key is small & so alternate bars of toe slab may be continued & bent down to form key steel.

Key may be provided under stem so that alternate bars of stem may be brought down to serve as key steel.

Steps →

① For 1m run of retaining wall, max BM at bottom of stem is found.

By equating this with max MR, deff can be found & so D.

base slab thickness = thickness of stem at bottom.

② Stability check → find $P_{max}/_{min}$, $P_{max\,(toe)}$ < Bearing capacity

③ Stem reinforcement ?

④ Toe slab →

max BM for cantilever slab due to upward soil pressure/reaction.

⑤ Heel slab →

Loads → ① self wt ② wt of soil above ③ surcharge ④ upward soil reaction.

⑥ Factor of Safety :- against sliding & overturning is investigated.

⑦ Shear key →

if factor of safety against sliding is insufficient then shear key is provided.

Q. Design the vertical stem of a T shaped retaining wall for a height 3m above the ground level. The top of earth retained is surcharged at an angle of 10° with horizontal. The angle of repose of earth = 29°, density of earth = 17 kN/m3. Safe bearing pressure = 100 kN/m2.

Sol - 1) Rankine formula, depth of foundation $= \frac{P}{\gamma}\left(\frac{1-\sin\phi}{1+\sin\phi}\right)^2$

$$= \frac{100}{17}\left(\frac{1-\sin 29}{1+\sin 29}\right)^2 = 0.71m \simeq 1m.$$

Total height of wall including base = 4m. (3+1)

$\left\{ \begin{array}{l} \theta = \text{angle of surcharge} \\ \phi - \text{angle of repose} \end{array} \right.$

Let base thickness = 40 cm

clear height of wall = h = 3.6m.

Pressure behind the wall $= \frac{1}{2} k_a \gamma h^2$ (acts at h/3 from base)

$$k_a = \cos\theta \left[\frac{\cos\theta - \sqrt{\cos^2\theta - \cos^2\phi}}{\cos\theta + \sqrt{\cos^2\theta - \cos^2\phi}}\right] = 0.365$$

$P_a = \frac{1}{2} \times 0.365 \times 17 \times 3.6^2 = 40 \text{ kN/m}.$

P_h = horizontal component of pre. $= P_a \cos\theta = 40 \cos 10 = 39.4$ kN/m.

BM at base of vertical wall $= P_h \frac{h}{3} = 39.4 \times \frac{3.6}{3} = 47.3$ kNm/m

Consider 1m width of vertical slab ⇒ its thickness.

from Fe 415, $\frac{x_{u\,max}}{d} = 0.48$, BM $= 0.138 f_{ck} b d^2$

using a partial safety factor 1.5 on horizontal earth pressure.

$1.5 \times 47.3 \times 10^6 = 0.138 \times 15 \times 1000 \times d^2$, d = 186mm (at base of stem)

∴ Let d ≃ 190mm ⇒ $\left\{ \begin{array}{l} \text{overall thickness of stem base} = 220\text{mm}. \\ \text{minimum stem base thickness} = 150\text{mm (top of wall)} \\ \text{Average thickness} = 190\text{ mm} \end{array} \right.$

Area of tension steel $= A_t = \frac{0.36 f_{ck} b x_{u\,max}}{0.87 f_y} = \frac{0.36 \times 20 \times 1000 \times (0.48 \times 190)}{0.87 \times 415} = 1364 \text{ mm}^2/\text{m}$

use 12 mm ϕ tor steel @ 80 mm c/c.

Ast provided $= \frac{113 \times 1000}{80} = 1412.5 \text{ mm}^2.$

Ast provided %ge $= \frac{113 \times 1000/80}{1000 \times 190} \times 100 = 0.74\% > 0.12\%$

Check for shear

Critical section ~~for~~ is at 'd' from stem bottom.

$\therefore h = 3.6 - 0.19 = 3.41m.$

$V = \frac{1}{2} K_a \gamma h^2 \cos\theta =$

$= \frac{1}{2} \times 0.365 \times 17 \times 3.41^2 \times 0.984 = 35.5 kN.$

Factored S.F. :, $V_u = 1.5 \times 35.5 = 53.25 kN$

Nominal shear stress, $\tau_v = \frac{V_u}{bd} = \frac{53.25 \times 10^3}{1000 \times 190} = 0.28 N/mm^2$

Permissible shear strength of concrete = 0.56 N/mm²] OK

Reinforcement curtailment

No bar can be curtailed in less than distance L_d from bottom of stem.

for 12φ bars, $L_d = \frac{\phi \sigma_s}{4 \tau_{bd}} = \frac{(0.87 \times 415) \times 12}{4 \times 1.6 \times 1} = 56 \times 12 = 672mm.$

Let us curtail bars at $\underset{h}{3.6} - 0.7 = 2.9m$ from top of wall.

$d = 176mm.$

$BM = \frac{1}{6} k_a \gamma h^3 \cos\theta = \frac{1}{6} \times 0.365 \times 17 \times 2.9^3 \times 0.984 = 24.8 kNm.$

$1.5 \times 24.8 \times 10^6 = 0.87 \sigma_y A_{st} \left(d - \frac{\sigma_y A_t}{\sigma_{ck} b}\right)$

$= 0.87 \times 415 A_t \left(176 - \frac{415 A_t}{15 \times 1000}\right)$

$\Rightarrow A_t = 650 mm^2.$

12 φ bars @ 160mm c/c, giving area = 678 mm² > 650mm².

* According to IS456, bar should extend ~~by~~ beyond the point it is no longer required to bear tension by 12φ or d, which is more.
* Actual curtailment should be done at 2.6m below top of vertical wall.

~~TOP~~ Development length

* The wall must be connected ~~by~~ to base by carrying 12mm bars inside full length of toe to form toe reinforcement, or
* Dowel of 12mm bars can be used upto a height of at least development length = 672 mm

inside wall & extended to full length of toe.

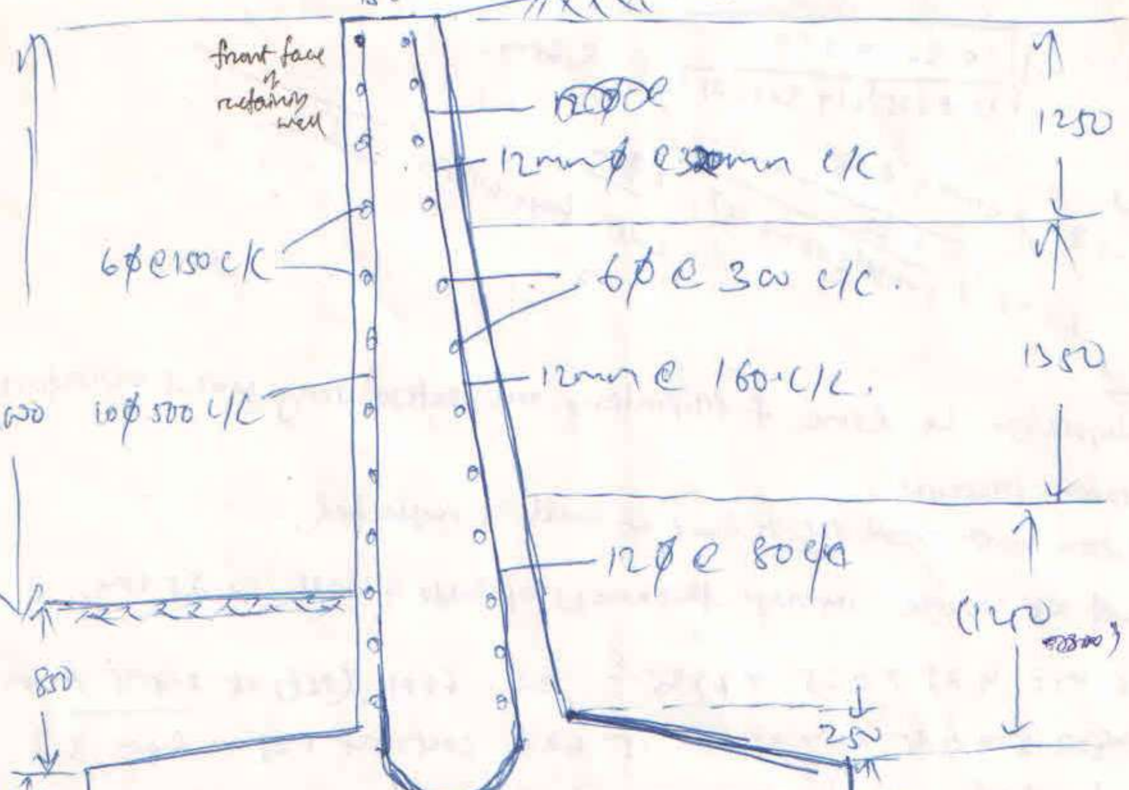
Temp & Shrinkage reinforcement →

Provide horizontal steel – 0.15% of vertical sectional area of wall

$$= \frac{0.15}{100} \times 3.6 \times 100 \left(\frac{22+15}{2}\right) = 10\,cm^2.$$

$$\text{No of 6mm MS bars} = \frac{10}{0.28} = 36 \text{ nos.}$$

* Since front ~~face~~ face is generally more exposed to temp. changes, more of this reinforcement is provided here.
* Normally it is suggested that 2/3 be placed on the front face & remaining on inner face.
* Provide 24 bars horizontally on front face at a spacing of 150mm c/c & 12 bars on inner face at a spacing of 300 mm c/c.
* To support these horizontal bars on front face 10mm ϕ bars @ 500mm c/c vertically.



Q/ Determine dimensions of a T shaped retaining wall for a height of $\overset{3.5m}{4m}$ above the ground level. The top of earth retained is surcharged at 20° with horizontal. Angle of repose of earth = 35°, density of earth = 19kN/m3, safe bearing capacity of soil = 80kN/m2, coefficient of friction betn conc & soil = 0.55

Sol → depth of foundation = $h_d = \frac{p}{\gamma}\left(\frac{1-\sin\phi}{1+\sin\phi}\right)^2 = \frac{80}{19}\left(\frac{1-\sin 35}{1+\sin 35}\right)^2 = 0.31$

depth of foundation below GL = 1m.

Total height of wall including base = 4.5m.

$$\frac{l}{H} = \sqrt{\frac{k_a \cos\theta}{(1-m)(1+3m)}}$$

Let base thickness = 25cm, height of wall = 4.25 m.

$$q = \frac{\gamma h}{p_s} = \frac{19 \times 4.25}{80} = 1, \quad m = 1 - \frac{3}{8q} = 0.625.$$

$$k_a = \cos\theta\left[\frac{\cos\theta - \sqrt{\cos^2\theta - \cos^2\phi}}{\cos\theta + \sqrt{\cos^2\theta - \cos^2\phi}}\right] \quad \theta = 20°, \phi = 35° \therefore k_a = 0.32.$$

$$l = 4.5\sqrt{\frac{0.32 \times 0.94}{(1-0.625)(1+3\times0.625)}} = 2.38m.$$

or $\frac{l}{H} = \sqrt{\frac{k_a}{3\lambda(2\beta-\lambda)}}$ Let $\beta = 0.7$, $\lambda = 1-m = 0.38$

$l = 4.5\sqrt{\frac{0.32}{3\times0.38(1.07-0.38)}} = 2.36m$

Let base width = 2.5M.

Proportions

* stability analysis can be done if proportions are satisfactory w.r.t overturning, sliding & bearing pressure.

* In this calculation earth above toe in front of wall is neglected.

Let us assume average thickness of base & wall as 25 cm.

wt of wall = $W_1 = 4.25 \times 0.25 \times 1 \times 25 = 26.6$kN, (acts at 0.875 from B)

wt of base $W_2 = 2.5 \times 0.25 \times 1 \times 25 = 15.6$kN (acts at 1.25m. from B)

wt of earth above heel = $W_3 = \left(\frac{4.25+4.525}{2}\right) \times 1 \times 0.75 \times 19 = 62.5$kN (acts at y from B)

$$y = \frac{0.75}{3}\left[\frac{2\times4.5 + 4.775}{4.5+4.775}\right] = 0.37 m.$$

Total earth pre = $P_a = \frac{1}{2}k_a\gamma H^2 = \frac{1}{2}\times0.32\times19(4.525+0.25)^2 = 69.3$ kN.

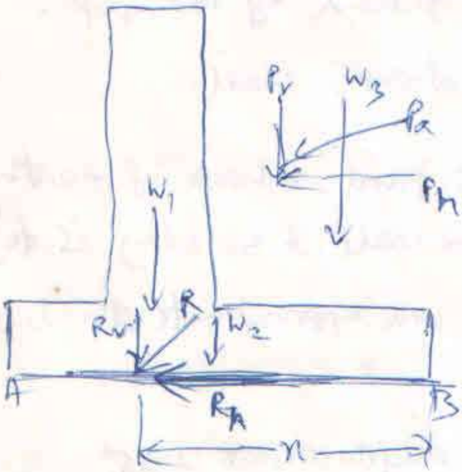
It acts at $\frac{H}{3}$, i.e. 1.59M above base of footing.

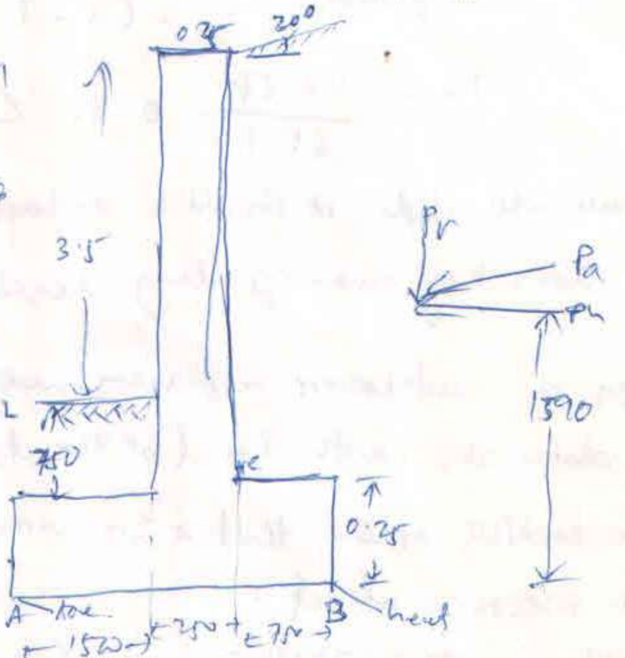
vertical component of earth pressure = $P_v = 69.3 \times \sin 20 = 23.7$ kN.

Horizontal " " " " = $P_h = 69.3 \cos 20 = 65.1$ kN

Let us assume resultant R passes through base at distance x from B.

Taking moment of all forces about B





$$x = \frac{\overset{w_1}{26.5 \times 0.875} + \overset{w_2}{15.6 \times 1.25} + \overset{w_3}{62.5 \times 0.370} + \overset{P_h}{65.1 \times \frac{4.77}{3}}}{26.5 + 15.6 + 62.5 + \underset{(P_v)}{23.7}} = 1.32\text{ m.}$$

(P_v is considered in denominator only. Becoz Pa is resolved into P_h & P_v vertically above B so that line of action of Pv passes through B. so its moment about B is zero) ①

* So resultant strikes the base within middle third. ①
* so entire base is in compression.

Eccentricity of resultant from Base centre $= 1.32 - \frac{2.5}{2} = 0.07$m.

Pressure on toe & heel, $P = \frac{W}{bl}\left(1 \pm \frac{6e}{l}\right) = \frac{128.3}{1 \times 2.5}\left(1 \pm \frac{6 \times 0.07}{2.5}\right)$ ②

$= 60$ kN/m² at toe < 80 kN/m²
$= 42.75$ kN/m² at heel. < 80 kN/m²
} safe bearing capacity

check for overturning →

* Resisting moment about A = $= Wx_1$

$= 26.5(2.5 - 0.875) + 15.6(1.25) + 62.5(2.5 - 0.37)$
$= 195.85$ kNm.

* overturning moment about A $= P_h \frac{H}{3} = 65.1 \times \frac{4.775}{3} = 103.6$ kNm.

$F\cdot S = \frac{195.85}{103.6} = 1.89 > 1.55$ ok

Check against sliding

Resisting force $\mu W = 0.55(26.5 + 15.6 + 62.5) = 57.58$

Sliding force $= P_h = 65.1$

$$FS = \frac{57.58}{65.1} = 0.88 < 1.55$$

∴ Wall will slide, it should be anchored into ground by means of, a shear key running along length of retaining wall.

Q. Design a cantilever retaining wall to support a bank of earth 5m high above the earth level at the toe of the wall. A building is to be built on the backfill. Assume that a 3m surcharge will approximate the lateral earth pressure effect.

Earth density = 17 kN/m3, angle of internal friction (repose) = 35°

coefficient of friction betn concrete & soil = 0.45

Bearing capacity = 150 kN/m2, M 20, Fe415.

Sol :- Depth of foundation $= h_d = \frac{P}{\gamma}\left(\frac{1-\sin\phi}{1+\sin\phi}\right)^2 = \frac{150}{17}\left(\frac{1-\sin 35}{1+\sin 35}\right)^2 = 0.65$ m.

Allowing 1.25m for frost penetration to the bottom of footing in front of the wall,

Total height $= 5 + 1.25 = 6.25$ m.

Let us assume, footing thickness = 10% × total height

= 60cm.

Base length, $l = H\sqrt{\frac{k_a\cos\theta}{(1-m)(1+3m)}}$, $m = 1 - \frac{4}{9q}$, $q = \frac{\gamma h}{P}$, $k_a = \frac{1-\sin\phi}{1+\sin\phi} = 0.27$.

$m = 1 - \frac{4}{9\times0.64} = 0.3$ ⇒ $q = \frac{17}{150}(6.25 - 0.6) = 0.64$

$l = 6.25\sqrt{\frac{0.27\times\cos\theta}{(1-0.3)(1+3\times0.3)}}$ $(\theta = 0)$ $= 2.83\text{m} \simeq 3\text{m}$.

$ml = 0.3\times3 \simeq 1$m

Total height = 6.25m.

base = 0.5 − 0.6H

= 3.125m − 3.75m

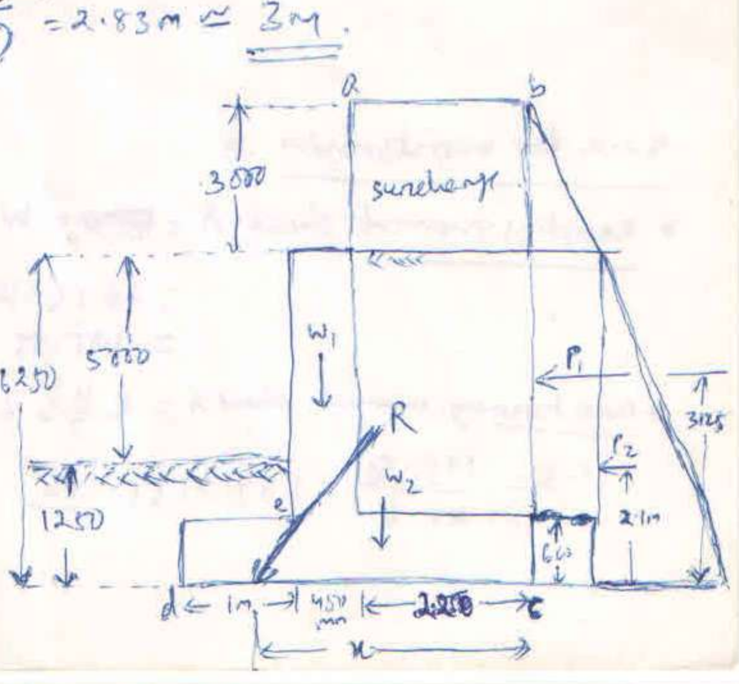
≃ 3.7m.

base thickness = 10% of total ht. = 0.1 × 6.25 = 60 cm

stem thickness = ~~base~~ thick = 450 mm

Toe Projection $= \frac{b}{3} = \frac{3.7}{3} \simeq 1.233 \simeq 1.00$m.

heel alone $= 3.7 - 1.00 - 0.45 = $ ~~2.00~~m 2.25M.



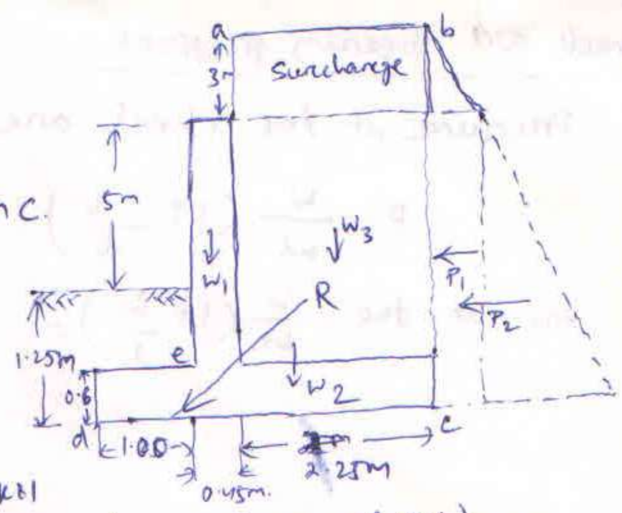
stability check

- wt of wall = $W_1 = (6.25 - 0.6)0.45 \times 1 \times 25 = \underline{63.5\text{ kN}}$

  it acts at $(2 + 0.225 = \underline{2.500\text{m}}$ from C.

- wt of base, $W_2 = 3.7 \times 0.6 \times 1 \times 25 = \underline{55.5}\text{ kN}$

  at $\underline{1.85\text{m}}$ from c.



(Dimensioning of RW)

- wt of earth over heel including surcharge,

  $W_3 = (3+6.25-0.6) \times \overset{2.25}{\cancel{2}} \times 1 \times 17 = \underline{331.0\text{ kN}}$ 331kN

  at $\underline{1.125}$m from c.

- Earth pre. due to surcharge, $P_1 = k_a \gamma h_1 h_2 = \left(\frac{1-\sin\phi}{1+\sin\phi}\right) \times 17 \times 3 \times 6.25 = \underline{86\text{ kN}}$

  at $= \frac{6.25}{2} = \underline{3.125\text{ m}}$ ~~from~~ above c.

- Earth pre. $P_a = \frac{1}{2} k_a \gamma h_2^2 = \frac{1}{2} \times 0.27 \times 17 \times 6.25^2 = \underline{90\text{kN}}$

  at $= \frac{6.25}{3} = \underline{2.1\text{m}}$ above c

Let x = distance from c where resultant force strikes the base :-

$$x = \frac{63.5 \times 2.500 + 55.5 \times 1.85 + 331 \times 1.125 + 86 \times 3.125 + 90 \times 2.1}{63.5 + 55.5 + 331.0}$$

= 2.40m from c.

Eccentricity $e = 2.4 - \frac{3.7}{2} = \underline{0.55\text{m}}.$

$\frac{b}{6} = \frac{3.7}{6} = \underline{0.62\text{ m}}$: $\therefore e < \frac{b}{6}$ (ok)

Factor of safety against overturning →

Resultant of vertical forces from c is at $= \frac{63.5 \times 2.5 + 55.5 \times 1.85 + 331 \times 1.125}{63.5 + 55.5 + 331} = \underline{1.4\text{m}}.$

Resisting moment about d = (63.5+55.5+331) × (3.7−1.4) = 450 × 2.3 = 1035.2 kNm.

overturning moment about d = 86 × 3.125 + 90 × 2.1 = 457.75 kNm.

Factor of safety $= \frac{1035.2}{457.75} = 2.26 > 1.55.$ ok

Factor of safety against sliding →

Forces causing sliding = 86 + 90 = 176 kN.

Frictional force = $\mu W = 0.45 \times \underset{}{\overset{\text{vertical force}}{450}} = 202.5$ kN

$FS = \frac{202.5}{176_{\text{— hori. force}}} = 1.15 < 1.55$

since resistance does not provide adequate safety against sliding, a shear key against sliding is required.

check for bearing pressure

Pressure at toe & heel are :-

$$P = \frac{W}{bl}\left(1 \pm \frac{6e}{l}\right)$$

$$\text{pre. at toe} = \frac{W}{bl}\left(1 + \frac{6e}{l}\right) =$$

Check for bearing pressure

pre. at toe & heel are given by (1m width retaining wall, b=1m

$P = \frac{W}{bl}\left(1 \pm \frac{6e}{l}\right) =$ b = unit width along length of wall
l = base length

$= \frac{450.1}{1 \times 3.7}\left(1 \pm \frac{6 \times 0.55}{3.7}\right) =$ 230 kN/m2 at toe > 150 kN/m2 (bearing ~~pre~~ capacity)

= 13 kN/m2 at heel

Since bearing capacity is 150 kN/m2, let us increase the width of base in front of wall by 0.6m. Total base bearing = 3.7+0.6 = 4.3 m.

Resultant striking base at

Resultant strikes base at 2.4m from C

New eccentricity = $\frac{b}{2}$ 2.4 − $\frac{4.3}{2}$ = 0.25m

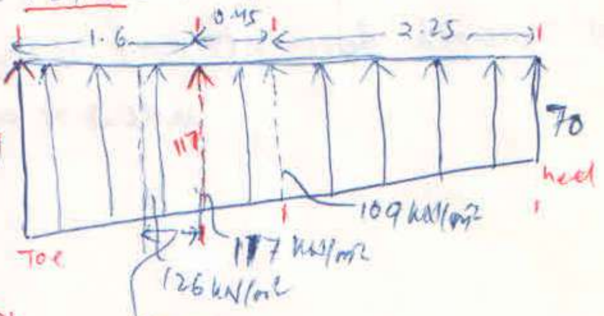
Pressure of heel/toe = $\frac{459.0}{4.3}\left(1 \pm \frac{6 \times 0.25}{4.3}\right)$

= 144 kN/m2 at toe
= 70 kN/m2 at heel } < 150 kN/m2 bearing capacity

OK,

Design of toe

- Toe is subjected to upward pre. varying from 144 kN/m2 to 117 kN/m2
- The downward load intensity due to self wt = 0.6×25 = 15 kN/m2
- The net upward pressure (upward - downward)
  = 129 kN/m2 & 102 kN/m2
- The toe is treated as a cantilever beam
- with critical sec. for shear at (d = 550mm) from front face of wall.



D = base slab depth (600)
d = 600 − 50 = 550mm

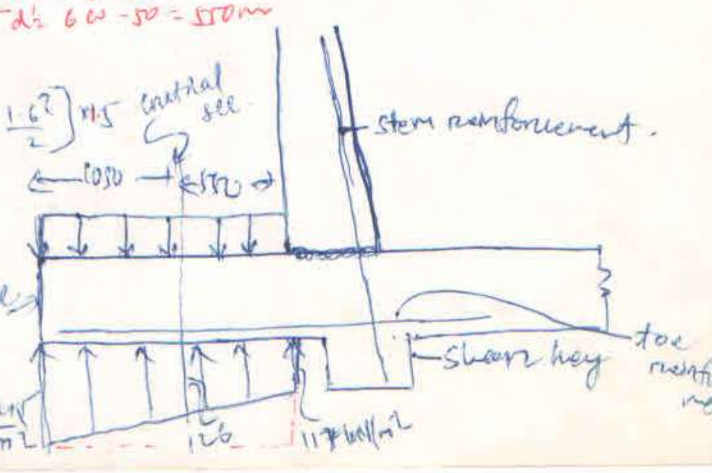
- $V_u = 1.5\left[\frac{144+126}{2} - 15\right] \times 1.05 = 189 kN.$

$M_u = \left[144 \times \frac{1.6^2}{2} - \frac{1}{2} \times 27 \times 1.6 \times \frac{1}{3} \times 1.6 - 15 \times \frac{1.6^2}{2}\right] \times 1.5$

or

$= 1.5\left[\frac{1}{2} \times 144 \times 1.6 \times \frac{2}{3} \times 1.6 + \frac{1}{2} \times 117 \times 1.6 \times \frac{1.6}{3}\right.$
$\left. - 15 \times \frac{1.6^2}{2}\right)$

= 230.4 kNm



$M_u = 0.138\, f_{ck}\, bd^2$

$$d = \sqrt{\frac{230.4 \times 10^6}{0.138 \times 25 \times 1000}} = 334\text{ mm}.$$

Let d_{eff} = 400 mm

∴ D = 450 mm.

Tension steel)

$$M_u = 0.87\, \sigma_y A_t \left(d - \frac{\sigma_y A_t}{f_{ck}\, b} \right)$$

$$\Rightarrow 230.4 \times 10^6 = 0.87 \times 415\, A_t \left(400 - \frac{415 \times A_t}{15 \times 1000} \right)$$

$\Rightarrow A_t = 1825\text{ mm}^2$

16 mm ϕ tor steel @ 100 mm c/c.

The anchorage length of 16mm bars into footing = at least development length = 90 cm.

check for shear

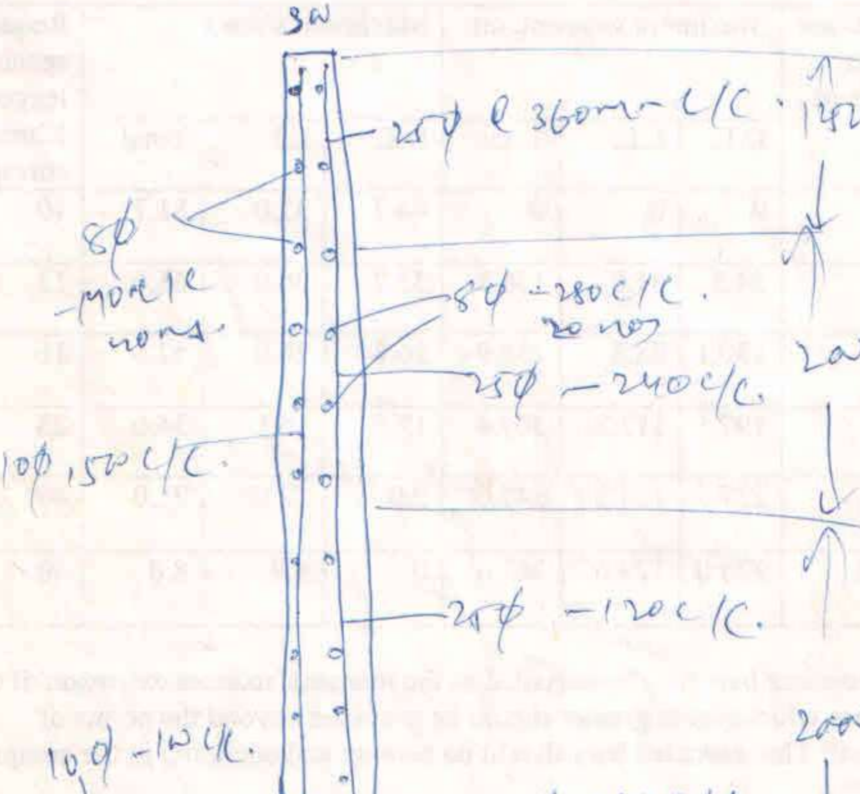
$$P = \frac{100 \times 2010}{1000 \times 550} = 0.36\%.$$

$\tau_c = 0.42\text{ N/mm}^2$

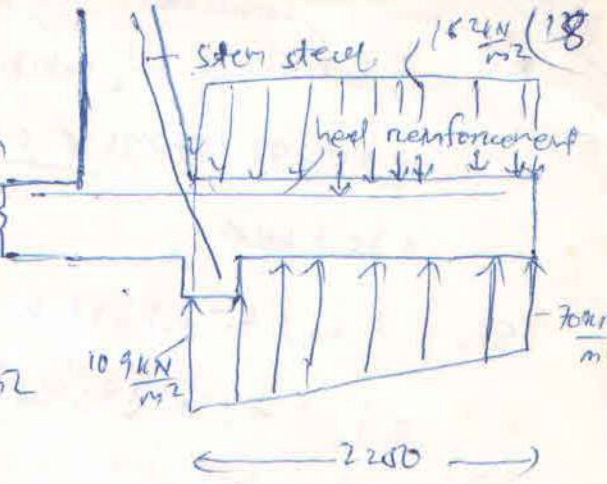
$$\tau_v = \frac{V_u}{bd} = \frac{189 \times 10^3}{1000 \times 400} = 0.39\text{ N/mm}^2$$

Let us increased d_{eff} = 450 mm & D = 520 mm.

So that it will be better & safer against shear.


300
25ϕ @ 360mm C/C.
1400
8ϕ
-190 C/C
20mm.
8ϕ -280 C/C.
20 mm
25ϕ -240 C/C
2000
10ϕ, 150 C/C.
25ϕ -120 C/C.
10ϕ -100 C/C.
2000
16ϕ -100 C/C.
600
400
1.9
340
2
10ϕ-100 C/C

Design of heel



heel is subjected to upward pre varying from 109 kN/m2 to 70 kN/m2.

The downward pre due to earth, surcharge & conc. wt $= 17(3 + \overset{(6.25-0.6)}{5.65}) + 0.6 \times 25 = 162$ kN/m2

Since downward load > upward pre

tension is induced in upper face of heel ~~⊕~~ where it joins stem.

⇒ Cl. 22.6.2, critical section for shear is at face of support.

The effect of shear key is neglected.

$$M_U = 1.5\left[\overset{\text{downward}}{162} \times 2.25 \times \frac{2.25}{2} - \frac{1}{2} \times 109 \times 2.25 \times \frac{2.25}{3} - \frac{1}{2} \times 70 \times 2.25 \times 2.25 \times \frac{2}{3}\right]$$

$= 300$ kNm

$V_U = 1.5\left(162 \times 2.25 - \frac{1}{2} \times 109 \times 2.25 - \frac{1}{2} \times 70 \times 2.25\right) = 245$ kN.

$$d_{eff} = \sqrt{\frac{300 \times 10^6}{0.138 \times 20 \times 1000}} = 380\text{mm}.$$

Let $d_{eff} = 550$ & $D = 600$mm.

Steel

$$M_U = 0.87 f_y A_t \left(d - \frac{f_y A_t}{f_{ck} b}\right) \Rightarrow 300 \times 10^6 = 0.87 \times 415 \times A_t \left(550 - \frac{415 A_t}{20 \times 1000}\right)$$

$\Rightarrow A_t = 1646 \text{mm}^2$

use 18mm tor steel @100mm c/c ($A_t = 2540 \text{mm}^2 > 1646 \text{mm}^2$)

Shear check

$$\tau_v = \frac{V_u}{bd} = \frac{245 \times 10^3}{1000 \times 550} = 0.44 \text{ N/mm}^2.$$

$p = \frac{100 \times 2540}{1000 \times 550} = 0.46\%$, $\tau_c = 0.47 \text{ N/mm}^2 > \tau_v$ OK

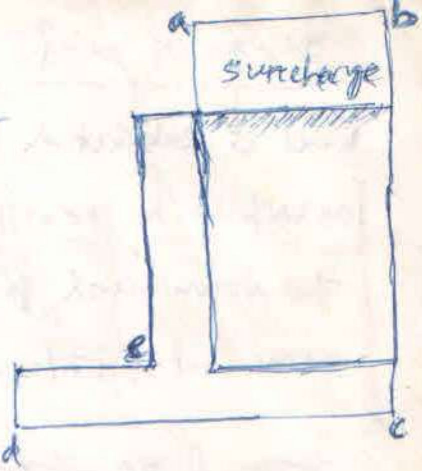
p(73)

Lateral earth   Surcharge

$BM_e = \frac{1}{2} k_a \gamma H^2 \times \frac{H}{3} + \frac{1}{2} k_a h_1 h^2$

$= \frac{1}{2} \times 0.271 \times 17 \times \frac{(6.25-0.6)^3}{3} + \frac{1}{2} \times 0.271 \times 3 \times 5.65^2 \times 17$

$= 359$ kNm



$SF_e = \frac{1}{2} k_a \gamma h^2 + k_a \gamma h_1 h$

$= \frac{1}{2} \times 0.271 \times 17 \times 5.65^2 + 17 \times 0.271 \times 3 \times 5.65$

$= 152$ kN.

Factored BM = 1.5 × 359 = 539 kNm

" SF = 1.5 × 152 = 228 kNm.

Effective ~~depth~~ thickness of wall at base = $d = \sqrt{\frac{539 \times 10^6}{0.138 \times 20 \times 1000}} = 510$ mm.

Let $d_{eff} = 510$ mm & D = 570 mm.

$A_t = \frac{0.36 f_{ck} b x_{um}}{0.87 f_y} = \frac{0.36 \times 20 \times 1000 \times (0.48 \times 510)}{0.87 \times 415} = 3660 \text{ mm}^2$.

use 25mm φ tor steel @ 120mm c/c (Ast provided = 4091 mm² > 3660 mm²)

shear check

$P = \frac{100 \times 491 \times 1000/120}{1000 \times 570} = 0.8\% \Rightarrow \tau_c = 0.55 \text{ N/mm}^2$.

$\tau_v = \frac{V_u}{bd} = \frac{228 \times 10^3}{1000 \times 570} = 0.45 < \tau_c$

~~Back ground~~

- ~~Need for the project.~~
- ~~Selection of the project.~~
- ~~Project components.~~
- ~~Design criteria & methodology of design.~~
- ~~Design details.~~
- ~~Detailed estimate~~
- ~~Concluding remarks:~~

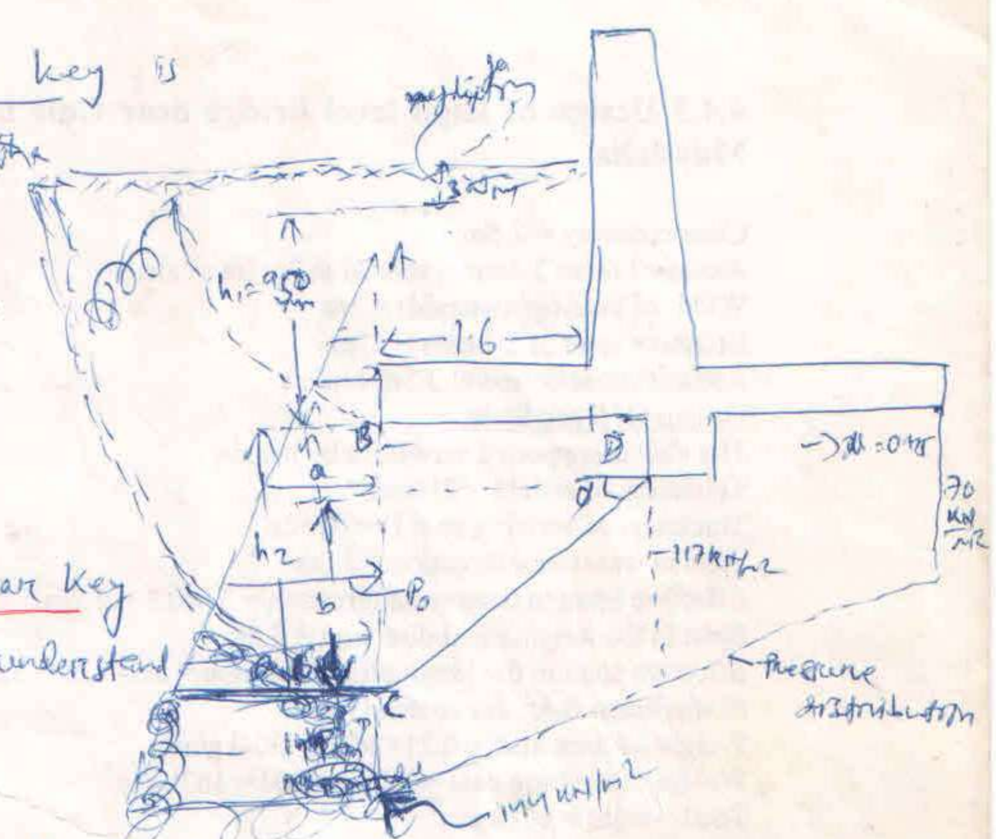
~~- Rehabilitation~~
~~New Bridge.~~
~~→ 5m × 7 spans~~
~~↔ 30 feet height river~~
~~HFL - 20ft~~
~~width of road - 7m~~

Design of shear key

* To check wall sliding, a shear key is required, ~~[illegible]~~ below the base all along the length of the retaining wall.

* A passive pressure in region in front and below bottom of retaining wall.

* The exact behavior of shear key is complex & difficult to understand



* Effect of shear key would be to develope passive resistance over depth BC.

So failure occurres along CC' instead of BB'

* To calculate passive resistance below toe, top overburden of 300mm is neglected.

$h_1 = 950$ mm, $b = \overset{(1.6+0.3)\tan 35 = 1.33m.}{=} 1.9 \tan\phi = 1.33$ m, $k_p = \frac{1+\sin\phi}{1-\sin\phi} = 3.69$, $\mu = 0.45$

$$P_0 = \frac{1}{2}\gamma K_p (h_1 + a + b)^2 - \frac{1}{2}\gamma k_p h_1^2$$

$$= \frac{17}{2} \times 3.69 (0.95 + a + 1.33)^2 - \frac{17}{2} \times 3.69 \times 0.95^2$$

$$= 31.4 a^2 + 143.8a + 135$$

Equilibrium of forces with factor of safety 1.4 gives

$$1.55 = \frac{R.\ \text{Force}}{\text{driving force}} = \frac{P_0 + \mu R_0}{P_1 + P_2} \quad 1.4(P_1 + P_2) = 0.9(P_0 + \mu R_0)$$

$$\Rightarrow 1.4(86 + 90) = 0.9\left(31.4a^2 + 143.8a + 135 + 0.45\frac{(117+70)}{2} \times 2.4\right)$$

or, $a = 0.25$ m.

Provide 40×40cm square shear key.

steel from vertical wall should be embeded in shear key.

Advanced Concrete Structures

DESIGN
OF
COUNTERFORT RETAINING WALL

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COUNTERFORT RETAINING WALL

Counterfort RW

- If ht of RW > 6 to 8m
- counterforts Connect vertical wall & heel slab.
- spacing of about $\frac{1}{3}$ to $\frac{1}{2}$ of wall height.
- It provides support to both vertical wall & heel slab.
- vertical wall
  - Acts as a Continuous slab supported by counterfort & base slab.
- It also exhibits some cantilever action at base.
- heel slab
  - Behaves as a continuous slab supported by counterfort & vertical wall.
- Counterfort
  - act as a T beam of varying section

Toe slab

- Behaves as cantilever being fixed with vertical wall & heel slab.
- may result large cantilever moment ⇒ large thickness of toe slab.

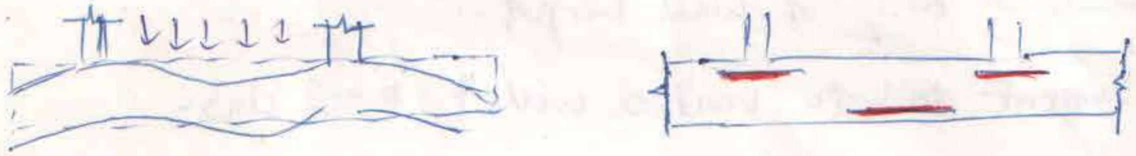
⇒ provided → front counterfort i.e buttress

- Behaviour of heel slab ≡ that of toe slab except

  toe slab is subjected to upward pre.

  heel slab " " " downward ".
- Buttress act as compression member under action of pressure transferred from vertical wall & toe slab.

horizontal sec. on vertical wall (A-A)



vertical sec. on Heelslab (B-B)

* vertical wall under lateral pre & heel slab under downward earth pre. induces separation of counterfort
* vertical wall & heel slab (for counterfort R.W) & toe slab ^{for} (buttress R.W) are supported on three sides & free on one side.

vertical wall → free at top & supported on two vertical side by counterfort & at base by heel slab.

heel slab → free at one side & supported by counterforts & vertical wall. • acted upon by downward earth pressure.

Toe slab – behaves as a cantilever if buttress is not provided

Buttress if provided behaves as similar to ~~toe~~ heel slab except pressure acts in upward direction.

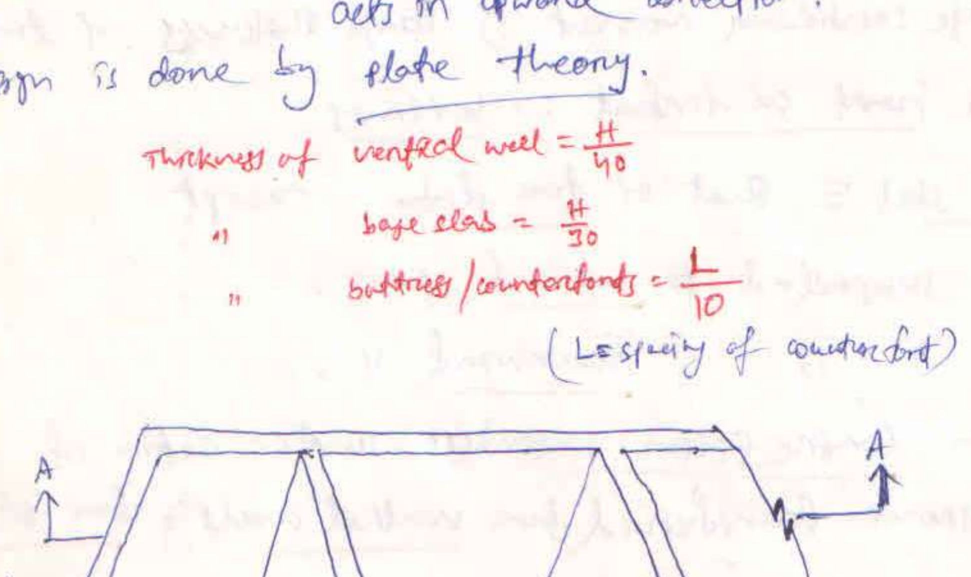
* Design is done by plate theory.

Thickness of vertical wall = $\frac{H}{40}$

" base slab = $\frac{H}{30}$

" buttress/counterforts = $\frac{L}{10}$

(L=spacing of counterfort)



counterfort R.W

Q Design a counter font retaining wall for retaining 7.5m high earth above ground level

wt of soil $\gamma = 15\ kN/m^3$

Angle of repose of soil $= \phi = 30^\circ$

coefficient of friction at base $= \mu = 0.5$

Allowable bearing pressure of ~~the~~ soil, $P = 150\ kN/m^2$

M20 Fe415

Sol → section of RW

depth of foundation $= \frac{P}{\gamma}\left(\frac{1-\sin\phi}{1+\sin\phi}\right)^2 = \frac{150}{15}\left(\frac{1-\sin 30}{1+\sin 30}\right)^2 = \underline{1.11m.}$

consider base of foundation is at a depth of 1.2m below which soil is not subjected to seasonal vol. change caused by alternate wetting & drying.

Total height of wall $= H = 7.5+1.2 = \boxed{8.7m.}$

Base width of foundation on safety against overturning

$$b = H\sqrt{\frac{k_a\cos\theta}{(1-m)(1+3m)}}$$

$k_a = \frac{1-\sin 30}{1+\sin 30} = \frac{1}{3}.$

$m = 1 - \frac{4}{9q}$

$q = \frac{\gamma h}{P_s} = \frac{15(7.8)}{150} = 0.78$

$m = 1 - \frac{4}{9\times 0.78} = \underline{0.47}$

Let footing thickness $= 8.7m \times 10\%$

$= 0.87m.$

$\therefore h = H - 0.87 = 8.7 - 0.87 = \boxed{7.8m.}$

Total height of RW $= 7.5+1.2 = 8.7m.$

base $= 0.5H - 0.6H = 4.35m - 5.22m.$

Let base $= \underline{4.5m.}$

Toe slab $= \frac{b}{3}$ to $\frac{b}{2} = 1.5$ to 2.25, let $= \underline{2.15m.}$

heel slab $= 2.35m.$

b. $b = 8.7\sqrt{\frac{1/3\times\cos 0}{(1-0.47)(1+3(0.47))}} = 4.255\ m$

Let $b \simeq \boxed{4.5m.}$

width of toe slab $=$ ~~m~~ $b = 0.47\times 4.5 = 2.149 \simeq \boxed{2.15m.}$

width of heel slab $= 4.5 - 2.15 = \boxed{2.35m.}$

Spacing of counterforts provided at back & front

$L = 0.8$ to $1.2\sqrt{H} = 0.8\sqrt{8.7}$ to $1.2\sqrt{8.7} = 2.36 - 3.54m.$

Let $\boxed{L = 3m.}$

Thickness of counterfort $= \frac{L}{10} = 0.3m.$ [thickness of counterfort = 10% spacing)

" " base slab $= \frac{H}{30} = \frac{8700}{30} = 290mm$

" " vertical wall $= \frac{H}{40} = \frac{8700}{40} = 217.5$

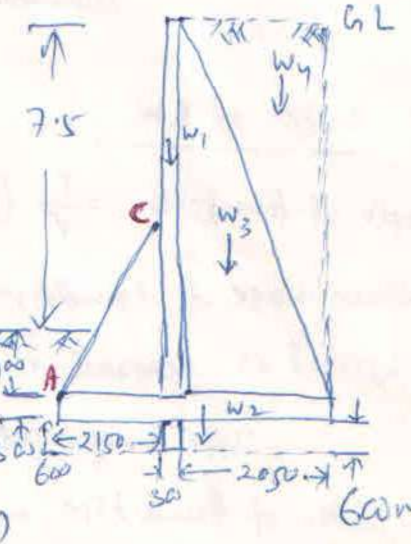
$\therefore$ Let us consider all the above three thickness $\simeq \boxed{300mm}$.

2/ Stability Analysis

(a) Stability against overturning →

→ earth on toe slab is neglected.
(because earth height on toe slab is small or may not exist during construction or may be eroded)

overturning moment $= M_{PA} = \frac{P_a H_a}{3} = \frac{1}{2} k_a \gamma H^2 \times \frac{H}{3}$



$M_{PA} = \frac{1}{6} \times 1 \times 15 \times 8.7^2 \times \frac{8.7}{3} =$

$= \underline{189.225} \times \frac{8.7}{3} = \underline{548.75 \text{ kNm}}.$

→ $W_1 = 0.3 \times 8.4 \times 25 = 63 \text{ kN}$
(at $2.15 + 0.15 = 2.3$ m from A)

∴ moment about A $= 63 \times 2.3 = 143.9$ kNm.

→ $W_2 = 0.3 \times 4.5 \times 25 = 33.75$ kN. (at 2.25 m from A)

moment about A $= 33.75 \times 2.25 = 75.938$ kNm.

→ $\underset{(\text{soil})}{W_3} = 2.05 \times 8.4 \times 15 = 258.3$ kN (at $2.15 + 0.3 + \overset{(2.05/2)}{1.025} = 3.475$ m from A)

moment at A $= 258.3 \times 3.425 = 897.593$ kNm.

Total vertical downward load $= W_1 + W_2 + W_3 = \underline{355.05 \text{ kN}}$

Total moment about A $= M_A = 1118.43$ kNm.

∴ $F_O = \frac{1118.43}{548.75} = 2.038 > 1.55$ (safe) ✓

(b) Stability against sliding →

Factor of safety $= \frac{\mu W}{\text{horizontal force} (P_a)} = \frac{0.5 \times 355.05}{189.225} = 0.938 < 1.55$
(unsafe)

Shear key is provided to ensure adequate safety against sliding.

Let us say depth of shear key = 600 mm, below foundation slab

* For passive earth pre., soil above top of foundation slab is neglected as it may not as it may not exist for some time during construction / may be eroded.

$$\therefore F_S = \frac{\mu W}{(P_a - P_p)}$$

$$P_p = \frac{1}{2} k_p \gamma h_p^2 = \frac{1}{2} \times \frac{1}{k_a} \gamma h_p^2$$
$$= \frac{1}{2} \times 3 \times 15 \times 1.975^2 = 87.764 \text{ kN/m.}$$

~~$\frac{1}{2} \times 3$~~

$$h_p = 300 + 600 + 2150 \sin 30 = 1975$$

$$\therefore F_S = \frac{0.5 \times 355.05}{(189.225 - 87.764)} = 1.749 > 1.55 \text{ (Safe)}$$

Foundation stability Analysis

Foundation should be under compression & max upward soil pressure should be within its permissible value.

$$P_{max/min} = \frac{W}{b}\left(1 \pm \frac{6e}{b}\right)$$
$$= \frac{355.05}{4.5}\left(1 \pm \frac{6 \times 0.6455}{4.5}\right)$$
$$= 146.8 \text{ and } 10.99 \text{ kN/mm}^2 < 150 \text{ kN/m}^2$$

✓ OK

* Below

$$e = 0.5b - \frac{\text{net moment}}{\text{net load}}$$
$$= 0.5 \times 4.5 - \frac{1118 - 548.753}{355.05}$$
$$= 0.6455 < \frac{b}{6} (= 0.75)$$

Hence foundation base is under compression & max upward soil pressure is within permissible capacity.

Vertical wall structural Design & foundation slab design →

~~Net~~ Net pressure on foundation slab = [upward pre on foundation base] − [downward pre on both heel & toe slab]

Downward pre. on heel slab = Pre. due to earth on heel + Self wt.

$$= 15 \times 8.4 + 0.3 \times 25 = 133.5 \text{ kN/m}^2$$

Down ward pre on toe slab = pre due to self wt. of toe slab = $0.3 \times 25 = 7.5$ kN/m²

* stability check

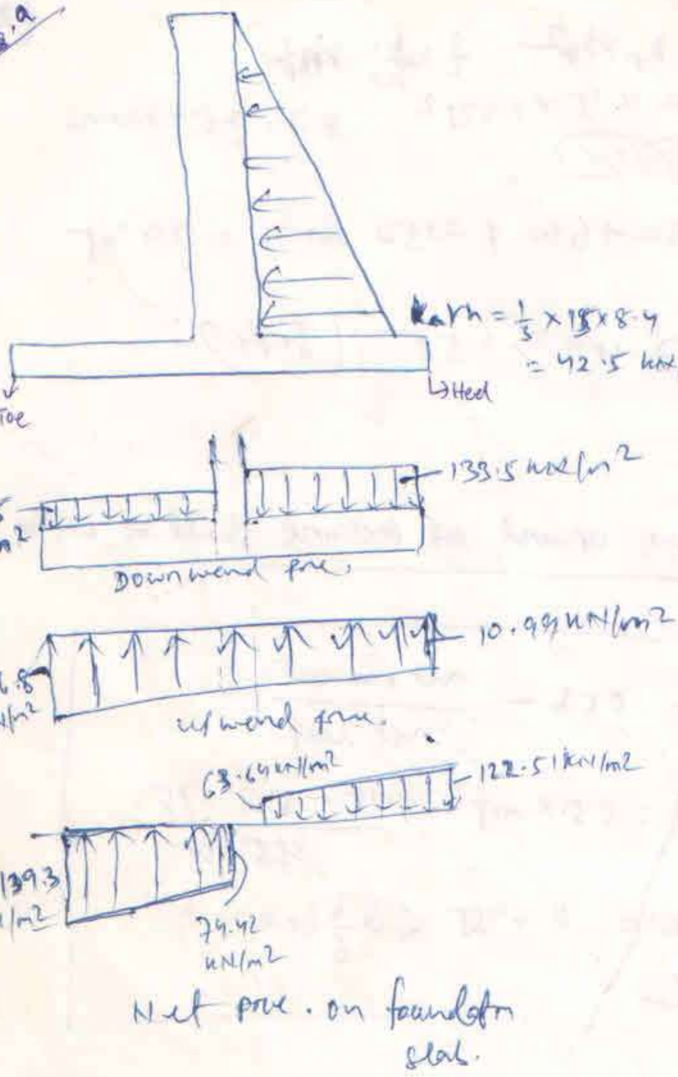
Resultant force let acts at x from B.

$$= \frac{63 \times 2.2 + 33.75 \times 2.25 + 258.3 \times 1.025 + 548.75}{63 + 33.75 + 258.3} = 2.9 \text{m.}$$

$$e = 2.9 - \frac{4.5}{2} = 0.65$$

$$\frac{b}{6} = 0.75 \text{m} \quad \therefore e < \frac{b}{6} \text{ ok } ✓$$

23.a



Net pre. on foundation slab.

Ultimate moment = $1.5 \alpha w l^2$

W = intensity of UDL / max intensity of triangular load

L = clear span betn edges,

α = moment coefficients.

Connection betn base slab - counterfort :

Consider one meter wide strip of heel slab near heel end.

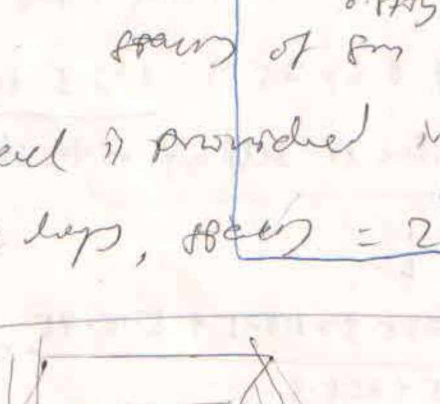
Net load on strip of heel slab = ( )

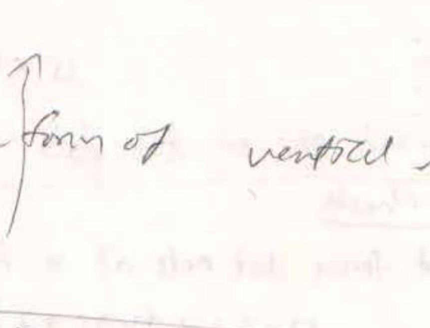
Tension = ( ) (3 - 0.3)

$\text{steel} = \frac{\text{Ten}}{0.87 f_y}$

spacing of 8m bar

Since steel is provided in the form of vertical links of 2 legs, spacing = 2 ×





Design of heel slab (heel slab is a continuous slab over counterfort.)

Net max pressure for heel slab = 122.5 kN/m2 (at heel slab end)

spacing of counterfort = 3 m.

Let us take heel slab of unit ~~length~~ strip width.

Net max moment $\boxed{M = \frac{wl^2}{12}} = \frac{122.5 \times 3^2}{12} = ..$

⊙ 'd' check ⊙ find Ast. ? ⊙ distribution steel (0.12% bD)

⊙ find S.F, V. ⊙ find τ_c & τ_v

⊙ shear check.

* Net pressure = deduction between the upward & downward pressure on the slabs.
* Toe slab design is same as heel slab design.

* Design of stem

* Stem acts as a continuous slab.

* Pressure $P_a = K_a \gamma H$ kN/m2

$$\boxed{BM = \frac{P_a \times (\text{spacing})^2}{12}}$$

* proceed as in heel slab.

* Counterfort →

23,C

* why counterfont wt., is not considered in stablity analysis for overturning (P-21)

Sol -> if wt. of conterfont & its moment is considered then factor of safety will increase i.e. structure becomes more safe. we must consider a critical section which is less safe i.e. less safe factor of safety.

If a structure is have factor of safety for overturning is 1·7 > 1·53

- by considering countenfont; the F.S becomes more than 1·7, so structure becomes more safe. We should design for the critical section, not a safer section.

~~Design of counterfort~~

~~Let us provide top width of counterfort = 42cm.~~

~~Total earth pre. transfered to counterfort above base slab level :~~

~~$P_a = Ka \frac{\gamma h^2}{2} L = \frac{1}{3} \times 15 \times 8.4^2 \times 3$.~~

~~max BM = $Pa \times \frac{8.4}{3}$.~~

~~width of counterfort at base = 2.35m.~~

~~d_{eff} =~~

Design of front counterfont (buttress, on toe slab side)

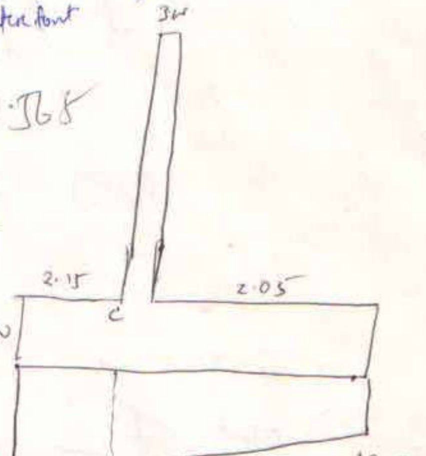
Total soil pre = $\frac{\overset{139.3}{P_{max}} + \overset{74.42}{P_{junction}}}{2} \times \overset{\text{spacing of counterfont}}{3} \times 1 = 320.568$

Thy acts at = $\frac{P_{jc} + 2 P_{max}}{P_c + P_{max}} * \frac{2.15}{3} = 1.18$

moment = $\overset{=320.56 \times 1.18}{378.24}$ kNm $\times$ 1.5 load factor

$0.138 f_{ck} bd^2$ = moment 300

d = (827.8)



eff. cover =

cover to centre of toe slab bar = 60
half dia of toe slab bar = 5
dia of distribution bar = 8 ✓
half dia of counterfond bar = 10

83.

overall depth = (827.8) + 83 = 910.8 m. = 920 mm, $\boxed{d = 837}$

$M = 0.87 f_y A_{st} (d - 0.42 x_u)$ ⇒ A_{st} = 2357.7 mm^2
no of bars 20mm, 8 nos

V = max shear = Total upward (average) soil pre − 0.3 × 3 × 1 × 25 × 10 = 298.065 kN

$\tau_v = \frac{V}{300 \times d}$ = 1.78 N/mm^2 , $P_{st} = \frac{100 A_{st}}{bd}$ ⇒ τ_c = 0.6

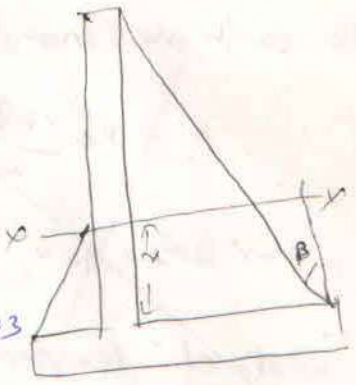
Shear to be resisted by stirrup = 291.2 kN
Spacing of stirrups = 104.7 $\frac{0.87 f_y A_{sv}}{V_s}$

~~35~~

<u>main counterfort</u>

critical sec for main at top of front counterfort
width of counterfort at critical sec x-x = 1.89 m

$$w_1 = 2.35 - \frac{2.35 - 0.3}{8.7} \times (\quad) = 1.9$$



- Total horizontal force transferred to counterfort above x-x

$$P_1 = \boxed{k_a \gamma h}^{(8.7-0.62)} \times \frac{8.7 - x}{2} \times 3 = k_a \gamma \frac{(8.7-x)^2}{2} \times 3$$

$$BM = P_1 \times \frac{h-x}{3}$$

deff available = ⓑ = $W_1 - 60$

Let β = inclination of main steel with vertical

$\tan\beta = \quad \Rightarrow \beta =$

Ast: $M = 0.87\, A_{st} f_y (d - 0.42 x_u) \times \cos\beta$

$\Rightarrow A_{st} = ?$

20mm φ, no of bars = ?

out of __ no of bars

$$\frac{A_{st_1}}{A_{2t}} = \frac{M_1}{M_2} = \frac{h_1^2}{h_2^2}$$

<u>Connection</u> (vertical slab – counterfort)

horizontal earth pre intensity at x-x

$$P_{h_1} = \frac{\quad}{\quad}$$

width of counterfort = 300

Tension transmitted to counterfort per meter height

$= P_{h_1} (3 - 0.3)$

horizontal steel required/m = $\frac{Tension}{0.87 f_y}$ =

spacing of 8m φ = ?

Steel is provided in form of horizontal links of 2 legs

spacing = 2x

255

Design of counterfort ⟹

Thickness of counterfort = 300mm

l = Spacing of counterfort = 3m. c/c.

* CF will receive earth pressure from width 3m (l)

and downward reaction from heel slab for a width 3m (l)

At any section depth 'h' below top,

* soil pressure on each counterfort = $k_a \gamma h \times 3$

ⓧ $= \frac{1}{3} \times 15 \times h \times 3 = 15h$ kN/m ⟹

* Net downward pre. on heel at C

$= 8.4 \times \cancel{1}15 + 0.3 \times 25 - 10.99 = 122.51$ kN/m²

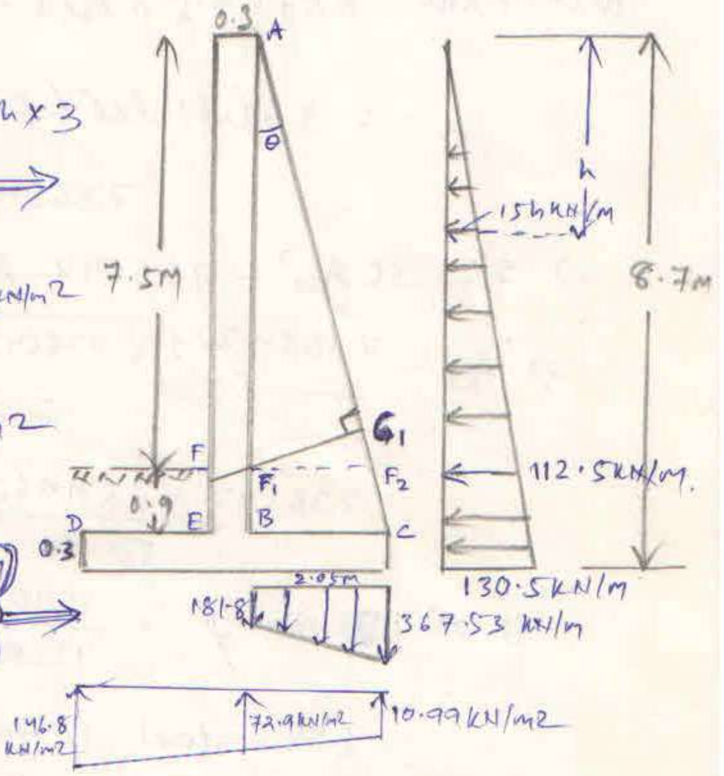
Net downward pre. on heel at B

$= 8.4 \times 15 + 0.3 \times 25 - 72.9 = 60.6$ kN/m²

∴ Reaction transferred to each counterfort

at C = 122.51 × 3 = 367.53 kN/m

at B = 60.6 × 3 = 181.8 kN/m } ⟹



* critical section for CF is at F,

as below this sufficient depth is available to resist bending.

ⓧ At F, Pressure intensity = 15h = 15 × 7.5 = 112.5 kN/m (Lateral force)

Shear force at F = ½ × 7.5 × 112.5 = 421.9 kN.

Bending moment at F = 421.9 × $\frac{7.5}{3}$ = 1054.7 kNm.

* Counterfort act as a T-beam.

Even as rectangular beam, required depth

$M = \cancel{0.87}\ 0.138 f_{ck} b d^2 \Rightarrow d = \sqrt{\frac{1054.7 \times 10^6}{20 \times 300 \times 0.138}} = \cancel{(97)}\ 1128.6\text{ mm} \simeq 1130\text{ mm}$

Total Depth = 1130.0 + 60 = 1190 mm

$\tan\theta = \frac{2.05}{8.4} = 0.244$, θ = 13.71, sin θ = 0.24, cos θ = 0.97.

~~tanθ~~ $\sin\theta = \frac{F_1C_1}{AF_1} \Rightarrow F_1C_1 = 7.5 \sin\theta = 1.8$ m.

∴ $FC_1 = 1.8 + 0.3 = 2.1$ m. (provided depth)

∴ Provided depth (2.1m) > Required depth (1.19m) (OK)

* Let reinforcement in $\overset{\text{counterfort}}{\text{CF}}$ is provided in ~~both~~ two layers.

Let clear cover is 50mm & 20mm ø bars are used,

$D = 2100$ mm, $d = 2040$mm.

$$M = 0.87 f_y A_{st} \left(d - \frac{0.42 \times 0.87 \times f_y A_{st}}{0.36 f_{ck} b}\right)$$

$$1054.7 \times 10^6 = 0.87 \times 415 \times A_{st} \left(2040 - \frac{0.42 \times 0.87 \times 415 \times A_{st}}{0.36 \times 20 \times 300}\right)$$

$$= 361.05 A_{st} (2040 - 0.07 A_{st})$$

$$= 736542 A_{st} - 25.35 A_{st}^2$$

$$\Rightarrow 25.35 A_{st}^2 - 736542 A_{st} + 1054.7 \times 10^6 = 0$$

$$\Rightarrow A_{st} = \frac{736542 \pm \sqrt{736542^2 - 4 \times 25.35 \times 1054.7 \times 10^6}}{2 \times 25.35}$$

$$= \frac{736542 - 659960.255}{50.7} = 1510.5 \text{ mm}^2$$

No of 12 mm ø $= \frac{1510.5}{113} = 13.36 \simeq 14$ nos (provided in 2 layers)

(This steel is provided in 2 rows each with 7 nos ‖le to direction of slope of counterfort.)

Design of Horizontal ties

vertical stem has a tendency to be separated from the counterfort.

* So horizontal ties bind CF + stem.

* At any height lateral pre. intensity $= \boxed{K_a \gamma h} = \frac{1}{3} \times 15 \times h = 5h$ KN/m2.

* To get force causing separation, ~~either so~~ soil pressure from either side of CF should be considered except CF.

* So $\overset{\text{tensile}}{\wedge}$ force causing separation $= 5h(3 - 0.3) = 5 \times 7.5 \times 2.7 = 101.25$ KN/m.

A_{st} required $= \frac{101.25 \times 10^3}{0.87 \times 415} = 280.432 \text{ mm}^2$/meter height

using 8mm ø 2legged ties. $A_\phi = 2 \times \frac{\pi}{4} \times 8^2 = 100.53 \text{mm}^2$

Spacing $= \frac{1000 \times 100.53}{280.432} = 358.5 \simeq 350$ mm c/c.

vertical ties

* Heel slab has a tendency to be separated from counterfort, due to net downward pressure.

Downward force at C = $\frac{367.53}{3} \times (3-0.3) = 330.78$ kN/m.

at B = $\frac{181.8}{3} \times (3-0.3) = 163.62$ kN/m.

* Near C, heel slab is tied to CF with main steel of CF.

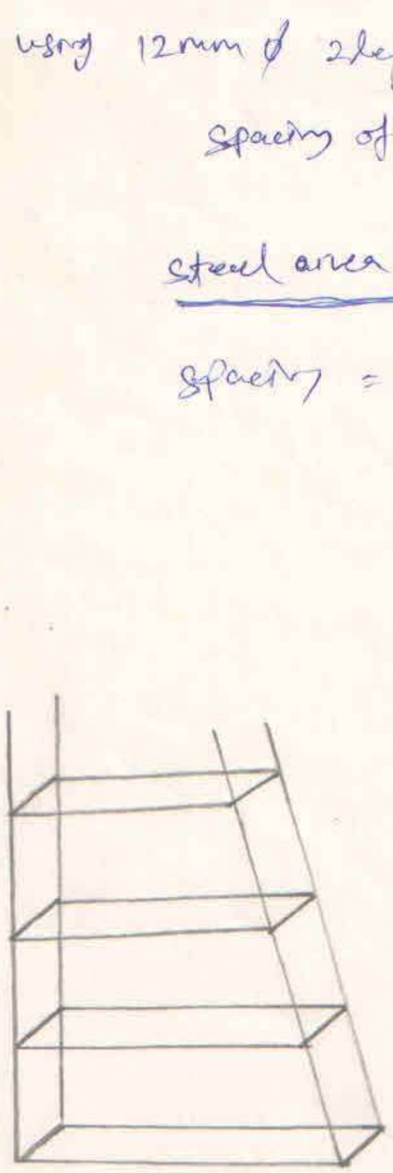
steel area at C = $\frac{330.78 \times 1000}{0.87 \times 415} = 916.16\ mm^2$

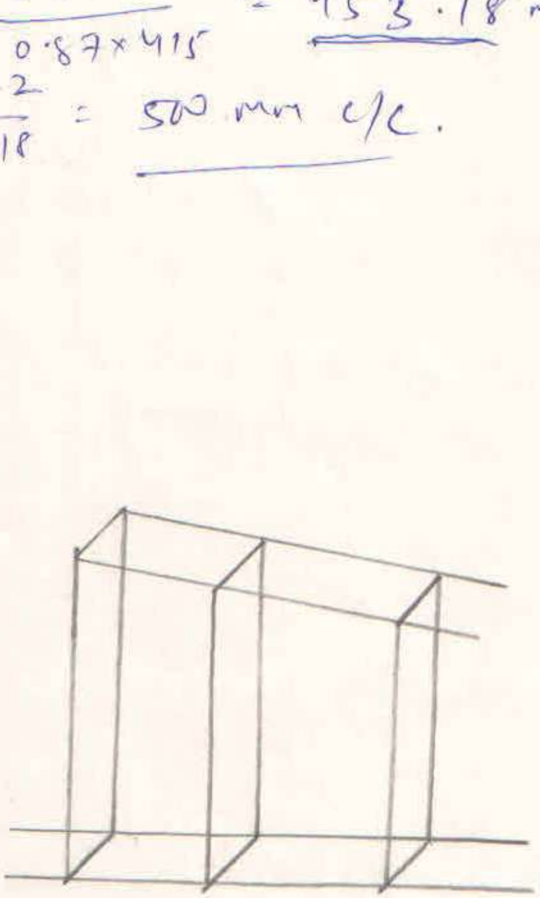
using 12mm ϕ 2legged ties, $A_\phi = 2 \times \frac{\pi}{4} \times 12^2 = 226.2\ mm^2$

spacing of ties = $\frac{1000 \times 226.2}{916.16} = 250$ mm c/c.

steel area at B = $\frac{163.62 \times 1000}{0.87 \times 415} = 453.18\ mm^2$

spacing = $\frac{1000 \times 226.2}{453.18} = 500$ mm c/c.





ADVANCED CONCRETE STRUCTURES (CE 15031)

Module-IV

Module IV Syllabus

Design of Water tanks: Design requirements

Design of On Ground Water Tanks

Design of Underground Water Tanks

Introduction to Prestressed Concrete:

- *Prestressing Methods*
- *Analysis of Prestressing Systems*
- *Losses in Prestressed Concrete*

Subject to Revision

Advanced Concrete Structures

DESIGN
OF
WATER TANK

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Associate Professor
Department of Civil Engineering
VSSUT, Burla

WATER TANK

15/04/10

* stores water for daily requirement.

* Water tank classified into
- ① Tank resting on ground
- ② Elevated tank resting on staging
- ③ under ground tank.

* From shape point of view, water tanks may be of several types :-
- circular tank
- Rectangular tank
- Spherical tank
- Circular tank with conical bottom
- Intze tank.

* To store water/liquid materials, Concrete structures should be impervious.

* For uniform & thoroughly compacted concrete, Permeability depends on Water/cement.

Permeability $\propto$ water/cement.

But very low w/c, may cause compaction difficult & harmful.

* For water retaining structures → highly compacted concrete

→ For thorough compaction, cement content should be high ⇒ low w/c ratio.

*

→ cement content $\nless$ 330 kg/m^3 of concrete.

→ cement content < 530 kg/m^3 of concrete to keep low shrinkage.

→ For thicker section, cement content should be low because to restrict temp. rise due to cement hydration

– Mostly M30 concrete, should be used.

* Design of liquid retaining structures are based on avoidance of cracking due to tensile strength.

* cracking may occur due to
① Restraint to shrinkage ② free expansion & contraction of concrete due to temp,
③ shrinkage & swelling due to moisture effect.

* crack can be minimized by using ① thin timber shuttering, which allows easy escape of heat of hydration from concrete mass.
② Reducing restraints on free expansion/contraction.

* For long wall/slabs founded at or below ground level,

Restraint can be minimized by founding structure on flat layer of concrete with interposition of sliding layer to break the bond & allow movement.

* causes of leakage from water retaining structures are:-
  ① cracking
  ② segregation
  ③ Honeycombing.

* source of leakage are the joints.

DESIGN REQUIREMENTS (IS 3370 - Part II, 2009) →

① Steel requirement ⇒

* minimum reinforcement (in wall, floor, roof in each of two direction at right angle)

= 0.3% of concrete section in that direction

[upto 100mm concrete thickness]

* For, 100mm < thickness < 450mm,

minimum reinforcement in each of two directions,

· shall be linearly reduced from $\underset{(t=100mm)}{0.3\%}$ to $\underset{(t=450mm)}{0.2\%}$

* For thickness ≥ 225 mm,

two layers of reinforcing bars be used [one at $\overset{\text{(min steel)}}{\text{outer face}}$ & one at $\overset{\text{(minimum steel)}}{\text{inner face}}$]

for minimum steel to be provided at each face

* For floor slab resting directly on ground,

minimum reinforcement = 0.15% of concrete section.

minimum cover →

* For face of structural parts, either in contact with water or enclosing the space above the liquid like inner face of slab,

$$\text{minimum cover to all reinforcement} = \left.\begin{cases} 25\text{mm} \\ \text{dia of main bar} \end{cases}\right\} \text{which is more.}$$

* In sea water, soil & water of corrosive character

cover is increased by 12mm

but the additional cover is not considered in design.

* For faces away from liquid (not in contact & not enclosing the space)

cover = same as normal RCC cover for corresponding structural element.

JOINTS in water tank →

① movement joint → contraction joint, Expansion Joint, sliding joint } — (Flexible Joint)

② construction Joint - - - - - - - - - - Rigid Joint

③ Temporary open Joint - - - - - - - - - - Flexible Joint.

① movement Joint → [Flexible Joint]

* uses special material to maintain water tightness.

* Allows relative movement betn sides of joints.

contraction Joint →

* allows contraction of concrete.

* contraction Joint → complete contraction joint
  → Partial contraction Joint.

* Complete contraction Joint →

Discontinuity of both concrete & steel.

Partial contraction Joint →

- Discontinuity of concrete
- There is continuity of steel i.e. reinforcement run through Joint.

* No initial gap is kept at Joint.

* In complete contraction Joint, a water bar i.e. pre formed strip of impermeable material like metal, PVC, rubber, etc, is inserted.

* In Partial contraction Joint,

the mouth of joint is filled ~~is filled~~ with Joint sealing compound i.e. impermeable ductile material providing a water seal by adhesion to conc.

(Materials are Asphalt, Bitumen, coal tar, etc with or without filler like limestone, slate dust, asbestos fibre, etc)

Expansion Joint →

* with complete discontinuity of both steel & concrete.

* Aim is to accomodate both expansion & contraction

* Initial gap is provided at Joint betn adjoining parts of structure.

* The initial gap closes & opens accomodating expansion & contraction

- * Initial gap is filled with joint filler.
- * Joint fillers are compressible sheets.
- * Joint filler are fixed to face of first placed concrete & then second placed conc. is cast
- * Initial gap = 30mm

  Max permissible expansion/contraction = 10mm.

SLIDING JOINT →

- * complete discontinuity of both steel + concrete.
- * Typical application is between wall & floor in cylindrical tank design.

CONSTRUCTION JOINT

Temporary open joint →

Hoop stress

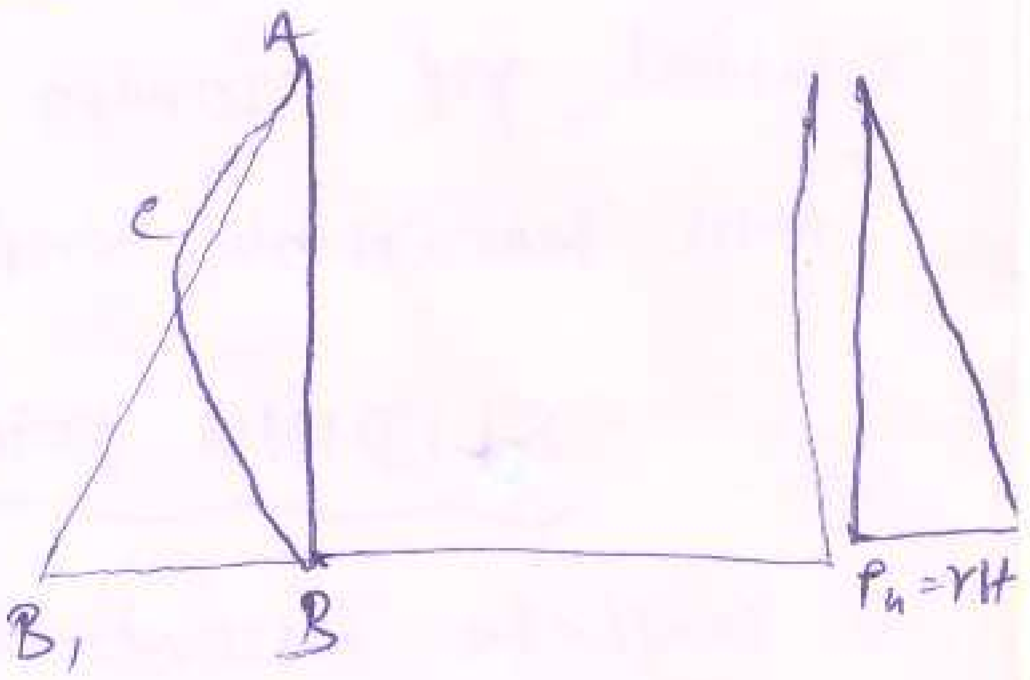
- * stress in a pipe wall
- * force inside cylinder acting towards the circumference
  ⊥r to length of pipe.
- *

Circular tank with flexible joint bet^n floor & wall →

- Due to hydrostatic pre, (P_h)

  the diameter of section will tend to increase.

- it depends on lower joint B.



- if B is flexible, B changes to B_1
- At A, $P_h = 0$, no change in dia,
- If B is flexible, increase of dia is linear from A to B (A B B_1 curve)
- If B is rigid, fixing moment is induced at B (ACB curve)

$$T_h = \text{Hoop tension} = \frac{\gamma H D}{2}$$

$$\text{Area of steel/meter height} = \frac{T_h}{\text{Per. stress.}}$$

- Ast is provided at centre if thickness < 225 mm.

0 - if thickness > 225mm.

  Ast is provided in each face, with cover 25mm.

- This is in the form of hoops.
- spacing of hoops < 3 × thickness of wall.

0 - Thickness of wall, $T = (30H + 50)$ mm,

  H = water height in metre.

- secondary steel in the form of vertical bars

0 @ 0.3% concrete section upto 100mm thickness.

  - 100mm < thickness < 450 mm, steel reduced from 0.3% to 0.2%.

- If floor slab is resting continuously on ground, min. thickness = 150mm.

0 Nominal steel in slab = 0.3% in each direction.

slab rest on 75mm thick lean concrete (M 10).

on 75mm lean concrete a layer of tar felt is provided.

14/04/10 (7

Q. Design a circular tank with flexible base for capacity 400000 litre. Depth of water = 4m, including free board of 200mm. M20, Fe415.

Sol ⇒ From capacity point of view, deff = 3.8m.

$\therefore \frac{\pi}{4} \times D^2 \times 3.8 \simeq 400000 \times 10^{-3}\ m^3$ [D = inside dia of tank]

$\Rightarrow D = 11.62\ m \simeq$ Provided $D = 11.7m$.

Design of section ⇒

$\gamma_{water} = 9.8\ kN/m^3$.

* max hoop tension/meter height at base $= \gamma H \frac{D}{2} = 9800 \times 4 \times \frac{11.7}{2} = 229320N$

* Area of hoop steel (i.e. circular steel rings acting as main reinforcement),

$$A_{sh} = \frac{229320}{0.87f_y} = \frac{229320}{0.87 \times 250} = 1054.345\ mm^2$$

Using 20mm ϕ bars, spacing of hoops $= \frac{1000 \times \frac{\pi}{4} \times 20^2}{1054.345} = 297.8$

$\simeq$ 290mm c/c. ——— ①

$\therefore$ Provided hoop steel $= \frac{1000 \times \frac{\pi}{4} \times 20^2}{290} = 1083\ mm^2$

* Wall thickness (empirical formula)

$T = 30H + 50 = 30 \times 4 + 50 = 170mm$.   (H = height of water in 'm')

Provide a wall thickness of 170mm throughout the height.

* Spacing of hoops can be increased near top.

Let us provide a minimum steel of 0.3%. at top.

$A_{sh} = \frac{0.3}{100} \times 1000 \times 170 = 510mm^2$.

Spacing $= \frac{1000 \times \frac{\pi}{4} \times 20^2}{510} = 615.7mm \simeq 600mm$ c/c.

But spacing of hoops ≯ 3 × wall thickness (i.e. 3×170 = 510mm)

Let's have hoop spacing of 500mm c/c at top. ——— ②

* Since wall thickness < 225mm, steel is provided at centre of thickness.

* Spacing of hoops at depth 2m below top, | $A_{sh} = \frac{\gamma H \frac{D}{2}}{0.87f_y} = 527.2\ mm^2$
| Spacing $= \frac{1000 \times \frac{\pi}{4} \times 20^2}{527.2} \simeq 500mm$ c/c ——— ③

* Spacing of hoops at depth 3m, below top, | $A_{sh} = 790.8\ mm^2$
| Spacing = 397mm $\simeq$ 390mm c/c ——— ④

vertical steel

→ Distribution & temperature steel is provided in vertical direction.

for 100mm — 0.3%.

for 450mm — 0.2%.

for 170mm, $A_{sd} = 0.3 - \frac{0.3-0.2}{450-100} \times (170-100) = 0.28\%$.

$$\therefore A_{sd} = \frac{0.28}{100} \times 170 \times 1000 = 476 mm^2$$

Provide 10mm ϕ bars, spacing $= \frac{1000 \times \frac{\pi}{4} \times 10^2}{476} = 164.9mm \simeq 160mm$ c/c.

∴ Provide distribution steel of 10mm ϕ @ 160mm c/c in vertical direction.

They can also be used for tieing hoop reinforcement.

Design of Tank floor

Let tank floor is on ground.

Let us provide a nominal thickness of 150mm.

min $A_{st} = \frac{0.3}{100} \times 150 \times 1000 = 450 mm^2$ in each direction.

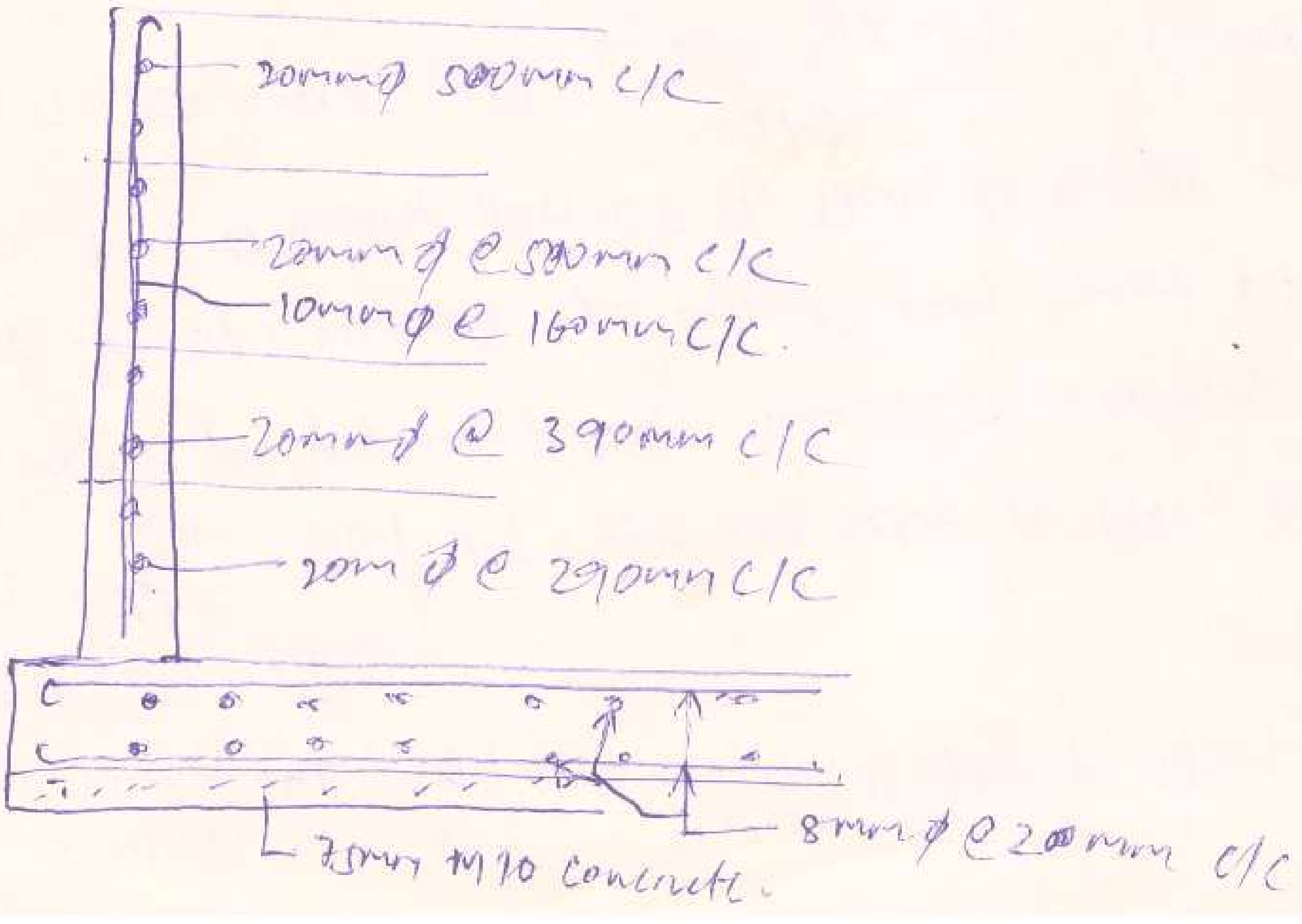
Provide half the reinforcement near each face, $A_{st} = 225 mm^2$

using 8mm ϕ bars, spacing $= \frac{1000 \times \frac{\pi}{4} \times 8^2}{225} \approx 222.22mm$ c/c.

$\simeq$ 200mm c/c.

∴ Provide 8mm ϕ bars @ 200mm c/c in both direction at top & bottom of floor slab.

∴ The floor slab will rest on 75mm thick layer of lean concrete covered with a layer of tar felt.



Underground Reservoir (UR) 23/11/16

- design depend on
  - nature of soil
  - SBC of sub soil
  - level of sub soil water.

- If subsoil is dry, well drained, no uneven settlement then

UR consists of four vertical walls forming sides and water tight floor.

- The walls nothing but RW.
- The joints are water tight.

~~Floor of reservoir~~

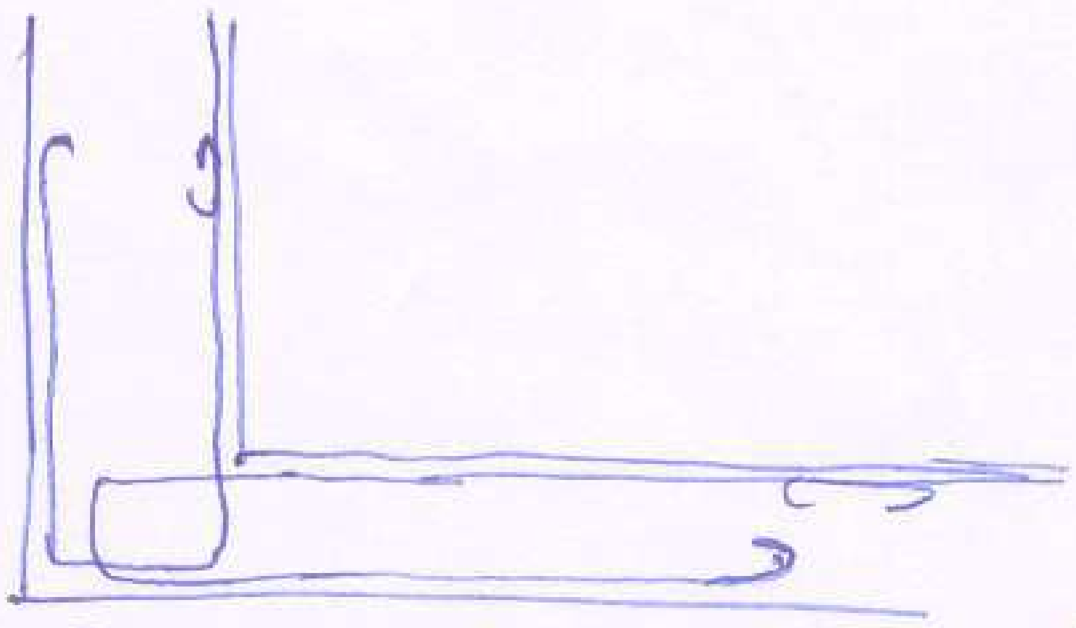
Reservoir wall

- walls are designed as cantilever RW
- They are designed to be stable in following two conditions:-

(i) when reservoir is empty & walls are loaded by lateral soil pressure.

(ii) when reservoir is full

(normally reinforcement on both faces are provided)



[horizontal reinforcement in corner of walls]

Based on head, tanks classification

- Tank rest on ground
- Elevated tank on staging
- Underground tank

various elevated tanks having various shapes

- circular tank
- Rectangular tank
- Spherical tank
- circular tank with conical bottom

Intze tank

- For large storage capacity. Overhead tanks, circular tanks are economical. But in flat bottom, the thickness & reinforcement is found heavy.
- in doomed bottom, though thickness & reinforcement in dome is ~~heavy~~ normal, reinforcement in ring beam is high.

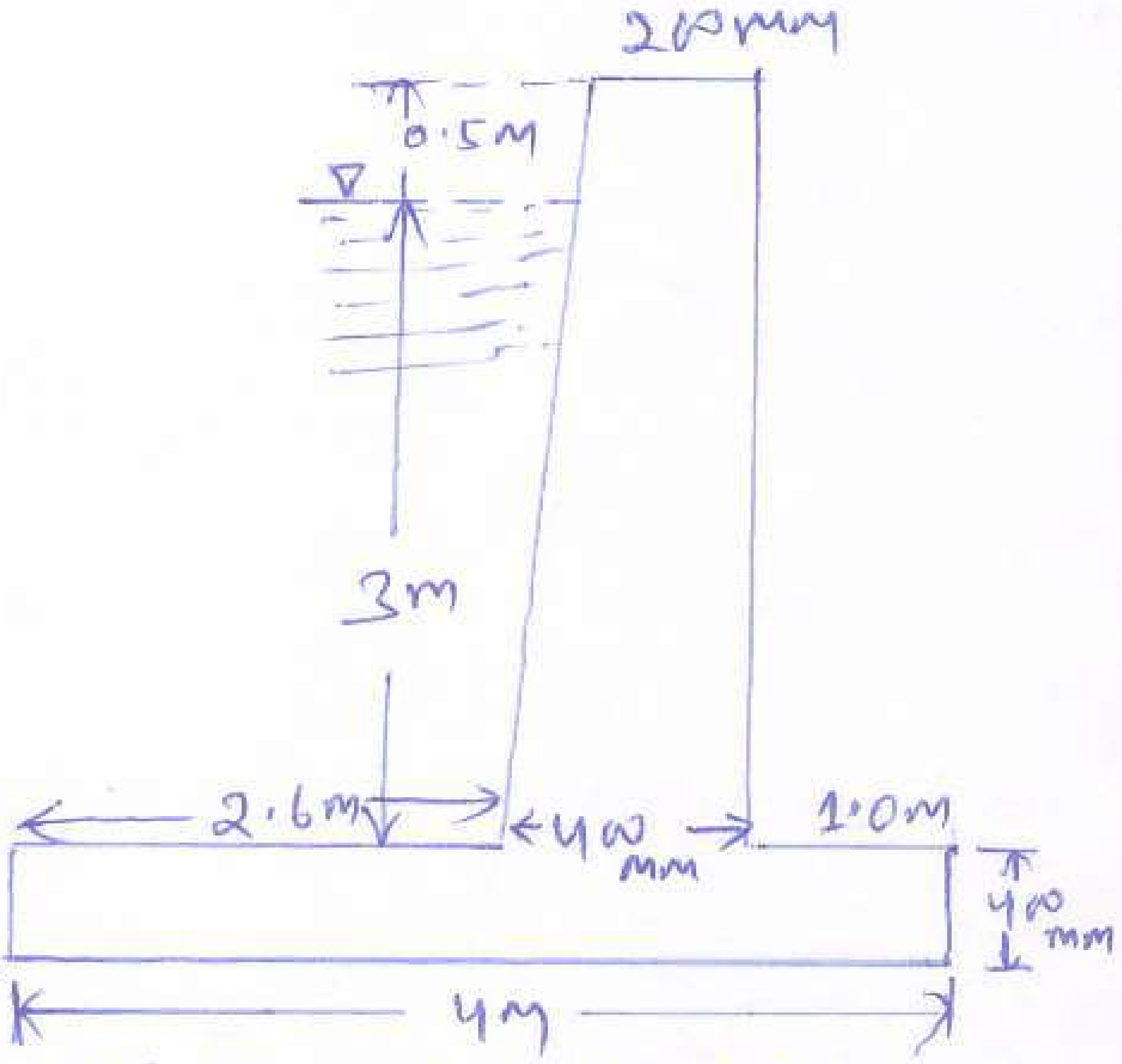
* The main advantage is the outward thrust from top of conical part is resisted by ring beam at bottom of cylindrical part.

Underground Reservoir

Ref -- Book: RC structures
by Ramamrutham & Narayan

Q ⇒ Design the wall and floor of a reservoir to be provided below ground level in good dry earth to hold 3m depth of water. (M20, Fe415)

- safe bearing capacity of soil = 160 kN/m2
- Angle of repose = 30°
- density of soil = 16 kN/m3



(Retaining wall of tank)

sol -) stability of wall / water

case - 1 : ⇒
when only water pressure acts

- load of stem = 0.2 × 3.5 × 25 = 17.5 kN (↓) (rectangular)
  acts at 2.9m from 'a'
  $= \frac{1}{2} \times 0.2 \times 3.5 \times 25 = 8.75$ kN (↓) (triangular)
  acts at $(2.6 + \frac{2}{3} \times 0.2 = 2.73)$ from a

- wt of water = 2.6 × 3 × 9.81 = 76.52 (kN)(↓)
  acts at $\frac{2.6}{2} = 1.3$m from 'a' (rectangular water downward load)

  $= \underbrace{\frac{1}{2}\left(\frac{0.2}{3.5} \times 0.5\right)}_{\text{width of water at surface level}} \times \underset{\text{height}}{3} \times 9.81 = 2.52$ kN (↓)

  acts at $\left[\frac{1}{3}\left(\frac{0.2}{3.5} \times 0.5\right) + 2.6\right] = 2.66$m from a

- wt of base slab = (2.6 + 0.4 + 1) 0.4 × 25 = 40 kN (↓)
  acts at 2m from a.

- Lateral water pressure $= \frac{1}{2} \times 9.8 \times 3^2 = 44.1$ kN (←)
  acts at $\frac{h}{3} = \frac{3}{3} = 1$m from 'a'

W = Total Vertical load = 17.5 + 8.75 + 76.52 + 2.52 + 40 = 145.29 kN
Total moment acting at 'a' = (17.5 × 2.9) + (8.75 × 2.73) + (76.52 × 1.3) + (2.52 × 2.66) + (40 × 2) + (44.1 × 1)
= 50.75 + 23.89 + 99.48 + 6.7 + 80 + 44.1 = 304.92 kNm.

* Resultant of all forces acts at $\frac{304.92}{145.29} = 2.1$m from a = Z

Eccentricity $e = Z - \frac{b}{2} = 2.1 - \frac{4}{2} = 0.1$m $< \frac{b}{6} \Rightarrow 0.1 < \frac{4}{6} \Rightarrow 0.1 < 0.67$ ok — ①

max/min pressure at base, $p = \frac{W}{b}\left[1 \pm \frac{6e}{b}\right] = \frac{145.29}{4}\left[1 \pm \frac{6 \times 0.1}{4}\right] = 41.77$ kN/m2

$\Rightarrow P_{max} = 41.77$ kN/m2 & $P_{min} = 30.87$ kN/m2 < 160 kN/m2 (ok) —— ②

* Total horizontal pressure by water $= P = \frac{\gamma h^2}{2} = 9.8 \times \frac{3^2}{2} = 44.1$ kN

max available friction $= \mu W = 0.5 \times 145.29 = 72.65$ kN.

Factor of safety $= \frac{\mu W}{P} = \frac{72.65}{44.1} = 1.6 > 1.55$ ok —— ③

case-2 → when tank is empty: →

- load of stem → 17.5 kN(↓) acts at 2.9m from A
  - → 8.75 kN(↓) acts at 2.73 from a
- load of base slab ⇒ 40 kN(↓) at 2m from a
- wt of soil $= 1 \times 3.5 \times 16 = 56$ kN(↓) acts at 3.5m from a
- Lateral soil pressure $= \frac{1}{2} k_a \gamma h^2 = \frac{1}{2}\left(\frac{1-\sin 30}{1+\sin 30}\right) \times 16 \times 3.5^2 = 32.34$ kN

acts at $\frac{3.5}{3}$ from a $= 1.17$m from A.

Resultant of all forces acts ~~at~~ from a $= \frac{(17.5 \times 2.9) + (8.75 \times 2.73) + (40 \times 2) + (56 \times 3.5) - (32.34 \times 1.17)}{17.5 + 8.75 + 40 + 56}$

$= \frac{50.75 + 23.89 + 80 + 196 - 37.84}{122.25}$

$= \frac{312.8}{122.25} = 2.55$ from 'a' = Z

* eccentricity, $e = Z - \frac{b}{2} = 2.55 - 2 = 0.55 < \frac{b}{6} = 0.55 < 0.67$ (ok) —— ①

max/min pressure at base $= P = \frac{W}{b}\left[1 \pm \frac{6e}{b}\right] = \frac{122.25}{4}\left[1 \pm \frac{6 \times 0.55}{4}\right]$

$\Rightarrow P_{max} = 55.77$ kN/m2, $P_{min} = 5.35$ kN/m2 < 160 kN/m2 (ok) —— ②

Reinforcement details :-

Case 1 :- Max BM at bottom of stem due to water pressure alone

$= \frac{1}{2}\gamma h^2 \times \frac{h}{3} = \frac{1}{2} \times 9.81 \times 3^2 \times \frac{3}{3} = 44.15$ KNm

→ This moment gets reduced by lateral soil pressure but here moment is maximised without reducing it as soil & water pressure cancel each other. and water pre > soil pressure

This BM produces tension on water face

$0.138 f_{ck} b d^2 = 44.15 \times 10^6$

$\Rightarrow 0.138 \times 20 \times 1000 \times d^2 = 44.15 \times 10^6 \Rightarrow d = 127$ mm (required)

provided D = 400mm

provided d = 400 − 40 = 360 mm.

$M = 0.87 f_y A_{st} \left(d - \frac{f_y A_{st}}{f_{ck} b}\right)$

$\Rightarrow 44.15 \times 10^6 = 0.87 \times 415 \times A_{st} \left(360 - \frac{415 \times A_{st}}{20 \times 1000}\right)$

$= 361 A_{st}(360 - 0.02 A_{st})$

$44.15 \times 10^6 = 129960 A_{st} - 7.22 A_{st}^2$

$\Rightarrow 7.22 A_{st}^2 - 129960 A_{st} + 44150000 = 0$

~~$A_{st} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$~~

$A_{st} = 1460$ mm²

spacing of 16mm ϕ bars $= \frac{201 \times 1000}{1460} = 138 \simeq 130$ mm/c/c

∴ on tension face of RW, 16 mm ϕ @ 130 mm c/c.

Temp & shrinkage steel

$= 0.12\%$ of vertical sectional area of wall

$= \frac{0.12}{100} \times (3.5 \times 10^3) \times \frac{1}{2}(200 + 400) = 1260$ mm²

No of 6mm ϕ bars $= \frac{1260}{\frac{\pi}{4} \times 6^2} = 45$ no.

- Since front face is more exposed to temp, hence at front face $\frac{2}{3} \times 45 = 30$ no bars & at inner face 15 no of bars are provided.

Case - 2

max BM at bottom of stem due to lateral pressure alone

$$= \frac{1}{6}\left(\frac{1-\sin 30}{1+\sin 30}\right) \times 3.5^3 \times 16 = 37.84 \text{ kN.m.}$$

This produces tension on outer face i.e. away from water face

$\therefore\ 0.138 f_{ck} b d^2 = 37.84 \times 10^6$ ~~Ast~~ $\Rightarrow d = ?$

$$37.84 \times 10^6 = 0.87 f_y A_{st}\left(d - \frac{f_y A_{st}}{f_{ck} b}\right)$$

$\Rightarrow A_{st} = 985 \text{ mm}^2$

spacing of 12mm ϕ bars $= \frac{113 \times 1000}{985} = 115$ mm ≃ 110mm

distribution steel = 0.12% of cross section of wall = 1260 mm²

similar to (Case - 1)

* Design of base i.e. heel slab (water slab) & toe slab is same as we did in Cantilever RW)

Ref- structural Design & Drawing
both, by; Krishna Raju

WATER TANK on GROUND / 24/11/16

Q Design a rectangular RC water tank resting on ground with an open top for a capacity of 80 000 litre. The inside dimension of the tank may be taken as 6 × 4 m.

- Design side walls of tank (M 25, Fe250)
- Draw –
  - c/s elevation of tank showing reinforcement detail in tank wall
  - plan of tank showing reinforcement details.

Sol =) Tank capacity = 80 000 lit = $80\,000 \times 10^{-3} = 80 m3$

Tank size = 6 × 4 m.

Free board = 150 mm

Table 2, cl 4.5.2.2. , $\sigma_{cbc} = 8.5 N/mm^2$.

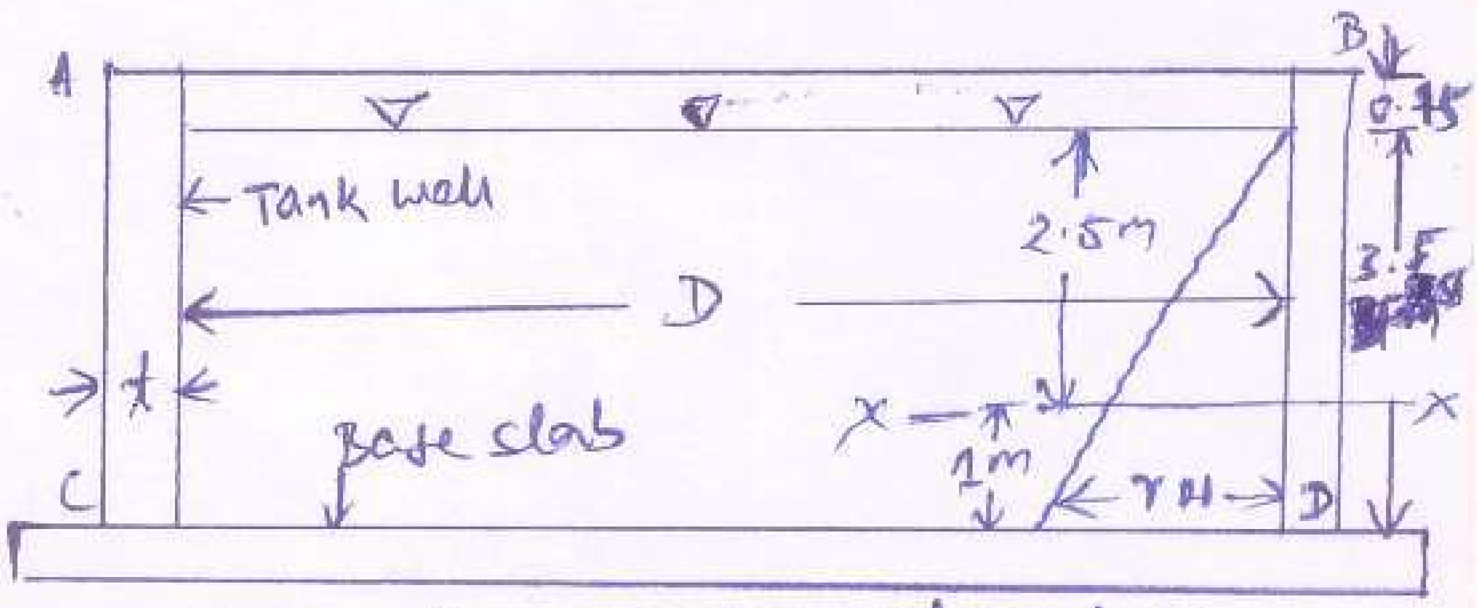
Table 4, cl 4.5.3.2 , $\sigma_{st} = 115 N/mm^2$

m = 13 , Q = 1.41 , j = 0.84

Tank dimension →

~~D = tank diameter height of water = 80000 × 10~~

height of water = $\frac{80}{6 \times 4} = 3.35$



Rectangular ~~circular~~ water tank

Free board = 150 mm

H = height of side wall = H = 3.35 + 0.15 = 3.5 m (H)

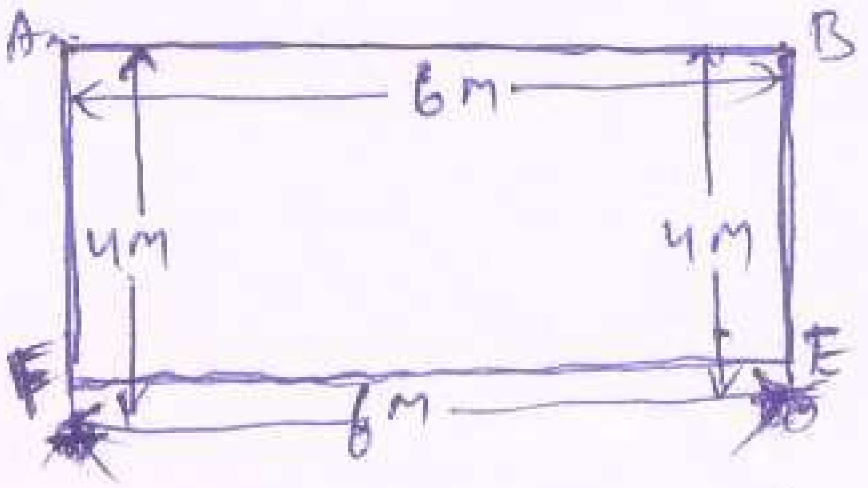
$\frac{\text{length}}{\text{width}} = \frac{6}{4} = 1.5 < 2$

* Assumed that the walls function as a continuous slab to water pressure above $\frac{H}{4} \approx 1 m$ from the bottom and as a cantilever for bottom 1m = h

* pressure intensity, P = γ(H − h) at X-X = 10 × 2.5 = 25 kN/m² =) $P_{X-X} = 25 kN/m^2$

moment in side walls →

moment in side walls are determined by moment distribution method



(Top view of rectangular water tank of size 6m×4m)

Relative stiffness of AB = $\frac{I}{6}$, Relative stiffness of AF = $\frac{I}{4}$

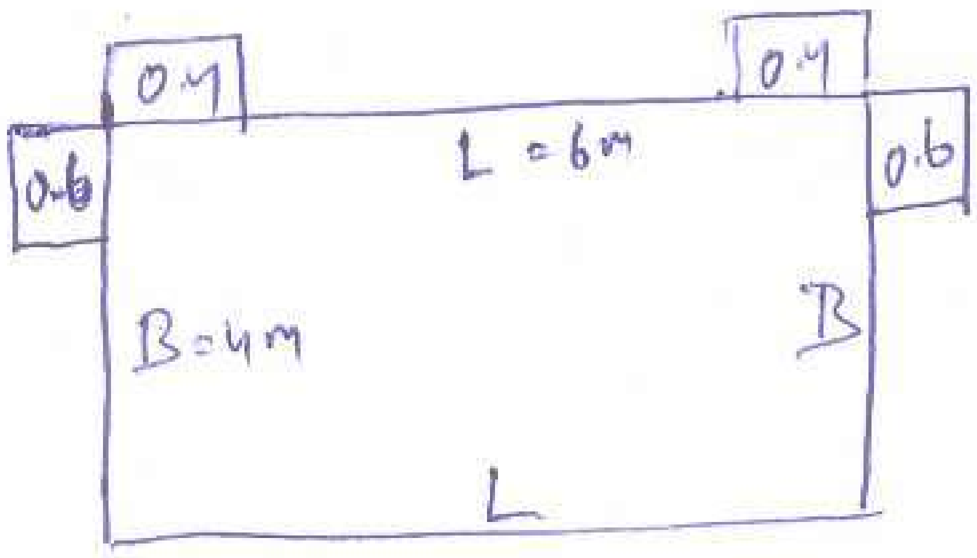
Sum of relative stiffness at joint A = $\frac{I}{6} + \frac{I}{4} = \frac{5}{12} I$

Distribution factor, $D_{AB} = \frac{I/6}{\frac{5}{12} I} = \frac{1}{6} \times \frac{12}{5} = \frac{2}{5} = 0.4$

Distribution factor, $D_{AF} = \frac{I}{4} / \frac{5}{12} I = \frac{1}{4} \times \frac{12}{5} = \frac{3}{5} = 0.6$

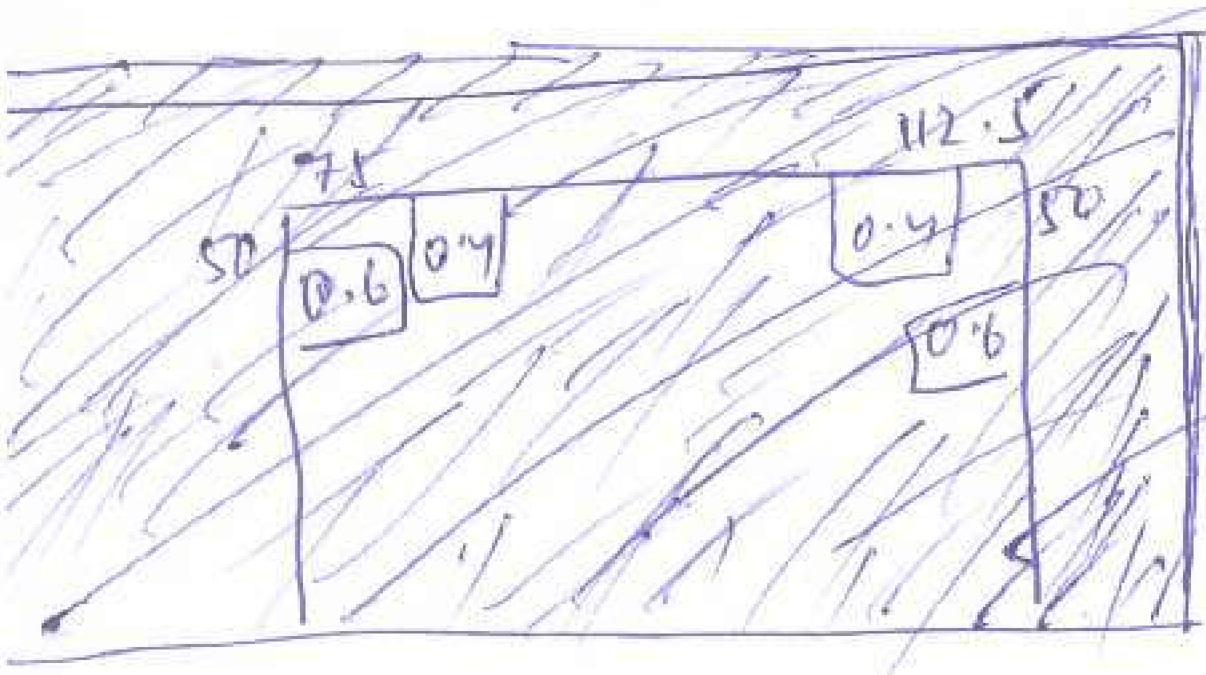
∴ $D_{AB} = 0.4$ and

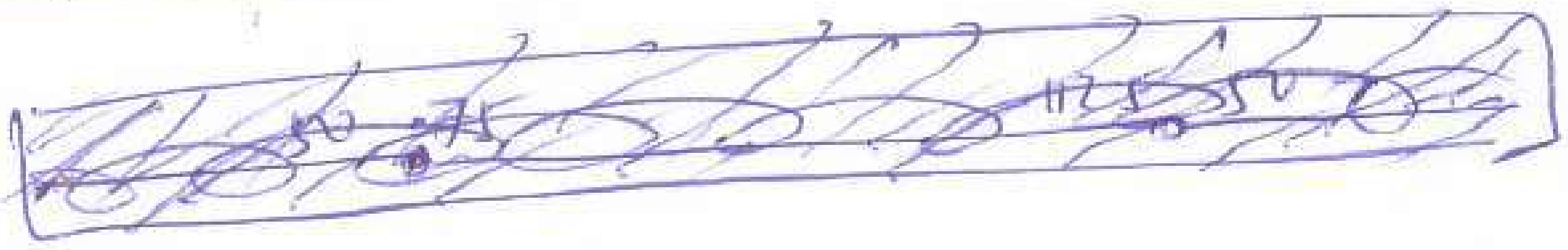
$D_{AF} = 0.6$



(Top view of water tank)

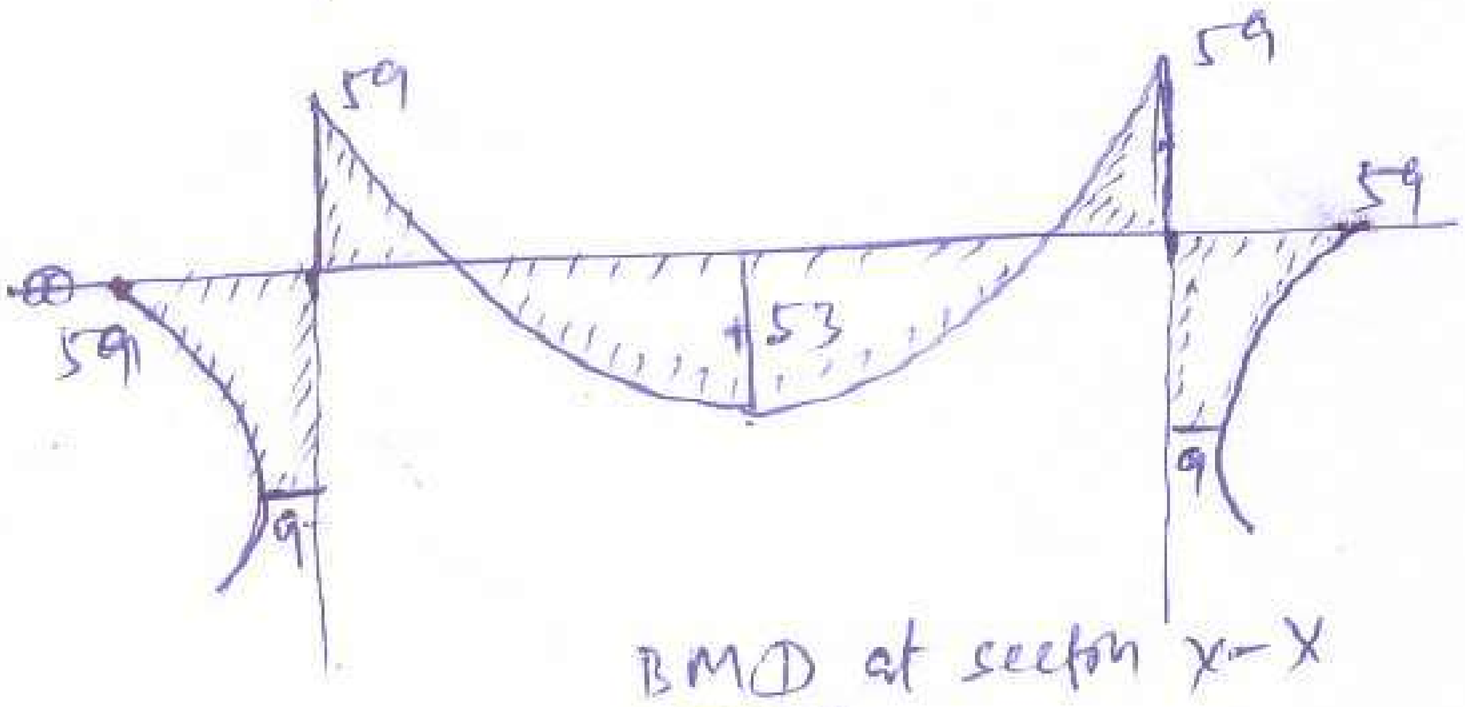
	support moments	mid span moments
(long span)	$\frac{PL^2}{12} = \frac{25\times6^2}{12} = \underline{75\ kNm}$ →	$\frac{PL^2}{8} = \frac{25\times6^2}{8} = \underline{112.5\ kN/m}$
(short span)	$\frac{PB^2}{12} = \frac{25\times4^2}{12} = \underline{34\ kNm}$ →	$\frac{PB^2}{8} = \frac{25\times4^2}{8} = \underline{50\ kNm}$





+34	−75
0.6	0.4
+25	+16
+59	−59

+75	−34
0.4	0.6
−16	−25
+59	−59



BMD at section x-x

moment at support = 59 kNm

moment at centre of long wall = 112 − 59 = 53 kNm

moment at centre of short wall

= 50 − 59 = −9 kNm

Design of long wall ~~and~~ short wall →

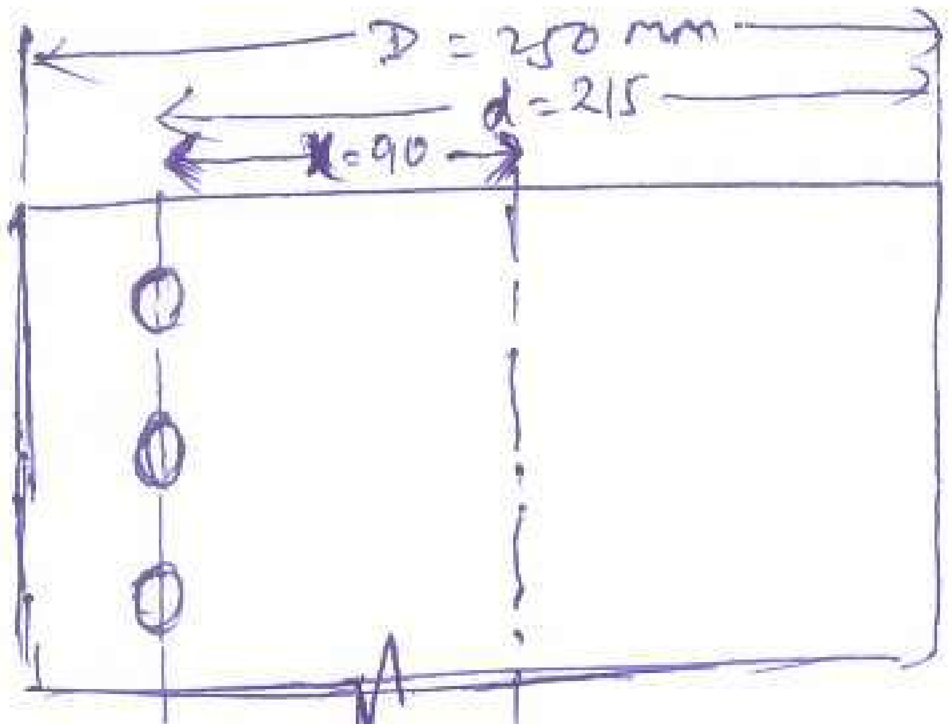
max design moment = 59 kNm

$$d = \sqrt{\frac{59\times10^6}{jd}} = \sqrt{\frac{59\times10^6}{0.84\times1000}} = 204\,\text{mm} \simeq \underline{\underline{215\,\text{mm}}}$$

$$D = \underline{\underline{250\,\text{mm}}}$$

Direct tension in long wall $T = \cancel{0.5}\ \frac{1}{2}\times P\times B = \frac{1}{2}\times25\times4 = 50\,\text{kN}$

Direct tension in short wall $T = \frac{1}{2}\times P\times L = \frac{1}{2}\times25\times6 = 75\,\text{kN}$



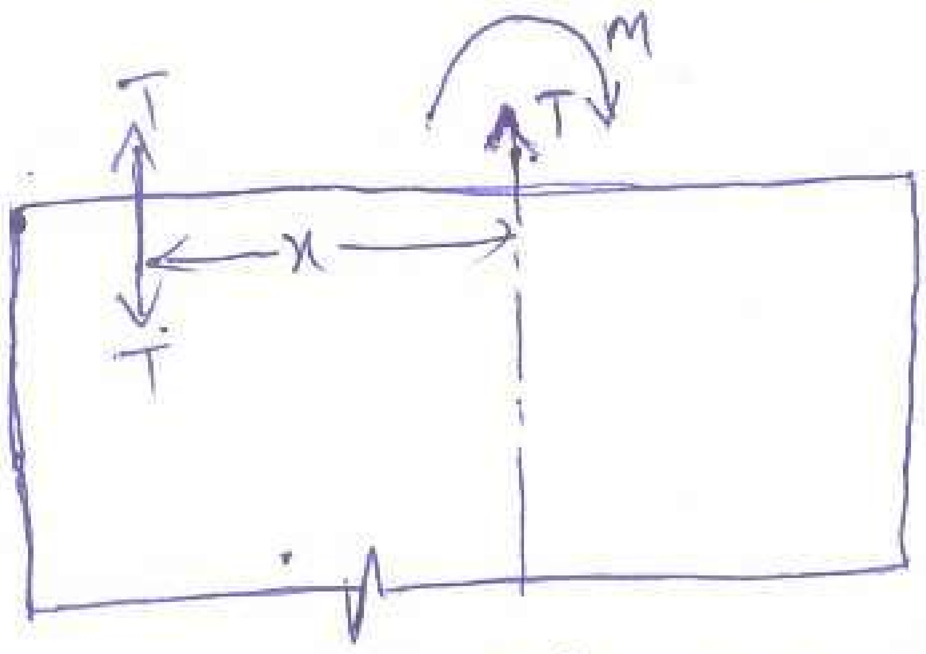
T = pull in steel
net moment = (M − Tx)

* At long wall corners, $A_{st} = \frac{M - Tx}{\sigma_{st}\,jd} + \frac{T}{\sigma_{st}}$

$$\Rightarrow A_{st} = \frac{(59\times10^6)-(50\times10^3\times90)}{100\times0.84\times215}\ \cancel{+80}\ + \frac{50\times10^3}{100} = 3480$$

$20\,\text{mm}\,\phi$ bars @ $\frac{1000\times314}{3480} = 86\,\text{mm c/c}$

Let $\boxed{20\,\text{mm}\,\phi \text{ bars @ } 80\,\text{mm c/c}}$ (Provided $A_{st} = 3928\,\text{mm}^2$)



moment & direct tension in in tank wall

* At long wall centre of span $= \frac{(53\times10^6)-(50\times10^3\times90)}{125\times0.84\times215} + \frac{50\times10^3}{125}$

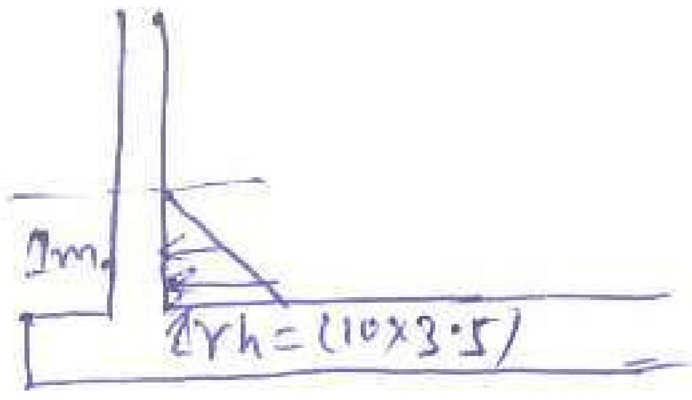
$= \underline{\underline{2500\,\text{mm}^2}}$

Half the bars from innerface at support are bent towards the outerface at centre, providing an area of (0.5 × 3928 = 1964 mm².

For remaining steel, 2500 − 1964 = 536 mm²,
Provide $\boxed{16\,\text{mm}\,\phi \text{ bars at } 150\,\text{mm c/c}}$.

For short wall bent 50% bars towards outerface ~~op~~ at centre.

Reinforcement for cantilever moment:-



(For 1m height from the bottom)

cantilever moment $= (\frac{1}{2}\times\gamma h\times 1)\times(\frac{1}{3})$

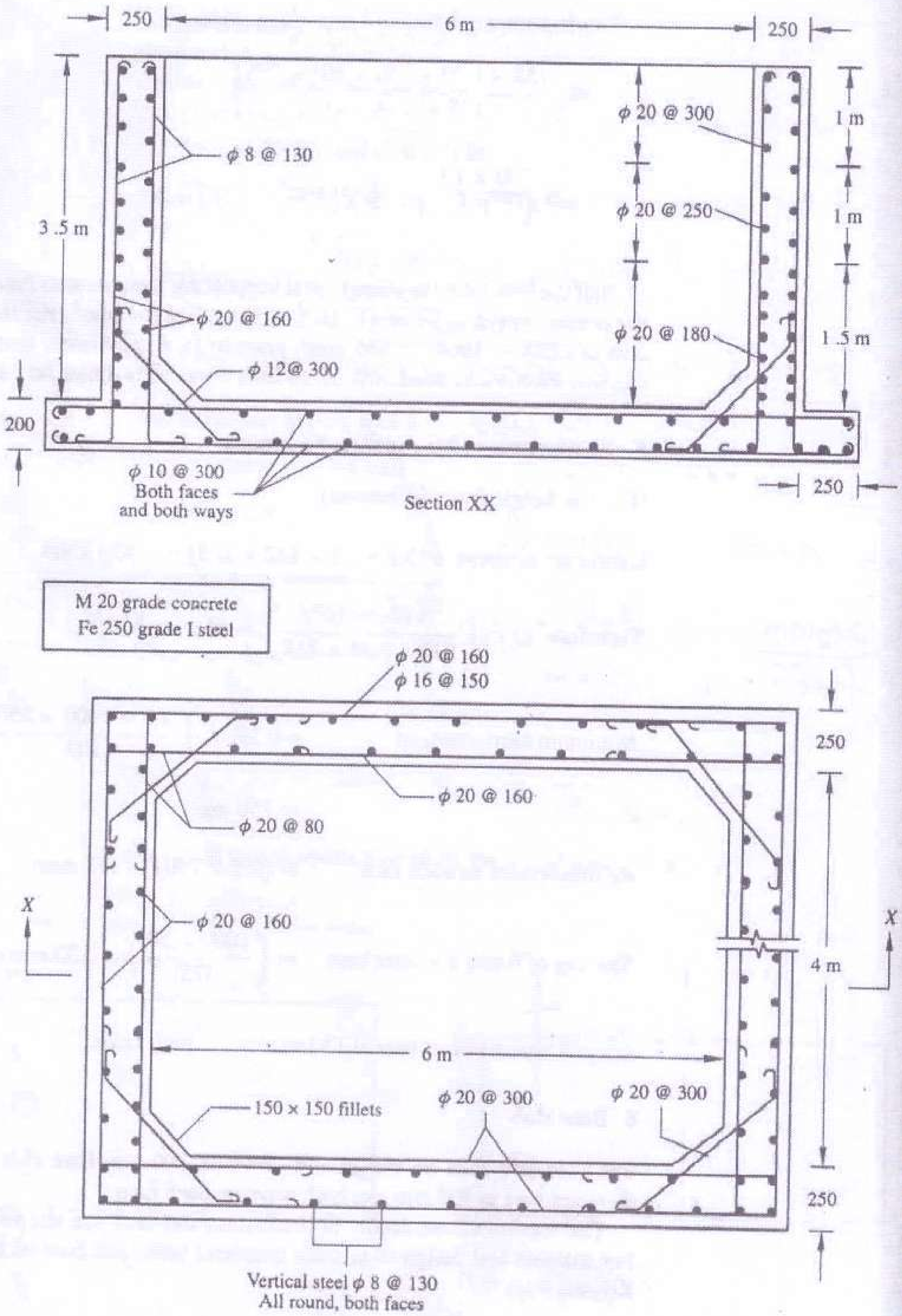
$= \frac{1}{2}(10\times3.5)\times1\times\frac{1}{3} = \frac{1}{2}\times10\times3.5\times\frac{1}{3} = 5.833$ kNm

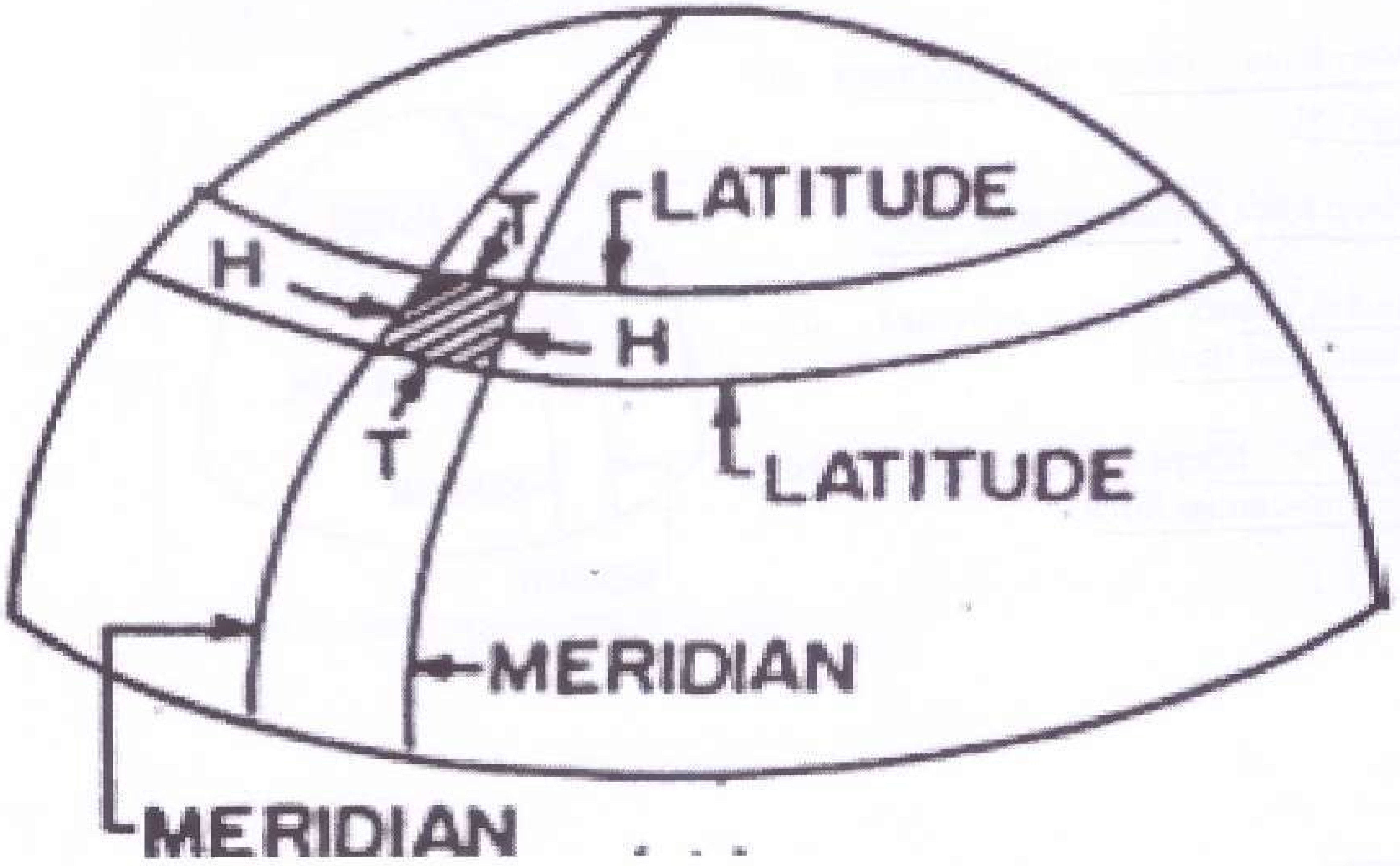
- $A_{st} = \frac{5.833\times10^{6}}{\sigma_{st}\, j d} = \frac{5.833\times10^{6}}{100\times0.84\times215} = 323\,mm^{2}$
- minimum reinforcement $= 0.3\% = \frac{0.3}{100}\times1000\times250 = 750\,mm^{2}$
- Reinforcement on each face $= 0.5\times750 = 375\,mm^{2}$
- Spacing of 8mm ϕ bars $= \frac{1000\times50}{375} = 130$ mm c/c

Provide $\boxed{\text{8mm } \phi \text{ bars @ 130mm c/c}}$ on both faces..

Base slab

- Base slab rests on ground
- provide 200mm base slab with $\boxed{\text{10mm } \phi \text{ bars @ 300 mm c/c}}$, both ways on each face.


250
6 m
250
3.5 m
φ 8 @ 130
φ 20 @ 160
φ 12@ 300
200
φ 10 @ 300
Both faces
and both ways
φ 20 @ 300
φ 20 @ 250
φ 20 @ 180
1 m
1 m
1.5 m
250
Section XX
M 20 grade concrete
Fe 250 grade I steel
φ 20 @ 160
φ 16 @ 150
φ 20 @ 160
φ 20 @ 80
φ 20 @ 160
X
X
250
4 m
6 m
150 × 150 fillets
φ 20 @ 300
φ 20 @ 300
250
Vertical steel φ 8 @ 130
All round, both faces


LATITUDE
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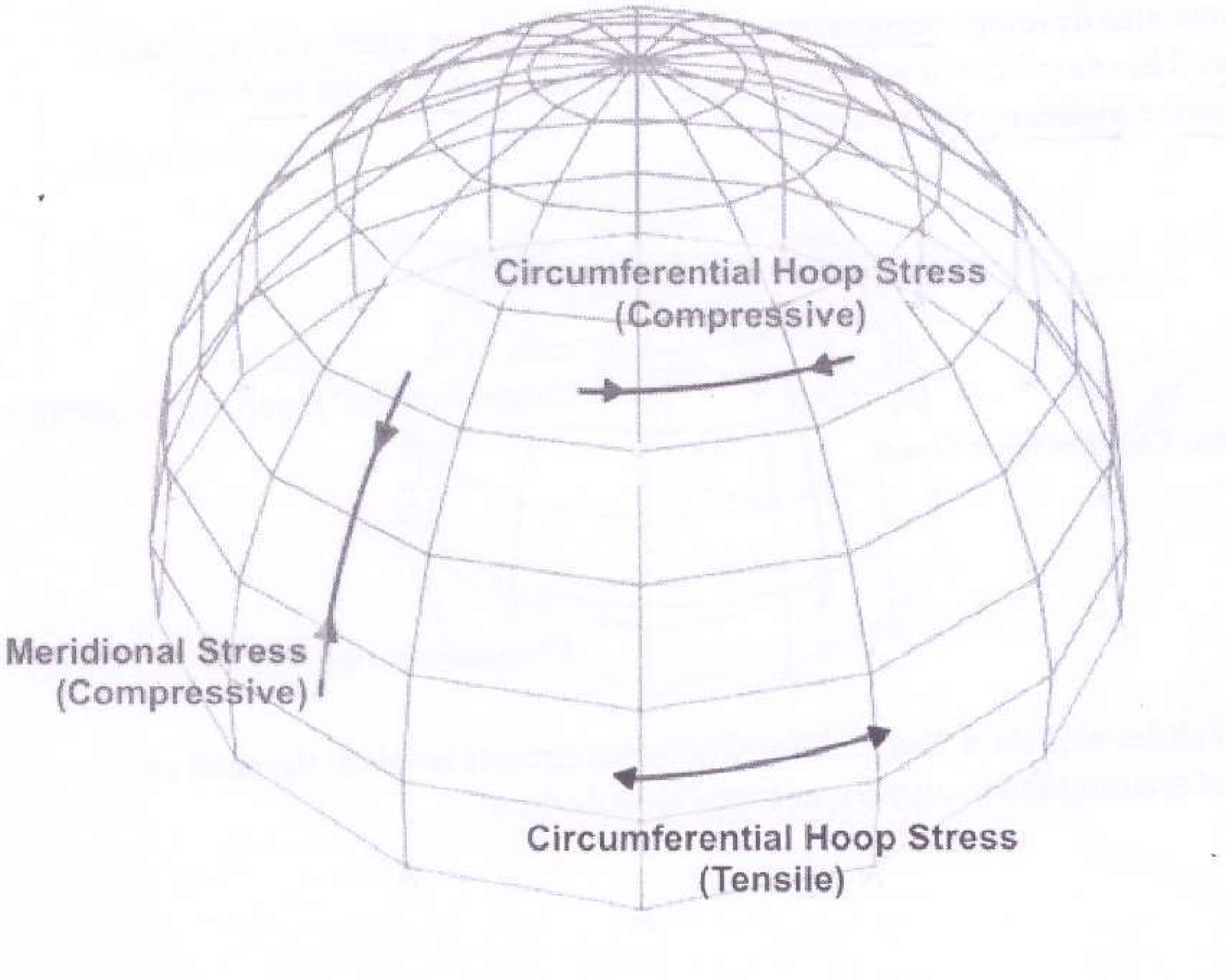
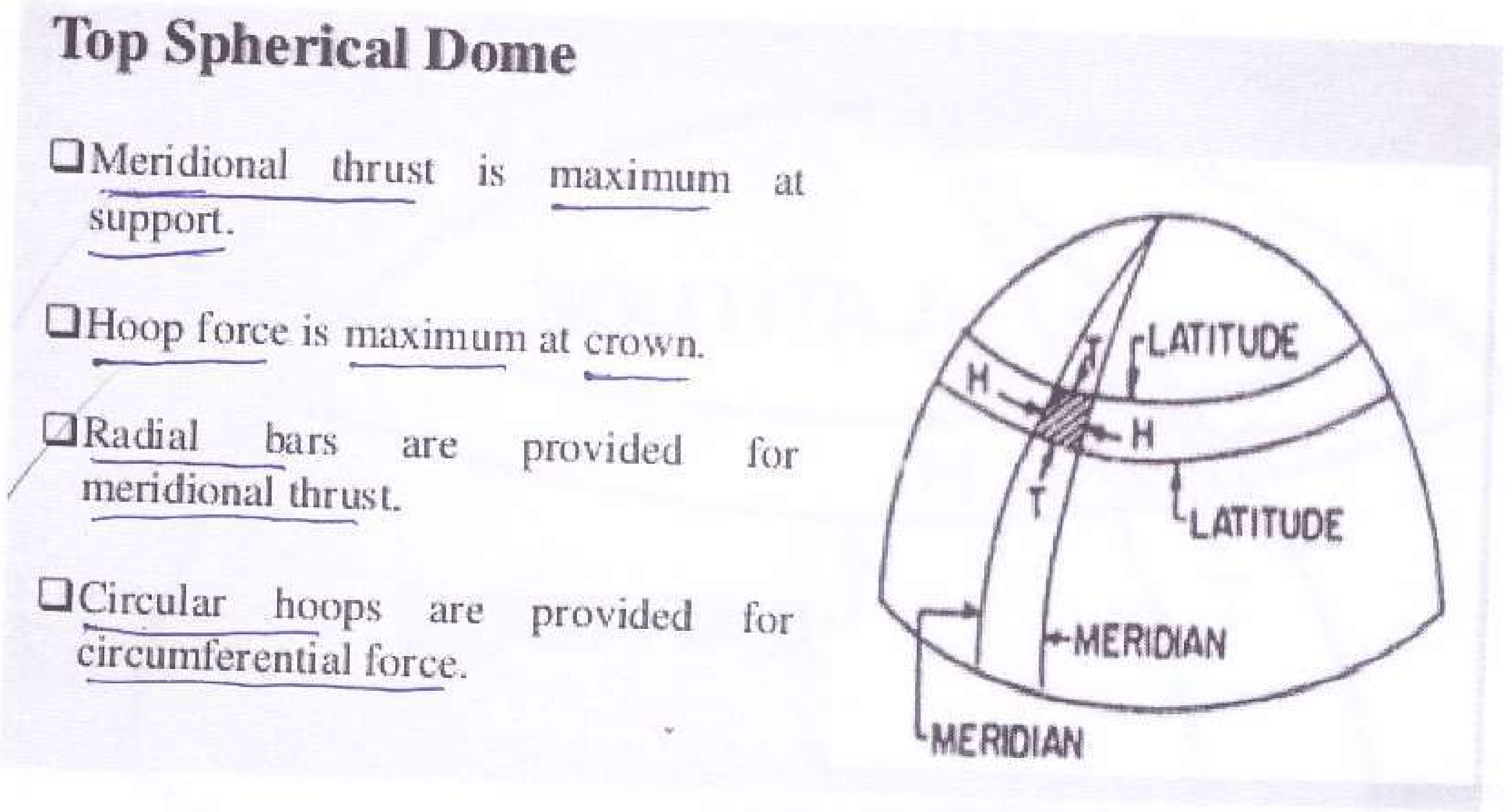
MERIDIAN


Circumferential Hoop Stress
(Compressive)
Meridional Stress
(Compressive)
Circumferential Hoop Stress
(Tensile)



Top Spherical Dome

☐ Meridional thrust is maximum at support.

☐ Hoop force is maximum at crown.

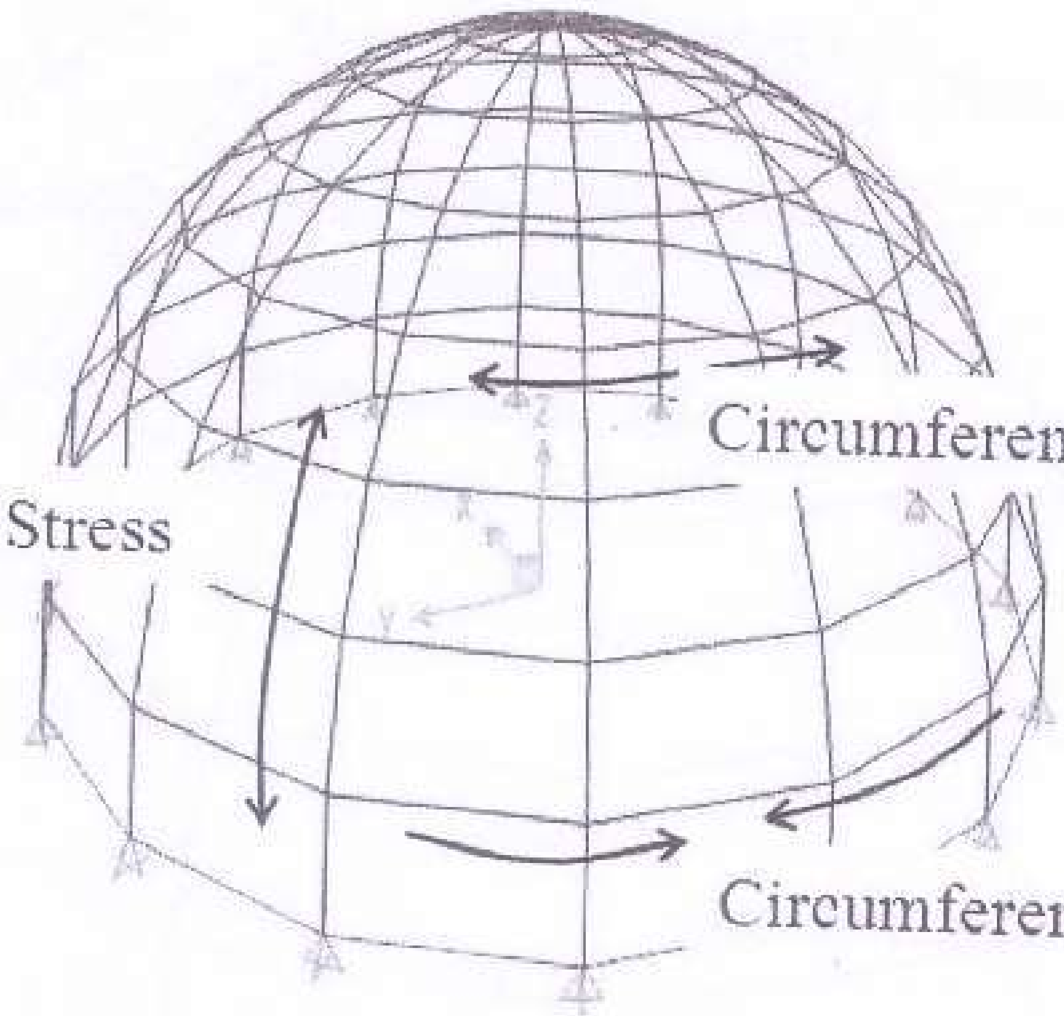
☐ Radial bars are provided for meridional thrust.

☐ Circular hoops are provided for circumferential force.

Domes

The primary response of a dome to loading is development of membrane compressive stresses along the meridians, by analogy to the arch.

The dome also develops compressive or tensile membrane stresses along lines of latitude. These are known as 'hoop stresses' and are tensile at the base and compressive higher up in the dome.



Circumferential Hoop Stress (comp.)

Meridional Compressive Stress

Circumferential Hoop Stress (tens.)

The next slides will show that additional bending stresses result in the shell as a result of restraint at the support, or unrestrained edges

Ref: Krishna Raju

Design of circular water tank (on ground)

L7

Design a RC circular water tank resting on ground with a flexible base and spherical dome for a capacity of 500000 litre. The storage depth = 4m. Free board = 200mm. M 20 & Fe 250.

(Permissible stress as in IS 456 & IS 3370 (part-II))

Design :- (1) cross section of tank showing reinforcement details in dome, tank wall, and floor slab. show the reinforcement details.

sol →

Advanced Concrete Structure

PRESTRESSED CONCRETE

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Introduction

23-3-02

* Development of structural materials can be classified as follows.

→ material resisting compression :-

stone & bricks
↓
concrete
↓
high strength concrete

→ material resisting tension :-

bamboo & ropes
↓
iron bars & steel
↓
high strength steel

→ material resisting both tension & compression (i.e. bending)

Timber
↓
structural steel
↓
RCC
↓
Prestressed concrete.

* The main diff bet^n RCC & PCC is the fact that :-

→ RCC combines concrete & steel bars by simply putting them together & letting them act together as they may wish.

→ // PSC combines high strength concrete & high strength steel in an "active" manner.

// This is achieved by tensioning the steel & holding it against concrete & thus putting concrete into compression.

// This active combination results in a much better behaviour of the two materials.

// steel is ductile & is used to act in high tension by prestressing.

// concrete is brittle with its tensile capacity is improved by being compressed without harming its compressive capacity.

* In old days, aim of prestressing was to create permanent compression in concrete to improve its tensile strength.

* Later it is understood that prestressing the steel is essential for the efficient utilization of high tensile steel.

- Prestressing means intentional creation of permanent stresses in a structure to improve its behaviour & strength under various service conditions.
- Prestressed concrete is the concrete in which internal stresses of suitable magnitude & distribution are introduced so that stresses resulting from external loads are counteracted to a desired degree.
- Prestressed concrete means, concrete reinforced with metal that had tensile stresses applied to it before it is loaded.
- In reinforced concrete member prestress is introduced by tensioning steel reinforcement.

example -)

Basic principle of prestressing was applied to Barrel construction, when metal bands were wound around wooden staves to form barrels.

when metal bands were tightened they were under tensile prestress which created compressive prestress bet^{n} staves & enabled them to resist hoop tension produced by internal liquid pressure. We can say that

Bands & staves were both prestressed before subjected to service loads.

- The application of prestress is based on the conception that concrete though strong in compression was quite weak in tension and prestressing the steel against concrete would put concrete under compressive stress which could be utilized to counterbalance any tensile stress produced by dead/live loads.
- Development of early cracks in RCC due to incompatibility in strains of steel & concrete, was the starting point of development of prestressed concrete.
- Application of permanent compressive stress to concrete increases the tensile strength of it, because subsequent application of tensile stress firstly nullifies the compressive prestress.
- Freyssinet of France is the first man for the modern development of prestressed concrete.

* Freyssinet used high strength steel wires for prestressing, whose

ultimate strength, $\sigma_U = 1725$ N/mm2

yield strength, $\sigma_y = 1240$ N/mm2

prestress, $\sigma = 1000$ N/mm2 , $E = 2 \times 10^5$ N/mm2

$\therefore$ strain $= \epsilon = \frac{\sigma}{E} = \frac{1000}{2 \times 10^5} = 0.005$

Assuming total loss due to shrinkage & creep of concrete = 0.0008

net strain = 0.005 − 0.0008 = 0.0042 (would left in wire)

equivalent stress of above strain = $0.0042 \times 2 \times 10^5 = 840$ N/mm2

*(1) ~~Prestre~~ Application of prestressed concrete → in

→ Tank, Bridge, Building

→ Dam by anchoring prestressed steel bars to foundation or by jacking the dam against the foundation.

→ Pipes, posts, piles.

→ principle of prestressing is not limited to structures in concrete, it has also been applied to steel structure.

② Purpose of applying prestressing force ⇒

[① to induce desirable strains & stresses in structure.

② to counterbalance undesirable strains & stresses.

* In prestressed concrete,

→ steel is pre-elongated to avoid excessive lengthening under service load

→ concrete is pre-compressed to prevent cracks under tensile stress.

Thus ideal combination of two materials is achieved.

③ Need for high strength concrete & steel →

The two main observations in PSC are

→ use of high strength concrete & steel

→ calculation of losses of prestress due to various causes.

* High strength steel →

→ Normal loss of stress in steel is about 100 N/mm2 to 240 N/mm2.

→ This amount (i.e. loss) should be small enough w.r.t the initial stress.

→ so the initial high range stress is of order 1200 N/mm2 to 2000 N/mm2 & is only possible in case of high strength steel.

* high strength concrete (HSC) → is essential for PSC because

→ HSC offers high resistance in tension, shear, bond & bearing.

→ HSC is generally preferred in zone of anchorage as bearing stress is higher in this zone & HSC minimizes cost.

→ HSC is less liable to shrinkage cracks.
has a higher modulus of elasticity.
has a smaller ultimate creep strain.
which results in smaller loss of prestress in steel.

→ use of HSC results in reduction of c/s dimension of PSC structural elements.

→ with reduced dead wt of material, longer span is practicable.

⑨ Terminology →

Tendon → * A synonym of prestressed reinforcement.
* A stretched element used in a concrete ~~element~~ member of structure to impart prestress to concrete.
* High tensile steel ~~bar~~ wires
Bars
cables
strands
— are used as tendons.

cable → A group of tendons.

strand → ~~# group of wires~~ Formed by twisting wires together in factories.
A single length of wire twisted ~~with~~ together with others.

Duct → A tube/passage way for inserting cables in a PSC structural element.

Anchorage → A device used to enable the tendons to impart & maintain prestress in concrete.

commonly used anchorages → Freyssinet, Magnel Blaton, Gifford-Udall
Le McCall, BBRV systems.

Pretensioning →
* A method of prestressing concrete where tendons are tensioned before concrete is placed.
* Here prestress is imparted to concrete by BOND betn steel & concrete.

Post tensioning →
* A method of prestressing concrete where tendons are tensioned against hardened concrete/after concrete is placed.
* Here prestress is imparted to concrete by BEARING.

Transfer → The transferring of prestress to concrete.
* In Pretensioned member, it takes place at the release of prestress from bulkheads.
* In Post Tensioned member, it take place after completion of tensioning process.

Untensioned Reinforcement → Reinforcement which is not tensioned w.r.t surrounding concrete before application of load in prestressed member.
* such reinforcements are used in partially prestressed members.

Bonded & unbonded reinforcement → Reinforcement bonded or unbonded throughout its length to the surrounding concrete.

Anchored & not-end-anchored reinforcement ⇒ Reinforcement anchored at its ends (not anchored) by mechanical devices capable of transmitting the tensioning force to concrete.

Prestressed & nonprestressed Reinforcement ⇒ Reinforcement in prestressed concrete members which are elongated (not elongated) w.r.t surrounding con- concrete. * Tendons are prestressed reinforcement.

Transmission length → Length of bond anchorage of prestressing wire from end of a pre-tensioned member to the point of full steel stress.

(i) Girder load → (The wt of beam or girder itself) + (wt on it at the time of transfer.)

(ii) working load/service load → max. total load which the structure is expected to carry.

(iii) cracking load → The total load in a prestressed-concrete member required to initiate cracks/for first visible crack.

(iv) Ultimate load → The total load which a member can carry upto total rupture.

Load factor → ratio of ultimate load/working load.

Reserve Strength → ratio of ultimate load/yield load

Shape factor → $K = \frac{\text{plastic M.R}}{\text{yield M.R}} = \frac{\text{plastic section mod}}{\text{elastic section mod}}$

Factor of safety → $F_s = \frac{\text{yield stress}}{\text{working stress}}$

$\boxed{\text{Load factor = shape factor × Factor of safety.}}$

creep → Time dependent progressive increase in inelastic deformation of concrete or steel resulting from sustained stress & it is function of stress.

creep coefficient → ratio of total creep strain to elastic strain in concrete.

shrinkage of concrete → contraction of concrete due to drying & chemical changes dependent on time but not directly dependent on stress induced by external loading.

Relaxation of steel → Decrease of stress in steel at constant strain.

Proof stress → Tensile stress in steel which produces a residual strain of 0.2% of original gauge length on unloading.

Debonding → prevention of bond bet^n steel wire & surrounding concrete.

Degree of prestressing ⇒ A measure of magnitude of prestressing force related to resultant stress occurring in structural member at working load.

cap cable ⇒ * A short curved tendon arranged at interior supports of a continuous beam.

* The anchors of cap cable are in compression zone.
* The curved portion of " is in tensile zone.

* Concordant & non concordant cables →

* Lineare Transformation →

Bonded prestressed concrete →

* concrete in which prestress is imparted to concrete through bond bet^n tendon & surrounding concrete.
* Pre-tensioned members belong to this group.

Non bonded prestressed concrete →

* In this method of construction, the tendons are not bonded to surrounding concrete.
* Tendons may be placed in ducts formed in concrete members or they may be placed outside the concrete section.

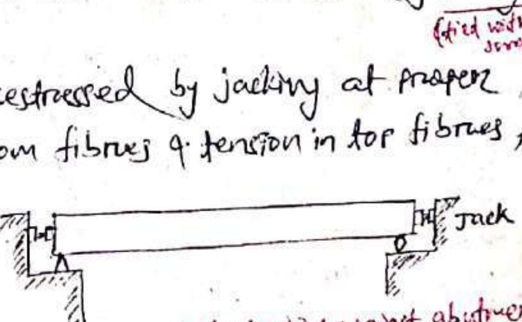
Bonded or unbonded Tendons in post-tensioned PSC structure → * Defination is already explained.

* Non-end-anchored tendons are necessarily bonded ones.
* End anchored tendones may be either bonded/unbonded to concrete.
* Bonding of post tensioned tendons is done by subsequent grouting.
* If such tendons are un-bonded, then protection of tendons from corrosion is provided by Galvanizing, Greasing, etc.

⑤ CLASSIFICATION OF PRESTRESSED CONCRETE STRUCTURE ⟹

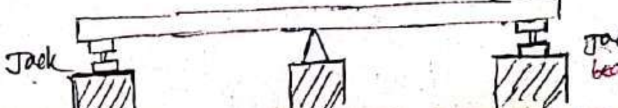
① Externally or internally prestressed →

* All the designs of present syllabus, are based on internally prestressed of PSC structures with high tensile steel.
* External prestressing is done by adjusting external reactions.
* In method of arch compensation, concrete arch is prestressed by jacking (tied with something) against abutments.
* S/s concrete beam is externally prestressed by jacking at proper places to produce compression in bottom fibres & tension in top fibres, with steel reinforcement at bottom.



(Prestressing a s/s by jacking against abutments)

* For indeterminate ~~beams~~ structures like continuous beam, by inserting jacks it is possible to adjust the level of supports to produce most desirable reactions.



prestressing a continuous beam by jacking its reactions.

(ii) Linear or Circular prestressing →

* circular prestressing is applied to prestressed circular structures, such as round tanks, silos, pipes where prestressing tendons are wound around in circles.
* Linear prestressing is applied to beams & slabs.

→ In linearly prestressed structure, tendons are not necessarily straight, they can be bent/curved, but they don't go round in circles as in circular prestressing.

(iii) Pre-tensioning & post tensioning →

* pre tensioning is a method of prestressing in which tendons are tensioned before concrete is placed.

→ Tendons are temporarily anchored against some abutments/stressing beds when tensioned & prestress is transferred to concrete after it has set.

→ This is employed in precasting plants/laboratories where permanent beds are provided & in field where abutments are economically constructed.

* post tensioning is a method of prestressing in which tendons are tensioned after concrete has hardened.

→ This type of prestressing is always performed against hardened concrete. & tendons are anchored against hardened concrete immediately after prestressing.

→ Post tensioning method is applied to members either precast/cast in place.

(iv) End anchored & non-end-anchored tendons ⇒

⊛ In post-tensioned PSC structure, tendons are anchored at ends by mechanical devices to transmit the prestress to concrete. such members are termed as end anchored.

→ In post-tensioned PSC structure, tendons may be held by grout. But end anchorage is necessary

⊛ In Pre-tensioning, prestress is transmitted by bond action at ends

→ Effectiveness of stress transmission is limited to

(a) wires of small size

(b) larger dia strands which possess better bond properties than smooth wires.

→ The most common type material for pretensioning is SEVEN WIRE STRAND

* seven wire strand is used both in pretensioning & post tensioning.

(v) Pre cast, cast-in-place, composite construction →

* Pre cast construction means casting is done at diff area away from construction site.
* cast-in-place means casting is done at construction site itself.

→ cast-in-place construction requires more falsework & form but saves transportation cost & erection & is used for large construction

* composite construction is done near or within the structure.

→ In composite construction, one part is precast & other part is cast-in-place.

→ In such construction, less false & less transportation is required.

(VI) Full, partial, moderate prestressing →

* These types of prestressing depend on degree of prestressing to which concrete member is subjected & mainly on magnitude of working load used in design.

Full prestressing →

when a member is designed so that under working load no tensile stresses are induced in it by sufficiently high prestress in members.

partial prestressing →

The degree of prestress applied to a concrete in which tensile stresses to a limited degree are permitted under working load.

* Here additional mild steel bars are provided to reinforce the portion under tension to limit the crack width.

* So in partially prestressed concrete, both tensioned steel & untensioned steel are provided.

moderate prestressing →

* In this type, no limit is imposed upon the magnitude of tensile stresses at working loads.

* This form of construction is not PSC but RCC with reduced cracking.

* Here section is analysed according to rules of RCC as a case of bending with axial force.

Above classification is based on degree of prestressing. But classification as above mainly depends upon the amount working load.

eg → Highway bridges are generally designed for full prestressing but they are subjected to tensile stresses during passage of heavy vehicles.

similarly roof beam are designed as partially prestressed beams but they never subjected to tensile stresses as assumed live loads may never act on them.

(VII) Axial & Eccentric prestressing →

* In axial prestressing, entire c/s is subjected to uniform comp prestress.

→ Here centroid of tendons coincides with the centroid concrete section.

* In eccentric prestressing, tendons are eccentric to centroid of conc section.

→ Here triangular or trapezoidal comp stress distribution is obtained.

(VIII) Uniaxial, Biaxial, Triaxial prestressing →

* uniaxial if concrete is prestressed in only one direction.

* Biaxial if concrete is prestressed in two mutually ⊥r directions.

* Triaxial if " " " " three mutually ⊥r directions.

(ix) Non-Distortional Prestressing →

- In this type combined effect of prestress & dead wt stress is such that deflection of axis of member is prevented.
- Here moments due to prestress & dead wt balance each other resulting only an axial force in the member.
- Concordant prestressing →

(6) General Principles of PSC ⇒

Three diff concepts are applied to explain & analyze the basic behaviour of PSC.

(a) Ist concept ⇒ (DIRECT METHOD OF ANALYSIS) (Prestressing to transform concrete into an elastic material)

- This concept is credited to Eugene Freyssinet which treats concrete as an elastic material.
- According to this concept PSC is an concrete which is transformed from brittle material to an elastic material by pre compression given to it.
- concrete which is weak in tension & strong in compression is compressed by steel under high ~~compression~~ tension so that brittle concrete would be able to resist tensile stresses.
- It is believed that if there is no tensile stresses in concrete, no cracks are there, & concrete is not a brittle but an elastic material.
- From this view, concrete is subjected to two systems of forces

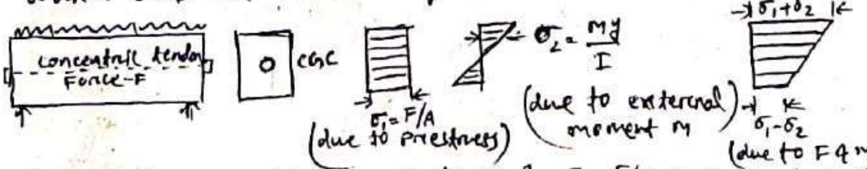
  (1) internal prestress (2) External load

with tensile stresses due to external load is counteracted by compressive stresses due to prestress.

- The cracking due to load of concrete is prevented by precompression in tendons ~~tend~~
- As long as there is no crack, stress, strain & deflection of concrete due to two systems of forces are senapately considered & superimposed

(case-1) ⇒ simple rectangular beam Prestressed by tendon through centroidal axis & loaded by external loads :-

- The tensile prestress force F in tendon produces equal comp force F in conc which acts at centroid of tendon ⇒ force F is at centroid of C/S of bear

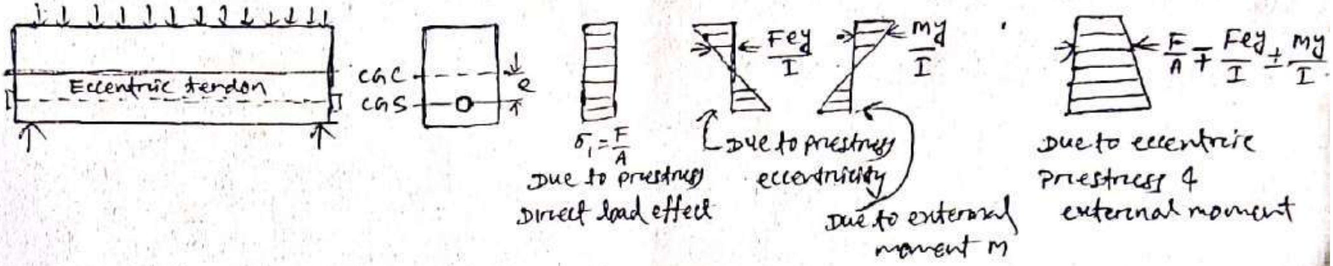


- Due to Prestress F, uniform comp stress of $\sigma_1 = F/A$ is produced across section of area A.
- External moment produced 'M' due to load & self wt, produced stress, σ_2 at any point of section, $\sigma_2 = \frac{My}{I}$, RESULTANT STRESS $\boxed{\sigma = \sigma_1 + \sigma_2 = (F/A) \pm (My/I)}$

* case – II → when tendon is placed eccentrically w.r.t centroid of conc

* Here resultant comp force in concrete acts at centroid of tendon which is at a distance 'e' from centre of gravity of concrete (CGC).

* Due to eccentric prestress conc is subjected to moment & direct load.

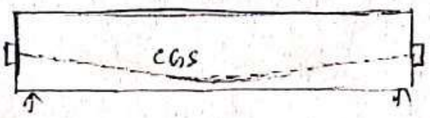


* The moment produced by prestress is 'Fe', stress due to this moment = $\frac{Fey}{I}$

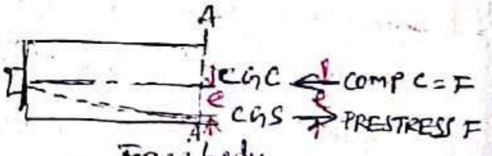
* The resulting stress distribution, $\boxed{\sigma = \frac{F}{A} \mp \frac{Fey}{I} \pm \frac{My}{I}}$

* when tendons are curved/bent, it is convenient to take either left/right portion of the member as a free body in order to evaluate the effect of prestressing force F.

* Resultant compression on concrete due to prestress alone is equal to tendon force 'F' acting at eccentricity 'e'



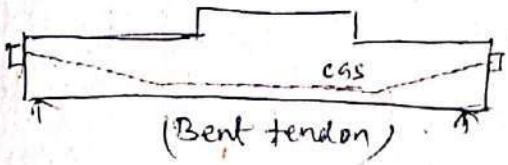
(curved tendon).



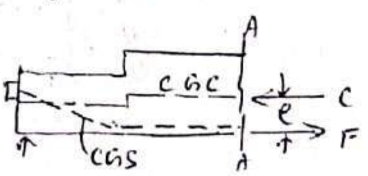
Free body

Equilibrium of horizontal forces indicate that comp in conc equal to prestress in steel F, & stress in conc due to eccentric force F, $\boxed{\sigma = \frac{F}{A} \pm \frac{Fey}{I}}$

* The conc stresses at a sec due to prestress are dependent on the magnitude & location of F at that section, regardless of how the tendon profile may vary along the beam.



(Bent tendon)



* Example-) If sec AA in above two figures are identical, the conc stresses due to prestress F with eccentricity e are identical for the two secs, regardless of (i) variation in shape of beam or
(ii) cable profile away from section.

* This is true only for statically determinate structures where External Reactions are not affected by internal prestressing.

(b) 2nd Concept → (Principle of internal resisting couple) (PRESSURE LINE/THRUST LINE ‖ CONCEPT) (C-LINE METHOD)

* In RCC, steel supplies a tensile force & concrete supplies comp force. The two forces forming a resisting couple with a lever arm betn them opposing the applied moment.

* In PSC high tensile steel is elongated to a great extent before its strength is fully utilized.

* If high tensile steel is simply buried in conc as in RCC, the surrounding concrete cracks seriously before full strength is developed.

* In PSC, by PRESTRETCHING & ANCHORING the steel against concrete, we can produce desirable compressive stresses & strains in concrete and desirable tensile stresses & strains in steel.

* This combined action permits safe & economical utilization of both materials which can't be achieved by simply burying the reinforcement in concrete as in RCC. so RCC undergoes cracks & excessive deflection but PSC has no cracks & small deflection.

* PSC is an extension & modification of the applications of RCC.

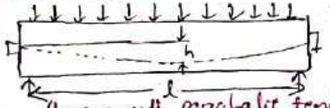
* In PSC the internal resisting couple is supplied by steel in tension & concrete in comp as in RCC. This concept is utilized to find the ultimate strength of PSC beams & is applicable to their elastic behavior.

(c) 3rd Concept ⇒ (Method of load balancing) → (Developed by T.Y.Lin the author)

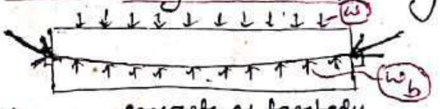
* According to this concept, prestress balances the loads on a member.

* In overall design of PSC structure, effect of prestressing is viewed as balancing the gravity loads so that members under bending (slab, beam, etc) are not subjected to flexural stresses under a given loading condition but subjected to DIRECT stresses.

* Application of this concept requires taking concrete as a free body & replacing tendons with forces acting on concrete along the span.



(beam with parabolic tendon)



concrete as freebody

example → A simple beam prestressed with parabolic tendon

If F = Prestressing force, L = span length, h = sag of parabola, w_b = upward uniform load

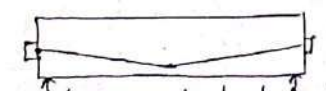
we know $F = \frac{w_b L^2}{8h}$ ⇒ $\boxed{w_b = \frac{8Fh}{L^2}}$

* For a given downward uniform load w, it is balanced by a uniform upward load, The beam is only subjected to axial force F, producing uniform stress $\boxed{\sigma = \frac{F}{A}}$ in conc.

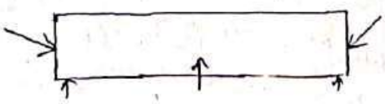
* The change in stresses from this balanced condition, is easily computed from $\sigma = \frac{My}{I}$

M = moment due to unbalanced load $(w - w_b)$

*



(Beam with bent tendon)



(Freebody of concrete with tendon replaced by forces)

* This approach may be used for continuous beam, rigid frame, flat & waffle slab, thin shells, self anchored PSC bridge,

problem A prestressed concrete rectangular beam of sec (20"×30") has a span of 24' & is loaded by a uniform load of 3k/ft including its self wt. The prestressing tendon is located as shown & produces an effective prestress of 360k. Compute fibre stresses in the concrete at midspan section.

sol Ist Concept →

$F = 360k$, $A = 20 \times 30 = 600 in^2$ (neglecting hole area for tendon)

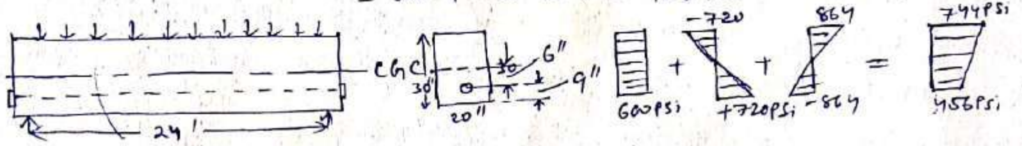
$e = 6''$, $I = \frac{bd^3}{12} = \frac{20 \times 30^3}{12} = 45{,}000 in^4$, $y = 15''$ for extreme fibres.

$M = \frac{wl^2}{8} = \frac{3 \times 24^2}{8} = 216$ k-ft

$$\sigma = \frac{F}{A} \mp \frac{Fey}{I} \pm \frac{My}{I} = \frac{360{,}000}{600} \mp \frac{360{,}000 \times 6 \times 15}{45{,}000} \pm \frac{216 \times 12000 \times 15}{45{,}000}$$

$= 600 \mp 720 \pm 864 = 600 - 720 + 864 = 744$ psi at top fibre

$= 600 + 720 - 864 = 456$ psi at bottom fibre.



Problem A concrete beam with same span, section, loading & prestress as above is parabolically curved tendon. compute extreme fibre stresses at midspan.

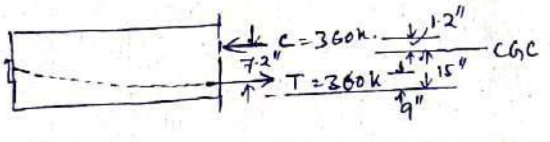
sol ⇒ same procedure as above. will be done & we will get the same answer.

problem ⇒ The same question as above & solve it by 2nd concept.

sol → Take one half of beam as a free body & so exposing the internal couple.

The external moment at sec is, $M = \frac{wl^2}{8} = \frac{3 \times 24^2}{8} = 216$ k-ft.

→ internal couple is furnished by forces $C = T = 360k$. which must act with lever arm of $\frac{216}{360} \times (12) = 7.2''$.



* since T acts at 9" from bottom, C acts at (9+7.2 = 16.2") from bottom
* The stress distribution in concrete is obtained by elastic theory.
* C = T = 360 ~~000~~ k = 360,000 lb.

$e = 16.2 - 15 = 1.2''$

$$\sigma = \frac{F}{A} \pm \frac{My}{I} = \frac{360\,000}{600} \pm \frac{(360\,000 \times 1.2) \times 15}{45{,}000} = 600 \pm 144$$

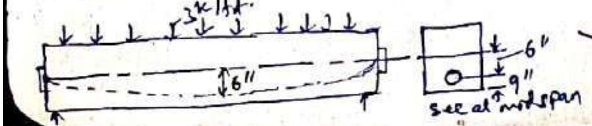
$= 600 + 144 = 744$ psi (at top fibre)

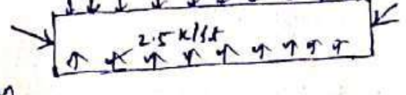
$= 600 - 144 = 456$ psi (at bottom fibre)

Problem solve the same problem by concept-3 taking the concrete as free body, isolated from tendon.

sol →) free body, isolated from tendon.

The upward uniform force, $w_b = \frac{8Fh}{L^2} = \frac{8 \times 360 \times 6/12}{24^2} = 2.5$ k/ft





* net downward (unbalanced) load on conc beam $= 3 - 2.5 = 0.5$ k/ft

* moment at mid span $= \frac{wl^2}{8} = \frac{0.5 \times 24^2}{8} = 36$ k-ft

* fibre stresses due to this moment, $\sigma = \frac{My}{I} = \frac{36 \times 12000 \times 15}{45000} = 144$ psi

* fibre stress due to direct load effect of prestress $= \frac{F}{A} = \frac{360\,000}{600} = 600$ psi

* Resulting stresses: $600 \pm 144 = 600 + 144 = 744^{psi}$ at top fibre

$= 600 - 144 = 456$ psi at bottom fibre

⑦ Stages of loading →

* For cast in situ structures, PSC is designed for two stages of loading.

(i) initial stage - during prestressing.

(ii) final stage — under external loading.

* For pre-cast members, PSC is designed for three stages of loading.

① initial stage

② intermediate stage — during handling & transportation.

③ final stage

→ shrinkage cracks will destroy the capacity of concrete to carry tensile stresses

→ concrete has not reached the proper age during ~~transfer of~~ prestress is maximum & so bearing strength of anchorage must be tested, because crushing of concrete at anchorages is possible if concrete quality is inferior & is honeycombed.

→ wt of s/s prestressed girder is expected to exert max +ve BM at mid span which counteracts the -ve moment due to prestressing. If girder is cast & prestressed on soft ground without suitable pedestals at ends, the expected +ve moment may be absent & prestressing may produce excessive tensile stresses on top fibres of girder resulting in its failure. If girder is cast & prestressed at proper place & proper ground then it becomes self supporting during & after prestressing.

* superimposed dead load → ROOFING & FLOORING.

Final stage →

Lateral loads → wind & earthquake forces.

strain loads ⟶ by settlement of supports & temperature.

sustained load -{ keep alive, keep going continuously load)

* The camber/deflection of a PSC member under its actual sustained load (often consists of dead load only), produces flexural creep.

working load ->

* During this load, excessive stresses & strains must be checked.

cracking load ->

* cracking in PSC member signifies a sudden change in bond & shearing stresses.

ultimate load =>

* ultimate strength of a structure is defined by the max load it can carry before collapsing.

* Before this load is reached, permanent yielding of some parts of structure may have developed.

* Any strength beyond the point of permanent yielding serves as additional guarantee against total collapse.

⑧ Advantages of PSC ->

⊛ In PSC, materials of high strength are used.

⊛ To utilize the full strength of high strength steel, we have to adopt prestressing to prestretch it.

(use of entire sec)

-> prestressing the steel & anchoring it against concrete, produces desirable stresses & strains, which eliminates/reduces ~~control~~ concrete cracks.

-> In such a way, entire section of concrete becomes effective in PSC where as portion of section above neutral axis in RCC is effective.

⊛ -> In PSC, a permanent dead load may be resisted simply by increasing the eccentricity of the prestressing force & thus saving of material occures in PSC.

(economy)

-> Due to utilization of concrete in tension zone, an extra saving of 15% to 30% in concrete is possible in PSC w.r.t. RCC.

-> Due to high permissible stresses in prestressing wires, there is a saving in steel of 60% to 80% in PSC w.r.t RCC

-> The saving of materials is not so significant due to the additional cost for high strength concrete & steel, anchorages & other hardware required for production of PSC members.

-> But an overall economy occures due to the reduction of dead wt further reduces design loads & cost of foundation.

(DYNAMICALLY LOADED STRUCTURE)

(*) PSC is <u>resilient</u> enough as it can recover from heavy effects of overloading without any serious damage.

→ The <u>temporary cracks</u> due to overloading close up completely when loads are removed.

→ <u>Fatigue strength</u> of PSC members is better than any other material. That's why PSC is recommended for <u>dynamically loaded structure</u>

eg → RAILWAY BRIDGE, MACHINE FOUNDATION, etc.

(SHEAR STRENGTH)

(4) <u>curved tendons</u> in PSC can carry some shear in a member & precompression in concrete reduces the principal tension & increases shear strength.

→ To carry same amount of shear in a beam, smaller sections can be used in PSC & so more efficient <u>I-shaped</u> sec with thin webs is desirable for PSC.

(High strength conc)

(7) <u>High strength concrete can't</u> be economically utilized in <u>RCC</u> as HSC results smaller section & requires more reinforcement leading to costly design.

→ In PSC HSC uses HSS to yield economic design.

→ HSC resists high stress at anchorages & give strength to thinner sections which is generally used in PSC.

Advantages & DIS Adv of PSC over RCC is discussed w.r.t

(a) SERVICEABILITY →

* PSC is suitable for <u>long span structures</u> & structures carrying <u>heavy loads</u> as higher strength materials are used.

* PSC structures are more <u>slender</u> & raises beauty.

* They <u>don't crack</u> under working load & cracks developed due to overload are closed when load is removed unless it is excessive.

* under <u>dead load</u>, deflection is reduced due to <u>cambering effect</u> of prestress.

* under <u>live loads</u>, deflection is smaller due to the <u>effectiveness</u> of entire <u>uncracked conc</u> sec which <u>M.I</u> <u>two to three times</u> that of <u>cracked section</u>.

* PSC members are more adaptable to <u>PRECASTING</u> due to their lighter wt.

*→ From serviceability point of view, the only dis adv of PSC is its <u>LIGHT WEIGHT</u>. There are situations where wt is important than strength ~~ab~~ There RCC can be used in place of PSC.

eg → { off shore structure
concrete gravity dam
machine foundation

* For same min cover, tendons have greater average cover because of the spread & curvature of individual tendons.

(b) SAFETY →

* safety of structure depends upon its design & construction.
* But there are some inherent safety features in psc. They are:-

→ There is partial testing of both steel & concrete during prestressing operation as during prestressing both materials are subjected to highest stresses which exists in them throughout their life. so if the material can sustain prestress, then they are strong enough for service loads.

* Resistance to corrosion is better in psc than RCC due to absence of cracks & high quality concrete. But if cracks occure, corrosion is more serious in psc.

* → From safety consideration, the disadv in psc are

① * They deflect more before failure giving less warning before collapse.

② * Regarding fire resistance RCC is better than PSC as high tensile steel is more sensitive to high temp.

③ * PSC members need more care in design, construction & erection because of high strength, smaller section & delicate design features.

(C) ECONOMY →

* Reduction in economy due to smaller need of materials due to high strength.
* saving in stirrup as curved tendons resist shear & prestress reduces the amount of diagonal tension.
* Little material required reduces its wt & depth.
* In precast members, reduced wt saves handling & transportion cost.

* → From economy consideration, dis adv. in psc are
  * stronger materials have higher unit cost.
  * more auxiliary materials are required for such as end anchorage, conduits & grouts.
  * more complicated formwork as in psc non rectangular elements are generally used.
  * more skilled labour, more supervision, are required.

conclusion →

* so psc design is economical when
  same unit is repeated many times &
  heavy dead load on long span is required.

MATERIALS IN PSC

① High strength concrete (HSC) →

* Q// why high strength concrete is used in PSC?

Sol → To minimize the cost of production of PSC members, commercial anchorages for ~~BSO~~ prestressing steel are always designed on the basis of high strength concrete.

* If weaker concrete is used, it may need special anchorage or, may fail under application of prestress.

* such failure may occur in bearing or
  in bond betn steel & concrete or
  in tension near the anchorages.

* HSC offers high resistance in Tension, Shear, Bond, Bearing & is desirabl for PSC whose various parts are under higher stresses.

* HSC is less liable to shrinkage cracks before application of prestress.

* HSC has higher modulus of elasticity & small creep strain resulting in smaller loss of prestress in steel.

* HSC use reduces c/s area of PSC members ⇒ reduces wt ⇒ long span is possible

* HSC is more Durable, Impermeable & Abrasion resistant.

---

* crushed rock aggregate being angular produces stronger concrete at the same age in comparision with gravel aggregate.

(*) strength requirement ⇒

* For Pre tensioned members, min 28day cube comp strength by IS 1343 = 40 N/m
* For Post-tensioned member, min 28day " " " by IS 1343 = 30 N/mm

* $\frac{\text{cube strength}}{\text{cylinder strength}} = 1.25$

* For HSC mixes, w/c ratio should be low & w/c < 0.45 by wt.

* For HSC, slump value of 51 mm to 102 mm is needed.

* Since excessive cement increases shrinkage, lower cement factor is neede

* Good vibration & proper admixure to increase workability is desirable.

* To avoid excess shrinkage, cement content in mix < 530 kg/m3.

* By use of Rapid Hardening Portland cement, cube strength of 40N/mm2 can be achieved within 7 days.

* more parts of PSC members are subjected to high stresses than RCC.

In a PSC member,

- Bottom fibres — at high comp — at transfer of prestress
- Top fibres — at high comp — under heavy external load.
- Mid span sec — resist heavy BM
- End sections — carry & distribute highly prestressing force.

→ Hence a prestressed member is needed to design for UNIFORMITY OF STRENGTH. But in RCC only critical sections are designed carefully.

* A lower concrete strength at transfer can be specified which is less than 28 day cube strength. It is desirable for early transfer of prestress. Because at transfer conc is not subjected to external loads & strength only is required to guard against anchorage failure & excessive creep & so lower factor safety is sufficient.

* modulus of rupture is higher than direct tensile stress in concrete.

permissible

(*) Permissible stresses in concrete →

* permissible compressive & tensile stress in concrete at stage of transfer & service loads are defined in terms of corresponding compressive strength of concrete at each stage.
* Indian code uses Reduction coefficient applied to compute design max permissible comp stress in flexure varies from

0.41 for M30 to 0.35 for M60

At transfer — {Permissible comp stress (N/mm2)} — varies from 0.54 to 0.37 f_{ci} (Post tensioned)
— varies from 0.51 to 0.44 f_{ci} (pre tensioned)

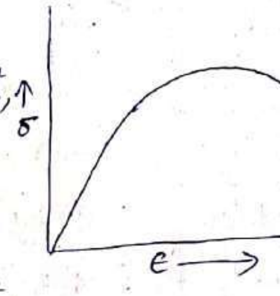
At service → {permissible comp stress (N/mm2)} — varies linearly from 0.41 to 0.35 f_{ck}
(depends upon strength of concrete)

⊛ strain characteristics in concrete →

- In PSC, it is required to compute the stresses & strains produced, from which the losses of prestress in steel can be computed.
- The strains produced in PSC structure can be classified into 4 ways.

① Elastic strain ⇒

- Elastic strain is a absurd term in concrete as stress-strain curve for concrete is not a straight line at normal stress level.
- Eleminating creep strain, lower portion of instantaneous stress-strain curve being straight, may be called elastic.
- mod of elasticity of concrete can be computed.
- modulus depends upon strength of concrete, age of concrete, properties of aggregate & cement.



- mod of elasticity of beam is greater than cylinder.
- Before cracking, mod of elasticity of concrete is same in tension & comp.

② Lateral strain ⇒

- Lateral strain is computed from poisson's ratio.
- From Poisson's ratio effect, loss of prestress is decreased in biaxial prestressing.
- Poisson's ratio for concrete varies from 0.15 to 0.22, average $\nu = 0.17$

③ creep strain → (creep or plastic flow of concrete)

- creep is defined as the time dependent deformation due to presence of stress / progressive increase in inelastic strain due to sustained stress is called creep strain
- Basically creep is due to the migration of water in capillaries of cement paste.
- creep is the deformation mainly due to the externally applied stress.
- creep is very important in PSC because increase in strain due to sustained stress is several times larger than strain on loading.

of the total 20 year creep, If creep after one yr of loading is unity creep after 10 yrs is 1.26 & after 30 yrs, creep is 1.36.	25% occures in first 2 weeks of loading. 55% occures within 3 months. 76% occures within 1 year.

- creep increases with w/c ratio & low aggregate-cement ratio, but it is not directly proportional to total water content of mix.
- creep is greatest for crushed sandstone aggregate & man effective against creep is LIMESTONE.
- older the specimen at the time of loading, the hydration of cement is more complete & creep is less.

* creep per unit stress is only slightly greater at high stresses ~~than~~ than at lower stresses.

* creep is inversely proportional with size of specimen.

* 20 year total creep strain is about 3 times the instantaneous ~~defor~~ deformation.

* when sustained stress is in excess of about $\frac{1}{3}$ of ultimate strength of concrete, the (rate of increase of strain with stress) tends to get higher.

* This ~~rate of~~ increase of strain rate becomes quite pronounced as stress approaches the ultimate strength of concrete..

* it take a long time to recover the creep than for the creep to take place.

* If creep is allowed to occure for a fixed length of time & then if it is allowed to recover the creep for the same fixed length of time then only 80% to 90% of the creep deformation can be recovered.

* If pre compression in concrete is high, then loss due to creep is significant.

* Factors influencing creep ~~are~~ of concrete are :-

  Relative humidity, stress level,
  strength of concrete, age of conc at loading.
  duration of stress, w/c ratio.
  type of cement & aggregate in concrete.

* ~~upto~~ For stresses upto half of crushing strength of concrete,

  creep $\propto$ stress,

  but for stresses > half of crushing strength

  creep increases very rapidly with increase of stress.

* According to IS 1343-80, loss of prestress due to creep is calculated by creep coefficient method -

$$\text{creep coefficient} = \frac{\text{Ultimate creep strain (i.e. instantaneous + creep strain)}}{\text{Elastic strain (i.e. instantaneous strain)}}$$

creep coefficient = 2.2 at 7 days of loading
= 1.6 at 28 days of loading
= 1.1 at 1 yr of loading.

* For pre tensioned member, where prestress is applied at early age, the creep coefficient is little more than for post tensioned member

where prestress is applied lately, the coefficient is little less.

* of the total amount of creep strain,

first $\frac{1}{4}$ take place within first 2 weeks after prestress application
and $\frac{1}{4}$ " " within 2 to 3 months
3rd $\frac{1}{4}$ " " within a year and
4th $\frac{1}{4}$ " " in the course of many years.

* For smaller members, creep & shrinkage occures faster than for large members.

⑤ shrinkage strain ⇒

* shrinkage of concrete in prestressed member is due to drying as a result of gradual loss of moisture & chemical changes, dependent on time & moisture condition but shrinkage is ~~ite~~ independent of stress.

* Due to shrinkage of concrete there is a reduction in volume of concrete.

* A portion of shrinkage resulting from drying of concrete is recoverable as a result of restoration of lost water.

* As contraction of cement gel increases with cement content, shrinkage of Rich mixes is greater than that of lean mixes.

* shrinkage depends upon aggregate type & quantity, w/c ratio in mix
Relative humidity, time of exposure &
degree of hardening ~~start~~ after drying starts.

* As exchange of moisture betn conc & atmosphere takes place through concrete surface, amount of shrinkage depend upon the ratio of surface area & volume of member.

* Aggregates of rock which are hard & dense with low absorption having high modulus of elasticity are more effective in restraining contraction of cement paste & reduces shrinkage of concrete.

* max effective against creep & shrinkage is LIMESTONE.

* The phenomenon of shrinkage is time dependent & so only anticipated or residual shrinkage strain is considered during calculation of losses of prestress

* According to IS 1343-80, the total residual shrinkage strain

for pre-tensioned member = $\boxed{3 \times 10^{-4}}$

post-tensioned member = $\boxed{\frac{2 \times 10^{-4}}{\log(t+2)}}$

t = age in days of concrete at transfer.

* Shrinkage recommended for member with pre-tensioned steel is higher than for members with post-tensioned steel because in pre-tensioned member, total shrinkage is considered & in post-tensioned member, shrinkage after transfer is only considered.

* where light wt aggregates are used, shrinkage is increased by 50%.

* If concrete is stored under water or under ~~very~~ very wet condition, shrinkage may be zero.

* If concrete is stored under very dry condition, max shrinkage may be <u>0.001</u>.

* shrinkage ∝ amount of water employed in mix.

⇒ shrinkage is min if w/c ratio & cement paste kept in mix is minimum.

* Larger aggregates & well graded aggregates with min void will need smaller amount of cement paste & thus shrinkage is [illegible] less.

* chemical composition of cement affects amount of shrinkage. Shrinkage is small for cement containing high C_3S & low alkali, oxides of sodium & potassium.

* If concrete is dry, most shrinkage occurs within first 2 to 3 months. If concrete is ~~all~~ kept always in wet condition, shrinkage is zero.

* Too early drying of concrete may cause shrinkage cracks before application of prestress.

* Admixtures like $CaCl_2$ accelerates the strength development in concrete, but it increases shrinkage & cause corrosion.

⊛ DEFORMATION CHARACTERISTICS OF CONCRETE ⇒

* Complete σ-ϵ characteristics of concrete in comp is not linear.

* For load < 30% of crushing strength, load-deformation behaviour may be assumed to be linear.

~~Deformation characteristics of concrete under short term & sustained load~~

* short term mod of elasticity, corresponds to secant modulus determined from experimental σ-ϵ relation under loads of $\frac{1}{3}$ of cube comp strength of concrete.

* For concrete $E_c \propto f_{ck}$ at a decreasing rate.

* ~~As per~~ IS 1343-1980 recommends the empirical formula

$$\boxed{E_c = 5000\sqrt{f_{ck}}}\ N/mm^2$$

* To achieve high strength of concrete, we should adopt lowest w/c ratio.

② HIGH STRENGTH STEEL →

* High tensile steel is the universal material for producing prestress & supplying the tensile force in PSC.

(1)* High tensile steel is produced by alloying for which carbon is mainly used and carbon content in steel is considerably increased as it is extremely economical element because it is cheap & easy to handle.

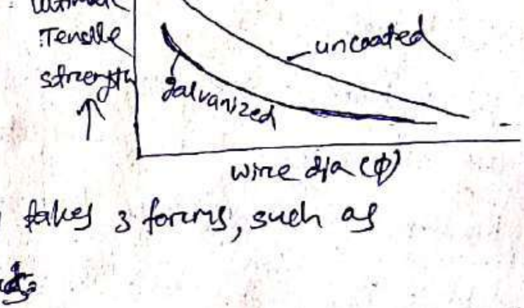
→ other alloys are manganese & silicon.

(2)* other methods to produce high tensile steel is by controlled cooling of steel after rolling & by heat treatment such as quenching & tempering.

→ second method is better.

* The most common method for increasing the tensile strength of steel for prestressing is by cold drawing, high tensile steel bars through a series of dyes to reduce the diameter & tensile strength is increased.

→* cold drawing realign the crystals & strength is increased by each drawing so that smaller the dia of wires, the higher is their ultimate strength. But ductility of wires is decreased by cold drawing.



* High tensile steel for prestressing usually takes 3 forms, such as wires, strands or Bars.

* For Post tensioning, wires which are grouped, ll, in cables are widely used.

In Post tensioning, strands & high tensile rods are also used.

(strands are formed by twisting wires together & so no of units handled decreases in tensioning operation.)

* For Pre tensioning, 7 wire strand is commonly used & hard draw steel wires which are indented are preferred because of their superior bond characteristics.

→ For the same tensile strength, strands are better than wires though strands are costly because they have better bonding characteristics.

* Ultimate strength of high tensile steel can be easily determined but its yield point can't determined easily. so 0.2% Proof stress is taken as its yield stress.

(a) steel wires →

- Steel wire are made by cold drawing rods which are [illegible] hot rolled from high carbon steel ingots
- cold drawing of wires reduces its durability & ductility.
- cold drawn wires are available in nominal sizes of 2.5mm, 3, 4, 5, 7 & 8mm dia.
- wires which are indented are preferred for pre tensioned element because of their superior bond characteristics.

(b) strands →

- small dia wires of 2mm to 5mm are mostly used in the form of strands which comprises of two, three or seven wires.
- Helical form of twisted wires in strand improve the bond strength.
- In (7 wire strand), there will be a centre wire of slightly larger dia than the outer six wires which enclose it tightly in a helix with a uniform pitch betn 12 to 16 times the nominal dia of strand.
- strands are used both for pre & post tensioned members.
- Nominal dia of 7-wire strand varies from 6.3mm to 15.2mm.

~~(c) steel Bars~~

- when strands are galvanized (i.e. zinc coated), their strength is reduced by 15% than ungalvanized strands.

(c) steel Bars →

- steel bars have nominal sizes of 10, 12, 16, 20, 22, 25, 28, 32 mm dia.
- ultimate tensile strength of bars does not depend upon diameter, because high strength of bar is due to alloying, rather than cold working as in case of wires.
- yield strength of high tensile bar is also defined by 0.2% proof stress,

In 0.2% proof stress, a line ‖ to initial tangent is drawn from the 0.002 strain & its intersection with σ-ε curve of steel is defined as the yield strength point.

Strength requirement ⇒

* ultimate tensile strength of a plain hard-drawn steel wire decreases with increase in the diameter of wire.

Nominal dia of wires (mm)	ultimate tensile strength of wires (N/mm²)
	2
2.5	2010
3	1865
4	1715
5	1570
7	1470
8	1375

For high tensile steel Bar,

1) min characteristic tensile strength — 980 N/mm²
2) proof stress — $\nless$ 80% of min tensile strength

For high tensile steel wire

* 0.2% proof stress — $\nless$ 85% of min tensile strengt

* An important requirement of steel used in PSC is the plasticity of steel at stresses near ultimate stresses, which is essential to achieve progressive failure of PSC member with sufficient warning before final failure.

* To avoid possibility of brittle failure, a minimum elongation at ~~ru~~ rupture is specified for high tensile steel.

* IS 1343 prescribes a minimum %age of elongation —

for wires, it is 2.5%.

for bars, it is 10%.

for strands, it is $\nless$ 3.5%. (on a gauge length $\nless$ 600 mm)

* IS 1343 specifies, modulus of elasticity

for high tensile wires — 2.1×10^5 N/mm²

for high tensile bars — 2×10^5 N/mm²

for high tensile strands — 1.95×10^5 N/mm²

Permissible stresses in steel →

* Tensile stress in steel at time of tensioning behind anchorages & after allowing for all losses are expressed as a fraction of ultimate tensile strength or proof stress.

IS 1343-1980 (Permissible stresses)

→ At time of initial tensioning — Initial prestress is less than 80% of characteristic ~~ten~~ tensile strength of tendons.

→ Final prestress after all losses of prestress — minimum 45% characteristic tensile strength of tendons.

Relaxation of stress in steel →

* when a high tensile steel wire is stretched & maintained at a constant strain, the initial force in the wire does not remain constant but decreases with time.

* The decrease of stress in steel at constant strain is termed Relaxation.

* In a prestressed member, high tensile wires betn anchorages are more/less in a state of constant strain.

* If stress is kept constant, the material exhibits plastic strain above initial elastic strain, the phenomenon is called CREEP.

* cold drawn steel creep more than heat treated steel due to their lower magnitude of 0.01% proof stress.

* The creep in steel is influenced by chemical composition, grain size & variables in the manufacturing process, ~~microstructure~~, which changes in internal crystal structure & micro structure in steel material.

* Tendons in PSC, neither be maintained at constant stress nor strain strictly.

* Most serious condition occurs generally at ~~initial~~ stage of initial stressing & subsequently strain in steel reduces as concrete deforms under prestressing force.

~~IS 1343 prescribes for Relaxation for wires & bars~~

* Indian code for wires & bars prescribe 1000hour relaxation test,

Relaxation < 5% of initial stress

one can accept, 100 hour relaxation test.

Relaxation < 3.5% of initial stress.

* Reduction in relaxation stress is possible by PRELIMINARY OVERSTRESSING.

→ A preliminary overstress of 5% to 10% maintained for 2 to 3 minutes considerably reduces the magnitude of Relaxation.

STRESS CORROSION ⇒

* Stress corrosion cracking is due to combined action of corrosion & static tensile stress which may be residual or externally applied.

* stress corrosion results in sudden brittle fractures.

* This type of attack in alloys is due to internal metallurgical structure which is influenced by composition, heat treatment & mechanical processing.

* causes of susceptibility of high tensile steel to stress corrosion is that heat treated wires are specially prone to stress corrosion fractures when compared to cold drawn wires.

* If ducts of post tensioned members are not grouted, there is possibility of stress corrosion leading to failure of structure.

* other types of corrosion encountered in PSC construction are:-
  - → Pitting Corrosion
  - → chloride corrosion.

* some protectives against stress corrosion are:-
  - → Protection ~~against~~ from chemical contamination
  - → protective coating for high tensile steel.
  - → Grouting of ducts immediately after prestressing
  - → Alkaline environment of portland cement concrete saves tendons.

▷ Residual stress ⇒

stress that exists in an elastic solid body in the absence of or, in addition to stress caused by an external load.

Resilience

▷ Resilience ⇒

power of an elastically strained body to spring back on removal of load.

Hydrogen Embrittlement ⇒

* Due to action of acid on high tensile steel, atomic hydrogen ~~is liber~~ is liberated.
* The liberated Atomic Hydrogen penetrates to steel surface & makes it brittle & fracture prone when subjected to ~~external~~ tensile stress.
* small amount of hydrogen can cause considerable damage to tensile strength of high tensile steel wire.
* If sulphide rich cement (eg- high alumina cement & blast furnace slag cement) are used in making PSC, then hydrogen embrittlement occurs.
* use of dissimilar metals such as Al & Zn for sheath (a close fitting cover) to house high tensile steel wires also cause hydrogen embrittlement.
* In order to prevent it, following measures can be taken.
  - → steel should be protected from action of acids.
  - → protective ~~coating~~ covering like bituminous crepe paper covering during transportation
  - → wires should be protected from rain water & excess humidity by storing them in dry condition.

Cover Requirement for PSC members →

* steel tendons must be provided with cover for their protection against corrosion.
* According to IS 1343-1980,

for Pre-tensioned member → a min clear cover of 20mm.

for Post tensioned member → it is 30mm/size of cable which is greater.

* If PSC members are subjected to aggressive environment,

cover requirement is increased by 10mm.

Auxiliary Materials (GROUTING) → (conduit = Duct = cable way)

* For post tensioning, a conduit to house tendons is necessary.
* Two types of conduits — one for bonded Prestressing.
  — other for unbonded prestressing.
* when tendons are to be bonded by grouting, the ducts are made up of ferrous metal which may be galvanized
* materials used for ducts → galvanized or longitudinally seamed steel strips with flexible or semirigid seams, rigid tubing, ~~corrugated pla~~ corrugated plastic ducts, etc.
* Ducts can also be formed by withdrawing steel tubing before concrete hardens, by withdrawing extractable rubber cores buried in concrete.
* when tendons are unbonded, plastic/heavy paper sheathing, properly greased tendons are used to avoid corrosion.
* In post tensioning for bonding tendons to concrete after tensioning, cement grout ~~is injected~~ is injected, which protects tendons against corrosion.
* For grout, either OPC/high early strength cement is used with water + sometimes fine sand is used.
* To achieve good bond for small duct, grouting under pressure is done.

methods of prestressing

* prestressing system comprises

method of stressing the steel + method of anchoring it to conc

are

methods of applying prestress + details of anchorages.

* prestressing by application of direct forces bet^{n} abutments is generally used for arches & pavements. and

flat jacks are definately used to impart the desired force.

* There are four methods of Tensioning the steel. They are

① mechanical prestressing by means of jacks.
② electrical prestressing by application of heat.
③ chemical prestressing by means of expanding cement.
④ miscellaneous

mechanical prestressing →

* In both systems of prestressing i.e. in pretensioning & post tensioning, the most common method of stressing the tendon is JACKING.

* post tensioning → Jacks are used to pull the steel with reaction acting against hardened concrete.

pre tensioning → Jacks pull the steel with reaction against end bulkheads.

* mechanical devices include

→ weights with or without lever transmission
→ Geared transmission in conjuction with pulley blocks
→ screw jacks with/without gear drives & wire winding machine

→ These devices are used when PSC structures are produced in mass scale in factories.

* Levers are suitable, when very small wires are tensioned individually.

* Hydraulic jacks are used to produce large prestressing force.

* Jacks are mounted (i.e. fixed in a support) on the end bearing plates.

* systems of jacking vary from pulling one or two wires up to over a hundred wires at a time.

* several commonly used patented hydraulic jacks are due to

All the jacks are used in post-tensioning.

{ Freyssinet , Magnel Blaton , Gifford udall
{ Lee McCall [Baur Leonhardt]

* During jacking (i.e. pulling the tendons), sometimes tendons are jacked a few %age above their specified initial prestress.

→ This overjacking is due to
- ✓ minimize creep in steel.
- ✓ to reduce frictional loss of prestress.
- ✓ compensate for slippage.

* when tendons are long & curved, jacking should be done from two ends.
* when several tendons are tensioned in succession, care should be taken so that no serious eccentric loading will result.

Electrical prestressing → ~~(Post tensioning method)~~

* steel wires are electrically heated ~~before~~ & anchored before placing concrete in place.
* Electrical prestressing is generally employed in post-tensioning method.
* Tendons are heated to elongate them & then tightened. When they are allowed to cool, prestress is developed.
* There are higher losses of prestress in steel & method is expensive. so it is an uneconomical prestressing method.

Chemical prestressing →

* Expansive cements are used & degree of expansion is controlled varying curing condition.
* when expansive action of cement is restrained, it induces tensile forces in tendons & comp stresses in concrete.

PRETENSIONING SYSTEM

* The tendons are tensioned or pulled bet^n two bulkheads, rigid anchor blocks cast on ground or in a column or unit mould type pretensioning bed, before the casting of concrete in mould.

* After concrete hardens, tendons are cut loose from bulkheads & prestress is transferred to concrete.

or

when concrete attains sufficient strength, the jacking pressure is released high tensile wires tends to shorten but are checked by bond bet^n conc & steel

* Here prestress is transferred by bond, mostly near the ends of beam, & no special anchorages are required as in post-tensioned member.

* For mass production of pretensioned elements, the Long Line process by Hoyer is generally used in factory.

* In long line process, tendons are stretched bet^n two bulk heads several hundred meters apart so that no of similar units are casted along the same group of tensioned wires.

→ Here tension is applied by hydraulic jack / by moveable stressing machine.

→ The wires/strands when stretched are generally anchored to abutments by steel wedges.

* Generally strands upto 18mm dia & high tensile wires upto 7mm dia anchor themselves satisfactorily by surface bond & interlocking with surrounding matrix

* strands have better bond characteristics than plain wires of same c/s.

* Devices for gripping pretensioning wires to bulkheads are usually made on "WEDGE & FRICTION PRINCIPLE".

* Small wires are generally used to ensure good anchorage to transmit prestress bet^n wires & conc by virtue of BOND.

* wires of dia >3mm are used if they are corrugated along their length.

* sometimes a length of ~~wire~~ transfer is required to develop bond.

* when insufficient length of transfer is provided, cracks may develop near the end of beam & bond may break & wire will slip.

POST TENSIONING SYSTEM ⇒

* when conc attains sufficient strength, high tensile wires are tensioned by jacks bearing on the end face of member & anchored by wedges/nuts.
* when tendons are straight, forces are transmitted to concrete by end anchorages.
* when tendons are curved, forces are transmitted to conc by radial pressure betn tendon & duct.
* Generally space betn tendon & duct is grouted after tensioning operation.

There are three principles by which tendons are anchored to concrete.

① principle of WEDGE ACTION producing a frictional grip on wires.
(Freyssinet, Gifford vddal, magnel Blaton, Anderson anchorages)
② by DIRECT BEARING from rivet/bolt heads formed at the end of wires.
(Le McCall
③ by Looping the wires around the concrete. (not widely used)

* several prestressing (post-tensioning) systems anchor their wires by wedge action. They are Freyssinet, Gifford-vddal, magnel Blaton, Anderson anchorages

Freyssinet system → (POST TENSIONING ANCHORAGES by WEDGE ACTION) →

* This system has been used through out all over the world using wedge principle.
* This system can accept 12 strands in a tendon.
* Adv → This system used double acting hydraulic jack which tensions all tendons at a time.
* The anchorage consists of a cylinder with conical interior through which tendons pass & against the wall of this cylinder wires are wedged by a conical plug lined longitudinally with grooves to house tendons.
* In this systems, tendons can be wires & strands.

POST TENSIONING ANCHORAGES FOR TENDONS by DIRECT BEARING →

* They employ cold formed rivet heads for direct bearing at end of stressing wires.
* Le McCall anchorage system uses tendons in the form of high tensile bars, of dia varying from 12 to 40mm which are threaded at ends.
* After tensioning each bar is anchored (fixed) by screwing a nut & washer tightly against end plates.
* Forces are transmitted by bearing of end blocks

Adv → This system eliminates loss of stress due to anchorage slip.
Dis adv → curved tendons can't be used in this system.

Post tensioning system	Type of Tendon	method of Tensioning	Type of anchorage	cable duct	Range of force
Freyssinet	wires & strands	Hydraulic jack tensioning all wires at a time	conical serrated cone wedge driven by jack into female cone embeded at end of beam	circular	medium & large
Gifford Udall (Britain)	wires	Hydraulic jack tensioning wires singly	split conical wedge & bush to each wire, bearing on anchor thrust plate & ring cast into end of beam	circular	small & medium.
Magnel Blaton (Belgium)	wires	Hydraulic jack tensioning two wires at a time.	pairs of wires held by flat steel wedges in sandwich plates bearing on distribution plates.	Rectangular	small, medium & large.
Anderson (USA)	strands	Hydraulic jack simultaneous tensioning of all wires	Steel socket with grooves & metal plug driven into the socket.	circular	medium & large.
Lee McCall (Britain)	Bars threaded at ends	Hydraulic jack screwed to threaded ends of bar	~~Hg~~ High strength nut & spacing washers bearing on steel plates at end of beam	Circular	small, medium & large.

small force = < 130KN, medium = 130 - 500 KN, large > 500 KN

→ PRE TENSIONING VS POST-TENSIONING →

Essential diff betn diff tensioning systems lies in following 3 features: -

① materials for producing the prestress

② Details of jacking process

③ method of anchoring.

④ choice betn pre & post tensioning ⇒

PRETENSIONING ⇒

⇒ When pre-tensioning plant is accessible & pre cast members can be conveniently transported, then pre tensioning is cheaper because of

- saving in end anchorage, conduit, grouting and
- centralization of production process.

⇒ Establishment of pretensioning plant is justified if it can render services to many jobs and jobs are of big in nature.

⇒ For Long & Heavy members, post-tensioning is best.

⇒ strands have better bond with concrete & so anchorage for pretensioning is usually not found.

⇒ A major disadvantage in pre tensioning →

Application of pretensioning is limited to use of straight tendons, tensioned betn two bulk heads & so advantages of bend or curved tendon can't be obtained in this method.

- POST_TENSIONING →

⇒ post tensioning is best for long & heavy ~~wo~~members.

⇒ This system can accept precast / cast in-situ structure and bonded/unbonded structures.

⊛ choice of proper Material for Prestressing →

⇒ proper material means wires, strands or bars.

Strand ⇒ strands possess higher strength than others, but close to wires.
⇒ Larger size strands or bars ⇒ fewer units of handling.

Bar ⇒ Bars possess least strength, but are easier to handle & cheaper to ~~bandle~~ anchor. Bars need splices for longer length.

Strand ⇒ But strands & wires can be supplied without splices.
⇒ Anchorage cost of strand is max, but %ge cost of anchorage decreases with length of tendons.

W ⇒ Wires are very common to be used as tendons than strands/bars.

⊛ Details of Jacking process →

⇒ when fewer wires are stretched per operation, smaller jacks are needed, they are easier to handle but take more time for total tensioning.

⇒ systems in which jacking is done all at once for all tendons, there jacks of high capacity are used, which are more costly & difficult to operate.

⊛ method of anchoring →

⇒ There are diff methods for anchoring, if proper anchorage system is ~~more~~ necessary then post-tensioning is used. If proper anchorage is not required then pre tensioning system is used.

Applications of post-Tensioning ->

* It is generally suited for medium to long span insitu works
* It is economical to use few cable/bars with large forces in each than a large number of small ones.
* The important advantage of post tensioned member is that it allows ~~the~~ the use of curve cables which help the designer to vary the prestress distribution at will from section to section so as to counter the external loads more efficiently.
* This system is used to strengthen concrete dams, circular prestressing of large concrete tanks & biological shields of nuclear reactors.
* It is suitable in concrete construction work involving stage prestressing.
* Long span bridge structures are constructed using this system.

Tendon splices ->

* In case of PSC continuous members involving long tendons, splicing of tendons is required to achieve continuity.
* The diff types of splices are:-

SCREW CONNECTOR ->

* It is required to splice large dia high tensile bars which can be threaded at ends.
* A sheet-metal sheath of enlarged dia & sufficient length is used to cover the splice.
* It is ~~req~~ not required to splice heat treated prestressing steel.

TORPEDO SPLICE ->

* It is used to splice cold drawn wires required ~~to splice~~ in circular prestressing.
* Adv -> There is no reduction in strength of wires.

CLAMP SPLICE ->

* Nuts & bolts with a series of clamp plates are required to have tendons betn them in such splices.
* Since there will be a considerable reduction in tensile strength up to 50%, these are used where prestressing force is considerably reduced by curvature of tendon due to friction.

WRAPPED SPLICE ->

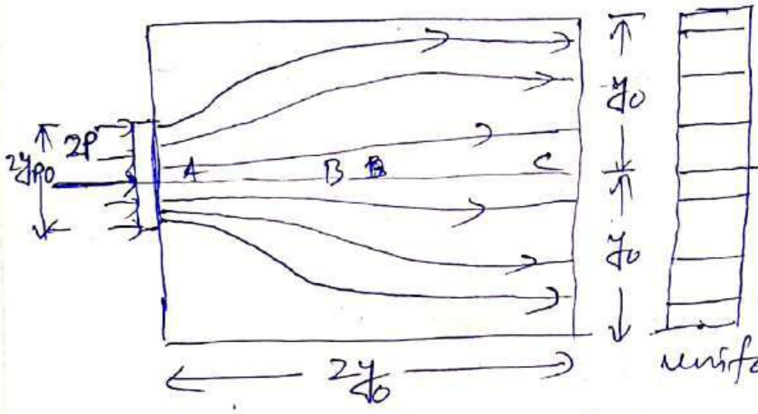
* These are used to splice small dia wires (3mm-6mm) by wrapping high tensile wires under high tension.
* Wrapping wires of 1mm dia is used to splice wires up to 6mm dia.
* used for wires of circular conc tanks & anchorage loops.
* Hence the splice strength $\simeq$ strength of normal wire [splice length 20-30cm]

Anchorage zone stresses in post tensioned members

- In anchorage zone/end block of a post tensioned PSC element, state of stress distribution is complex & 3D in nature.
- In most post tensioned PSC members, prestressing wires are introduced in cable holes/ducts, i.e. pre-formed in the members.
- They are stressed & anchored at end faces.
- Hence large forces, concentrated over small area are applied over small area.
- These discontinuous forces applied at beam ends changes progressively to continuous linear distribution, develops transverse & shear stresses.
- As per St venant's principle, stress distribution at a distance far away from loaded face (normally at a distance ≥ depth of beam) computed from simple bending theory
- Zone betn beam end & section where longitudinal stress exists is called anchorage zone/end block.
- Transverse stresses developed in anchorage zone are tensile over a large length and because concrete is weak in tension proper tensile reinforcement should be provided to resist tension.
- so stress distributions in anchorage zone should be properly done so that adequate steel should be properly distributed to resist transverse tensile stress.

Stress distribution in end block →

* Forces in end block of post-tensioned is shown.



Transmission of forces in end block

← (Single Anchor plate)

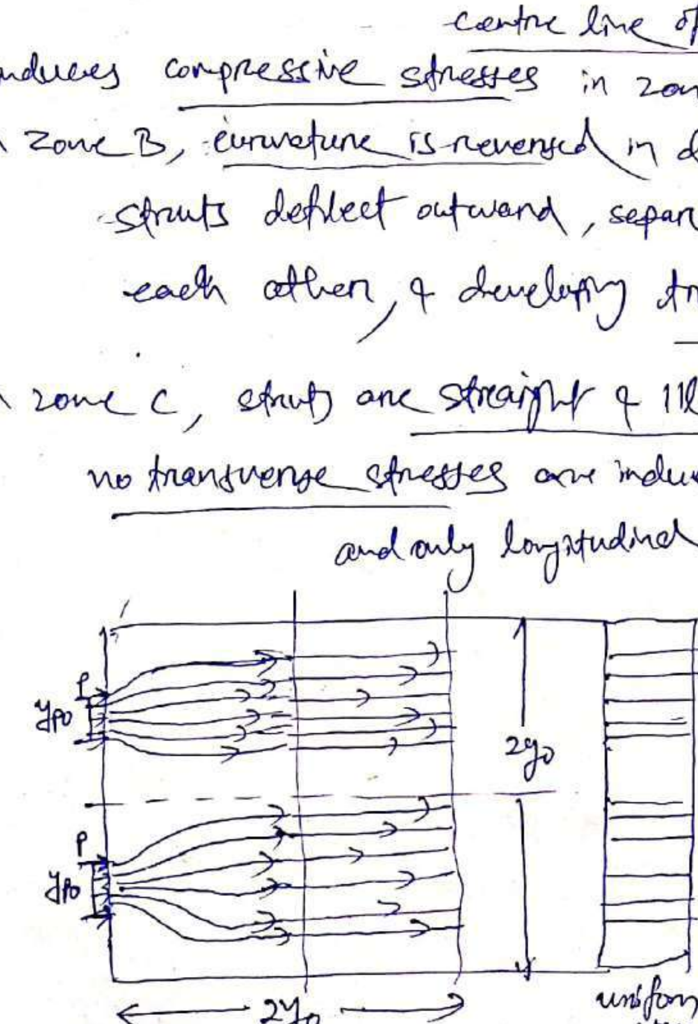
uniform stress.

* A physical concept of state of stress in transverse direction, i.e. normal to planes ∥ with top & bottom beam surfaces, may be obtained by considering these force lines as individual fibres acting as curved struts inserted between end force 2P and main body of beam.

* The strut curvature, being convex towards centre line of block, induces compressive stresses in zone A.

* In zone B, curvature is reversed in direction and struts deflect outward, separating from ~~each other~~ each other, & developing transverse tensile stresses.

* In zone C, struts are straight & ∥ & hence no transverse stresses are induced. and only longitudinal stresses develops.



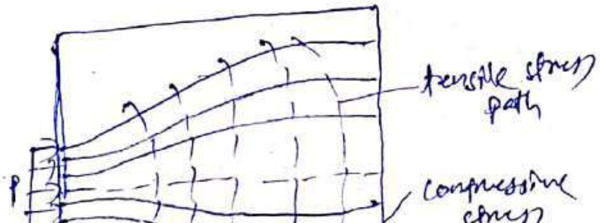
Transmission of forces in end block

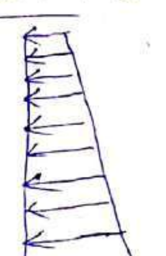
Double anchor plate

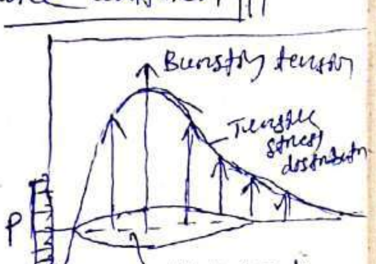
- In end figure same end block subjected to same total load through two zones, symmetrically disposed in upper & lower halves of beam

- Since line of forces follow same pattern with half of radius of curvature, length of anchorage zone is halved.

- Transverse tension is proportionally reduced
- Hence greater the no of point of application of prestressing force, stress distribution is more uniform




compressive stress distribution



- Idealised stress distribution in end block with comp & tensile path is shown.
- Effect of transverse tensile stress :- to develop a zone of bursting tension in direction $\perp$r to anchorage force, resulting horizontal cracking.
- As concrete is weak in tension, suitable reinforcement is provided in transverse direction to resist bursting tension.

- Distribution of transverse stresses in anchorage zone subjected to a symmetrically placed prestressing force, distributed over a small area, for increasing ratio of (y_{po}/y_o) varying from zero to 0.5 is developed by Guyon.
- ISOBARS – Lines of equal transverse tensile stress
- Figure shows influence of anchorplate height on comp & tensile stress distribution in transverse direction.
- Ratio of transverse tensile stress to avg comp. stress gradually decreases with increase in ratio of depth of anchor plate to that of end block.

← Investigation on Anchorage Zone stresses →

A no of investigators studied stress distribution in anchorage zone using empirical equations / theoretical solutions based on 2D/3D elasticity or experimental technique.

They are Magnel, Guyon, Zielinski & Rowe, etc.

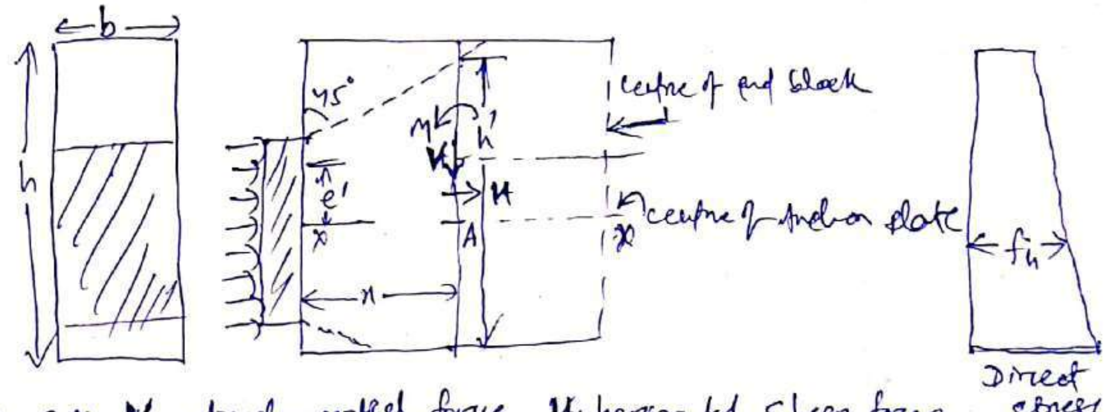
* Aim of anchorage zone stress analysis is to obtain transverse tensile stress distribution in end block from which total transverse bursting tension could be computed.

- Magnel's method →

← MAGNEL METHOD →

* end block is considered as a deep beam subjected to concentrated loads due to anchorages on one side and to normal & tangential distributed loads from linear direct stress and shear stress distribution from the other side.

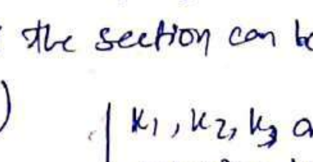
* Forces acting on end block & stresses at any point on horizontal axis ‖ to beam are :-



M = BM, V = direct vertical force, H = horizontal shear force

M = B.Moment

σ_v = vertical stress
σ_h = direct stress
τ = shear stress



stress distribution at A

* Stress distribution across the section can be approximated by (Point A)

point 'A'

$$\begin{cases} \sigma_v = K_1\left(\frac{M}{bh^2}\right) + K_2\left(\frac{V}{bh}\right) \\ \tau = K_3\left(\frac{H}{bh}\right) \\ \sigma_h = \frac{P}{bh}\left(1 + 12\frac{e'^2}{h'^2}\right) \end{cases}$$

K_1, K_2, K_3 are from Table at varying distance from the end face of beam

* f_h is computed assuming that concentrated load disperses at 45°

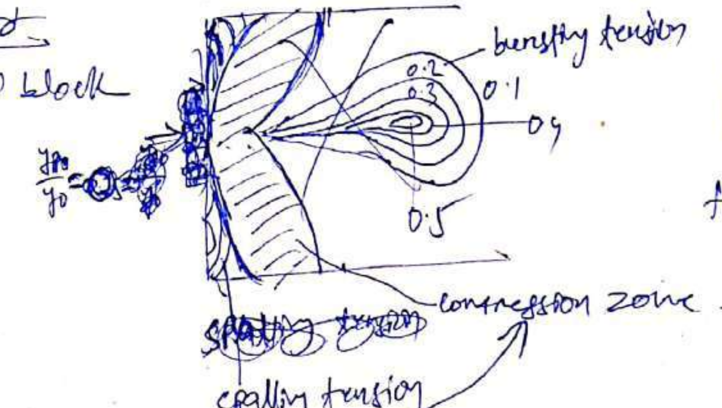
* principal stresses at point

$$\begin{cases} f_{max/min} = \frac{f_h+f_v}{2} \pm \frac{1}{2}\sqrt{(f_h-f_v)^2+4\tau^2} \\ \tan 2\theta = \frac{2\tau}{f_v-f_h} \end{cases}$$

* Bursting tension computed from distribution of principal tensile stress on required axis & suitable reinforcement are designed to take up tension

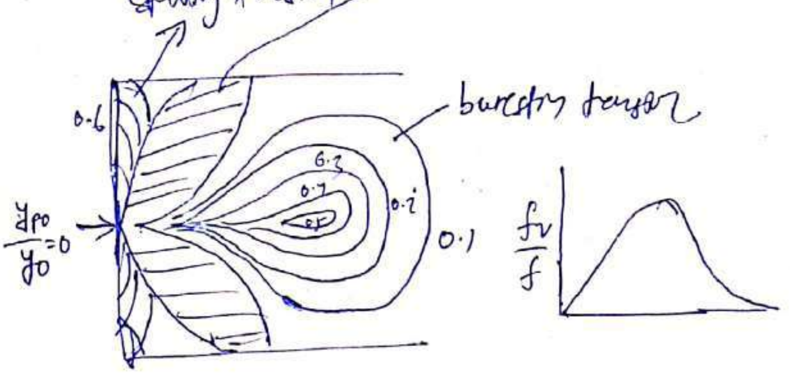
~~Eg~~

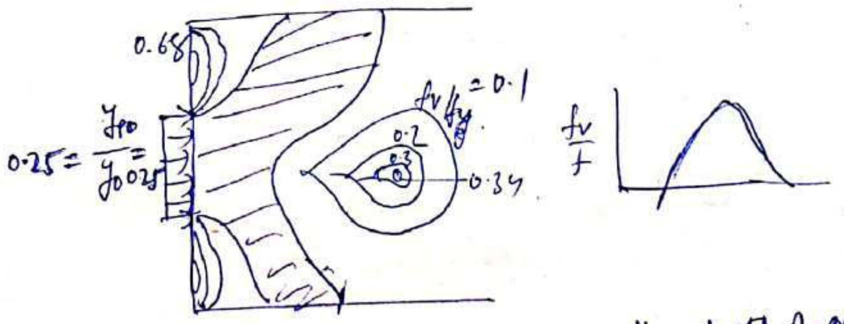
End block



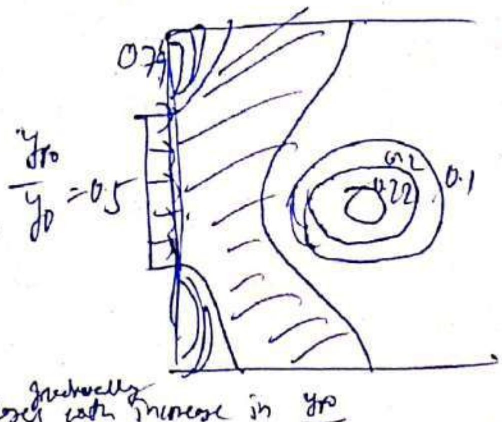
$\frac{f_v}{f} = \frac{\text{vertical tensile stress}}{\text{average stress}}$

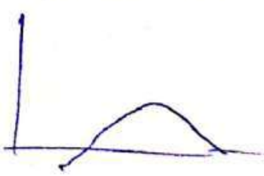
f_v = transverse tensile stress / vertical tensile stress





y_{po} = depth of anchor plate
y_o = Depth of end block





* $\frac{f_v}{f}$ decreases gradually with increase in $\frac{y_{po}}{y_o}$

① End block size in PSC beam is (100×200 mm). Prestressing force is 100kN transmitted to concrete by a distribution plate of 100mm wide & 50 mm deep, concentrically located at ends. Calculate position & magnitude of max. tensile stress on horizontal section through the ~~section~~ centre (& edge of anchor plate). compute bursting tension on these ~~given~~ horizontal plane.

Sol-)

$P = 100 \text{ kN}$

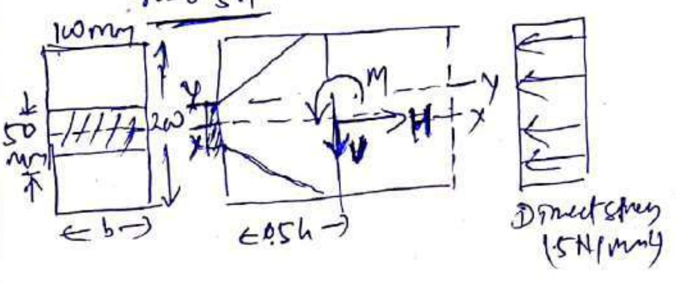
$h = 200 \text{ mm}$

$b = 100 \text{ mm}$.

(a) Direct stress, $\sigma_h = \dfrac{\overset{P}{100\times10^3}}{\underset{h\to}{200}\times\underset{b}{100}} = 5 \text{ N/mm}^2$

Vertical stress σ_v & principal tensile stress are critical at

$x = 0.5h$



For section X-X

$$\left[\text{At } \frac{x}{h} = 0.5,\ k_1 = -5,\ k_2 = 2,\ k_3 = 1.25\right]$$

$$M = \Big[\underbrace{\overset{\sigma_h}{5}\times\overset{h/2}{100}\times\overset{b}{100}\times\overset{h/4}{\frac{100}{2}}}_{\text{moment of }\sigma_h\text{ at }x\text{-}x} - \underbrace{\overset{P/2\ \ 1\text{kN}=10^3\text{N}}{\frac{100\times10^3}{2}}\times\overset{(\frac{50}{2}/2)}{\frac{50}{4}}}_{\text{moment of 'P' at }x\text{-}x}\Big] = 1875\times10^3 \text{ N.mm}.$$

* $H = 0\ [5\times100\times100\ (\sigma_h) - 50\times10^3\ (P/2)] = 0$ (horizontal shear)
* $V = 0$ i.e. vertical force is zero

$\rightarrow \sigma_v = \overset{k_1}{-5}\left[\dfrac{\overset{M}{1875\times10^3}}{\underset{b\to}{100}\times\underset{h\to}{200^2}}\right] = -2.35 \text{ N/mm}^2$

$\rightarrow \sigma_h = +5 \text{ N/mm}^2$

→ principal tensile stress at 0.5h = 100 mm from end is

$$\sigma_{min} = \frac{5-2.35}{2} - \frac{1}{2}\sqrt{(5+2.35)^2+0} = -2.35 \text{ N/mm}^2$$

∴ Total splitting tension, assuming parabolic distribution of stress

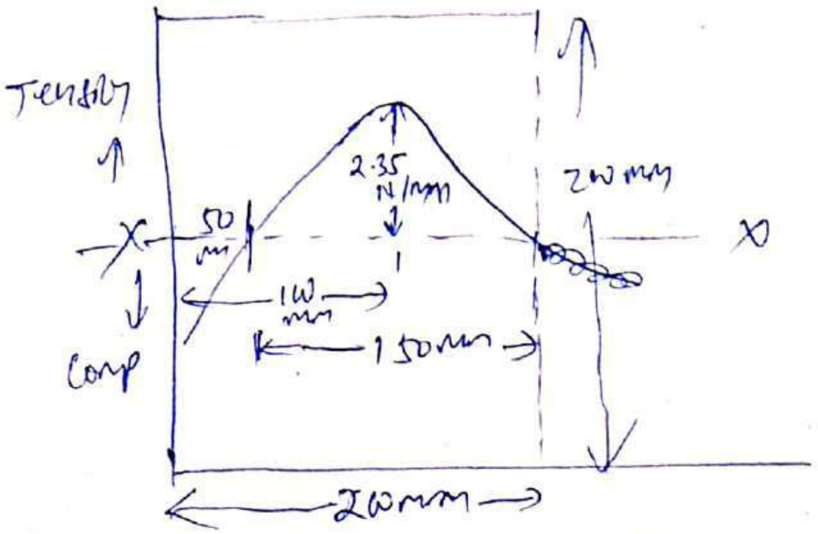
*∴ $F_{bst} = \left(\frac{2}{3}\times150\times2.35\right)\times\overset{b}{100} = 23500 \text{ N}$

b) For section yy (passing through edge of plate)

stresses at $x = 0.5h = 100\text{mm}$ from end

$\rightarrow M = \overset{b}{100}\times\overset{h}{75}\times\overset{\sigma_h}{5}\times\overset{h/2}{75/2} = 14\times105 \text{ N mm}$

$\rightarrow H = -(\underset{b}{100}\times\underset{h}{75}\times\underset{\sigma_h}{5}) = -37500 \text{ N}$ (acting towards end of beam)



Distribution of tensile stress.

$\rightarrow V = 0$

$\rightarrow \sigma_v = -\overset{K_1}{5}\left(\frac{\overset{M}{14}\times10^5}{100\times200^2}\right)+0 = -1.75\ N/mm^2$

$\rightarrow \tau = \underset{K_3}{1.25}\times\left(\frac{-37500^{4}}{\underset{b}{100}\times\underset{h}{200}}\right) = -2.35\ N/mm^2$

$\rightarrow \sigma_h = +5\ N/mm^2$

$\rightarrow$ Principal tensile stress $= \left(\frac{5-1.75}{2}\right)-\frac{1}{2}\left(\sqrt{(5+1.75)^2+4(-2.35)^2}\right)$

$= -2.475\ N/mm^2$

Angle of inclination of plane of principal stress w.r.t vertical plane is

$$\tan 2\theta = \left(\frac{2\tau}{\sigma_v-\sigma_h}\right) = \frac{-2\times2.35}{-1.75-5} = 0.7$$

$$\theta = 17.5^\circ$$

$\rightarrow$ Tensile stress component in vertical direction

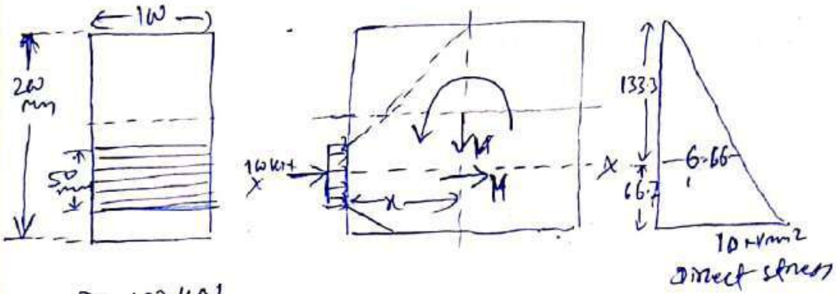
$= 2.475 \sec 17.5^\circ = 2.6\ N/mm^2$

Bursting tension $= F_{bst} = \left(\frac{2}{3}\times150\times2.6\right)100 = 26000\ N$ in plane xx

—x—

2

The end block of a PSC beam, (100 × 200 mm deep), supports an eccentric prestressing force of 100 kN, the line of action of which coincides with bottom kern of section. Anchor plate depth is is 50 mm. Estimate the magnitude and position of principal tensile stress on a horizontal plane passing through the centre of anchorage plate.



$P = 100$ kN

$h = 200$ mm

$b = 100$ mm

$$\text{Direct stress} = \frac{1}{2} \times \frac{\overset{P}{100 \times 1000}}{\underset{h'}{50} \times \underset{b'}{100}} = 10\ \text{N/mm}^2$$

At $\frac{x}{h} = 0.5$, from table $\Rightarrow$ $x = 100$ mm from free end face of beam

$$\left[K_1 = -5,\ K_2 = 2,\ K_3 = 1.25\right]$$

Stresses at section X-X

$$M = \left[\left(\frac{1}{2} \times \underset{\sigma_h'}{6.66} \times \underset{h'}{133.3} \times \underset{b'}{100}\right) \times \left(\frac{1}{3} \times 133.3\right)\right] - \left[\frac{100 \times 10^3}{2} \times \frac{50}{4}\right] = 1345 \times 10^3\ \text{N mm}$$

$$H = \left(\frac{1}{2} \times 6.66 \times 133.3 \times \overset{\text{width}}{100}\right) - \left(\frac{100}{2} \times 10^3\right) = -5612\ \text{N}$$ (Horizontal shear)

$V = 0$ (vertical force)

$x = 0.5h = 100$ mm from free end face.

$\rightarrow \sigma_h = 6.66\ \text{N/mm}^2$

$$\rightarrow \sigma_v = \underset{K_1}{-5}\left[\frac{\overset{M}{1345 \times 10^3}}{\underset{b'}{100} \times \underset{h'}{200}^2}\right] \qquad \left(\sigma_v = K_1 \frac{M}{bh^2} + K_2 \frac{V}{bh}\right)$$

$= -1.66\ \text{N/mm}^2$

$$\rightarrow \tau = K_3\left(\frac{H}{bh}\right) = 1.25\left[\frac{-5612}{100 \times 200}\right] = -0.35\ \text{N/mm}^2$$

$$\sigma_{min} = \cancel{6.66\ \sigma 1.66}\left(\frac{\sigma_v + \sigma_h}{2}\right) - \left(\frac{1}{2}\sqrt{(\sigma_h - \sigma_v)^2 + 4\tau^2}\right)$$

$$= \left(\frac{-1.66 + 6.66}{2}\right) - \left(\frac{1}{2}\sqrt{(6.66 + 1.66)^2 + 4(-0.35)^2}\right)$$

$= -1.7\ \text{N/mm}^2$

* Assuming the magnitude of tensile stresses in vertical direction also 1.7, bursting tension

$$F_{bst} = \left(\frac{2}{3} \times 150 \times 1.7\right) \times 100 = 17000\ \text{N}.$$

11-5-07 End block

* End block means portion of a PSC beam surrounding the anchorages of the tendons.

* Large concentrated forces are transmitted
on the bearing surfaces at the ends of of the beam
by anchorages.

* The prestress is transferred through out the length of the end block
from nearly concentrated areas.

* These stresses spread out into the concrete
setting up complicated stress-patterns.

* The stress distribution close to the anchorage is different from
the stress distribution at section away from the anchorage.

* Consider a beam of width b & depth d.

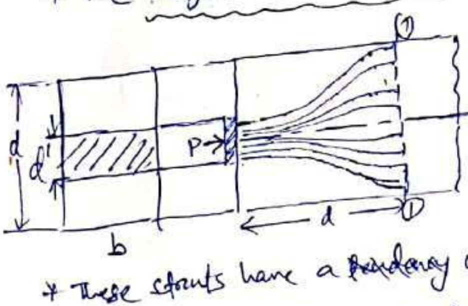
F= prestressing force applied on bearing area bd' on the end face of beam.

* At sections, close to end face, stress on section $> \frac{F}{bd}$

* There exists a section 1-1, beyond which stress is $\frac{F}{bd}$

∴ The part of beam from end face to sec-1-1 is called END BLOCK.

* The length of end block is usually taken equal to 'd' (depth of section)

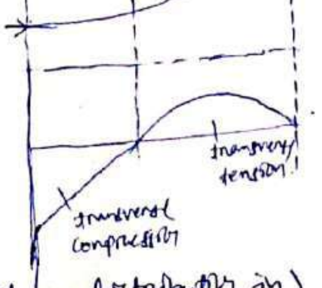


* This shows line of pressure transfer which spread out from area bd' to area bd in end block.

* These lines of pressure may be taken to act as individual slender struts.

* These struts have a ~~tendency~~ curvature which changes gradually from
convexity towards centroidal axis near end face to
concavity towards CA at sufficient distance from end face.

* Transverse compressive stresses are developed for a certain length,
transverse tensile stresses are developed beyond this zone.



(stress distribution in end block)

Guyon's method to study stress distribution in end block

The theory behind is:-

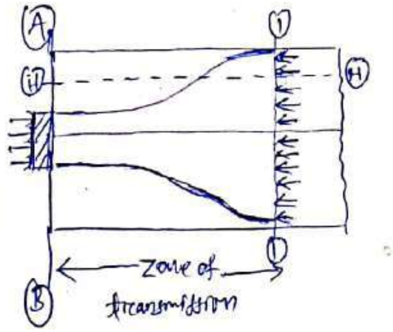
* When prestress force is applied through anchor plate, the force is dispersed in a certain length of beam to the entire C/S.

* The length of beam within which the dispersion of prestressing force takes place is called (TRANSMISSION ZONE.)

* Within transmission zone, the stress distribution can be analysed as follows.

* In a dirn transverse to the axis of this concentrated force, tensile forces are developed called (BURSTING FORCES.)

* At end section, the surface adjacent to anchor plate is also subjected to a tensile force called (SPALLING FORCE.)



→ prestressing force F is transmitted through centrally placed anchor plate.

AB = end face of beam.

1-1 = end of transmission zone.

→ horizontal sec (H-H) is subjected to transverse stress & shear stress.

* The depth of anchor plate, influences the length of transmission.

* Length of transmission & bursting force (transverse tensile force) depend on $\left[\frac{\text{depth of anchor plate } (d')}{\text{depth of beam } (d)}\right]$

* usually -)

Length of transmission zone = Length of End block = depth of beam.
(transmission length)

Bursting force, $\boxed{F_b = 0.3F\left[1-\frac{d'}{d}\right]}$

F_b = bursting force

F = prestressing force

d' = depth of anchor plate

d = depth of beam.

* Bursting stress distribution depends on $\frac{d'}{d}$.

Q A PSC beam (250 × 600 mm) is subjected to an axial prestressing force of 1500 kN. Design the end block.

Sol → Let size of anchor plate, $b' = 0.8b = 0.8 \times 250 = 200$ mm.

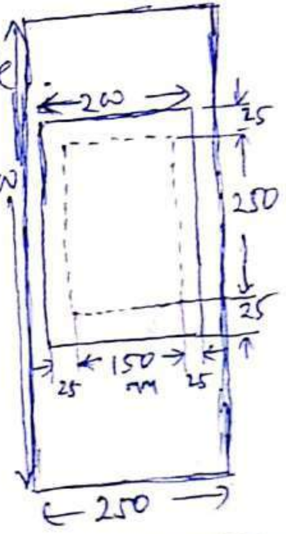
$d' = 0.5\,d = 0.5 \times 600 = 300$ mm.

Design of anchor plate →

* Let us assume that various cables passes through one duct & are anchored to one anchor plate with an overhang at 25mm on all sides.

Induced bearing pressure intensity on anchor plate $= \frac{1500 \times 1000}{200 \times 300} = 25$ N/mm2

consider 1 mm wide cantilevering strip of anchor plate



$BM_{max} = 25 \times \frac{25^2}{2} = 7812.5$ N mm.

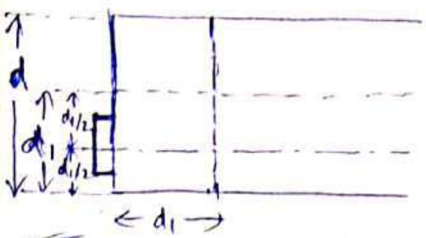
permissible bending stress $= 0.66 f_y = 165$ N/mm2.

$M = \sigma Z \Rightarrow 7812.5 = 165 \times \frac{1 \times t^2}{6} \Rightarrow t = 16.8$ mm.

∴ provide 18mm thick anchor plate.

Anchor plate (cable) placed eccentrically →

- where anchor plate is placed eccentrically at end face of beam:-
- Here an equivalent prism is considered whose axis is same as axis of prestressing force.
- Depth of prism = 2 × distance of nearer edge from axis of prestressing force.



Q A PSC beam (250 × 600 mm) & is subjected to a prestressing force of 1000 kN at an eccentricity of 100 mm. The anchoring plate (200 × 320 mm deep) calculate bursting stress & reinforcement required in zone of transmission.

Sol →) distance betn axis of cable & nearer edge = 200 mm.

$$\frac{d'}{d} = \frac{320}{400} = 0.8$$

Bursting force on horizontal plane $= 0.3F\left(1 - \frac{d'}{d}\right)$

$$= 0.3 \times 1000(1 - 0.8) = 60 \text{ kN}.$$

Bursting force/mm width $= \frac{60 \times 10^3}{200} = 300$ N/mm

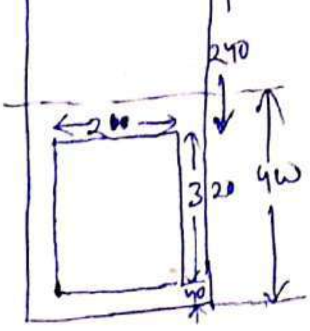
- Bursting tensile stress is zero & max. at distances $0.24d_1$ & $0.48d_1$.

$$0.24d_1 = 0.24 \times 400 = 96 \text{ mm}$$
$$0.48d_1 = 0.48 \times 400 = 192 \text{ mm}.$$

To calculate max bursting stress.

$$\frac{2}{3}(96 + 192)\,\sigma_{max} = 300 \;\Rightarrow\; \sigma_{max} = 1.48 \text{ N/mm}^2 < 1.8 \text{ N/mm}^2.$$

(OK)



Advanced Concrete Structure

PRESTRESSED CONCRETE

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21-4-02

LOSSES OF PRESTRESS

* prestressing force used to in making stress calculation is not remaining constant.
* stress during various stages of loading varies since concrete strength & modulus of elasticity increases with time.
* The most common stages to be checked for stresses & ~~behaviour~~ behavior are:-

-> Immediately after transfer of prestress force to conc sec, stresses are evaluated as a measure of behavior.

-> At service load after all losses of prestress have occured & a long term effective prestress level is reached stresses are checked for behavior & strength.

* certain structures may be prestressed in stages to match loading which may be ~~subsequently~~ sequentially added to the structure.
* Prestress loss depends upon time elapsed,
  exposure conditions &
  size of member, etc.

* The PCI committee recommendations for to estimate prestress losses are:-

① Determination of stress loss in PSC members is a complicated problem because rate of loss due to one factor (such as relaxation of steel) is altered by changes in stress due to other factors (such as creep of concrete).

similarly rate of creep is altered by change in tendon stress.

It is very difficult to separate net loss due to each factor under diff conditions of stress, environment, loading, etc.

② In addition to above uncertainties due to interaction of shrinkage, creep & relaxation, physical conditions such as variation in actual properties of concrete can vary the total loss. so prestress loss calculation is not exact.

③ Errors in computing prestress loss can affect service conditions such as CAMBER, DEFLECTION, & CRACKING.

④ It has no effect in ultimate strength of flexural members unless
tendons are unbonded or
final stress after losses is less than 0.5 fpu.

* Detailed analysis of prestress loss is required where deflections are critical.

-> slender beams are more sensitive to moment from prestress which balances moment from applied loading to control deflection,

* so Initial prestress in concrete undergoes a gradual reduction with time from stage of transfer due to various causes.

Types of losses of prestress

Pre tensioning	Post tensioning
① Elastic deformation of concrete	① No loss due to elastic deformation if all wires are simultaneously tensioned. If wires are successively tensioned there will be loss due to above cause.
② Relaxation of stress in steel	② Relaxation of stress in steel.
③ Shrinkage of concrete.	③ Shrinkage of concrete
④ Creep of concrete	④ Creep of concrete.
	⑤ Friction
	⑥ Anchorage slip.

* There will also be loss of prestress if temp changes suddenly, in case of pre tensioned element.

* Rise in temp causes a partial transfer of prestress due to elongation of tendons in long line process which causes large creep if conc is not cured properly.

① Loss due to Elastic deformation/Elastic shortening of Concrete ⇒

(a) pre tensioned concrete →

* As prestress is transferred to concrete, the member shortens & prestressed steel shortens with it, and so there is a loss of prestress in steel.

* The loss depends upon
- the modular ratio (α)
- average stress in concrete at the level of steel (σ_c)

If σ_c = prestress in concrete at the level of steel.
E_s, E_c are modulus of elasticity of steel & concrete.

$\boxed{\alpha = \frac{E_s}{E_c}}$ (modular ratio)

~~stress~~ :
strain in concrete at steel level $= \frac{\sigma_c}{E_c}$ = strain in steel at same level

stress in steel $= E_s \times$ strain in steel $= E_s \times \frac{\sigma_c}{E_c} = \sigma_c \times \frac{E_s}{E_c} = \alpha \sigma_c$

$\therefore \boxed{\text{Loss of stress in steel} = \alpha \sigma_c}$

* { The elastic shortening is instantaneous at the time of transfer & is independent of other sources of loss that occures after this time }.

* Loss due to elastic shortening can be compensated by tensioning the steel higher than the stress desired at transfer (both in pre & post tensioning case).

(b) Post - Tensioned member →

* In post tensioning if we have a single tendon or all the tendons are jacked at the same time, concrete shortens as tendons are jacked, against concrete.

→ Since force in tendon is measured after elastic shortening of concrete has taken place, no loss of prestress has been observed.

* But if tendons are successively tensioned, then prestress is gradually applied to concrete & shortening of concrete increases as each tendon is tightened against it & loss of prestress is diff in diff tendons.

→ Tendon which is first tensioned suffers max loss as shortening of concrete by subsequent prestress application from all other tendons.

→ Tendon tensioned lastly won't suffer any loss due to elastic shortening of concrete as all the shortenings have already taken place.

* example:-

In most bridge girders, cables are curved with max eccentricity at centre of span & in such cases average stress in concrete at steel level is calculated to find out the loss of stress due to elastic deformation.

Q-1 A pretensioned conc beam (100mm wide × 300 mm deep) is prestressed by straight wires carrying an initial force of 150kN at an eccentricity of 50mm. Estimate the percentage loss of stress in steel due to elastic deformation of concrete if area of steel wire is 188mm².

$$E_s = 210\ kN/mm^2 \quad , \quad E_c = 35\ kN/mm^2$$

Sol → $P = 150\ kN.$ $\quad A = 100 \times 300 = 3 \times 10^4\ mm^2$ $\quad I = \frac{bd^3}{12} = \frac{100 \times 300^3}{12} = 225 \times 10^6\ mm^4$

$e = 50mm$ $\quad A_s = 188 mm^2$

$y = 50mm$ $\quad \alpha = \frac{E_s}{E_c} = \frac{210}{35} = 6$

Initial stress in steel $= \frac{P}{A_s} = \frac{150 \times 10^3}{188} = 800\ N/mm^2$

stress in concrete at the level of steel, $\sigma_c = \frac{P}{A} \pm \frac{Pey}{I}$

$$\Rightarrow \sigma_c = \frac{150 \times 10^3}{3 \times 10^4} \pm \frac{150 \times 10^3 \times 50 \times 50}{225 \times 10^6} = 6.66\ N/mm^2$$

Loss of stress due to elastic deformation of concrete $= \alpha \sigma_c$

$$= 6 \times 6.66 = 40 N/mm^2$$

Percentage loss of prestress $= \frac{40}{800} \times 100 = 5\%.$

| Ans)

Q-2 A rectangular concrete beam (200mm wide x 300mm deep) is prestressed by fifteen 5mm dia wires located at 65mm from bottom of beam & 3 nos of 5mm dia wires located at 25mm from top of beam. If wires are initially tensioned to a stress of 840N/mm², calculate %age loss of prestress in steel immediately after transfer, allowing for loss of stress due to elastic deformation of concrete only. $E_s = 210\overset{k}{}N/mm^2$, $E_c = \overset{31.5}{\cancel{35}}kN/mm^2$

sol→ position of centroid of wires from soffit of beam $= \frac{(15\times 65)+(3\times 275)}{15+3} = 100mm.$

$e = 150 - 100 = 50mm.$, $\underbrace{\alpha = \frac{E_s}{E_c} = 6.68}$

Area of concrete $= A = 200\times 300 = 6\times 10^4 mm^4$

$I = \frac{bd^3}{12} = \frac{200\times 300^3}{12} = 45\times 10^7 mm^4$

prestressing force, $P = 840\times(18\times 19.7) = 3\times 10^5$ N. = 300kN.

Stress in conc at level of top wires $= \frac{300\times 10^3}{6\times 10^4} - \frac{(300\times 10^3)(50)(150-25)}{45\times 10^7} = 0.83 N/mm^2$

Stress in conc at level of bottom wires $= \frac{300\times 10^3}{6\times 10^4} + \frac{(300\times 10^3)(50)(150-65)}{45\times 10^7} = 7.85 N/mm^2$

Loss of stress in wires at top $= \alpha\sigma_c = 6.68\times 0.83 = 5.55 N/mm^2$

Loss of stress in wires at bottom $= \alpha\sigma_c = 6.68\times 7.85 = 52.5 N/mm^2$

%age loss of prestress in steel for top wires $= \frac{5.55}{840}\times 100 = 0.66\%$.

%age loss of prestress in steel for bottom wires $= \frac{52.5}{840}\times 100 = 6.25\%$.

Q-3 A post tensioned conc beam (100mm wide x 300mm deep) is prestressed by 3 cables. C/s area of each cable is 50mm² & initial stress of 1200 N/mm². All the three cables are straight & located at 100mm from soffit of beam. Calculate loss of stress in 3 cables due to elastic deformation of concrete

(a) simultaneous tensioning & anchoring of all the 3 cables &

(b) successive tensioning of 3 cables one at a times. $(\alpha = 6)$

sol→ Force in each cable, $P = 50\times 1200 = 60\times 10^3 N = 60kN.$

$A = 100\times 300 = 3\times 10^4 mm^4$, $e = 50mm$, $y = 50mm$, $I = \frac{bd^3}{12} = 225\times 10^6 mm^4$

stress in conc at steel level, $\sigma_c = \frac{60\times 10^3}{3\times 10^4} + \frac{(60\times 10^3)(50)(50)}{225\times 10^6} = 2.7 N/mm^2$

(a) under simultaneous tensioning & anchoring of all the 3 cables, there will be no loss due to elastic deformation of concrete.

(b) $\sigma_c = 2.7$ N/mm2 , $\alpha = 6$

when cable-1 is tensioned, no loss due to elastic deformation.

when cable-2 is tensioned, loss of stress in cable-1 $= \alpha\sigma_c = 6 \times 2.7 = 16.2$ N/mm2

when cable-3 is tensioned, loss of stress in cable-1 & cable-2 is 16.2 N/mm2 each

Total loss of stress due to elastic deformation of concrete in

cable-1 = 16.2 + 16.2 = 32.4 N/mm2

cable-2 = 16.2 N/mm2

cable-3 = 0.

Average loss of stress considering all the 3 cables = 16.2 N/mm2

If no of tendons are large, the loss due to elastic shortening approaches but does not exceed one half the corresponding loss with pre-tensioning,

i.e. $\boxed{\text{loss of stress} = \frac{1}{2}\alpha\sigma_c}$

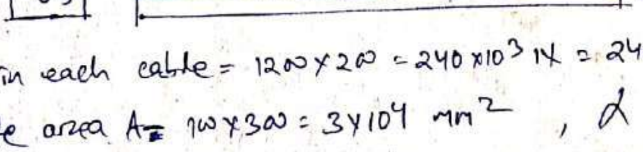
σ_c = stress in concrete at steel level when all cables are simultaneously tensioned.

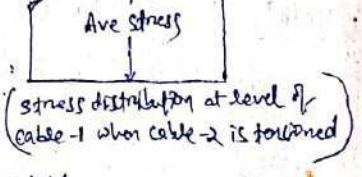
Applying this principle to present case

$$\text{Loss of stress} = \frac{1}{2}(6 \times 3 \times 2.7) = 24.3 \text{ N/mm2}$$

Q-4 A post tensioned conc beam (100mm wide × 300 mm deep) has span 10M is stressed by successive tensioning of 3 cables respectively. The c/s area of each cable is 200 mm2 & initial stress in cable is 1200 N/mm2, $\alpha = 6$. The Ist cable is parabolic with an eccentricity of 50mm below centroidal axis at centre of span & 50mm above centroidal axis at support section. The 2nd cable is parabolic with zero eccentricity at supports & 50mm at centre of span. The 3rd cable is straight with uniform eccentricity of 50mm below centroidal axis. Calculate %age loss of stress in each cable if they are successively tensioned & anchored.

sol ⇒





(stress distribution at level of cable-1 when cable-2 is tensioned)

Force in each cable = 1200 × 200 = 240 × 10^3 N = 240 kN.

concrete area A = 100 × 300 = 3 × 10^4 mm2 , $\alpha = 6$

$$I = \frac{bd^3}{12} = 225 \times 10^6 \text{ mm}^4$$

when cable-1 is tensioned, no loss of stress due to elastic deformation of conc.

when cable-2 is tensioned, stress at level of cable-1 is given by

stress at support section $= \sigma_c = \frac{240 \times 10^3}{3 \times 10^4} = 8$ N/mm2 (as e = 0 & y = −50mm)

stress at centre of span, $\sigma_c = \frac{240 \times 10^3}{3 \times 10^4} + \frac{240 \times 10^3 (50)(50)}{225 \times 10^6} = 10.7$ N/mm2

Ave stress in concrete $= 8 + \frac{2}{3}(10.7-8) = 9.8$ N/mm2

Loss of stress in cable-1 $= \alpha \sigma_c = 9.8 \times 6 = 58.8$ N/mm2 ——— ①

when cable-3 is tensioned & anchored, stress distribution at level of cable 1 & 2 & ave ~~loss~~ stress & loss of stress is obtained in below.

	cable-1	cable-2
stress at support	$\frac{240\times10^3}{3\times10^4} - \frac{240\times10^3\times50\times50}{225\times10^6} = 5.3$ N/mm2 $(e=50, y=-50)$	$\frac{240\times10^3}{3\times10^4} = 8$ N/mm2 $(e=50, y=0)$
stress at centre of span	$\frac{240\times10^3}{3\times10^4} + \frac{240\times10^3\times50\times50}{225\times10^6} = 10.7$ N/mm2 $(e=50, y=50)$	$\frac{240\times10^3}{3\times10^4} + \frac{240\times10^3\times50\times50}{225\times10^6} = 10.7$ N/mm2 $(e=50, y=50)$
Ave stress in concrete	$5.3 + \frac{2}{3}(10.7-5.3) = 8.9$ N/mm2	$8 + \frac{2}{3}(10.7-8) = 9.8$ N/mm2
Loss of stress in concrete	$6\times8.9 = 53.4$ N/mm2	$6\times9.8 = 58.8$ N/mm2

%age loss of stress

Total loss, in cable-1 $= 58.8 + 53.4 = 112.2$ N/mm2, $\frac{112.2}{1200}\times100 = 9.35\%$

in cable-2 $= 58.8$ N/mm2 ——— $\frac{58.8}{1200}\times100 = 4.9\%$

in cable-3 → no loss of stress ——— 0 %

Q-5 A s/s conc beam of uniform sec is post tensioned by 2 cables, both of which have an eccentricity of 100 mm below the centroid of sec at mid span. The Ist cable is parabolic & is anchored at eccentricity of 100 mm above centroid at each end. The 2nd cable is straight & ‖ to line joining the supports. If c/s area of each cable is 100 mm^2, the conc beam has c/s area 2×10^4 mm^2 & rad of gyration of 120 mm, calculate loss of stress in Ist cable when 2nd is tensioned to stress of 1200 N/mm2. $\alpha = 6$.

Sol→ constant $e = 100$ mm, $A = 2\times10^4$ mm^2, $r = 120$ mm, $I = Ar^2 = 288\times10^6$ mm^4

$P = 1200\times100 = 120\times10^3$ N $= 120$ kN.

when cable 2 is tensioned, stress at level of cable-1 is given by

stress in concrete at support $= \frac{120\times10^3}{2\times10^4} - \frac{(120\times10^3)(100)(100)}{288\times10^6} = 1.8$ N/mm2

$(e = 100$ mm, $y = -100$ mm$)$

stress in concrete at central sec $= \frac{120\times10^3}{2\times10^4} + \frac{(120\times10^3)(100)(100)}{288\times10^6} = 10.2$ N/mm2

$\sigma_c =$ Ave stress in concrete $= 1.8 + \frac{2}{3}(10.2 - 1.8) = 7.4$ N/mm2

Loss of stress in cable-1 $= \alpha\sigma_c = 6\times7.4 = 44.4$ N/mm2 (Ans)

TIME DEPENDENT LOSSES →

* Prestress loss due to creep & shrinkage of concrete & relaxation of steel are time dependent & interdependent.

* After transfer of prestress, a sustained stress is imposed on both steel & concrete which changes with time.

* The stress in a PSC member is changing with time due to losses in the prestressing force.

* The time dependent losses can't be counterbalanced.

* It is not possible to over tension the tendons excessively to allow for such losses because it would mean very high initial stresses in steel which might

→ increase its ~~to~~ relaxation loss ~~as approaches its yield point~~

→ approache its yield point stress

→ increase the creep loss significantly (concrete)

* If steel is unbonded, it is possible to retension the steel after some losses have taken place, but it is expensive & undesirable.

There are cases where prestressing is done by stages to match additional loading.

② Loss due to shrinkage of concrete →

* Shrinkage of concrete in PSC members results in shortening of tensioned wire & so contributes to loss of stress.

* High strength conc with low w/c ratio, results in reduction in shrinkage & loss of prestress is reduced.

* Rate of shrinkage is higher at surface of member.

* The differential shrinkage betn interior & surface of concrete results in strain gradient leading to surface cracking.

* So proper curing is required to prevent shrinkage cracks.

* In pretensioned member, shrinkage takes place after the time of transfer because moist curing is done for these members.

* Total residual shrinkage strain for pre tensioned member is greater than post tensioned member, ~~because~~ after transfer of prestress as in pre-tensioned member moist curing is done to prevent shrinkage until time of transfer &

in post tensioned member a portion of shrinkage occurred before the transfer of prestress.

For pretensioned member, $\boxed{\epsilon_{cs} = 300\times10^{-6}}$

For post tensioned member, $\boxed{\epsilon_{cs} = \frac{200\times10^{-6}}{\log_{10}(t+2)}}$

where,

ϵ_{cs} = total residual shrinkage strain

t = age of concrete at transfer in days.

* The shrinkage strain for post tensioned member may be increased by 50% in dry atmospheric condition, but max value can be 300×10^{-6}.

* The loss of stress in steel due to shrinkage of concrete = $\boxed{E_s\times\epsilon_{cs}}$

E_s = modulus of elasticity of steel.

Q-6 A concrete beam is prestressed by a cable carrying an initial prestressing force of 300KN. The c/s area of wires in the cable is 300mm². Calculate the %age loss of stress in the cable only due to shrinkage of concrete using IS 1343 recommendations assuming that beam to be a (a) pre tensioned & (b) post tensioned:

Assume $E_s = 210\ kN/mm^2$ & age of conc at transfer = 8da

Sol-) Initial stress in wires = $\frac{Force}{area} = \frac{300\times10^3}{300} = 1000\ N/mm^2$

(a) pre tensioned ->

$\epsilon_{cs} = 300\times10^{-6}$

stress loss = $E_s\epsilon_{cs} = 210\times10^3\times300\times10^{-6} = 63\ N/mm^2$

%age loss of stress = $\frac{63}{1000}\times100 = 6.3\%$

(b) post-tensioned ->

$\epsilon_{cs} = \frac{200\times10^{-6}}{\log_{10}(8+2)} = 200\times10^{-6}$

stress loss = $E_s\epsilon_{cs} = 210\times10^3\times200\times10^{-6} = 42\ N/mm^2$

%age stress loss = $\frac{42}{1000}\times100 = 4.2\%$.

(3) Loss due to creep of concrete

- creep is assumed to occur with superimposed permanent dead load added to the member after it has been prestressed.
- part of initial comp strain induced in concrete immediately after transfer is reduced by tensile strain because of superimposed permanent dead load.
- The sustained prestress in concrete causes creep in conc which reduces stress in high tensile steel.
- stress loss in steel due to creep of concrete can be estimated by

(a) ultimate creep strain method →

ϵ_{cc} = ultimate creep strain for a sustained unit stress

σ_c = comp stress in concrete at steel level

stress loss in steel due to creep of concrete = $\boxed{\epsilon_{cc}\sigma_c E_s}$

(b) creep coefficient method →

ϕ = creep coefficient

ϵ_c = creep strain

ϵ_e = elastic strain

α = modular ratio

E_c & E_s = mod of elasticity for conc & steel.

$$\phi = \frac{\epsilon_c}{\epsilon_e} \Rightarrow \epsilon_c = \phi\epsilon_e = \phi\frac{\sigma_c}{E_c}$$

stress loss in steel $= E_s\epsilon_c = E_s \times \phi\frac{\sigma_c}{E_c} = \boxed{\phi\alpha\sigma_c}$

$$\left[\left.\begin{aligned}\phi &= 2.2 \text{ (7 days loading)}\\ &= 1.6 \text{ (28 days loading)}\\ &= 1.1 \text{ (1 yr. loading)}\end{aligned}\right\}\left\{\begin{aligned}\phi &= 1.5 \text{ for watery condition}\\ &= 4 \text{ for dry condition.}\\ &\text{with relative humidity of 35\%.}\end{aligned}\right.\right]$$

Q-7 A concrete beam of rectangular sec (wide 100 mm × depth 300 mm) is prestressed by 5 wires of 7mm dia located at an eccentricity of 50mm, the initial stress in the wires is 1200 N/mm2. Estimate stress loss in steel due to creep of concrete using both methods.

$E_s = 210$ kN/mm^2 $\quad\epsilon_{cc} = 41\times10^{-6}$ mm/mm per N/mm^2

$E_c = 35$ kN/mm^2 $\quad\phi = 1.6$

Sol → $A = 100\times300 = 3\times10^4$ mm^2, $\alpha = \frac{E_s}{E_c} = 6$.

$P = 5\times38.5\times1200 = 23\times10^4$ N.

$I = \frac{bd^3}{12} = \frac{100\times300^3}{12} = 225\times10^6$ mm^4

$\sigma_c = \frac{23\times10^4}{3\times10^4} + \frac{23\times10^4\times50\times50}{225\times10^6} = 10.2$ N/mm^2

(a) ultimate creep strain method →

stress loss in steel $= \epsilon_{cc}\sigma_c E_s = 41\times10^{-6}\times10.2\times210\times10^3 = 88$ N/mm^2

(b) creep coefficient method →

stress loss in steel $= \phi\alpha\sigma_c = 1.6\times6\times10.2 = 97.92$ N/mm^2.

Q-8 A post tensioned concrete beam of rectangular Sec (100×300 mm) is stressed by a parabolic cable with zero eccentricity at supports & an eccentricity of 50mm at centre of span. The cable area is 200 mm² & initial stress in cable is 1200N/mm². Compute stress loss in steel by creep in concrete. $\epsilon_{cc} = 30\times10^{-6}$ mm/mm per N/mm², $E_s = 210$ KN/mm² | .48

Sol → $A = 100\times300 = 3\times10^4$ mm² $P = 1200\times200 = 240$ kN.

$I = \frac{bd^3}{12} = 225\times10^6$ mm⁴ $e = 50$mm.

At support sec, $\sigma_{c_1} = \frac{P}{A} = \frac{240\times10^3}{3\times10^4} = 8$ N/mm² (as e=0)

At central span sec, $\sigma_{c_2} = \frac{P}{A} + \frac{Pey}{I} = \frac{240\times10^3}{3\times10^4} + \frac{240\times10^3\times50\times50}{225\times10^6} = 10.7$ N/mm²

Average stress at Steel level, $\sigma_c = 8 + \frac{2}{3}(10.7-8) = 9.8$ N/mm².

stress loss in cable due to creep of concrete

$= \epsilon_{cc}\sigma_c E_s = 30\times10^{-6}\times9.8\times210\times10^3 = 62$ N/mm²

(4) Loss due to Relaxation of stress in steel ⇒ (cl. 18.5.2.3)(P.33)

* Relaxation test is done at constant strain over a period of time.
* The test result gives that prestress force will gradually decrease.
* Amount of decrease depends upon ⇒ time duration.
* The loss of prestress force at constant strain is called Relaxation.
* According to Indian code loss of prestress is varying from 0-90N/mm² for stress in wires varying from 0.5fpu to 0.8fpu.
* some times temporary overstressing by 5 to 10%. for a period of 2min is used to reduce this loss.

initial stress	Relaxation loss	Initial	loss
0.5 fpu	0	0.7fpu	70N/mm²
0.6 fpu	35 N/mm²	0.8fpu	90 N/mm²

(5) Loss of stress due to Friction ⇒ (for post tensioned members)

* There is friction in 'Jacking & anchorage' system so that stress existing in tendon is less than that indicated by pressure gage.
(This is true for some systems whose wires change direction at the anchorage.
* overtension can be applied to jack so that calculate prestress will exist in the tendon, but it must be limited to stay within the yield point of the wires.
* ACI code ⇒ Jacking force < 0.8fpu
* In Post tensioned members, tendons are housed in ducts performed in concrete. The ducts are either straight or follow a curved profile depending on design.
* Frictional loss occurs betn tendon & its surrounding material, whether concrete or sheathing & whether lubricated or not.

* This frictional loss can be considered in two parts.

(a) WOBBLING EFFECT / LENGTH EFFECT →

* This is the amount of friction that would be encountered if tendon ~~is~~ is a straight one, i.e. tendon is not intentionally bent or curved.

* But in practice the duct for tendon can't be perfectly straight, some friction exist bet^n the tendon & surrounding material even though the tendon is meant to be straight. This is called wobbling effect of duct / wave effect of duct.

* This effect is the result of accidental misalignment.

* This depends on { length & stress of tendon.
  coefficient of friction bet^n the contact materials.
  methods used in aligning & obtaining duct.
  local deviation in alignment of cable.

(b) Curvature Effect ⇒

* The loss of prestress due to curvature effect results from the [intended curvature of tendons + unintended wobble of duct].

* This loss depends on { coefficient of friction bet^n contact materials
  pressure exerted by the tendon on the concrete.

→ coefficient of friction depends on { smoothness & nature of surfaces in contact.
  the amount & nature of lubricants &
  the length of contact.

⇒ Pressure bet^n tendon & concrete depends on { stress in tendon &
  total change in angle.

* For above two effects, two coefficients are there →

{ K → wobble coefficient / friction coefficient for wave effect.
  μ → curvature coefficient / coefficient of friction bet^n cable & duct.

* K & μ will depend on

  type of steel use (wire/strand/bar)
  kind of surface (indented/corrugated)
  (rusted/cleaned/galvanized)

* Amount of vibration used in placing the concrete will affect the straightness of the duct ⇒ affects the overall size of duct

* For unbonded reinforcement, lubricants can be used

* For bonded reinforcement, if lubricants are used, they must be applied carefully so that the bond formation by grouting is not affected

* There are several methods to overcome frictional loss in tendons.

(a) overtensioning ⇒ when friction is not excessive, amount of overtension is equal to the max frictional loss.

overtensioning ≯ 0.8-0.85 fpu

→ Amount of overtension required to overcome friction is not cumulative over that required for overcoming anchorage loss / minimizing creep in steel.

→ But max of the three required is taken & overtensioning is done

Because in all cases of overtensioning, overstretching & release back is done

If most of the friction exist in jacking end, then overtensioning to balance that friction will not produce any overstretching of the main portion of tendon & so creep can't be minimized.

→ If friction loss is high %age of initial prestress, then it can't be overcome by overtensioning because overtensioning must be less than the yield strength of tendons.

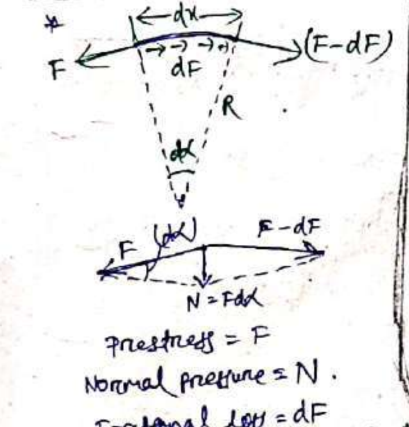
(b) Jacking from both ends ⇒

* It is adopted when tendons are long / angle of bending is large.

Derivation for frictional loss formula → (curved cable)

(a) curvature effect →

* Consider an infinitesimal length 'dx' of a prestressing tendon whose centroid follows the arc of an circle of radius R.



Prestress = F

Normal pressure = N.

Frictional loss = dF

(frictional loss along length 'dx')

* change in angle of tendon as it goes round length 'dx'

$$\boxed{d\alpha = \frac{dx}{R}} \text{ --------- } ①$$

* For an ~~infinitesimal~~ infinitesimal length 'dx', ~~stress~~ stress in tendon is constant (i.e. F)

* Normal component of pressure produced by F bending around an angle 'dα' is, $\boxed{N = F d\alpha = \frac{F dx}{R}}$ — ②

* Frictional loss dF around length dx, ~~dF=~~

$$dF = -\mu N = -\mu \frac{F dx}{R} = -\mu F d\alpha \text{ ------ } ③$$

* $\frac{dF}{F} = \frac{-\mu F d\alpha}{F} = -\mu d\alpha$ integrating both sides

$$\int_{F_1}^{F_2} \frac{dF}{F} = \int_0^{\alpha} -\mu d\alpha \Rightarrow [\log_e F]_{F_1}^{F_2} = [\mu\alpha]_0^{\alpha} \Rightarrow \log_e\left(\frac{F_2}{F_1}\right) = -\mu\alpha \Rightarrow \frac{F_2}{F_1} = e^{-\mu\alpha} \Rightarrow \boxed{F_2 = F_1 e^{-\mu\alpha}} \text{ — } ④$$

* ~~For section~~ For tendons with a succession of curves of varying radii, this formula is applied to diff sections to get the total loss.

(b) wobble effect :-)

* As this is length effect, so length of cable (L) is used here.

* In place of 'μα' we can use KL in eq ④

$$\therefore \boxed{F_2 = F_1 e^{-KL}} \text{———— ⑤}$$

* combining the above curvature & wobble effects, we get
Previously, $\left[\log_e \frac{F_2}{F_1} = -\mu\alpha\right]$, now $\left[\log_e \frac{F_2}{F_1} = -KL\right]$ combining these two, [illegible] $\left[\log_e \frac{F_2}{F_1} = -\mu\alpha - KL\right]$

$$F_2 = F_1\left[e^{-\mu\alpha} \times e^{-KL}\right] = F_1\, e^{-\mu\alpha - KL} \text{———— ⑥}$$

* If P_0 = prestressing force at jacking end

P_x = prestressing force at a distance 'x' from tensioning end

μ = coefficient of friction betn tendon & duct

K = friction coefficient for wave effect per unit length.

α = cumulative angle in radians through which tangents to cable profile has turned betn any two points under consideration.

e = 2.7183

Using P_0 & P_x in eq ⑥ we get, $\boxed{P_x = P_0 e^{-(\mu\alpha + Kx)}}$ ———— ⑦

According to Indian code

μ = 0.55 for steel moving on smooth concrete

= 0.35 " " " " steel fixed to duct

= 0.25 " " " " steel fixed to concrete

= 0.25 " " " " lead

= 0.18 to 0.3 for multi-layer wire rope cable in rigid rectangular steel sheath

K = 0.15 /100 m for normal condition.

= 1.5 /100 m for thin wall duct where heavy vibration is observed

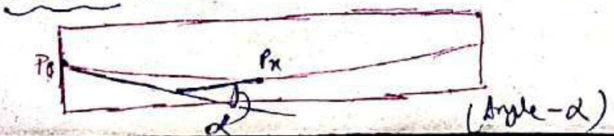
= 0 where clearance betn duct & cable is sufficiently large to ~~[illegible]~~ eliminate wave effect

* μ can be reduced by using lubricants such as grease, oil, paraffin [illegible] & graphite mixture & Teflon.

* Teflon & paraffin are best lubricants.

* paraffin coating gives lowest coefficient of friction ~~[illegible]~~ with high contact pressure.

-) paraffin ~~is~~ harmless to concrete & grout



Prob-9. A concrete beam of 10m span, 100mm wide & 300mm deep is prestressed by 3 cables. The area of each cable is 200 mm² & initial stress in the cable is 1200 N/mm². cable-1 is parabolic with eccentricity of 50mm above the centroid at supports & 50mm below at centre of span. cable-2 is also parabolic with zero eccentricity at supports & 50mm below the centroid at the centre of span. cable-3 is straight with uniform eccentricity of 50mm below the centroid. If cables are tensioned from one end only, estimate the %age loss of stress in each cable due to friction.

Assume $\mu = 0.35$ & $K = 0.0015$/meter.

sol→ Eq of parabola is given by, $\boxed{y = \frac{4h}{l^2} x(l-x)}$

$$\frac{dy}{dx} = \frac{4h}{l^2}(l-2x) \text{ —— (slope eq)}$$

slope at ends $(x=0)$ → $\boxed{\frac{dy}{dx}_{(x=0)} = \frac{4h}{l}}$

for cable-1 → $h = 50+50 = 100 \text{ mm} = 10 \text{ cm}.$

$$\therefore \frac{dy}{dx} = \frac{4 \times 10}{10 \times 100} = 0.04$$

cumulative angle betⁿ tangents, $\alpha = 2 \times 0.04 = 0.08$ radians.

for cable-2 → $h = 50 \text{ mm} = 5 \text{ cm}.$

$$\frac{dy}{dx} = \frac{4 \times 5}{10 \times 100} = 0.02, \quad \alpha = 2 \times 0.02 = 0.04 \text{ radian}.$$

As cumulative angle betⁿ supports in each cable is calculated, so loss betⁿ supports should be taken.

$\boxed{\text{initial prestressing force, } P_0 = 1200 \times 200 = 24,0000 \text{ N.}}$

At farther end $(x=l)$, $P_x = P_0\left[e^{-(\mu\alpha + kx)}\right] = P_0 e^{-(\mu\alpha + kl)}$

For small values of $(\mu\alpha + kl)$, $P_x = P_0[1 - (\mu\alpha + kl)]$

$$= P_0 - P_0(\mu\alpha + kl)$$

$$\rightarrow \boxed{P_0 - P_x = P_0(\mu\alpha + kl)} \rightarrow \text{stress loss } (\Delta P)$$

cable-1 → $\Delta P = P_0(0.35 \times 0.08 + 0.0015 \times 10) = 0.043 P_0$

cable-2 → $\Delta P = P_0(0.35 \times 0.04 + 0.0015 \times 10) = 0.029 P_0$

cable-3 → $\Delta P = P_0(0.35 \times 0 + 0.0015 \times 10) = 0.015 P_0$ (for straight tendon, $\alpha = 0$)

initial / Jacking stress, $P_0 = 1200 \text{ N/mm}^2$

cable-1, $\Delta P = 0.043 P_0 = 51.6 \text{ N/mm}^2$, %age loss $= \frac{51.6}{1200} \times 100 = 4.3\%$.

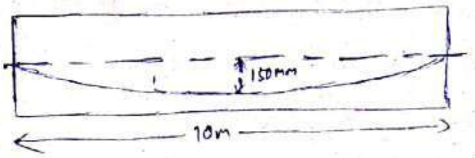
cable-2, $\Delta P = 0.029 P_0 = 34.8 \text{ N/mm}^2$, %age loss $= \frac{34.8}{1200} \times 100 = 2.9\%$.

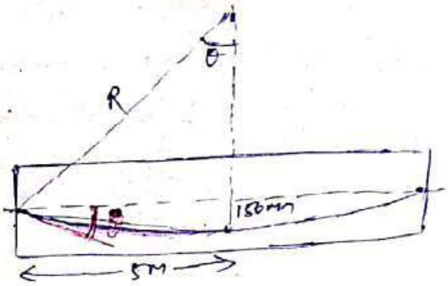
cable-3, $\Delta P = 0.015 P_0 = 18 \text{ N/mm}^2$, %age loss $= \frac{18}{1200} \times 100 = 1.5\%$.

Prob-10 — A post tensioned concrete beam, 200 mm wide, 450 mm deep is prestressed by a circular cable (total area = 800 mm²) with zero eccentricity at ends & 150 mm at centre. The beam span is 10m. The cable is stressed from one end such that an initial stress of 840 N/mm² is available in the unjacked end immediately after anchoring. Determine the stress in wires at jacking end & %age loss of stress due to friction.

$\mu = 0.6$, $K = 0.003$/meter.

Sol:)





R = radius of circular cable.

$(R-0.15)^2+5^2=R^2 \Rightarrow R = 84m.$

θ = angle betn horizontal & tangent to cable at support

$\sin\theta = \frac{5}{84} \Rightarrow \theta = 0.06$ radian

cumulative angle betn tangents to cable at supports $\alpha = 2\times0.06 = 0.12$ rad

As cumulative angle is taken betn supports, so prestress loss betn supports is to be evaluated.

P_x = stress at unjacked end = 840 N/mm² (at x = l)

P_0 = initial stress at jacking end (x = 0)

$P_x = P_0 e^{-(\mu\alpha+kl)} = P_0[1-(\mu\alpha+kl)]$

$\Rightarrow 840 = P_0[1-(0.6\times0.12+0.003\times10)] = 0.898P_0 \Rightarrow P_0 = 940$ N/mm²

loss of stress = 940 − 840 = 100 N/mm²

%age loss of stress = $\frac{100}{940}\times100 = 10.6\%$. (Ans)

Prob-11 A cylindrical concrete tank 40m external dia, is to be prestressed circumferentially by a high strength steel wires ($E_s = 210$ kN/mm²) jacked at 4 points, 90° apart. If minimum stress in wires immediately after tensioning is to be 600 N/mm² & $\mu = 0.5$ calculate

(a) max stress to be applied to the wires at the jack &

(b) the expected extension ~~of~~ at the jack.

Sol-) If P_x = prestressing force at farther end

P_0 = prestressing force at jacking end

* we can neglect wobble effect as it is declared that it is a cylindrical structure & so K can be neglected

so $\boxed{P_x = P_0 e^{-\mu\alpha}} \Rightarrow 600 = P_0 e^{-(0.5\times\pi/2)}$ where e = 2.7183 & $\alpha = \frac{\pi}{2}$

$\Rightarrow P_0 = 1320$ N/mm²

Average stress in wires = $\frac{1320+600}{2} = 960\ N/mm^2$

Length of wires = $\frac{\pi \times 40 \times 1000}{4} = 10^4 \pi$ mm.

Extension at jack = $\frac{\sigma_{wire}}{E_s} \times$ length of wire = $\frac{960}{210 \times 10^3} \times 10^4 \pi = 144$ mm (Ans)

(6) Loss due to Anchorage slip ⇒

* In post-tensioning systems, when cables are tensioned & jack is released to transfer prestress to concrete, the friction wedges employed to grip the wires, slip over a small distance before the wires are firmly housed betn the wedges.

* The anchorage fixtures subjected to stresses at the transfer of prestress, will tend to deform & allowing the tendon to slacken slightly.

* The magnitude of slip for friction wedges depend on
type of wedge & stress in the wire.

* When anchor plates are used, small settlement of plate is necessary to be allowed

* For direct bearing anchorages, the heads & nuts are subjected to slight deformation, at the release of jack.

* The loss during anchoring which occurs with wedge type grips is normally allowed in site by over extending the tendon by prestressing operation by the amount of draw-in before anchoring.

* But the momentary overstress here ≯ 80% - 85% of fpu.

* Hard & smooth wires may not immediately grip the steel before it was slipped & so large anchorage slip is possible for such wires.

* If Δ = anchorage slip in mm
l = length of cable in mm
A = c/s area of cable in mm²
Es = mod of elasticity of steel, N/mm²
P = Prestressing force in cable, N.

$\boxed{\Delta = \frac{Pl}{AE_s}}$ ——— ⑧

Loss of stress due to anchorage slip

$= \frac{P}{A} = \boxed{\frac{E_s \Delta}{l}}$

* Since loss of stress is caused by a definite amount of shortening, the %age loss of prestress is higher for small members.

* In long-line pre tensioning system, slippage at anchorage is very small w.r.t length of tendon & so generally ignored.

Q-12 A cone beam is post-tensioned by a cable carrying an initial stress of 1000 N/mm2. The slip at jacking end is 5mm. $E_s = 210 \text{ KN/mm}^2$ Estimate %age loss of stress due to anchorage slip if length of beam is 30m.

sol →) loss of stress due to anchorage slip $= \frac{E_s \Delta}{\lambda} = \frac{210\times10^3\times5}{30\times10^3} = 35 \text{ N/mm}^2$

∴ %age loss of stress $= \frac{35}{1000}\times100 = 3.5\,\%$.

Q-13 A post tensioned cable of beam 10m long is initially tensioned to a stress of 1000 N/mm2 at one end. If tendons are curved so that slope is 1 in 24 at each end, with an area of 600 mm2, calculate the loss of prestress due to friction. $\mu = 0.55$, $K = 0.0015$/meter; $\Delta = 3$mm

calculate the final force in cable & the %age loss of prestress due to friction & slip. $E_s = 210 \text{ KN/mm}^2$.

sol →) total change in slope from end to end, $\alpha = 2\times\frac{1}{24} = \frac{1}{12}$

Loss of prestress due to friction $= P_0(\mu\alpha + kx)$

$= 1000\left(0.55\times\frac{1}{12} + 0.0015\times10\right) = 61 \text{ N/mm}^2$

* prestressing force in each cable $= 1000\times600\times10^{-3} = 600$ KN.

Force in cable corresponding to slip of 3mm

$\Delta = \frac{P\lambda}{AE} \Rightarrow P = \frac{\Delta A E}{\lambda} = \frac{3\times600\times210\times10^3}{10\times1000} = 37800 \text{ N} = 37.8 \text{ KN}$

Loss of force due to friction $= 61\times600 = 36600 \text{ N} = 36.6$ KN.

Total loss of force due to friction & slip $= 36.6 + 37.8 = 74.4$ KN

Final force in cable $= 600 - 74.4 = 525.6$ KN

%age loss of prestressing force $= \frac{74.4}{600}\times100 = 12.4\,\%$.

(7) LOSS / GAIN OF PRESTRESS DUE TO BENDING

- Loss of prestress due to elastic deformation of concrete is because of a UNIFORM SHORTENING of member is done under AXIAL STRESS.
- when member bends, loss or gain of prestress may occur depending on
  - the direction of bending (i.e. concave/convex upward)
  - & the location of tendon.
- If several tendons are located at different levels, then change in prestress in them differs & so CENTROID OF ALL THE TENDONS (THE C.G.S LINE) is only considered to get an average value of change in prestress.
- The change in prestress due to bending depends on TYPE OF PRESTRESSING (i.e. pre or post tensioned, bonded or unbonded)

For Bonded members → (i.e. pre tensioned member/post tensioned grouted member)

- For Bonded members, bending of beam only affects local change in tendon stress, but prestress applied is not changed.
- For a S/S PSC beam (bonded), before application of external load, it cambers, & after application of exter load it deflects downward as external load creats B.M in beam.

Bending in beam changes stresses & strains in tendons. stresses in tendons near midspan changes rapidly but at ends don't change as no change in B.M occures there.

If 'prestress' from steel on concrete is considered to be a force applied at ends, then stress will change along its length but not 'PRESTRESS'. Because for bonded PSC structures, steel & concrete form one section & any change in stress due to bending of section is computed by transformed sec method.

For bonded members, when beam bends upward after transfer of prestress, the tendons shorten due to bending ~~from the~~ but prestress won't be lost & similarly lengthening of tendons when beam bends downward due to external load, prestress can't be gained due to above reason. In both cases, the prestress ~~co~~ is considered at the ends of the members which does not change under bending.

For unbonded beam → (post tensioned beam before grouting)

- Due to bending of beam, tendon will be stretched out along its entire length & prestress in the tendon will be uniformly modified & got affected by bending.
- If tendons are tensioned one by one, the beam cambers upward gradually as more tendons are tensioned. Tendons that are tensioned first will lose prestress due to bending & elastic deformation of concrete due to axial pre compression.

when camber is appreciable, it is required to

→ retension the tendons after completing the Ist round of tensioning, or

→ allow for these losses in design.

* creep in concrete increases curvature of beam & as curvature at the time of grouting determines the length of tendons & so creep is considered when change in prestress is allowed

* { when beam is curved upward by prestressing, prestress in beam may be lost.
  when beam is curved downward by external loading, prestress may be gained.

* If tendon does not remain at a constant distance from the C.G.C LINE (centre of gravity of concrete section), then computation of change in length of tendons on bending of beam will be complecated when tendons are permitted to slide freely within the concrete.

* Loss ore gain of prestress due to bending ≯ 2% to 3% of initial prestress.

Prob-14 => A concrete beam of sec (200 mm x 450 mm) with an unbonded tendon is prestressed through the lower third point with a prestressing force of 654 KN. Compute the prestress loss in tendon due to bowing up of beam under prestress neglecting the wt of the beam. Beam is simply supported.

$E_s = 207\ KN/mm^2$, $E_c = 27.6\ KN/mm^2$

Sol => Due to eccentric prestress, beam is under uniform BM.

$$M = \frac{654\times10^3\times(225-150)}{10^6} = 49.05\ KNm$$

concrete fibre stress at the level of cable due to bending, $\sigma_c = \frac{My}{I}$

$$I = \frac{200\times450^3}{12} = 1518.75\times10^6\ mm^4, \quad y = 225-150 = 75\ mm.$$

$$\therefore \sigma_c = \frac{My}{I} = 2.4\ N/mm^2, \quad \alpha = \frac{E_s}{E_c} = 7.5$$

* Stress due to axial prestress is not included only due to eccentricity is taken.

prestress loss due to bending $= \alpha\sigma_c = 7.5\times2.4 = 18\ N/mm^2$

* If beam is left under prestress alone, the creep of concrete tends to increase camber & results further prestress loss.

* If prestress in tendon is measured after the bowing of beam has taken place, then loss due to bending of beam need not be considered.

(8) Total Amount of Losses. →

- Effective prestress/Design prestress = Initial prestress − Losses (in steel)
- In design of PSC members, losses of stress is assumed a %age of initial stress
- Temporary max jacking stress → It is the stress to which a tendon may be subjected to minimizing creep in steel/to balance frictional losses.

Jacking stress (normal) → A slight release from max jacking stress give rise to normal jacking stress.

Initial prestress → stress at anchorage after release from jacking stress or when prestress is transferred to concrete, anchorage loss occurs.

[Initial prestress = Jacking stress − Anchorage loss]

- In post tensioning, if tendons are successively tensioned, then losses due to elastic shortening will gradually take place.

Elastic shortening of concrete
  - due to direct axial shortening.
  - due to elastic bending.

- In pre tensioning, entire prestress loss due to elastic shortening will occur at transfer of prestress.
- Depending on defination of ~~[illegible]~~ "initial prestress", amount of losses to be deducted will differ to get effective prestress from initial prestress

If ~~at~~

(a) Initial prestress = Jacking stress − anchorage loss

→ losses to be deducted from initial prestress to get effective prestress are

- Elastic shortening, creep & shrinkage in concrete + creep in steel.

(b) If initial prestress = Jacking stress

→ losses to be deducted from initial prestress to get effective prestress are

- only Anchorage loss.

(c) If initial prestress = stress after elastic shortening of concrete.

→ losses to be deducted from initial prestress to get effective prestress are

- creep & shrinkage in concrete + Relaxation in steel.

- For points away from jacking end, frictional loss is also considered.
- magnitude of losses is expressed in different ways :-

(a) In unit strains → This is convenient for losses due to creep, shrinkage & elastic deformation of concrete.

(b) In total strains → This is convenient for anchorage loss.

(c) In %age of prestress → This is convenient for losses due to creep in steel & frictional loss.

* It is difficult to calculate the exact amount of prestress loss, since prestress loss depends upon several factors such as

Properties of concrete & steel, method of curing

Degree of prestress, method of prestressing.

* For average steel & concrete properties, cured under average air conditions the total losses of stress is as follows.

Type of Loss	Pretensioning (%age)	Post-tensioning (%age)
Elastic shortening & bending of concrete	4%	1%
creep of concrete	6%	5%
shrinkage of concrete	7%	6%
Relaxation of steel	8%	8%
TOTAL LOSS	25%	20%

* The above table assumes that proper overtensioning is applied to reduce creep in steel & to overcome friction & anchorage loss.

If f_{pe} = effective stress in tendons after losses

f_{pi} = stress in tendon at transfer,

η = reduction factor for loss of prestress, $\boxed{\eta = \frac{f_{pe}}{f_{pi}}}$

generally, $\eta = 0.75$ for pretensioned member

$= 0.8$ for post tensioned member.

Q-15 A pretensioned beam (200 mm wide × 300 mm depth) is prestressed by 10 wires of 7 mm dia initially stressed to 1200 N/mm², with their centroids located 100 mm from the soffit. Find max stress in concrete immediately after transfer allowing only for elastic shortening of concrete.

If concrete undergoes a further shortening due to creep & shrinkage & relaxation loss of 5% of steel stress, estimate the final %age loss of stress in wires using IS 1343-1980.

$E_s = 210\ kN/mm^2$, $E_c = 36.9\ kN/mm^2$, $f_{cu} = 42\ N/mm^2$, $\phi = 1.6$, $\epsilon_{cs} = 3\times10^{-4}$

Sol→ $A = 200\times300 = 6\times10^4\ mm^2$, $I = \frac{bd^3}{12} = \frac{200\times300^3}{12} = 45\times10^7\ mm^4$, $\alpha = \frac{E_s}{E_c} = 5.7$

Initial prestressing force in wires, $P = 1200\times(10\times38.5) = 462\times10^3\ N = \underline{462\ kN}$.

stress in conc at the level of steel, $\sigma_c = \frac{462\times10^3}{6\times10^4} + \frac{462\times10^3\times50\times50}{45\times10^7} = 10.3\ N/mm^2$

stress loss due to elastic deformation of concrete $= \alpha\sigma_c = 10.3\times5.7 = 58.8\ N/mm^2$

Force in wires immediately after transfer $= (1200-58.8)(10\times38.5)\times10^{-3} = \underline{440\ kN}$

After loss for elastic deformation, $\sigma_c = \frac{440\times10^3}{6\times10^4} + \frac{440\times10^3\times50\times50}{45\times10^7} = 9.78\ N/mm^2$

Loss of prestress

(a) elastic deformation - - - - - - - - - - - - - $= 58.8\ N/mm^2$

(b) creep of concrete $= \phi \alpha \sigma_c = 1.6 \times 5.7 \times 9.78$ = ——— - - $= 89.2\ N/mm^2$

(c) shrinkage of concrete $= E_s \times \epsilon_{cs} = 210 \times 10^3 \times 3 \times 10^{-4}$ = —— $= 63\ N/mm^2$

(d) Relaxation of steel stress $= 5\% \times 1200$ = ——— - - — $= 60\ N/mm^2$.

$\therefore$ Total loss $= 271\ N/mm^2$.

Final stress in wires: $1200 - 271 = 929\ N/mm^2$.

%age loss $= \frac{271}{1200} \times 100 = 22.58\%$. (Ans)

Q-16 A prestressed concrete pile, 250mm square, contains 60 pre-tensioned wires, each of 2mm diameter, uniformly distributed over the sec. The wires are initially tensioned on the prestressing bed with a total force of 300kN. calculate the final stress in concrete & the %age loss of stress in steel after all losses.

$E_s = 210\ kN/mm^2$, $E_c = 32\ kN/mm^2$

Shortening due to creep, $\epsilon_{cc} = 30 \times 10^{-6}$ mm/mm per N/mm^2 of stress

Total shrinkage, $\epsilon_{cs} = 200 \times 10^{-6}$ /unit length

Relaxation of steel stress = 5% of initial stress

Sol -) Average initial stress in concrete, $\sigma_c = \frac{P}{A} = \frac{300 \times 10^3}{250 \times 250} = 4.8\ N/mm^2$

$\alpha = \frac{E_s}{E_c} = \frac{210}{32} = 6.58$

initial stress in steel wires $= \frac{300 \times 10^3}{60 \times \frac{\pi}{4} 2^2} = 1590\ N/mm^2$

Losses of stress

(a) Elastic deformation $= \alpha \sigma_c = 6.58 \times 4.8 = 31.5\ N/mm^2$

(b) creep of concrete $= \epsilon_{cc} \sigma_c E_s = (30 \times 10^{-6}) \times (4.8) \times (210 \times 10^3) = 30\ N/mm^2$

(c) Shrinkage of concrete $= \epsilon_{cs} \times E_s = (200 \times 10^{-6}) \times (210 \times 10^3) = 42\ N/mm^2$

(d) Relaxation stress $= \frac{5}{100} \times 1590 = 79.5\ N/mm^2$

Total loss $= 31.5 + 30 + 42 + 79.5 = 183\ N/mm^2$

Effective prestress $= 1590 - 183 = 1407\ N/mm^2$ in steel

Final stress in concrete $= 1407 \times \frac{\text{steel area}}{\text{conc area}} = 1407 \times \frac{60 \times \frac{\pi}{4} \times 2^2}{250 \times 250} = 4.26\ N/mm^2$

%age loss of stress in steel $= \frac{183}{1590} \times 100 = 11.6\%$.

Q-17 A prestressed concrete beam (200 mm wide × 300 mm deep), is prestressed with wires (area = 320 mm²) located at an eccentricity of 50 mm & carrying an initial stress of 1000 N/mm². Beam span = 10 m. calculate the %age loss of stress in wires if beam is (a) pretensioned (b) post tensioned.

$E_s = 210\ kN/mm^2$, $E_c = 35\ kN/mm^2$

Relaxation of steel stress = 5% of initial stress

Shrinkage of concrete = $\epsilon_{cs} = 300 \times 10^{-6}$ (pretensioning) & 200×10^{-6} (post tensioning)

creep coefficient $\phi = 1.6$, slip at anchorage = 1 mm.

Friction coefficient for wave effect $K = 0.0015$/meter.

sol → prestressing force $= 1000 \times 320 \times 10^{-3} = 320\ kN$.

c/s area, $A = 200 \times 300 = 6 \times 10^4\ mm^2$, $\alpha = \frac{E_s}{E_c} = 6$

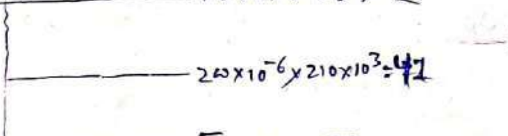
$I = \frac{200 \times 300^3}{12} = 45 \times 10^7\ mm^4$

stress in concrete at the level of steel, $\sigma_c = \frac{320 \times 10^3}{6 \times 10^4} + \frac{320 \times 10^3 \times 50 \times 50}{45 \times 10^7} = 7\ N/mm^2$

CALCULATION OF LOSSES IN

Type of loss	Pre tensioned beam (N/mm²)	post tensioned beam (N/mm²)
1) Elastic deformation (concrete)	$\alpha\sigma_c = 6 \times 7 = 42$	—
2) creep of conc	$\phi\alpha\sigma_c = 1.6 \times 6 \times 7 = 67.2$	$1.6 \times 6 \times 7 = 67.2$
3) shrinkage of conc	$\epsilon_{cs} E_s = 300 \times 10^{-6} \times 210 \times 10^3 = 63$	$200 \times 10^{-6} \times 210 \times 10^3 = 42$
4) Relaxation of stress (steel)	5% of 1000 = 50	$\frac{5}{100} \times 1000 = 50$
5) slip of anchorage	—	$\frac{E\Delta}{l} = \frac{210 \times 10^3 \times 1}{10 \times 10^3} = 21$
6) Friction loss	—	$P_0 K x = 1000 \times 0.0015 \times 10 = 15$
Total loss	$= 222.2\ N/mm^2$	$= 195.2\ N/mm^2$
%age loss	$= \frac{222.2}{1000} \times 100 = 22.22\%$	$= \frac{195.2}{1000} \times 100 = 19.52\%$

Q-18 A conc beam AB of span 12m is post tensioned by a cable which is concentric at supports A & B & has an eccentricity of 200mm in mid third span with a linear variation towards the supports. If cable is tensioned at jacking end A, what should be the jacking stress in the wires if the stress at B is 1000 N/mm²? ~~Also~~ $\mu = 0.55$ & $K = 0.0015$/meter.

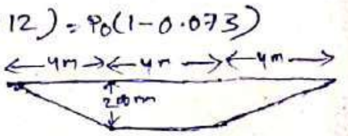


sol

~~slope~~ st. slope of cable at $A = \frac{200}{4000} = 0.05$

Total change of slope of cable from A to B $= \alpha = 2 \times 0.05 = 0.1$

$P_x = P_0(1 - \mu\alpha - Kx) \Rightarrow 1000 = P_0(1 - 0.55 \times 0.1 - 0.0015 \times 12) = P_0(1 - 0.073)$



$\Rightarrow 1000 = 0.927 P_0 \Rightarrow P_0 = \frac{1000}{0.927} = 1078.7\ N/mm^2$ (Ans)

Q-19 A conc beam AB of effective span 20m is post tensioned by a cable which is concentric at supports A & B with eccentricity of 400mm for a length of 10m in mid span zone. The cable is horizontal in the middle 10m portion & has a parabolic profile in remaining 5m near supports. calculate stress in cable at B if jacking stress at A is 1200 N/mm2. what will the minimum stress in cable if it is tensioned from both ends with a jacking stress of 1200 N/mm2.

$\mu = 0.35$, $K = 0.002$/meter.

Sol →) ~~slope of cable at A & B = $\frac{4h}{\lambda^2}x(L-x)$, at x = 0, slope =~~

slope of cable at A & B $= \frac{4h}{\lambda} = \frac{4 \times 400}{20 \times 10^3} = 0.16$

In middle 10m portion, the cable is horizontal & so no change of slope.

Total change of slope of cable from A to B $= \alpha = 2 \times 0.16 = 0.32$

Loss of stress from A to B $= P_0(\mu\alpha + kx) = 1200(0.35 \times 0.32 + 0.002 \times 20) = 182.4$ N/mm²

Stress in cable at B $= 1200 - 182.4 = 1017.6$ N/mm²

ⓑ If cable is tensioned from both ends, loss of stress is reduced by 50%.

So minimum stress at centre of span $= 1200 - 0.5 \times 182.4 = 1108.8$ N/mm²

Advanced Concrete Structure

PRESTRESSED CONCRETE

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ANALYSIS FOR FLEXURE.

- Analysis of PSC section means determination of stresses in steel & concrete when the form & size of a section is already given or assumed.
- Design of the section involves the choice of a suitable section out of many possible shapes & dimensions.
- Generally design is first by assuming the section, then analysis is done of the assumed section.
- But for study purpose, Analysis is done first & then Design is done.
- In this chapter, analysis of section for beam & slab under flexure i.e. bending is done only due to moment, but effect of shear & bond is not considered.

Assumptios in the Analysis of section –)

- concrete is homogeneous elastic material.
- within working stresses, both conc & steel behave elastically not resisting creep in both materials under sustained loading.
- plane sec remains plain before bending & after bending ⇒ linear strain distribution across the depth of the member.

- modulus of rupture → corresponds to stage of visible cracking of concrete.

The stress at which a particular beam fails in bending when tested to destruction.
Such stress = B.M at failure / calculated section modulus at c/s where failure occurs.

- until tensile stress in concrete < mod of rupture,

any change in loading of member results in change of stress in concrete only, only function of prestressing tendon is to impart & maintain prestress in concrete.
∴ change in stress of steel being small is not considered in calculation.

Analysis of Prestress –)

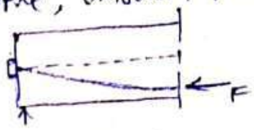
- stresses due to prestressing = stress due to direct load + bending from eccentricity of applied prestress.

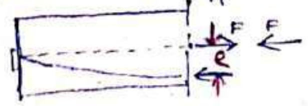
If, P = prestressing force (+ve if produces compression)
e = eccentricity of P measured from the centroidal axis of the section.
M = Pe moment produced by P.
A = C/S area of conc section of member.
I = M.I of sec about its centroid.
Z_t, Z_b = section modulus at top & bottom fibres of sec.
σ_{sup} & σ_{inf} = developed prestress in conc at top & bottom fibre (+ve = comp, -ve tensile)
y_t / y_b = distance of top & bottom fibre from centroid of section.
r = radius of gyration.

(a) concentric ~~section~~ tendon –)

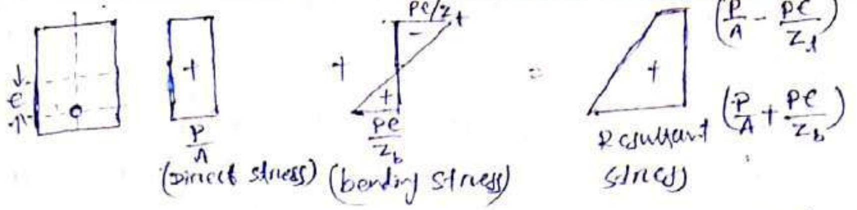
- by St venant's principle, uniform prestress in conc = $\frac{P}{A}$ (comp = +ve)

(b) eccentric tendon –)





- Eccentric tendon balances tensile stresses developed near soffit by external applied load & dead load of beam.



$$\sigma_{inf} = \frac{P}{A} + \frac{Pe}{Z_b} = \frac{P}{A} + \frac{Pey_b}{I} = \frac{P}{A} + \frac{Pey_b}{Ar^2} = \frac{P}{A}\left[1 + \frac{ey_b}{r^2}\right]$$

$$\sigma_{sup} = \frac{P}{A} - \frac{Pe}{Z_t} = \frac{P}{A} - \frac{Pey_t}{I} = \frac{P}{A} - \frac{Pey_t}{Ar^2} = \frac{P}{A}\left[1 - \frac{ey_t}{r^2}\right]$$

(c) Resultant stress at a section (Prestress + applied load + dead load)

* Due to external load of intensity w_1 & dead load w_2 & eccentric prestressing force P at eccentricity e, the resultant stress is given by,

$$\boxed{\sigma_{sup} = \left(\frac{P}{A} - \frac{Pey_t}{I}\right) + \frac{M_1}{Z_t} + \frac{M_2}{Z_t}}, \quad \boxed{\sigma_{inf} = \left(\frac{P}{A} + \frac{Pey_b}{I}\right) - \frac{M_1}{Z_b} - \frac{M_2}{Z_b}}$$

$$\boxed{M_1 = \frac{w_1 l^2}{8}}\ \boxed{M_2 = \frac{w_2 l^2}{8}}$$

Q-1 A rectangular concrete beam of c/s (300mm deep × 200mm wide) is prestressed by 15wires of 5mm ϕ located at 65mm from bottom of beam & 3 wires of 5mm dia located at 25mm from the top. If prestress in steel is 840 N/mm², calculate the stresses at extreme fibres of mid span sections when the beam is supporting its own wt over a span of 6m. If a UDL (live load) of 6kN/m is imposed, evaluate the max. working stress in concrete. Density of concrete is 24 kN/m³.

sol → Distance of ~~prestre~~ centroid of prestressing force from the base

$y = \frac{15 \times 65 + 3 \times 275}{15+3} = 100\,mm$, $A = 200 \times 300 = 6 \times 10^4\,mm^2$

$\therefore e = 150 - 100 = 50mm$ $\quad I = \frac{200 \times 300^3}{12} = 45 \times 10^7\,mm^4$

$P = 840 \times 18 \times (\frac{\pi}{4} \times 5^2) = 3 \times 10^5\,N.$ $\quad Z_t \text{ or } Z_b = \frac{I}{y} = \frac{45 \times 10^7}{150} = 3 \times 10^6\,mm^3$

self wt of beam, $w_1 = 24 \times (0.3 \times 0.2) = 1.44\,kN/m$, $M_1 = \frac{w_1 l^2}{8} = \frac{1.44 \times 6^2}{8} = 6.48\,kNm.$

$w_2 = 6\,kN/m$, $M_2 = \frac{w_2 l^2}{8} = \frac{6 \times 6^2}{8} = 27\,kNm.$

Direct stress due to prestress $= \frac{P}{A} = \frac{3 \times 10^5}{6 \times 10^4} = 5\,N/mm^2\ (+)$

Bending stress due to prestress, $\frac{Pe}{Z} = \frac{3 \times 10^5 \times 50}{3 \times 10^6} = 5\,N/mm^2\ (\mp)$

stress due to self wt $= \frac{M_1}{Z} = \frac{6.48 \times 10^6}{3 \times 10^6} = 2.16\,N/mm^2\ (\pm)$

live load stress $= \frac{M_2}{Z} = \frac{27 \times 10^6}{3 \times 10^6} = 9\,N/mm^2\ (\pm)$

Resultant stress at top fibre $= 5 - 5 + 2.16 + 9 = 11.16\,N/mm^2$ (comp)

at bottom fibre $= 5 + 5 - 2.16 - 9 = -1.16\,N/mm^2$ (tension)

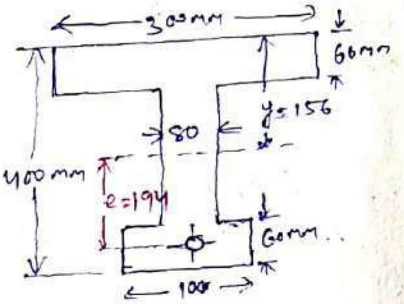
∴ Max working stress in concrete is 11.16 N/mm² which is compressive.

Q-2 An unsymmetrical I-sec beam is used to support an imposed load of 2kN/m over a span of 8m. The sectional dimensions of beam is given. At central span, the effective prestressing force of 100kN is located at 50mm from the soffit of the beam. Estimate the stresses at central span section of beam for →

(a) prestress + self wt (b). prestress + self wt + live load.

Sol → $P = 100\,kN.$

$A = 300 \times 60 + 100 \times 60 + 80 \times 280 = 46400\,mm^2$



Distance of centroid of sec from top fibre

$$y = \frac{300 \times 60 \times 30 + 100 \times 60 \times 370 + 80 \times 280 \times 200}{46400} = 156\,mm$$

$e = 400 - 156 - 50 = 194\,mm.$

$I = 75.8 \times 10^7\,mm^4$

$$Z_t = \frac{75.8 \times 10^7}{y_t} = \frac{75.8 \times 10^7}{156} = 485 \times 10^4\,mm^3, \quad Z_b = \frac{75.8 \times 10^7}{400 - 156} = 310 \times 10^4\,mm^3$$

Self wt, $w_1 = 24 \times (46400 \times 10^{-6}) = 1.12\,kN/m.$

$$M_1 = \frac{wl^2}{8} = \frac{1.12 \times 8^2}{8} = 8.96\,kNm$$

$$M_2 = \frac{2 \times 8^2}{8} = 16\,kN/m.$$

stresses at centre of span

Type of stress	At top fibre (N/mm²)	At bottom fibre (N/mm²)
prestress	$\frac{P}{A} = 2.15$	$\frac{P}{A} = 2.15$
	$\frac{Pe}{Z_t} = -4$	$\frac{Pe}{Z_b} = 6.25$
self wt stress	$\frac{M_1}{Z_t} = 1.85$	$\frac{M_1}{Z_b} = -2.9$
Live load stress	$\frac{M_2}{Z_t} = 3.3$	$\frac{M_2}{Z_b} = -5.15$

Resultant stress, due to

(a) prestress + self wt $= 2.15 - 4 + 1.85 = 0$ (at top)

$= 2.15 + 6.25 - 2.9 = 5.5\,N/mm^2$ (at bottom)

(b) prestress + self wt + live load $= 0 + 3.3 = 3.3\,N/mm^2$ (at top)

$= 5.5 - 5.15 = 0.35$ (at bottom)

Q-3 A rectangular conc beam (250mm wide, 600mm deep) is prestressed by 4, 14mm ϕ high tensile bars located 200mm from the soffit of the beam. If the effective prestress in the wires is 700 N/mm², what is the max BM that can be applied to the section without causing tension at the soffit of the beam.

Sol → $A = 250 \times 600 = 15 \times 10^4\,mm^2$, $A_s = 4 \times \frac{\pi}{4} \times 14^2 = 616\,mm^2$

$Z = \frac{bd^2}{6} = \frac{250 \times 600^2}{6} = 15 \times 10^6\,mm^3$, $e = 200\,mm$

$P = 700 \times 616 = 431200\,N.$, $\frac{P}{A} = 2.87\,N/mm^2$, $\frac{Pe}{Z} = 2.87\,N/mm^2$

prestress at the soffit of the beam $= 2.87 + 2.87 = 5.74\,N/mm^2$

If M = max moment on section for zero tension at the bottom face

$\frac{M}{Z} = 5.74 \Rightarrow M = Z \times 5.74 = 15 \times 10^6 \times 5.74 = 86.1 \times 10^6$ N.mm

$\Rightarrow M = 86.1$ KNM. (Ans)

Q-4 A PSC beam (200mm wide × 300mm deep) is used for effective span of 6m to support an imposed load of 4kN/m. concrete density = 24 kN/m3. At mid span sec of the beam, find (a) the concentric prestressing force required for zero fibre stress at soffit when beam is fully loaded (b) the eccentric prestressing force located at 100mm from bottom of beam which would nullify the bottom fibre stress due to loading.

sol → $A = 200 \times 300 = 6 \times 10^4$ mm^2 , $Z_b = Z_t = \frac{bd^2}{6} = \frac{200 \times 300^2}{6} = 3 \times 10^6$ mm^3

self wt, $w_1 = 24 \times 0.2 \times 0.3 = 1.44$ kN/m

$M_1 = \frac{w_1 l^2}{8} = \frac{1.44 \times 6^2}{8} = 6.48$ kN/m , $M_2 = \frac{w_2 l^2}{8} = \frac{4 \times 6^2}{8} = 18$ kNM

Tensile stress at bottom due to dead & live loads $= \frac{(6.484 + 18) \times 10^6}{3 \times 10^6} = 8.16$ N/mm^2 $\left(\frac{M}{Z}\right)$

(a) if P = concentric force for zero fibre stress at bottom of beam

then $\frac{P}{A} = 8.16 \Rightarrow P = 8.16 \times 6 \times 10^4 \times 10^{-3} = 489.6$ kN.

(b) if P = eccentric prestressing force for e = 50mm for zero stress at soffit

then $\frac{P}{A} + \frac{Pe}{Z} = 8.16 \Rightarrow \frac{P}{6 \times 10^4} + \frac{P \times 50}{3 \times 10^6} = 8.16 \Rightarrow P = 244.8$ kN.

* From the problem, advantage of eccentric prestressing in flexural members subjected to transverse loads is clearly indicated.

Q-5 A PSC beam (150mm wide × 300mm deep), prestressed by 4, 5mm ϕ high tensile wires stressed to 1200 N/mm². Eccentricity = 50mm. The stresses developed at soffit of beam will be examined by considering the

(a) nominal concrete (b) equivalent concrete section.

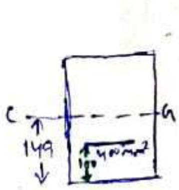
sol → (a) $A = 150 \times 300 = 45 \times 10^3$ mm^2 , $I = \frac{bd^3}{12} = \frac{150 \times 300^3}{12} = 3375 \times 10^5$ mm^4

$P = 1200 \times (4 \times \frac{\pi}{4} 5^2) = 96 \times 10^3$ N., e = 50mm, y = 150,

∴ stresses at soffit of section $= \frac{P}{A} + \frac{Pey}{I} = \frac{96 \times 10^3}{45 \times 10^3} + \frac{96 \times 10^3 \times 50 \times 150}{3375 \times 10^5} = 4.27$ N/mm^2

(b) For equivalent concrete section, let $\alpha = 6$

$A_e = A_g + (m-1) A_{st} = 45000 + (6-1)\left[4 \times \frac{\pi}{4} 5^2\right] = \overline{45000} + \overline{400} = 45{,}400$ mm^2



position of centroid from soffit of the equivalent section

$\bar{x} = \frac{45000 \times 150 + 400 \times 100}{45400} = 149$ mm

$I_e = 3375 \times 10^5 + [(150 \times 300) 1^2] + [400 \times (149 - 100)^2] = 3385 \times 10^5$ mm^4

stress at soffit $= \frac{P}{A_e} + \frac{Pey}{I_e} = \frac{96000}{45400} + \frac{96000 \times 49 \times 149}{3385 \times 10^5} = 4.2$ N/mm^2

diff in stresses betn two methods at soffit = 0.07.

%age diff $= \frac{0.07}{4.27} \times 100 = 1.64$ %.

Pressure line / Thrust line & Internal Resisting couple →

① Analysis of prestressed beams → Direct method of Analysis

Case-1 PSC beam prestressed by tendon provided through its CG axis

- Stress due to Prestress = $\boxed{\frac{P}{A} = \sigma_a}$

bending stress due to external load = $\boxed{\pm \frac{M}{Z} = \sigma_b}$

Q An PSC beam 400mm × 600mm in section has a span of 6m & is subjected to UDL of 16kN/m including self wt of beam. The tendons are located along centroidal axis providing an prestressing force of 960kN. Calculate the extreme fibre stresses at mid span section.

Sol →
$$A = 400 \times 600 = 2.4 \times 10^5 \text{ mm}^2$$
$$Z = \frac{bd^2}{6} = \frac{400 \times 600^2}{6} = 2.4 \times 10^7 \text{ mm}^3.$$
$$M = \frac{wl^2}{8} = \frac{16 \times 6^2}{8} = 72 \text{ kNm}.$$
$$\sigma_a = \frac{P}{A} = \frac{960 \times 10^3}{2.4 \times 10^5} = 4 \text{ N/mm}^2$$
$$\sigma_b = \pm \frac{M}{Z} = \pm \frac{72 \times 10^6}{2.4 \times 10^7} = 3 \text{ N/mm}^2$$

final stress at top fibre = 4+3 = 7 N/mm²

" " " bottom " = 4−3 = 1 N/mm².

Case-2 → PSC beam where tendon placed at eccentricity →

- direct stress due to prestressing force = $\frac{P}{A}$
- stress due to 'e' of prestress = $\mp \frac{Pe}{Z}$
- flexural stress due to applied UDL = $\pm \frac{M}{Z}$

Stress at top = $\frac{P}{A} - \frac{Pe}{Z} + \frac{M}{Z}$, Stress at bottom = $\frac{P}{A} + \frac{Pe}{Z} - \frac{M}{Z}$.

- The flexural stresses due to eccentricity on prestressing force counter-balances stresses due to external B.M.

Q A PSC beam 400mm × 600mm has a span of 6m, having UDL of 16kN/m including self wt. prestressing tendons are located at lower third point & P = 960 kN. Calculate fibre stresses at mid span.

Sol →
$$A = 400 \times 600 = 2.4 \times 10^5 \text{ mm}^2$$
$$Z = \frac{bd^2}{6} = \frac{400 \times 600^2}{6} = 2.4 \times 10^7 \text{ mm}^3$$

BM due to enternal loading = $M = \frac{wl^2}{8} = \frac{16\times6^2}{8} = 72$ kNm

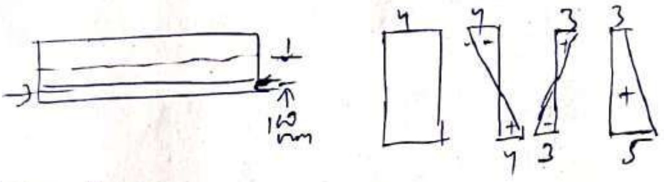
Direct stress due to P = $\frac{P}{A} = \frac{960\times10^3}{2.4\times10^5} = 4$ N/mm²

stresses due to e of P: $\mp\frac{Pe}{Z} = \mp\frac{960\times10^3\times100}{2.4\times10^7} = \mp 4$ N/mm²

Stresses due to enternal BM = $\frac{M}{Z} = \pm\frac{72\times10^6}{2.4\times10^7} = \pm 3$ N/mm²

∴ final stresses at top = 4 − 4 + 3 = 3 N/mm²

at bottom = 4 + 4 − 3 = 5 N/mm²

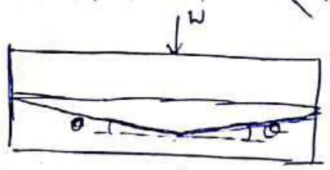


Case-3 → Prestressed beam with bent tendons

* By providing bent tendons tendons will exert an upward pressure on concrete beam.

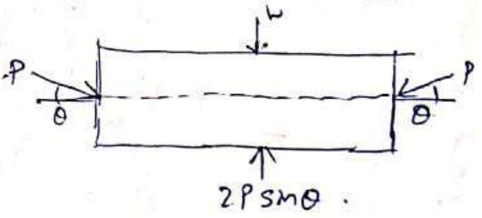
* This upward pre. counteracts a part of enternal down load.

* If tendons have a sharp bend,
friction force along the tendon = Zero.



* Axial longitudinal force by tendon = $P\cos\theta \approx P$

Direct stress on section ≃ $\frac{P}{A}$.



Net BM, $M = \frac{(W - 2P\sin\theta)l}{4} + \frac{wl^2}{8}$ (w = dead load/unit length of beam)

extreme stress = $\frac{P}{A} \pm \frac{M}{Z}$.

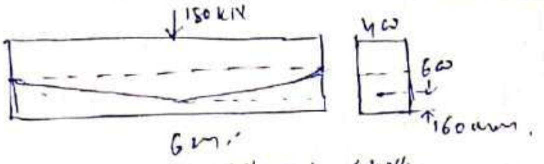
* profile of tendons should follow shape of BMD for given enternal load to offer effective upward force.

* If load is UDL, tendons should have parabolic profile
if load is concentrated, " " " sharp bend shape.

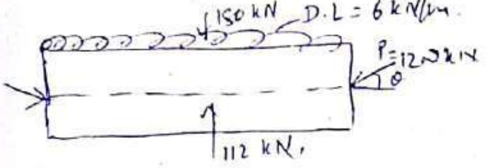
Case-3
- (a) bent straight tendon
- (b) bent parabolic tendon

Q ① (a) A PSC beam 400mm × 600mm, span 6m, tendon shape shown. concentrated load of 180KN at centre, if P = 1200 KN

Calculate extreme stresses at mid span.



$\tan\theta = \frac{140}{3000} = \frac{7}{150}$



* inclined tendon will provide an upward force at mid span

$= 2P\sin\theta = 2P\tan\theta = 2 \times 1200 \times \frac{7}{150} = 112 kN.$

Net vertical load at mid span = 180 − 112 = 68 KN

BM due to vertical load = $\frac{68 \times 6}{4}$ = 120 kNm.

Dead load of beam = 0.4 × 0.6 × 25 = 6 kN/m.

BM due to dead load = $\frac{6 \times 6^2}{8}$ = 27 kNm.

Total BM = 120 + 27 = 129 kNm.

$A = 400 \times 600 = 24 \times 10^4 \ mm^2$

$Z = \frac{bd^2}{6} = 2.4 \times 10^7 \ mm^3.$

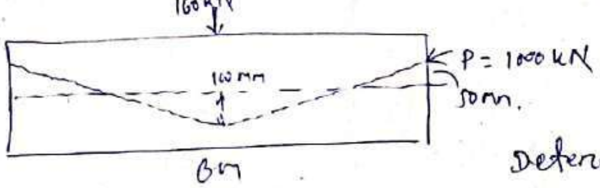
Direct stress due to prestressing force, $\frac{P}{A} = \frac{1200 \times 10^3}{2.4 \times 10^5} = 5 \ N/mm^2$

extreme fibre stress due to prestress = $\pm \frac{M}{Z} = \frac{129 \times 10^6}{2.4 \times 10^7} = \pm 5.4 \ N/mm^2$

extreme stresses, at top = 5 + 5.4 = 10.4 N/mm²

bottom : 5 − 5.4 = −0.4 N/mm².

(Q-2)



central load = 160 kN.

D.L = 6 kN/m.

Determine stresses at end & mid sec.

sol → $A = 400 \times 600 = 2.4 \times 10^5 \ mm^2$, $Z = \frac{bd^2}{6} = 2.4 \times 10^7 \ mm^3$

θ = inclination of tendon with horizontal.

$\sin\theta = \tan\theta = \frac{150}{3000} = \frac{1}{20}.$

Analysis of end section

extreme stresses = $\frac{P\cos\theta}{A} \pm \frac{P\cos\theta \times e}{Z} \simeq \frac{P}{A} \pm \frac{Pe}{Z}$

$= \frac{1000 \times 10^3}{A} \pm \frac{1000 \times 10^3 \times 50}{Z} = 4.17 \pm 2.08$

at top : 4.17 + 2.08 = 6.25 N/mm²

bottom : 4.17 − 2.08 = 2.09 N/mm²

Analysis of mid section

Downward load at centre = 160 kN.

Upward load by tendon = $2P\sin\theta = 2\times1000\times\frac{1}{20} = 100$ kN.

⇒ Net downward load = 160 − 100 = 60 kN.

(1) → BM due to Downward = $\frac{60\times6}{4}$ = 90 kNm ✓

(2) → BM " " dead load = $\frac{6\times6^2}{8}$ = 27 kNm ✓

(3) → BM " " eccentricity of P at end = $1000\times\frac{50}{1000}$ = 50 kNm.

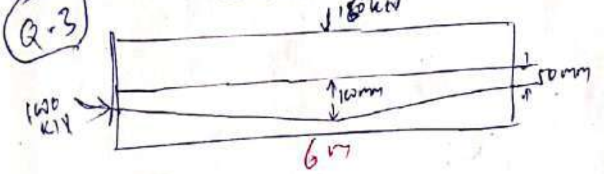
Total BM = 90 + 27 + 50 = 167.

stresses = $\frac{P\cos\theta}{A} \pm \frac{M}{Z} = \frac{1000\times10^3}{A} \pm \frac{167\times10^6}{Z} = 4.17 \pm 6.96$

at top = 4.17 + 6.96 = 11.13 (comp)

bottom = 4.17 − 6.96 = −2.79 (tensile)

(Q.3)



Point load central = 160 kN,
D.L = 6 kN/m.
Calculate end stresses at end & mid sec.

Sol → $A = 400\times600 = 2.4\times10^5$ mm², $Z = \frac{bd^2}{6} = 2.4\times10^7$ mm³

θ = inclination of tendon with horizontal

$\tan\theta \simeq \sin\theta = \frac{50}{3000} = \frac{1}{60}$.

Analysis of end sec

$$\frac{P\cos\theta}{A} \mp \frac{P\cos\theta\, e}{Z} = \frac{1000\times10^3}{A} \mp \frac{P\times50}{2.4\times10^7} = 4.17 \mp 2.08$$

top = 4.17 − 2.08 = 2.09 N/mm²

bottom = 4.17 + 2.08 = 6.25 N/mm².

Analysis of mid sec →

upward load by tendon = $2P\sin\theta = 2\times1000\times\frac{1}{60} = \frac{100}{3}$ kN.

Net downward load = $160 - \frac{100}{3} = \frac{380}{3}$ kN.

→ BM due to downward load = $\frac{380}{3}\times\frac{6}{4}$ = 190 kN/m

→ BM " " D.L = $\frac{6\times6^2}{8}$ = 27 kNm.

→ BM " " e of P at end = $-1000\times\frac{50}{1000}$ = −50 kNm. → ~~effective eccentricity at~~ midspan

Total BM = 190 + 27 − 50 = 167 kNm.

extreme stresses = $\frac{P\cos\theta}{A} \pm \frac{M}{Z} = \frac{1000\times10^3}{A} \pm \frac{167\times10^6}{Z} = 4.17 \pm 6.96$

= 11.13 N/mm² at top

= −2.79 N/mm² at bottom.

ⓑ Tendon with Parabolic Profile

- If load is UDL, tendon should be parabolic.
- When cable has got parabolic profile, it exerts uniform upward pressure/run on beam. & so net force is downward = applied load - upward uniform load = w
- From cable carrying UDL on full span,

horizontal reaction at each end : $\boxed{P_h = \frac{wl^2}{8h}}$

h = deep of cable at centre & $P_h \simeq P$

$\Rightarrow \boxed{w = \frac{8hP}{l^2}}$

$\Rightarrow$ Parabolic tendon carrying P provides upward UDL $\boxed{w = \frac{8hP}{l^2}}$ which counteracts the applied UDL.

Q — A PSC beam (400×600mm) with parabolic tendon, w = 35 kN/m

P = 1000 kN, calculate extreme stresses at mid span.
l = 6m, h = 0.1m.

Sol → $A = 400 \times 600 = 2.4 \times 10^5 \text{ mm}^2$

$Z = \frac{bd^2}{6} = 2.4 \times 10^7 \text{ mm}^3$

$w = \frac{8Ph}{l^2} = \frac{8 \times 1000 \times 0.1}{6^2} = 22.22 \text{ kN/m}$.

net downward load, $w = 35 - 22.22 = 12.78$ kN/m.

max BM, $M = \frac{12.78 \times 6^2}{8} = 57.51$ kNm.

extreme stresses at mid sec $= \frac{P}{A} \pm \frac{M}{Z} = \frac{1000 \times 10^3}{A} \pm \frac{57.51 \times 10^6}{Z}$

$= 4.17 \pm 2.4 \text{ N/mm}^2$

at top $= 4.17 + 2.4 = 6.57 \text{ N/mm}^2$

bottom $= 4.17 - 2.4 = 1.77 \text{ N/mm}^2$.

Load balancing method →

- Cable profile can be adjusted so that cable may exert upward force counteracting to some extent the downward external loading.
- If beam is so designed that upward force by cable neutralizes external applied loading, the method of design is called load balancing method.

Q -) Determine profile of a load balancing cable for a beam of span 6m carrying an all inclusive load of 40kN/m. P = 1200 kN, beam size = 400mm x 600mm.

Sol -) Let dip = h.

upward force by cable $= \frac{8Ph}{l^2} = \frac{8 \times 1200 \times h}{6^2} = \frac{800h}{3}$ kN/m

The upward force to balance downward load

$$\frac{800h}{3} = 40 \Rightarrow h = \frac{40 \times 3}{800} = 0.15\text{m}.$$

∴ net uniform stress in beam sec $= \frac{P}{A} = \frac{1200 \times 10^3}{400 \times 600} = 5\ \text{N/mm}^2$

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* ② method of Concept of Load Balancing

selects tendon profile

A suitable cable profile is selected such that transverse component of cable force balances external loads.

* This can be done by ~~rep~~ considering the free body of concrete with tendons replaced by forces acting on concrete beam.

* The type of reactions of a cable on concrete member depends on the shape of cable profile.

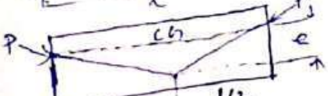
* straight cable don't induce reaction but at ends induces reactive moments.

* curved cable induces UDL reactions.

* sharp angles in a cable induces concentrated loads.

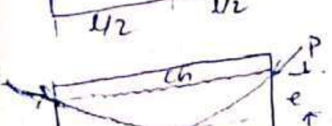
* The concept of load balancing is useful in selecting CABLE PROFILE.

* As cable profile balances external loads, the load balancing will be satisfied if cable profile in PSC member corresponds to BMD from external loads.



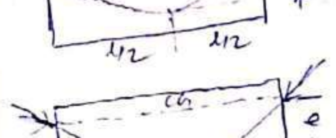
$M = Pe$

$\Delta = \frac{Ml^2}{8EI}$



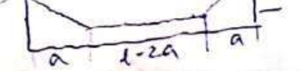
$W = \frac{4Pe}{l}$

$\Delta = \frac{wl^3}{48EI}$



$w = \frac{8Pe}{l^2}$

$\Delta = \frac{5}{384}\frac{wl^4}{EI}$



$W = \frac{Pe}{a}$

$\Delta =$

Pressure line / Thrust line & Internal Resisting couple

* At a given section of PSC beam, the combined effect of prestressing force & externally applied load results a distribution of concrete stress which can be resolved to a single force.

* PRESSURE LINE / THRUST LINE →

The locus of the points of application of this resultant force in any structure is called so.

* The concept of Pressure line is very useful in understanding the load carrying mechanism of a PSC section.

* The location of pressure line depends on

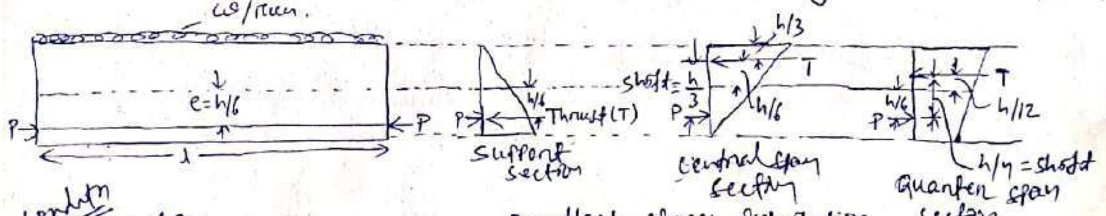
~~magnitude & distribution of moments~~

→ magnitude & dirn of moments applied at C/S

→ magnitude & distribution of stress due to prestressing force.

Example →

Let us consider concrete beam prestressed at eccentricity e, loaded + self wt = w/run



Resultant stress distribution

condition

* The load is of such magnitude that the bottom fibre stress at central span section of beam is zero and

→ resultant stress developed is max at top fibre.

→ so at central span shift of pressure line towards top fibre is h/3 w.r.t its initial position at support.

* At support section since there are no flexural stresses from external load, pressure line coincides with centroid of steel at h/6 from geometric centre.

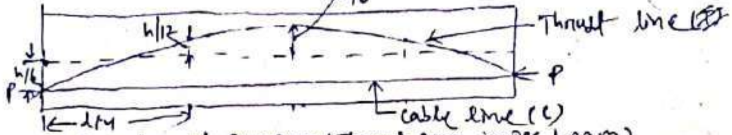
* As external moment at quarter span section being smaller in magnitude, shift of pressure line (=h/4) is also smaller than that at centre.

conclusion

* Larger the UDL, larger is the shift of pressure line at every section.

* within elastic range of PSC beam,

a change in external moments results in shift of pressure line rather than increase in the resultant force in the beam.



(Location of Pressure / Thrust line in PSC beam)

* But in RCC beam, increase in external moment results increase in tensile force & compressive force.

* The resultant comp/tensile forces increases at constant lever arm betn forces.

→ Load carrying mechanism of RCC beam.

* In PSC beam, increase in external moment results increase in shift of pressure line/resultant thrust line i.e. resultant forces remains constant but lever arm/shift changes.

→ Load carrying mechanism of PSC beam.

* But if a PSC member cracks, it behaves like a RCC beam.

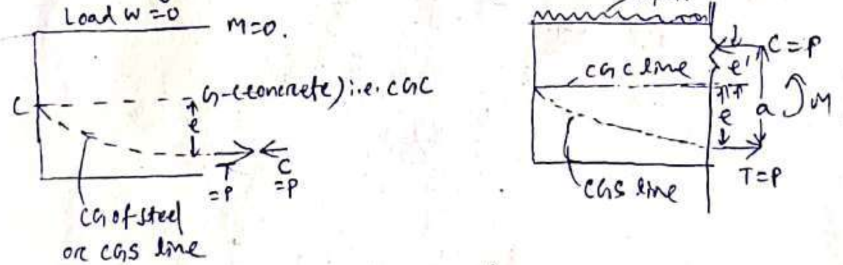
③ Internal Resisting couple method of stress Analysis ⇒

* Like direct method of stress analysis, pressure line/thrust line concept can also be used to calculate stresses.

* This is called internal Resisting couple method / C-line method, where prestressed beam is analysed as a plain concrete elastic beam using basic principles of statics.

* The prestressing force P is considered as an external comp. force with a constant tensile force T in the tendon throughout the span.

* Then at any sec. of a loaded prestressed beam, equilibrium is maintained satisfying equations $\Sigma H = 0$ & $\Sigma M = 0$.



Free body diagram of forces & moments at a sec of PSC beam when load is present and absent.

* When W=0, C & T line coincide since moment at section is zero.

* Under transverse load, C-line/centre of pressure line/thrust line is at a varying distance 'a' from T-line/tendon force/tensile force line.

M = BM at section due to dead & live load

$T = P$ = Prestressing force in tendon.

* Considering equilibrium of moments:- $M = Ta = Ca = Pa$ ⇒ $\boxed{a = \frac{M}{P}}$

e' = shift of pressure line from CG = $a - e$ ⇒ $\boxed{e' = \frac{M}{P} - e = a - e}$

* Resultant force at top & bottom fibres of section

$\boxed{\sigma_{sup} = \frac{P}{A} + \frac{Pe'}{Z_t}}$, $\boxed{\sigma_{inf} = \frac{P}{A} - \frac{Pe'}{Z_b}}$

Q-1

A prestressed concrete beam with a rectangular section (120 X 300mm) $w = 4$ KN/m (including self wt), $l_{eff} = 6$m.

The beam is ~~con centrally Xi~~ concentrically prestressed by a cable carrying a force of 180KN Locate position of pressure line in beam.

Sol →) $P = 180$KN, $e = 0$, $A = 36 \times 10^3$ mm2, $Z_t = Z_b = 18 \times 10^5$ mm3

At centre, $M = \frac{wl^2}{8} = \frac{4 \times 6^2}{8} = 18$ KNM.

Direct stress $= \frac{P}{A} = \frac{180 \times 10^3}{36 \times 10^3} = 5$ N/mm2

Bending stress, $= \frac{M}{Z} = \frac{18 \times 10^6}{18 \times 10^5} = 10$ N/mm2

Resultant stress at centre of span section

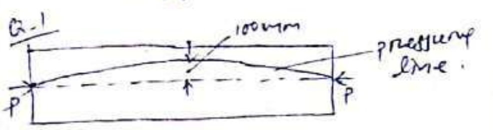
At top, $5 + 10 = 15$ N/mm2 (C). At bottom, $5 - 10 = -5$ (T)

if N = resultant thrust & e = corresponding eccentricity

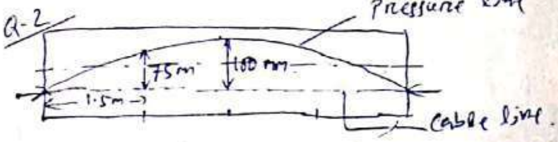
then $\frac{N}{A} + \frac{Ne}{Z} = 15 \Rightarrow \frac{180 \times 10^3}{36 \times 10^3} + \frac{180 \times 10^3 \times e}{18 \times 10^5} = 15 \Rightarrow e = 100$ mm.

∴ shift of pressure line, e = 100mm. (Ans)

Q-1



Q-2



Q-2 A PSC beam sec (120 X 300mm), $l_{eff} = 6$m., $w = 4$ KN/m (inclusive self wt)
The beam is prestressed by straight cable carrying a force of 180KN with eccentricity of 50mm. Determine position of thrust line & find out its position at centre & quarter span.

Sol → $P = 180$KN, $e = 50$mm, $A = 36 \times 10^3$ mm3, $Z = 18 \times 10^5$ mm3

stress due to prestressing force, $\frac{P}{A} = \frac{180 \times 10^3}{36 \times 10^3} = 5$ N/mm2

$\frac{Pe}{Z} = \frac{180 \times 10^3 \times 50}{18 \times 10^5} = \mp 5$ N/mm2

→ Bending moment at centre $= \frac{wl^2}{8} = \frac{4 \times 6^2}{8} = 18$KNM.

Bending stress at top & bottom, $= \frac{18 \times 10^6}{18 \times 10^5} = \pm 10$ N/mm2

Resultant stresses at centre, top $= 5 - 5 + 10 = 10$ N/mm2
bottom $= 5 + 5 - 10 = 0$

∗ shift of pressure line from cable line $= \frac{M}{P} = \frac{18 \times 10^6}{18 \times 10^4} = 100$mm.

→ Bending moment at quarter span $= \frac{3wl^2}{32} = 13.5$ KNm.

Bending stress at top & bottom $= \pm \frac{13.5 \times 10^6}{18 \times 10^5} = \pm 7.5$ N/mm2

Resultant stress at quarter span, top $= 5 - 5 + 7.5 = 7.5$ N/mm2
bottom $= 5 + 5 - 7.5 = 2.5$ N/mm2

Shift of pressure line from cable line $= \frac{M}{P} = \frac{13.5 \times 10^6}{18 \times 10^4} = 75$ mm.

Q-3 A rectangular PSC beam (250×300 mm), P = 540 kN, e = 60 mm. concentrated central load = 68 kN, l = 3 M, Determine the location of pressure line at centre, quarter span & support section of beam. Neglect self wt.

Sol → $P = 540\ kN$, $e = 60 mm$, $A = 75 \times 10^3 mm^2$, $Z = 375 \times 10^4 mm^3$

At centre, $M = \frac{wl}{4} = \frac{68 \times 3}{4} = 51 kNm$,

At quarter span, $M = \frac{wl}{8} = \frac{68 \times 3}{8} = \underline{25.5}\ kNm$.

stresses due to P, $\frac{P}{A} = \frac{54 \times 10^4}{75 \times 10^3} = 7.2 N/mm^2$, $\frac{Pe}{Z} = \frac{54 \times 10^4 \times 60}{357 \times 10^4} = \mp 8.6 N/mm^2$

* Due to external load

at centre, $\pm\frac{M}{Z} = \frac{51 \times 10^6}{375 \times 10^4} = 13.6 N/mm^2$,

at quarter, $\pm\frac{M}{Z} = \frac{25.5 \times 10^6}{375 \times 10^4} = 6.8 N/mm^2$

At centre

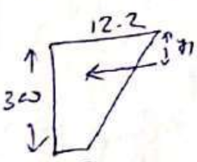
top = 7.2 − 8.6 + 13.6 = 12.2 N/mm2

bottom = 7.2 + 8.6 − 13.6 = 2.2 N/mm2

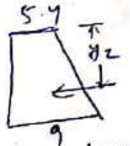
Quarter span

top = 7.2 − 8.6 + 6.8 = $\overset{5.4}{\cancel{9}}$ N/mm2

bottom = 7.2 + 8.6 − 6.8 = 9 N/mm2



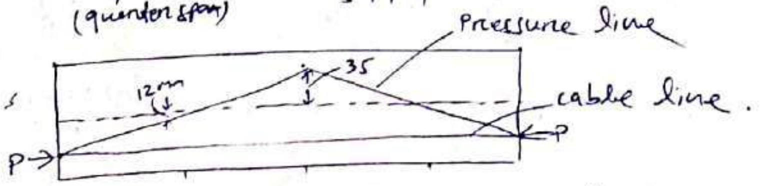
(central span)



(quarter span)

$y_1 = \frac{12.2 + 2 \times 2.2}{12.2 + 2.2} \times \frac{300}{3} = 115 mm$

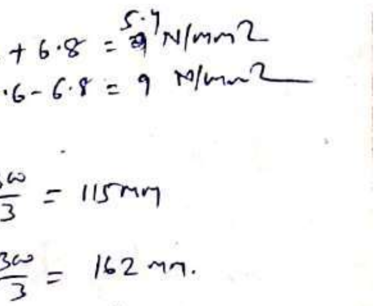
$y_2 = \frac{5.4 \times 2 \times 9}{5.4 + 9} \times \frac{300}{3} = 162 mm.$



Q-4 A prestressed concrete bridge deck comprise unsymmetrical I sec. prestressed by 7 wires each carrying a force of

l = 20 m, P = 660 kN, at 200 mm from Soffit at middle span.

At centre BM_{max} = 3600 kN.m, estimate resultant stress at section using internal resisting couple method.



Sol → $A = 62 \times 10^4$, $y_t = 646 mm$, $y_b = 854 mm$,

$I = 1727 \times 10^8 mm^4$, $Z_t = 2.67 \times 10^8$, $Z_b = 2.02 \times 10^8$

$P = 7 \times 660 = 4620 kN$, $e = 854 - 200 = 654 mm$.

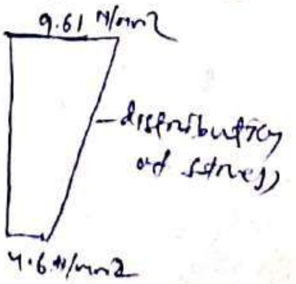
$M = 3600 kNm$.

Lever arm, $= a = \frac{M}{P} = \frac{3600 \times 10^3}{4620} = 779 mm$.

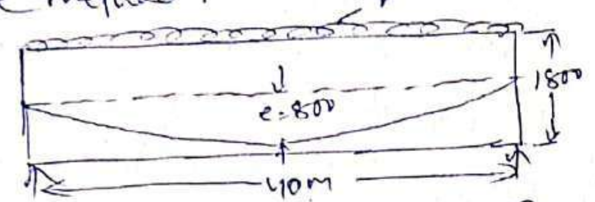
* shift of pressure line above CG = $e' = a - e = 779 - 654 = \underline{125 mm}$.

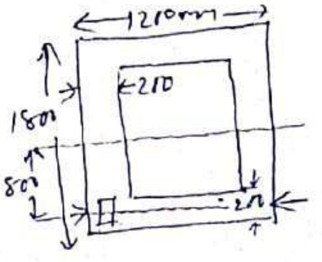
The resultant stress at centre of span, $\sigma_{sup} = \frac{P}{A} + \frac{Pe'}{Z_t} = \frac{4620 \times 10^3}{A} + \frac{P \times 125}{Z} = 9.61 (C)$

$\sigma_{inf} = \frac{P}{A} - \frac{Pe}{Z_b} = 4.6 N/mm^2 (C)$



Q-5 A box grider of PSC bridge of span 40m, uniform wall thickness = 210mm. max live load moment is 2000 kN m at centre of span. Beam is prestressed by parabalic cables with effective prestressing force of 7000kN. The cables which are concentric at supports have an eccentricity of 800mm at centre of span. compute resultant stresses at centre of span section using internal resisting couple method.





Sol ->

$A = 1.2 \times 1.8 - (0.8 \times 1.4) = 1.04 m^2$

$g = 1.04 \times 24 = 25 kN/m$.

$P = 7000 kN, \quad e = 800mm, \quad l = 40m.$

$I = \frac{1}{12}\left[(1200 \times 1800^3) - (800 \times 1400^3)\right] = 40 \times 10^{10} mm^4$

$Z_b = Z_t = Z = \frac{40 \times 10^{10}}{900} = 444 \times 10^6 mm^3$

$M_g = \frac{25 \times 40^2}{8} = 5000 kNm.$

$M_q = 2000 kNm.$

$M = M_g + M_w = 5000 + 2000 = 7000 kNm.$

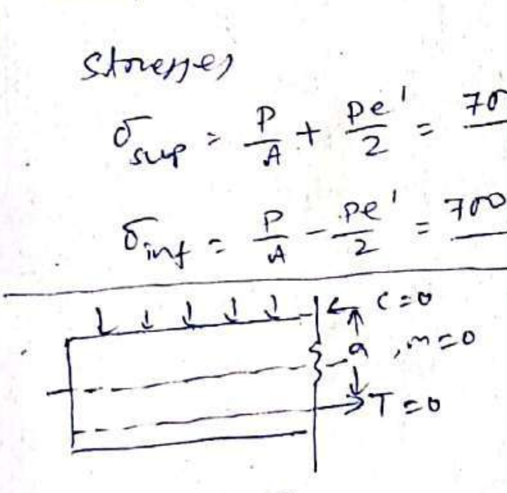
lever arm $a = \frac{M}{P} = \frac{7000 \times 10^3}{7000} = 1000 mm$

Shift of pressure line $= e' = a - e = 1000 - 800 = 200mm$

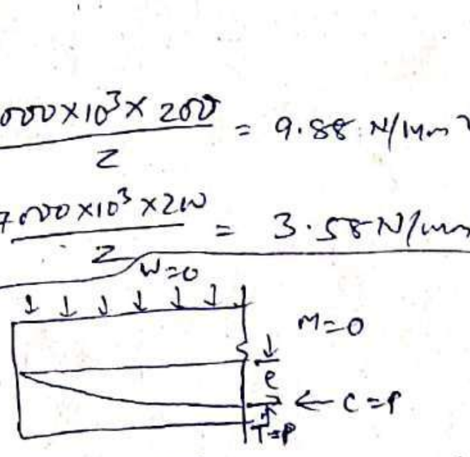
Stresses

$\sigma_{sup} = \frac{P}{A} + \frac{Pe'}{Z} = \frac{7000 \times 10^3}{A} + \frac{7000 \times 10^3 \times 200}{Z} = 9.88 N/mm^2$

$\sigma_{inf} = \frac{P}{A} - \frac{Pe'}{Z} = \frac{7000 \times 10^3}{A} - \frac{7000 \times 10^3 \times 200}{Z} = 3.58 N/mm^2$



RCC beams



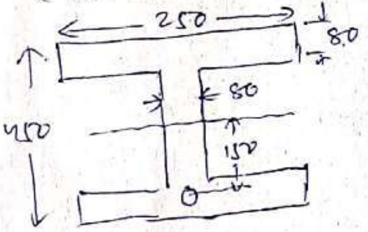
PSC beam.

Q-6 A beam of symmetrical I-section of span 8m. The beam is prestressed by parabolic cable with e = 150mm at centre & zero at supports. live load = 2.5 KN/m.

(a) Determine effective force in cable for balancing DL & LL

(b) what is resultant stress at centre of span section.

(c) calculate shift of pre. line from tendon-centre-line.



$A = 0.063\ m^2$

$I = 1.553 \times 10^9\ mm^4$

$Z = 6.9 \times 10^6\ mm^2$

$e = 150\ mm$

$g = 1.57\ KN/m.$

$q = 2.5\ KN/m.$ $l = 8m$

BM at centre, $M_g = \frac{1.57 \times 8^2}{8} = 12.56\ KNm.$

$M_q = \frac{2.5 \times 8^2}{8} = 20\ KN/m.$

$M = M_g + M_q = 32.56\ KNm.$

If P = tendon force for load balancing

$P = \frac{M}{e} = \frac{32.56 \times 10^3}{150} = 217\ KN.$

shift of pressure line at centre $= \frac{M}{P} = \frac{32.56 \times 10^6}{217 \times 10^3}$

$= 150\ mm.$

The centre of span sec. is subjected to direct stress of intensity $\left(\frac{P}{A}\right) = \frac{217 \times 10^3}{A} = 3.44\ N/mm^2$

∴ pre. line. coinciding with CG of beam.

$A = 0.25 \times 0.08 \times 2 + 0.08 \times 0.24$

$= 0.0592\ m^2 =$

$I = \left[\frac{250 \times 400^3}{12} - \frac{170 \times 240^3}{12}\right] \times 10^{-12}$

$= \left[\frac{25 \times 64}{12} - \frac{17 \times 2.4^3}{12}\right] \times 10^{-5}$

$= [133.33 - 19.584] = 113.75 \times 10^{-5}\ m^4.$

$I = 1.14 \times 10^9\ mm^4$

$A = 5.92 \times 10^4\ mm^2$

$Z = 5.69 \times 10^6$

$\frac{450}{160}$
290

$g = 25 \times 0.0592 = 1.48\ KN/m,$

$M_g = 11.84\ KNm.$

$M_q = 20\ KNm.$

$M = 31.84\ KNm.$

$P = \frac{31.84 \times 10^3}{150} = 212.27\ KN$

shift $= 150\ mm.$

$\delta_a = \frac{P}{A} = \frac{212.27 \times 10^3}{5.92 \times 10^4} = 3.6 \frac{N}{mm^2}$